Annals of Operations Research
https://doi.org/10.1007/s10479-023-05249-x

ORIGINAL RESEARCH

A valuation of a corn ethanol plant through a compound options model under skew-Brownian motions

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Accepted: 20 February 2023

Abstract
In the last decades, the production of fuel ethanol from corn has spread as a valid renewable alternative to pursue sustainability goals. However the uncertain nature of both input (corn) and output (gasoline) prices, together with price dependent operational decisions, combine to make this difficult plant valuation require a real options approach. Moreover, this project is characterized by various sequential stages that contribute to increase its valuation difficulties. The purpose of this paper is to provide a reliable valuation methodology of a corn ethanol plant project able to consider the characteristics of the project. We apply the compound Real Options Approach to price a corn ethanol plant project considering that the corn and gasoline prices both follow a skew-geometric Brownian motion. We also propose a case study to show a real implementation of our theoretical model. The results show that the corn ethanol plant is financially attractive as renewable investment since the uncertainties inherent in the project add value, via managerial flexibility, to the real option valuation.

Keywords Corn ethanol plant · Compound real options approach · Skew-Brownian motion · Multi-stage investments

1 Introduction

There is no doubt that renewable energy investments represent a new frontier in the pursuit of sustainability goals. Projects that involve wind farms, biomass, solar panels, tidal or hydro

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Published online: 09 March 2023

power are some of the most widespread investments in the renewable energy sector. Another important renewable energy process is the conversion of corn to fuel ethanol (Commodity Research Bureau, 2007). Although this conversion process has received many criticisms related to the excessive amount of energy required to produce fuel ethanol (Kim & Dale, 2005; Patzek et al., 2005) or related to the increase of food prices as a consequence of production change from food to fuel (Pimentel, 2003), it has become environmentally attractive as renewable investment.

However, in general the valuation of renewable investments is not a simple task since they are characterized by various types of uncertainties according to the different technologies used. For example, solar panels performance depends on solar irradiation, materials and technology of cells, and location of cells (Duong et al., 2019). Wind farm investments suffer from the difficult predicting wind power availability, as power cannot be produced when the wind does not blow (Boyle, 2007) making them unreliable power producers. Corn ethanol plant performance depends on the uncertain nature of process input (corn) and output (gasoline) prices. If input prices fluctuate to exceed the price to be received for the corresponding output, conversion becomes financially attractive and can be halted.

For these reasons, the classical valuation approaches such as Net Present Value (NPV) are not adequate to make a reliable valuation of these projects characterized by uncertainty (Ross, 1995). Conversely, the Real Options Approach (ROA) has spread as one of the most useful approaches to price uncertain projects by including in the valuation managerial flexibility, also called optionality (Trigeorgis, 1993). This managerial flexibility allows certain project risks to be mitigated and gives a potential investor the possibility of changing investment decisions during the project lifetime. For example, Di Bari (2020) applied the ROA to value solar energy projects by considering the uncertainty of meteorological conditions, the unpredictable behavior of government that could encourage or not these renewable investments and managerial flexibility. Venetsanos et al. (2002) combined a framework with ROA to assess the wind farm investments affected by the unstable conditions of the deregulated energy market by explaining the case of Greece. Again, Pederson and Zou (2009) applied real options analysis and Monte Carlo simulation to valuate ethanol plant projects by considering historical price data and representative operational parameters. Maxwell and Davison (2014) used the ROA as support to valuate the impact of model variables on the decision of pursuing the project given its financial performance and on the decision of switching between an idled an operating facility states, focusing on the cost of switching between these states.

In the current paper, we propose a valuation methodology to price a corn ethanol plant project by considering the particular stochastic nature of corn and gasoline prices that represent respectively the input and the output prices of the plant, since sometimes the historical prices of some commodities follow skew-t distributions rather than having normal returns (Orlando & Bufalo, 2021). The skew Brownian motions can represent the best solution to provide a realistic study of corn ethanol plant performance during its lifetime. Pasricha and He (2022) formulated a closed-form pricing formula for European Exchange Options, introduced by Margrabe (1978) under skew-Brownian motion. Moreover, since the corn ethanol plant project can be viewed as a multi-stage process from the engineering of the plant to its operation stage, we adopt the compound ROA to valuate this project. In fact, the compound ROA is often used in the literature to price renewable investments characterized by sequential stages. For example, Loncar et al. (2017)) adopted the compound ROA to assess a wind farm in Serbia by using a reliable wind farm project valuation to divide the investment from the operating period. This work contributes to the existing literature by using the compound ROA to valuate a corn ethanol plant project considering that corn and gasoline prices follow a skew-geometric Brownian motion. The paper is organized as follows: Sect. 2 provides the methodology to valuate a typical corn ethanol plant project. Section 3 provides a case study by using representative data. Section 4 provides the conclusive remarks.

