

## A review on the flow instability of nanofluids\*

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**Abstract** Nanofluid flow occurs in extensive applications, and hence has received widespread attention. The transition of nanofluids from laminar to turbulent flow is an important issue because of the differences in pressure drop and heat transfer between laminar and turbulent flow. Nanofluids will become unstable when they depart from the thermal equilibrium or dynamic equilibrium state. This paper conducts a brief review of research on the flow instability of nanofluids, including hydrodynamic instability and thermal instability. Some open questions on the subject are also identified.

**Key words** nanofluid, thermal instability, hydrodynamic instability, review

**Chinese Library Classification** O358, O359

**2010 Mathematics Subject Classification** 76E09, 76T20

### 1 Introduction

Nanofluids have aroused significant interest over the past few decades for their wide applications in energy, machinery, transportation, and healthcare. For example, low concentration of particles causes viscosity changes<sup>[1–2]</sup>, decreases viscosity with increasing shear rate<sup>[3]</sup>, changes friction factors and pressure drops<sup>[4–6]</sup>, and improves heat transfer<sup>[7–9]</sup>. Moreover, the unique flow properties of nanofluids are determined by the flow pattern. Heat transfer<sup>[10]</sup> and pressure drop<sup>[11]</sup> are also much lower and higher, respectively, in laminar flow than in turbulent flow. By adding particles to the fluid, the thermal entropy generation and friction are of the same order of magnitude as in turbulent flow, while the effect of heat transfer entropy generation strongly outweighs that of the friction entropy generation in laminar flow. Therefore, the transition from laminar to turbulent flow is an important issue. The flow instability of nanofluids is often the result of changes in thermal and mechanical equilibrium caused by changes in the space and time structure. In this paper, a brief review of research on hydrodynamic instability and thermal instability is conducted.

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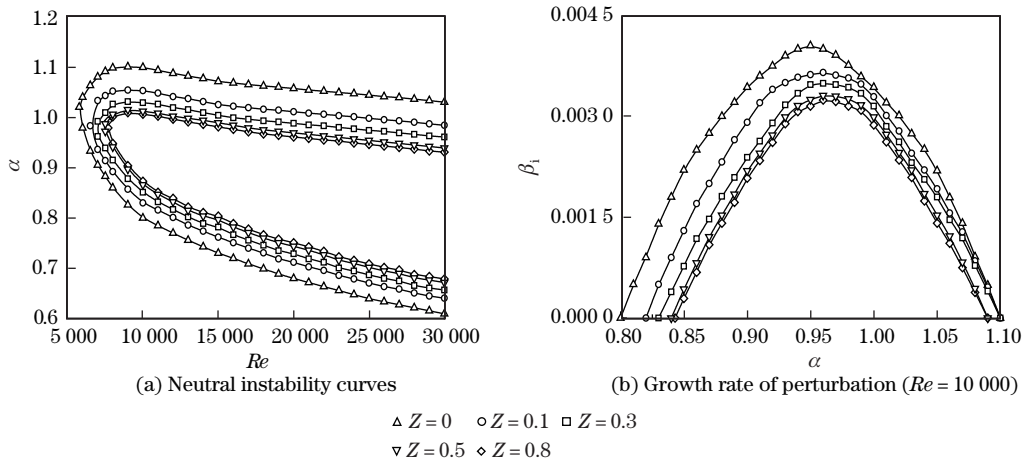
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## 2 Hydrodynamic instability

Hydrodynamic instability induced by mechanical non-equilibrium is very common. Research on hydrodynamic stability has focused on determining whether a given flow is stable or unstable, and the critical parameters that separate the laminar and turbulent categories. Nayak et al.<sup>[12]</sup> showed that flow stabilities were enhanced in nanofluids, and the degree of suppression was dependent on the nanoparticle concentration<sup>[13]</sup>. Lin et al.<sup>[14]</sup> studied the linear instability of nanofluids in a channel flow. The instability theory was applied to derive the governing equations with the following parameters: the average particle mass loading, the Stokes number, the Reynolds number, and the Knudsen number. A perturbation proportional to  $\exp(i\alpha(x - c_t))$  was added to a parallel flow in the  $x$ -direction. The governing equations were linearized based on which a closed eigenvalue problem was formed and the corresponding equations were solved numerically. Figure 1 shows the variations of  $\alpha$  with  $Re$  and the growth rate of perturbation  $\beta_i$  with  $\alpha$ , respectively, for different values of  $Z$ . In Fig. 1(a), the smallest  $Re$  is the critical  $Re$ , and the addition of nanoparticles leads to an increase in the critical  $Re$ . In Fig. 1(b), the largest  $\beta_i$  which controls instability decreases as  $Z$  is increased. The results show that as  $St$  and  $Kn$  decrease, the critical  $Re$  increases. The nanoparticles restrained the instability, but could not completely eliminate it. The instability was modified by increasing the particle mass loading. As  $Kn$  and  $St$  decreased, the flow stability was reinforced. Larger particles attenuated the flow instability.



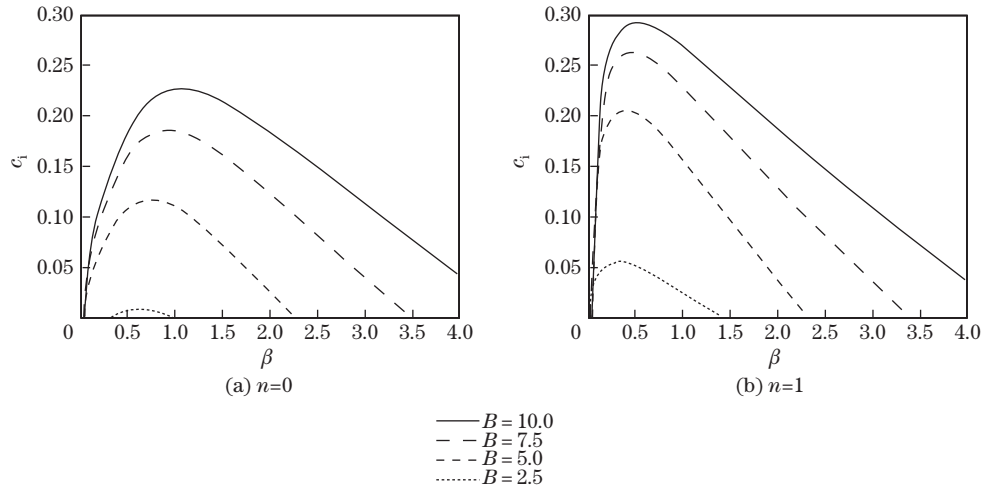
**Fig. 1** Relationship between the wavenumber and  $Re$ , and the growth rate of perturbation with the wavenumber for different  $Z$  when  $St = 0.001$ , and  $Kn = 0.8$ <sup>[14]</sup>

Xia et al.<sup>[15]</sup> studied the linear instability of round jet flow of nanofluids. The governing equations, which were the same as those in Ref. [13], were obtained based on the linear instability theory. A perturbation related to  $\exp(in\theta + i\beta(z - c_t))$  ( $n$  is the azimuthal mode,  $\beta$  is the wavenumber,  $z$  is the distance from the jet exit,  $\theta$  is the circumferential angle, and  $c = c_r + ic_i$ , in which  $c_r$  is the phase speed,  $c_i$  is the amplification factor,  $c_i > 0$  and  $c_i < 0$  indicate unstable and stable flows, respectively, and  $c_i = 0$  means a neutral perturbation) was added to a mean flow with a normalized velocity profile given by Parthasarathy<sup>[16]</sup>. The velocity profile contains the shape factor

$$B = R/\theta_0,$$

where  $\theta_0$  and  $R$  are the momentum loss thickness of the boundary layer and middle line of the shear layer, respectively. A smaller  $B$  corresponds to a larger axial position away from the exit.

Xia et al.<sup>[15]</sup> obtained variations of  $c_i$  with  $\beta$  for different  $n$  and  $B$ , shown in Figs. 2(a) and 2(b). Additionally, their work also showed that  $c_i$  is reduced to zero when  $Z$  is equal to 1.25, i.e., instability never occurs. There exist turning points  $St_c$  and  $Kn_c$  at which  $c_i$  is the smallest and largest, respectively. The major conclusions are that the addition of nanoparticles restrained flow instability, and there is a critical particle concentration below which flow stability remains. As  $Re$  increases, flow instability occurs. However, the flow stabilizes with the increasing particle concentration. The flow is most unstable and relatively stable when  $Kn_c = 1$  and  $St_c = 1$ , respectively.