1.1 Contributions and novelties

A strand of literature applies real options models to biofuels (corn-ethanol plant) production investment. Our main novelty on this topic is the use of skew-Brownian motions that makes more realistic the corn ethanol plant evaluation differently from the quoted literature. In fact, in contrast to real option models that use a geometric Brownian motion, in our case we have verified that corn and gasoline prices follow a skew-Brownian motion that performs a more realistic evaluation about the corn ethanol plants.

Now, we explicitly discuss and compare our paper with the most relevant ones on this topic. Pederson and Zou (2009), using the binomial option pricing approach, state that the investment decision of a corn-ethanol plant can be valued as an American option assuming the production capacity plant constant. In addition, Pederson and Zou (2009) use a two-period plan start-up, where the investor has the right to start it any time before period 2. We connect to Pederson and Zou (2009) for the sequential investment approach but we differ form them since we provide a generalization assuming $n + 1$ engineering-construction investments and then m operational phases. We improve their approach since we introduce a time-dependent efficiency function γ_t, that describes how the plant can lose efficiency over time in converting corn into gasoline. In addition, we consider the skew-Brownian motion and consequently the real option evaluation under continuous setting.

Schmit et al. (2009) conduct a real options analysis of entry-exit decisions (based on Dixit & Pindyck, 1994) for dry-grind corn ethanol plants in order to incorporate the impact of rising volatility in market prices. In their approach, the stochastic gross margin follows a geometric Brownian motion. We differentiate since we use some of more restrictive assumptions, as skew-Brownian motions, that reflects accurately the risk and uncertainty in the ethanol industry and we prove that under the skew-geometric Brownian motions the project value increases its value.

Kirby and Davison (2010) model ethanol production as a discrete sum of spark spread options by considering the possibility that widespread ethanol production might cause a correlation between the corn and gasoline prices. Moreover, Kirby and Davison (2010) consider that ethanol production is highly subsidized and they value the real option investment opportunity as a simple exchange option (see Margrabe, 1978). Our paper connects to Kirby and Davison (2010) for the use of spread options in order to estimate the project value and for the real option methodology that overcomes the limitations of the discounted cash flows approaches. We differ from Kirby and Davison (2010) since we implement a compound option strategy (Kirby & Davison, 2010 use a simple exchange option) in order to evaluate a corn-ethanol plants organized in phased manner. The use of compound options is widely employed for valuation of R&D investments and therefore in the field of corn-ethanol plants. In addition, as seen above, we apply the skew-geometric Brownian motions assumption that, as analyzed in the case study, makes the gasoline and corn prices more realistic with respect to geometric Brownian motion considered in Margrabe (1978) approach.

Li et al. (2015) present a ROA for valuing the investment of a new technology for producing biofuels subject to construction lead times and uncertain fuel price. Their result indicates that the project profitability changes if the plant-investment is realized immediately or postponed. However, the sequential structure of the production process that requires the use of compound options is not addressed. So, we differentiate since the compound real option valuation is able to capture the value of flexibility that arises in the decision-making and operational processes.

Summing up, although previous works can represent a good starting point to propose a valuation methodology of corn-ethanol plant, they can be inadequate if the projects are characterized simultaneously by a high level of uncertainty, multistage nature and unpredictable future performance of corn and ethanol fuel prices. By using the compound exchange options model with skew-geometric Brownian motion, our valuation methodology can perform better than the existing literature for several reasons. First, the proposed model allows to monitor the uncertainty by including the operational flexibility to pursue the investment only if the revenues, adjusted for a time-dependent decreasing efficiency function, are higher than costs. Second, the compound options approach and its numerical valuation makes the model more suitable to incorporate potential managerial flexibility to proceed with the next investment stage only if the previous one is profitable. Third, we model adequately the corn and ethanol fuel prices by adopting the skew-geometric Brownian motion that performs a more realistic evolution of prices and fits better the relationship between corn and gasoline prices. This third point is an important novelty in the compound option pricing. Some recent papers, such as Bufalo et al. (2022), Zhu and He (2018), Pasricha and He (2022) apply the skew-geometric Brownian motion theory to portfolio optimization, or to simple or exchange option pricing, but to date we have seen no works which use these processes for compound option pricing. The major benefit brought by the skew-geometric Brownian motions consists of a better fitness with respect to the real data observed (see the statistical tests of Sect. 2.3), which results in a more accurate prevision about the positive outcome from the plant investment. Last but not least, our procedure provides an out of sample valuation methodology which allows an ex-ante investment decision in a corn-ethanol plant by forecasting its future performance. This is possible thanks to an accurate probabilistic assumption on the underlying processes which are modeled as skew-geometric Brownian motions, according to the statistical properties of the related time series observed. In this way, the out-of sample valuation is needed to make predictions on the processes and, consequently, on the future performances of the corn-ethanol plant.