**Fig. 2** Relationship between the wave amplification and the wavenumber at different  $n$  and  $B$ , when  $Z = 0.01$ ,  $Re = 1000$ ,  $St = 1$ , and  $Kn = 1$ <sup>[15]</sup>

Avramenko et al.<sup>[17]</sup> studied the instability of Taylor-Couette flows with an inner cylinder rotating in a curved channel between two concentric cylinders. Based on the derived differential equations, they used the collocation method to determine the critical Taylor number related to flow instability, and then analyzed the effects of  $Sc$ ,  $Pr$ , the distance between the concave and convex walls, the nanoparticle density, and the thermophoretic and Brownian diffusion on the critical Taylor number. Table 1 shows the dependence of the critical Taylor number  $Ta_{cr}$  and wavenumber  $\gamma_{cr}$  on the parameter  $N$ ,

$$N = \alpha Pr \Delta T,$$

where  $\alpha$  is the thermal expansion coefficient, and  $\Delta T$  is the temperature difference between two concentric cylinders.

It can be seen that the decrease in the width of the radial gap causes the effects of all parameters to become negligible. The positive and negative temperature gradients increase the instability and stability of the flow field, respectively. An increase in the thermophoretic to Brownian diffusion ratio and the Schmidt number destabilized the flow despite the positive or negative temperature gradients. Higher  $Pr$  destabilized and stabilized the flow subject to positive and negative temperature gradients, respectively.

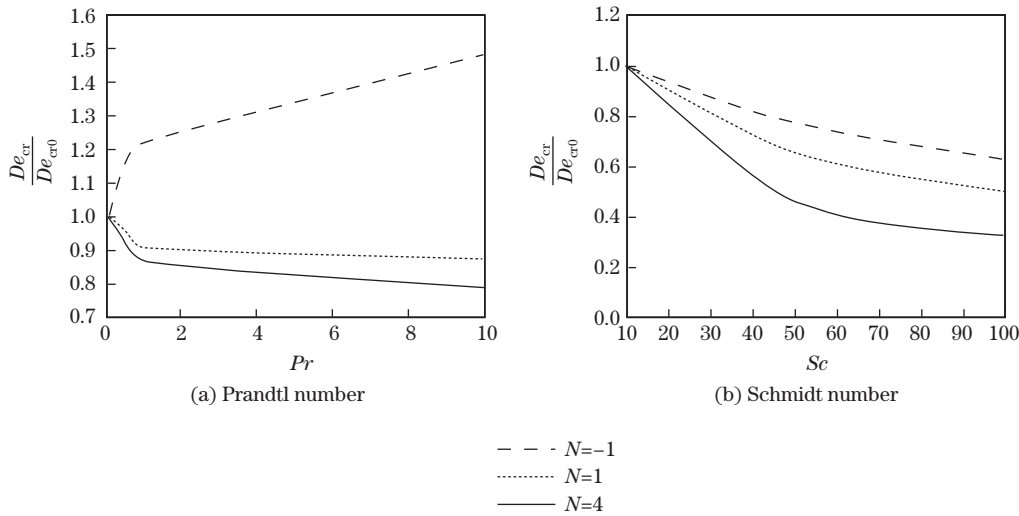
Avramenko et al.<sup>[18]</sup> investigated the instability of Dean flow in a curved channel between two cylinders produced by a pressure gradient in the azimuthal direction. The critical Dean number  $De_{cr}$  related to flow instability was found numerically using the collocation method. The dependence of  $De_{cr}$  on the Prandtl number, the Schmidt number, the temperature gradient, the distance between the concave and convex walls, the particle density, the Brownian diffusion, and the thermophoretic diffusion was revealed. The effects of the Prandtl number and the

Schmidt number on the Dean number for different  $N$  are shown in Fig. 3.  $N$  is defined in last part. The stabilizing effect of  $Pr$  when  $N < 0$  is stronger than the destabilizing effect when  $N > 0$ . The decrease in  $De_{cr}$  depending on  $Sc$  is rather significant until  $Sc \approx 50$ . Afterwards, the decrease in  $De_{cr}$  slows down and becomes linear.