2 Methodology

In this section we present a valuation methodology of a corn ethanol plant considering the basic characteristics of the project. First of all, we address our analysis to a general formulation in which there are: one engineering investment at time t_0; n construction investments at time $t_1, t_2, \ldots, t_n$ and subsequently m operational phases. After that, to emphasize our results, we describe a corn ethanol plant project in which there are three main sequential phases related to different investments. These phases are not independent: this means that a potential investor should proceed to the following investments only if the previous stages are profitable. We have identified three main stages that characterize corn ethanol plant implementation: the first is the engineering design activities at time t_0, then there is the construction of the plant at time t_1, and finally the investor can proceed to the operating stage in which the plant produces fuel ethanol from corn at time t_2. Different investment amounts are required for each stage. Specifically, I_0 is the capital required to start the project with the engineering activities. If the engineering investment is successful, the investor should construct the plant by investing I_1, and in sequence, if the construction phase is achieved without any failure, the investor can proceed to the operating stage by investing I_2.

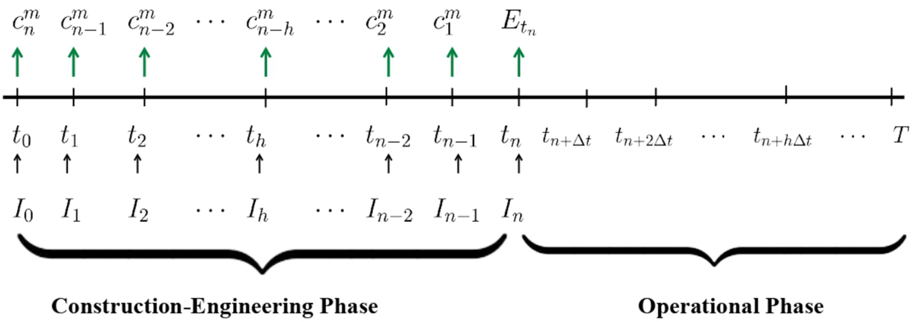

Fig. 1 Corn ethanol plant as compound options—general case

We propose a valuation methodology giving the potential investor the possibility to continue the project in each stage only if it is not characterized by some failures that could make the project unprofitable in the future. For example it might happen that the project turns out to be unfeasible in the engineering phase, or there can be some failures of procedural tasks in the construction phase, or again an increase of corn prices can makes the conversion financially unprofitable in the operating period. We model the valuation methodology of a corn ethanol plant project as a compound options model. In fact, the compound options model, also called compound ROA in the case of infrastructure investment, is used to price the investment characterized by various stages organized in phased manner (Villani, 2021; Cortelezzi & Villani, 2009) . The goal of our valuation methodology is to allow the investor to make an ex-ante valuation of a corn ethanol plant investment after making a projection of the project in the future by embedding in the model skew-geometric stochastic motions. Although previous studies valuated the ethanol plants by using the real options approach with fluctuating corn and gasoline prices e.g., Kirby and Davison (2010), none of them embeds the exchange options model into a multi-stage framework that affects these investments. We proceed by adopting the compound ROA to valuate a corn ethanol plant taking into account the sequential logic of projects and the unpredictable nature of corn and gasoline prices included in the model by stochastic motions.

2.1 A compound options approach for corn ethanol plant valuation

The focus of this section is the value the corn ethanol project as a compound option with stochastic input and output prices following a skew-Brownian motion. The compound options logic is shown graphically in the Fig. 1 where c_n^m represents the operational flexibility value considered as a compound option, E_{t_n} represents the project value, and I_n represents the the investments required in each stage.

Specifically, implementing the engineering investment (I_0), the investor obtains the compound option c_n^m so that he can make a reliable valuation of project at time t_0 considering both the option both to abandon the project and the unpredictable possibility of corn conversion in fuel ethanol. This value is called real option value RO, namely:

$$\mathrm{RO} = -I_0 + c_n^m \tag{1}$$

where c_n^m is the n-fold compound option c whose value at time t_0 is function of the next compound option exercisable $n-1$ times (as underlying asset), the investment I_1 (as strike price), and deadline $\tau_1 = t_1 - t_0$. So we can write that:

$$c_n^m = c\left(c_{n-1}^m, I_1, \tau_1\right)$$

In particular c_n^m represents, at time t_0, the value to realize the first construction phase investing the amount I_1 at time t_1, that gives the $(n-1)$-fold compound option. Recursively, c_{n-1}^m denotes, at time t_1, the value to implement the second construction investment I_2 at time t_2 in order to receive the $(n-2)$-fold compound option. So:

$$c_{n-1}^m = c\left(c_{n-2}^m, I_2, \tau_2\right)$$

with $\tau_2 = t_2 - t_1$.

Generalizing, the h-fold compound option, with $h = 2, \ldots, n$ can be write as:

$$c_h^m = c\left(c_{h-1}^m, I_{n-(h-1)}, \tau_{n-(h-1)}\right) \tag{2}$$

In addition, when $h = 1$, we obtain the evaluation at time t_{n-1} of the last construction phase that can be evaluated as a simple (1-fold) option denoted as s. So c_1^m, values the opportunity to implement the last construction investment I_n at time t_n in order to receive the operating benefits $\mathcal{V}_{t_n}^m$:

$$c_1^m = s\left(\mathcal{V}_{t_n}^m, I_n, \tau_n\right) \tag{3}$$

with $\tau_n = t_n - t_{n-1}$. Below we describe in more detail our approach.

Let $(\Omega, \mathbb{P}, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0})$ be a filtered probability space. The project value is calculated at time t_n starting from the initial date $t_0 \geq 0$, once the construction phase is completed and the operating phase represented by the real conversion starts. By selecting m equal time steps between t_n and the project maturity T each of length $\Delta t = \frac{T-t_n}{m}$, by denoting the risk-free rate by r, and by assuming that $h = 1, 2, \ldots m$, we can compute the project value as[1]

$$E_{t_n} = \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\mathcal{V}_{t_n}^m\right], \tag{4}$$

where $\mathbb{Q}$ is an equivalent martingale measure under which the (discounted) corn and gasoline processes are martingales (see Sect. 2.2),

$$\mathcal{V}_{t_n}^m = \sum_{h=1}^{m} e^{-r\cdot h\Delta t} \cdot V_{t_n+h\Delta t}, \tag{5}$$

and $V_{t_n+h\Delta t}$ is the project value represented by the revenues derived from the conversion of corn to gasoline at all instants after time t_n. For the sake of notation we shorten $(t_n + h\Delta t)$ with $t_{n,h}$. So, $V_{t_{n,h}}$ is calculated as follows:

$$V_{t_{n,h}} = \max\{0; (F_{t_{n,h}} \cdot \gamma_{t_{n,h}} - C_{t_{n,h}}) \cdot q\}, \tag{6}$$

where $F_{t_{n,h}}$ and $C_{t_{n,h}}$ are respectively the fuel and corn prices at time $t_{n,h}$, $\gamma_{t_{n,h}}$ is an efficiency factor at $t_{n,h}$, q is the expected quantity of fuel to be sold.

Notice that γ_t is the product of a suitable constant $\bar{\gamma}$ (see Sect. 3.1) by the decreasing function $f(t, \boldsymbol{b})$, i.e., $\gamma_t = \bar{\gamma} \cdot f(t, \boldsymbol{b})$, where $\boldsymbol{b} = [b_1, b_2]$ is a vector of some parameters. More specifically, $f(t, \boldsymbol{b})$ is defined as:

$$f(t, \boldsymbol{b}) = 1 - b_1\left(\frac{t - t_n}{T - t_n}\right)^{b_2} \quad (t \in [t_n, T]). \tag{7}$$

[1] Fixed $t \geq 0$, we denote by $\mathbb{E}_t[\cdot]$ the conditional expectation $\mathbb{E}[\cdot \mid \mathcal{F}_t]$.

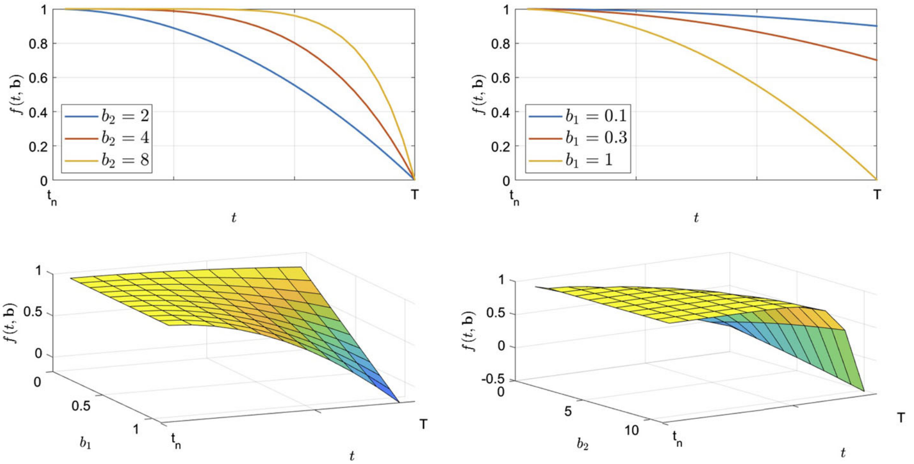

Fig. 2 From the top left to the right, first row: **a** $f(t, \boldsymbol{b})$ when $b_1 = 1$ and $b_2 \in \{2, 4, 8\}$; **b** $f(t, \boldsymbol{b})$ when $b_2 = 2$ and $b_1 \in \{0.1, 0.3, 1\}$. From the top left to the right, second row: **a** $f(t, \boldsymbol{b})$ when $b_2 = 2$ and $b_1 \in [0, 1]$; **b** $f(t, \boldsymbol{b})$ when $b_1 = 1$ and $b_2 \in [1, 10]$