**Table 1** Dependence of  $Ta_{cr}$  and  $\gamma_{cr}$  on the parameter  $N$ <sup>[17]</sup>

$N$	$Ta_{cr} (\gamma_{cr})$									
	-3	-2.7	-2.5	-2	-1	0	1	2	4	6
$\eta = 0.99$	98.04	82.19	74.63	62.2	48.79	41.41	36.57	33.13	28.41	25.28
	3.187	3.132	3.138	3.122	3.122	3.127	3.134	3.134	3.137	3.135
$\eta = 0.95$		82.19	78.04	64.45	50.16	42.42	37.41	33.84	28.97	25.75
		3.135	3.121	3.112	3.122	3.129	3.134	3.135	3.138	3.137
$\eta = 0.7$			112.16	86.63	63.25	52.02	45.17	40.46	34.24	30.21
			3.143	3.122	3.127	3.138	3.146	3.154	3.159	3.164
$\eta = 0.5$				130.1	86.21	68.19	58.76	51.4	42.92	37.59
				3.18	3.148	3.162	3.178	3.181	3.191	3.197
$\eta = 0.3$					152.79	111.79	91.9	79.76	65.22	56.51
					3.02	3.216	3.231	3.245	3.256	3.262
$\eta = 0.1$						421.78	316.17	262.64	206.01	174.97
						3.339	3.351	3.404	3.432	3.445

$\eta$  is the ratio of radii of the inner cylinder and outer cylinder



**Fig. 3** Relationship between the critical Dean number and the Prandtl number as well as the Schmidt number<sup>[18]</sup>

The major conclusions are as follows. The influence of all parameters on instability is weaker when the radial gaps are smaller. The positive and negative temperature gradients reduce and enforce flow stability, respectively. An increase in the Brownian diffusion to thermophoretic diffusion ratio, particle density, and  $Sc$  causes instability subject to the temperature gradients. The increase in  $Pr$  deteriorates and enhances the stability for the positive and negative temperature gradients, respectively.

Rudyak and Bord<sup>[19]</sup> studied the stability of plane and cylindrical Poiseuille flows. They found that the addition of nanoparticles led to flow destabilization, and the smaller the particle size and the greater the particle concentration, the stronger the degree of flow instability. Hos-sain and Khan<sup>[20]</sup> showed that the flow became unstable owing to the presence of nanoparticles in a wall-driven flow through a tube, and flow instability was dependent on the particle concentration, the particle density, and the Reynolds number. Moatimid et al.<sup>[21]</sup> studied the flow stability in a porous medium for the Oldroyd elasto-viscous magnetic fluid. Rudyak et al.<sup>[22]</sup> found that the degree of destabilization was enhanced by an increase in the particle concentration and a decrease in the particle diameter in the pipe. Dastvareh and Azaiez<sup>[23]</sup> found that in porous media flows, particles cannot cause stable flow to become unstable, but can destabilize or stabilize the flow that was originally unstable.

### 3 Thermal instability

Nanofluids are often used to enhance heat transfer which is dependent on the flow regime, i.e., laminar or turbulent flow. Thus, it is necessary to clarify the critical parameters that divide the flow into laminar and turbulent regimes in order to select the suitable correlations for both regimes. The thermal instability of nanofluids has been investigated by several researchers. Kim et al.<sup>[24]</sup> studied the flow instability driven by heat transfer and buoyancy properties. They applied the Brinkman model to the viscosity and expressed the Rayleigh number as  $Ra_{nf} = fRa$ , where  $f$  includes the effect of volume fraction  $\phi$  and the shape factor  $n$  of nanoparticles, and the ratio of the thermal conductivity  $\gamma$ , the density  $\delta_1$ , and the heat capacity  $\delta_2$  of nanoparticles to that of a base fluid. Figure 4 illustrates  $f$  as a function of  $\phi$  for different  $\gamma$  and  $n$ , respectively. The effects of  $\gamma$  and  $n$  on the convective instability decreased with the increase in  $\gamma$ . Consequently, nanoparticles suppressed the flow instability, and rod-like nanoparticles were more effective. As the particle density and the nanoparticle heat capacity to fluid heat capacity ratio increased, the flow became more unstable. The ratio of nanoparticle thermal conductivity to fluid thermal conductivity acted as stabilizers for the flow.