where $b_1 \in [0, 1]$ denotes the ratio of industrial plant deterioration and b_2 measures the efficiency intensity of the plant. Figure 2 shows the trend of plant efficiency $f(t, \boldsymbol{b})$ by varying the parameters b_1 and b_2. We remark that, when b_2 rises, the plant efficiency remains almost constant for a long time, after which it tends to rapidly decrease. Instead, when the value of $b_1 = 1$, the plant efficiency tends to zero at final time T. For other values of b_1, the plant efficiency remains positive in T.

The expression in Eq. (6) explains that the investor gains by the difference between the price of the converted fuel $F_{t_{n,h}}$ and the price of corn $C_{t_{n,h}}$ during their time evolution, but if the price of corn becomes higher than fuel the investor should avoid to produce fuel by obtaining 0.

Finally, the simple (s) and compound (c) options are defined as

$$\begin{cases} s\left(\mathcal{V}_{t_n}^m, I_n, \tau_n\right) = e^{-r\tau_n} \cdot \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\max\{\mathcal{V}_{t_n}^m - I_n; 0\}\right]. \\ c\left(c_{h-1}^m, I_{n-(h-1)}, \tau_{n-(h-1)}\right) = e^{-r\tau_{n-(h-1)}} \cdot \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\max\left\{c_{h-1}^m - I_{n-(h-1)}; 0\right\}\right], \end{cases} \tag{8}$$

with the following boundary conditions

$$\begin{cases} s\left(\mathcal{V}_{t_n}^m, I_n, 0\right) = \max\{E_{t_n} - I_n; 0\} \\ c\left(c_{h-1}^m, I_{n-(h-1)}, 0\right) = \max\left\{\mathbb{E}_{t_0}^{\mathbb{Q}}\left[c_{h-1}^m\right] - I_{n-(h-1)}; 0\right\}. \end{cases} \tag{9}$$

We assume that the operational prices parameters change only slowly over time, in contrast to the rapid fluctuations in corn and gasoline commodity prices. As such, it appears a reasonably assumption that this cost set I_h can be modelled as constant within this period. In fact, previous studies such as Pederson and Zou (2009) does not consider the stochasticity in the investment values I_h by adopting them as constant. However, for a generalization of Eqs. (2) and (3) to stochastic costs I_h, refer to Appendix C.

We proceed using backward induction to the starting time t_0. The value of c, calculated at time t_{n-h} for $h \in [1, n]$ becomes the new underlying asset value from which is subtracted the investment required to build the plant ($I_{n-(h-1)}$). We therefore have an n-fold that allows us to capture the sequential investment characterized by various stages. The value of such compound option at t_0 is given by c_n^m through Eqs. (2) and (3).

2.2 Gasoline and corn stochastic dynamics

As showed in Sect. 2.3, the historical gasoline and corn prices do not exhibit normal returns. The same occurs in many project involving renewable energies whose price do not follow the usual geometric Brownian motion, see e.g., Chinhamu et al. (2021), Nunes et al. (2021).

In contrast to claims by Andersen et al. (2001), and/or Rogers (2018) the asset returns do not seem to be unconditionally normally distributed, but often show a significant amount of skewness and extra-curtosis. As investigated by Orlando and Bufalo (2021), note that a "more realistic work hypothesis is that time series follow a t-skew distribution". The t-skew distribution can be seen as a mixture of skew-normal distributions Kim (2001) which generalizes the normal distribution thanks to an extra parameter regulating the skewness.

The main properties of the skew-normal distribution, first introduced by Azzalini (1985) and Henze (1986), are summarized in Appendix A.

First of all, recall the definition of a skew-Brownian motion.

Definition 1 The stochastic process Y_t is said to be a (standard) skew-Brownian motion if the following conditions hold true

(i) $Y_0 = 0$;
(ii) for any $t \geq 0$, Y_t has continuous sample paths;
(iii) for any $t \geq 0$, Y_t is skew-normally distributed with location and scale parameters equal to 0 and t, respectively.