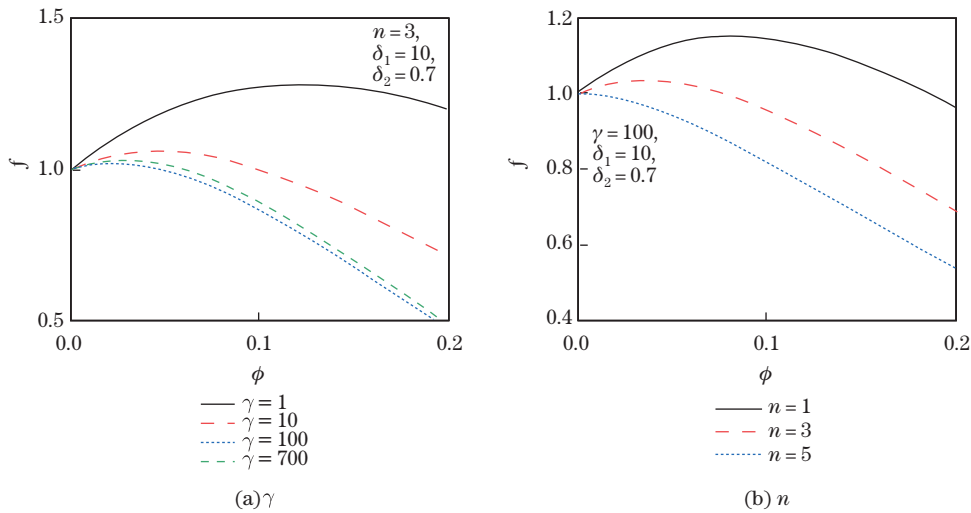
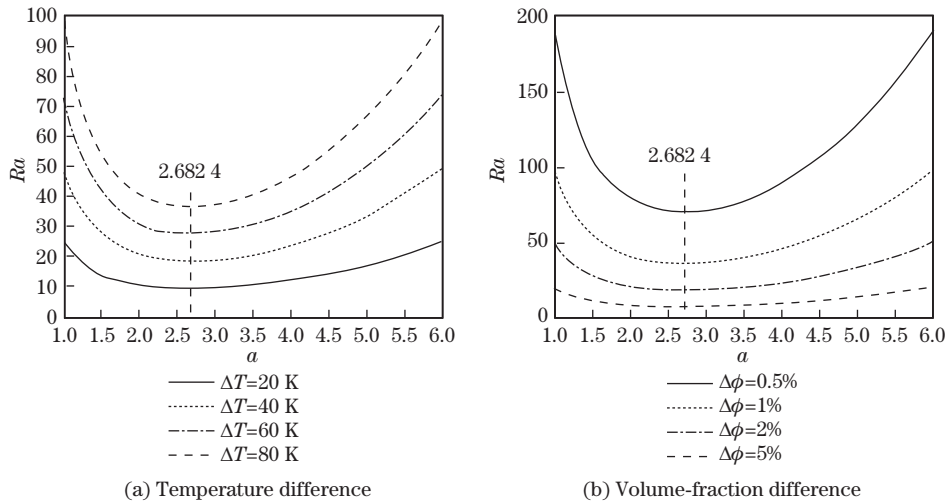


Fig. 4 The addition factor  $f$  as a function of  $\phi$  for different  $\gamma$  and  $n$ <sup>[24]</sup>

Tzou and Pfautsch<sup>[25]</sup> and Tzou<sup>[26]</sup> studied the Benard instability of nanofluids, taking into account the influence of the Brownian motion and thermophoresis of nanoparticles on natural convection. The critical Rayleigh number  $Ra_{cr}$  was obtained using eigenfunction expansions with the weighted residual approach. Figure 5 shows the influence of the temperature difference

$\Delta T$  and the volume fraction difference of nanoparticles  $\Delta\phi$  on the Rayleigh number  $Ra$ . We can see that  $Ra_{cr}$  decreases as  $\Delta T$  decreases and  $\Delta\phi$  increases. The critical value of  $Ra_{cr}$  occurs at  $a_c = 2.6824$ , and is two orders of magnitude lower than that of pure fluid. A larger  $\Delta T$  suppresses the growth of perturbations, and larger  $\Delta\phi$  corresponds to stronger Brownian motion. The combined behavior of Brownian motion and thermophoresis has strong influence of destabilizing the flow, reducing  $Ra_{cr}$  by two orders of magnitude compared with that of pure fluids. The effect of thermophoresis on instability is stronger than that of Brownian motion. A larger temperature difference leads to a more stable property. However, a larger volume fraction of nanoparticles, heavier nanoparticles, and a larger Brownian diffusion coefficient increase flow instability.