Following Corns and Satchell (2007), we characterize a (standard) skew-Brownian motion Y_t as the sum of a standard Brownian motion and a reflected Brownian motion, i.e.,

$$Y_t = \sqrt{1-\delta^2} \cdot W_t + \delta |U_t|, \tag{10}$$

where W_t and U_t are two independent (standard) Brownian motions and δ is the rescaled shape parameter. In the light of the above, we assume that both the gasoline and corn price follows a skew-geometric Brownian motion, i.e.,

$$\begin{cases} F_t = F_0 \, e^{\mu t + \sigma Y_t^F} \\ C_t = C_0 \, e^{\theta t + \eta Y_t^C}, \end{cases} \tag{11}$$

where $(F_0, C_0) \in \mathbb{R}_+^2$ and Y_t^F and Y_t^C are two correlated (standard) skew-Brownian motions. In order to correlate the two processes in Eq. (11) we follow (Bufalo et al., 2022, Proposition 4). More precisely, the next proposition generalizes such a result which is valid in the case of centered skew-Brownian motions, while our statement holds true for any skew-Brownian motion with non-zero expectation. So, let us define the process Y_t^C as

$$Y_t^C = \rho Y_t^F + \sqrt{\left(1 - \frac{2\delta^2}{\pi}\right)(1-\rho^2)} \cdot B_t, \tag{12}$$

where $\rho \in (-1, 1)$ is the correlation coefficient, $\delta = \frac{\beta}{\sqrt{1+\beta^2}}$ is the rescaled shape parameter of Y_t^F and B_t is a (standard) Brownian motion independent from Y_t^F.

Proposition 1 *The stochastic process Y_t^C is a (standard) skew-Brownian motion with shape parameter*

$$\alpha = \frac{\beta}{\sqrt{1 + (1+\beta^2)\left(1 - \frac{2\delta^2}{\pi}\right)\left(\frac{1}{\rho^2} - 1\right)}}, \tag{13}$$

such that $Corr(Y_t^C, Y_t^F) = \rho$.

Proof See Appendix B.1. □

To evaluate the project at the starting time t_0 through the compound option c_n^m (see Eq. (1)) we have to determine an equivalent martingale measure under which we compute the (conditioning) expectations.

We consider a market comprising two assets F_t and C_t with dynamics as in Eq. (11) together with a zero-coupon non-defaultable bond D_t with deterministic dynamics given by

$$dD_t = rD_t dt, \quad D_0 = 1,$$

where r is the risk-free rate. According to Eq. (10), Y_t^F may be written as

$$Y_t^F = \sqrt{1-\delta^2} \cdot W_t + \delta |U_t|.$$

Then, following Zhu and He (2018), there exists an equivalent martingale measure $\mathbb{Q}$, under wich the process

$$F_t = F_s\, e^{\left(r - \frac{\sigma^2}{2}\right)(t-s) + l(t-s) + \sigma \bar{Y}_{t-s}^F} \quad (s \geq 0), \tag{14}$$

with

$$l(t-s) = \ln\left[\Phi\left(\frac{(t-s)\delta\sigma^2 + \delta|U_s|}{\sqrt{t-s}\cdot\delta\sigma}\right) + e^{-2\delta|U_s|}\Phi\left(\frac{(t-s)\delta\sigma^2 - \delta|U_s|}{\sqrt{t-s}\cdot\delta\sigma}\right)\right], \tag{15}$$

and

$$\bar{Y}_t^F = \sqrt{1-\delta^2} \cdot \left(W_t + \frac{(\mu - r + \frac{\sigma^2}{2})(t-s)}{\sqrt{1-\delta^2}\cdot\sigma}\right) + \delta|U_t|, \tag{16}$$

is a martingale if discounted at risk-free rate r, i.e., $\mathbb{E}_s^{\mathbb{Q}}\left[e^{-rt}F_t\right] = e^{-rs}F_s$ $(0 \leq s < t)$. The same occurs for C_t, more specifically, we set[2]

$$\bar{Y}_t^C = \rho \bar{Y}_t^F + \sqrt{\left(1 - \frac{2\delta^2}{\pi}\right)(1-\rho^2)} \cdot \bar{B}_t, \tag{17}$$

where $\bar{B}_t$ is a (standard) Brownian motion independent of $\bar{Y}_t^F$.

From here on we express the processes F_t, C_t under the risk-neutral measure $\mathbb{Q}$.

With refer to Sect. 2.1, in order to evaluate any (compound) option c_h^m $(h \in [1, n])$ we give the next result. The next Proposition 2 and its related considerations help to implement the Monte Carlo technique for simulating the processes involved in our model.

[2] Note that, for the sake of clarity, we use the bar symbol to denote the stochastic part of our variables, when they are defined under the measure $\mathbb{Q}$.

First of all, let us introduce R_t^F and R_t^C the (log-)returns of the processes F_t, C_t, respectively; i.e.,

$$R_t^F = \ln\left(\frac{F_t}{F_s}\right), \qquad R_t^C = \ln\left(\frac{C_t}{C_s}\right) \qquad (0 \le s < t); \tag{18}$$

and denote by X_t^F, X_t^C the stochastic part of such processes. Notice that the function $\Phi(\cdot)$ denotes, hereinafter, the (standard) normal CDF.

Proposition 2 *The conditional densities of the processes X_t^F, X_t^C are:*

$$f_{X_t^F|X_s^F}(x_1|y,z) = \frac{1}{\sqrt{2\pi(t-s)}\sigma}\left(e^{-\frac{(x_1-y-z)^2}{2\sigma^2(t-s)}}\Phi(a_1) + e^{-\frac{(x_1-y+z)^2}{2\sigma^2(t-s)}}\Phi(a_2)\right) \quad (x_1 \in \mathbb{R}_+), \tag{19}$$

and

$$f_{X_t^C|X_s^C}(x_2|\tilde{y},\tilde{z}) = \frac{1}{\sqrt{2\pi(t-s)}\eta}\left(e^{-\frac{(x_2-\tilde{y}-\tilde{z})^2}{2\eta^2(t-s)}}\Phi(\tilde{a}_1) + e^{-\frac{(x_2-\tilde{y}+\tilde{z})^2}{2\eta^2(t-s)}}\Phi(\tilde{a}_2)\right) \quad (x_2 \in \mathbb{R}_+), \tag{20}$$

respectively, where

$$a_{1,2} = \frac{\delta^2(x_1 - y \mp z) \pm z}{\delta\sigma\sqrt{(t-s)(1-\delta^2)}}, \quad \tilde{a}_{1,2} = \frac{\delta^2\rho^2(x_2 - \tilde{y} \mp \tilde{z}) \pm \tilde{z}}{\delta\eta\rho\sqrt{(t-s)}\cdot\left[\sqrt{1-\delta^2} + \sqrt{\left(1-\frac{2\delta^2}{\pi}\right)(1-\rho^2)}\right]},$$

$$y = \sqrt{1-\delta^2}\cdot W_s, \quad \tilde{y} = \rho y + \sqrt{\left(1-\frac{2\delta^2}{\pi}\right)(1-\rho^2)}\cdot\bar{B}_s, \quad z = \delta|U_s|, \quad \tilde{z} = \rho z.$$

Proof See Appendix B.2 □

Now, we can compute the density of the process $\mathcal{V}_{t_n}$ defined in Eq. (5) by the convolution of the densities of each variable $e^{-r\cdot h\Delta t}V_{t_{n,h}}$, due to the independence of the skew-Brownian motion increments. But $V_{t_{n,h}}$ is a function of two correlated logarithmic skew-normal variables (see, e.g., Lin & Stoyanov, 2009), whose explicit density is unknown in literature.

To avoid this issue, we will use the densities (19), (20) to simulate under $\mathbb{Q}$ the stochastic part of the processes $F_{t_{n,h}}$, $C_{t_{n,h}}$ and, consequently, those of $V_{t_{n,h}}$ and $\mathcal{V}_{t_{n,h}}$. These considerations, allow us to evaluate the simple and compound options in Eqs. (2) and (3) through the Monte Carlo approach.

2.3 Empirical analysis on the distribution of gasoline and corn returns

This subsection provides a statistical analysis on the gasoline and corn returns. Our dataset is referred to a period from January 1994 to September 2021 (see Fig. 3), and it consists of monthly global price of corn in U.S. Dollars per Bushel and the U.S. all grades all formulations monthly retail gasoline prices in U.S. Dollars per Gallon.[3]

Among our tests, we mention the moments (see Table 1), the histograms (see the first picture of Figs. 4 and 5) and the quantile–quantile (Q–Q) plot Wilk and Gnanadesikan (1968) where are considered the normal distribution versus the skew-normal distribution. In order to prove that such returns are skewed and not normally distributed, we perform:

- the Kolmogorov–Smirnov (K–S) test Kolmogorov (1933), that is a nonparametric test of the equality of probability distributions, used to compare a sample with one reference probability distribution;

[3] The information about the dataset used are given in Sect. 3.1.

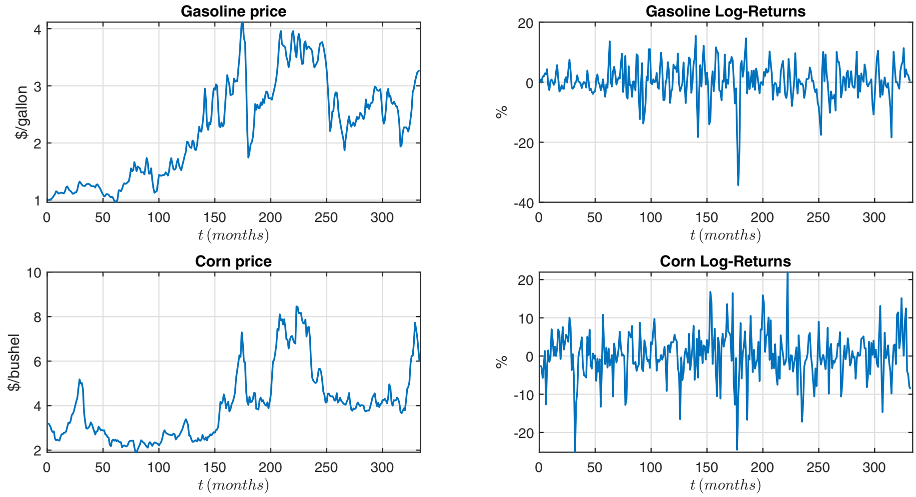

Fig. 3 From the top left to the right, first row: **a** Gasoline prices ($/gallon), **b** Gasoline log-returns (%). From the top left to the right, second row: **a** Corn prices ($/bushel), **b** Corn log-returns (%)