Nield and Kuznetsov<sup>[27]</sup> revealed that  $Ra_{cr}$  was increased or decreased by nanoparticles in a porous medium saturated by nanofluids, depending on the distribution of nanoparticles. Yadav et al.<sup>[28]</sup> studied nanofluid convection with two free boundaries, and analyzed the overstability. Nield and Kuznetsov<sup>[29]</sup> studied the instability in a nanofluid layer with finite depth, and showed that  $Ra_{cr}$  was increased or decreased by an abundant amount based on the distribution of nanoparticles. They also investigated the onset of double-diffusive nanofluid convection, and determined the stability boundary for cases of non-oscillation and oscillation<sup>[30]</sup>. The results were restricted to the case with large Lewis and Prandtl numbers for nanoparticles. Yadav et al.<sup>[31]</sup> investigated the instability in a nanofluid layer with rotation and two free boundaries, and found that the temperature difference and rotation stabilized the flow, and the particle Brownian motion and thermophoresis had the opposite effect in the non-oscillatory convection case. Tzou<sup>[32]</sup> also showed that  $Ra_{cr}$  was two orders of magnitude lower compared with that for pure fluids under the actions of thermophoresis and Brownian motion. Kang et al.<sup>[33]</sup> analyzed the influence of the Peclet number  $Pe$ , the Lewis number  $Le$ , and the power-law index  $n$  on instability by considering the effects of thermophoresis, shear-thinning, and particle Brownian diffusion on a porous layer saturated by nanofluids. They found that  $Ra_{cr}$  decreased or increased with increasing  $n$ , depending on  $Pe$ . Decreasing  $Le$  can suppress or promote the onset of convection, depending on the distribution of particles. Govender<sup>[34]</sup> studied analytically the thermal instability in a porous layer saturated by a nanofluid, and found that instability is dependent on the conservation of nanoparticles and buoyancy, and is not affected by thermophoresis and particle Brownian motion.



**Fig. 5** Effects of the temperature difference and the volume-fraction difference on  $Ra$ <sup>[26]</sup>

Yadav et al.<sup>[35]</sup> investigated numerically the thermal instability in rotating  $\text{Al}_2\text{O}_3$ -water and Cu-water nanofluid layer, which was heated from below. The rigid lower boundary and either rigid or free upper boundaries were considered. The eigenvalue problem was solved numerically based on linear analysis. Tables 2 and 3 show  $Ra_{cr}$  and the critical wavenumber  $a_c$  as functions of the Taylor number  $T_c$ , the temperature difference  $\Delta T$ , and the volume fraction  $\Delta\phi$ . The results showed that  $Ra_{cr}$  decreased with the increase in the volumetric fraction difference of nanoparticles  $\Delta\phi$  and the decrease in the temperature difference  $\Delta T$ . The rotation and temperature difference between two boundaries suppressed the instability. The critical Rayleigh number was reduced and the flow became more unstable with increasing difference of particle volume concentration between two plates, Brownian diffusivity to thermophoretic diffusivity ratio, nanoparticles to fluid density ratio, and Brownian diffusivity to thermal diffusivity ratio. The joint behavior of the thermophoresis of nanoparticles and Brownian motion made the flow more unstable.

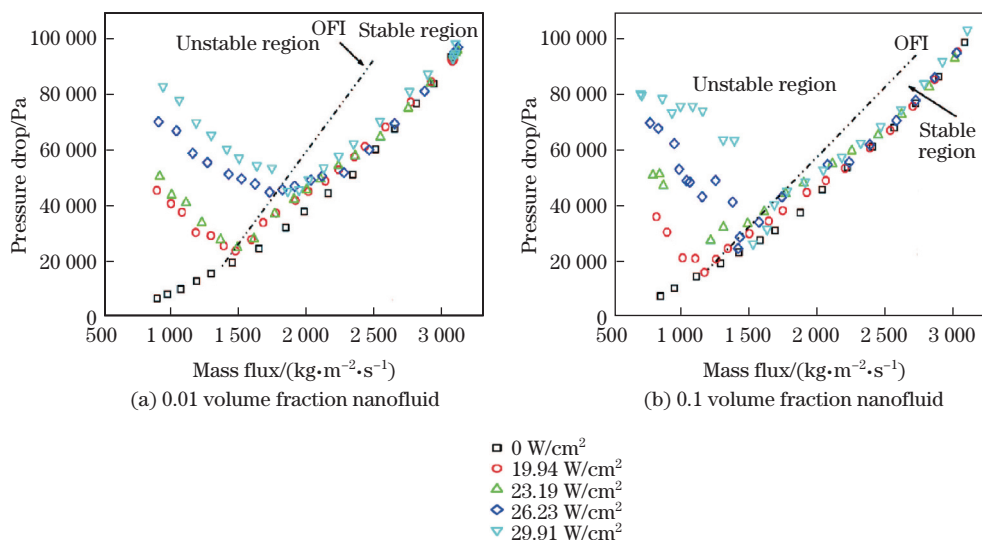
**Table 2**  $Ra_{cr}$  and  $a_c$  as the functions of  $T_c$  and  $\Delta T$ <sup>[35]</sup>

$T_c$	$\Delta T$	Rigid-rigid		Rigid-free	
		$Ra_{cr}$	$a_c$	$Ra_{cr}$	$a_c$
0	20	14.274 9	3.116	9.200 9	2.682
	60	42.217 7	3.116	27.216 0	2.682
	100	69.379 4	3.116	44.733 0	2.682
	140	95.792 3	3.116	61.772 4	2.682
$10^2$	20	14.681 0	3.161	10.090 4	2.854
	60	43.418 8	3.161	29.846 7	2.854
	100	71.353 3	3.161	49.056 6	2.854
	140	98.517 6	3.161	67.743 0	2.854
$10^4$	20	39.387 4	4.785	42.984 1	5.359
	60	116.487 2	4.785	127.126 3	5.359
	100	191.431 8	4.785	208.919 1	5.359
	140	264.310 1	4.785	288.459 2	5.359

**Table 3**  $Ra_{cr}$  and  $a_c$  as the functions of  $T_c$  and  $\Delta\phi$ <sup>[35]</sup>

$T_c$	$\Delta\phi$	Rigid-rigid		Rigid-free	
		$Ra_{cr}$	$a_c$	$Ra_{cr}$	$a_c$
0	0.005	108.276 4	3.116	69.806 0	2.682
	0.01	55.894 2	3.116	36.035 4	2.682
	0.03	19.043 2	3.116	12.277 4	2.682
	0.05	11.476 6	3.116	7.399 1	2.682
$10^2$	0.005	111.356 8	3.161	76.553 3	2.854
	0.01	57.484 3	3.161	39.518 5	2.854
	0.03	19.584 9	3.161	13.464 1	2.854
	0.05	11.803 1	3.161	8.114 3	2.854
$10^4$	0.005	298.756 3	4.785	326.044 9	5.359
	0.01	154.223 2	4.785	168.310 2	5.359
	0.03	52.544 0	4.785	57.343 4	5.359
	0.05	31.666 3	4.785	34.558 8	5.359

Yu et al.<sup>[36]</sup> studied experimentally the forced heat convective flow and  $\text{Al}_2\text{O}_3$ -water flowing through a mini channel. The decrease in flow rates from the maximum to the minimum values was obtained for each given heat flux. The pressure drop  $P_d$  versus the mass flux  $M$  under different heat fluxes for different volumetric fractions of nanoparticles is shown in Fig. 6. The dashed lines, which are defined as the onset of flow instability (OFI), divide the flow into stable and unstable regions. The dashed lines are closer to the single-phase case as the nanoparticle concentration increases, demonstrating a decreasing stable region. Consequently, the addition of nanoparticles suppressed the flow instability which is more obvious at high nanoparticle concentrations.



**Fig. 6** The pressure drop versus the mass flux<sup>[36]</sup>, where stable and unstable regions are divided by dashed lines

Agarwal and Bhadauria<sup>[37]</sup> studied the influence of rotation on the thermal instability of a horizontal nanofluid layer by considering the effect of thermophoresis and particle Brownian motion, and found that a rotating system has a stabilizing effect compared with a non-rotating system. Yadav et al.<sup>[38]</sup> found that rotation stabilized the flow in a nanofluid layer. Kahveci<sup>[39]</sup> studied convection instability in an annulus between two cylinders, and showed that the flow became unstable at high Rayleigh numbers for a narrow-gap annulus, while flow was stable for all Rayleigh numbers if the rotational effects were small for a wide-gap annulus. Das et al.<sup>[40]</sup> studied the influence of wall thermal conductance on a fully mixed convective flow between two vertical plates by considering the thermal radiation. They found that the nanoparticle concentration influenced the flow stability, and the thermal radiation had stabilizing influence on the flow field. Kiran and Narasimhulu<sup>[41]</sup> found that rotational speed modulation can be used effectively to control the instability of nanofluids with rotation and saturated porous medium. Zhang et al.<sup>[42]</sup> showed that nanoparticles can stabilize the flow in a rectangular cavity. Waki et al.<sup>[43–44]</sup> showed that flow stability depends on the nanoparticle concentration and size as well as the magnetic Chandrasekhar number for a flow between two impermeable boundaries and heated underneath by considering an external magnetic force. Dastvareh and Azaiez<sup>[45]</sup> concluded that the thermophoresis hinders particle Brownian diffusion and leads to less displacement compared with the case in which thermophoresis in homogeneous porous media is not considered. Zargartalebi and Azaiez<sup>[46]</sup> showed that the flow instability was controlled by the properties of nanoparticles that behave differently at various temperatures. Akbarzadeh



and Mahian<sup>[47]</sup> proved that in a layer of porous medium with rough walls, the porosity parameters and boundary roughness stabilized the occurrence of convective heat transfer, while the thermophoresis parameter, Rayleigh number of nanoparticle concentration, Lewis number, and increment with modified nanoparticle-density destabilized the occurrence of Rayleigh-Benard convection. Mahajan and Sharma<sup>[48]</sup> found that in a thin nanofluid layer heated by considering the magnetic force and gravity field, the system was most unstable for the free-free boundary and most stable for the rigid-rigid boundary.

## 4 Conclusions

The studies on the hydrodynamic instability of straight and curved channel flows of nanofluids, round jet, and a Taylor-Couette flow and thermal instability caused by heat transfer, buoyancy, and rotation are reviewed. The effects of parameters including the particle concentration, the Knudsen number, the Stokes number, the shape factor of velocity profile, the Prandtl number, the Schmidt number, the particle volume fraction, and the shape factor, thermal conductivity, heat capacity, and temperature difference on the critical Reynolds number, the Dean number, and the Rayleigh number are analyzed. The relations between the parameter variations and flow stabilization or destabilization are given. The following conclusions can be drawn.

Hydrodynamic instability is restrained significantly by low particle concentration. With the decrease in the Knudsen number and Stokes number, the stability is reinforced. Larger particles attenuate the flow instability more obviously. Increasing the thermophoretic diffusion to Brownian diffusion ratio destabilizes the flow despite the positive or negative temperature gradients.

For thermal instability, nanoparticles suppress the flow instability, and a rod-like nanoparticle is more effective. As the density of nanoparticles and the ratio of particle heat capacity to fluid heat capacity increases, the flow becomes more unstable. The particle thermal conductivity to fluid thermal conductivity ratio stabilizes the flow. The comprehensive behavior of thermophoresis and particle Brownian motion has strong destabilizing influence on the flow, and the influence of the latter is stronger than that of the former. A larger volume fraction of nanoparticles, heavier nanoparticles, and a larger Brownian diffusion coefficient make the flow more unstable.

In perspective, however, some points need to be examined. One open problem is the effects of particle coagulation and amalgamation as well as nonuniform distribution of nanoparticles on flow instability. The analytical results for oscillatory stability in a nanofluid layer are limited to flows with large Prandtl numbers and Lewis numbers. In the future, the flow convective instability will be explored by applying optimal models to the existing approach.

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