- the Dvoretzky–Kiefer–Wolfowitz (DKW) inequality Dvoretzky et al. (1956) which provides a measure of the distance between the empirical and normal CDF. This method is based on the Glivenko–Cantelli Theorem Tucker (1959) that estimates the tail probability of the Kolmogorov–Smirnov statistic.
  As in Orlando and Bufalo (2021), we introduce the variable, named "DKW exceeds", which enumerates the percentage of points of the theoretical CDF that exceed the DKW upper and lower bounds.

To be thorough, we compare the normal and skew-normal distribution with the following ones:

- Generalized hyperbolic (GH) distribution,

$$f(x; a_1, a_2, a_3, a_4, a_5) = \frac{(a_2^2 - a_3^2)^{a_1/2} \cdot a_2^{a_1+1/2} \sqrt{a_4^2 + (x - a_5)^2}\, K_{a_1 - 1/2}}{\sqrt{2\pi (a_2^2 - a_3^2)}\, a_4^{a_1+1} K_{a_1} \cdot (a_4^2 + (x - a_5)^2)^{1/4 - a_1/2}} \quad (x \in \mathbb{R}), \tag{21}$$

  where $a_3 \in \mathbb{R}$ is the asymmetry parameter, $a_4 \in \mathbb{R}$ is the scale parameter, $a_5 \in \mathbb{R}$ is the location, $a_2 \in R$, $(a_2^2 > a_3^2)$, and K_{a_1} $(a_1 \in \mathbb{R})$ denotes the modified Bessel function of the second kind.
- Generalized Pareto (GP) distribution,

$$f(x; \alpha_1, \alpha_2, \alpha_3) = \frac{(1 + \alpha_3 (\frac{x - \alpha_1}{\alpha_2}))^{-(1/\alpha_3 + 1)}}{\alpha_2} \qquad (x > \alpha_1), \tag{22}$$

  where $\alpha_3 \in \mathbb{R}$ is the shape parameter, $\alpha_2 \in \mathbb{R}_+$ is the scale parameter, and $\alpha_1 \in \mathbb{R}$ is the location.

The second row of Figs. 4 and 5 displays the Q–Q plots. These graphs represent the distribution quantiles comparing the CDF of the observed time series, which is unknown, a priori, with that of a specified distribution, chosen as benchmark. If the observed variable follows the

Table 1 First four central moments for Gasoline and Corn prices and their log-returns

Time series	Mean	SD	Skewness	Kurtosis
Gasoline	2.2602	0.8716	0.1972	1.8736
Corn	4.0098	1.5883	1.0050	3.2779
Gasoline (log-returns)	0.0036	0.0579	− 1.0614	7.9150
Corn (log-returns)	0.0019	0.0604	− 0.3563	5.1987

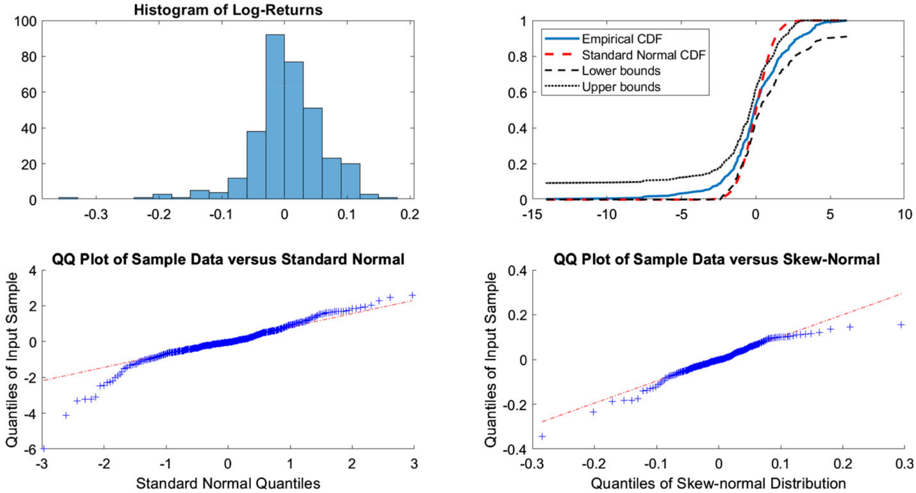

Fig. 4 From the top left to the right, first row: **a** Gasoline log-returns histogram, **b** Empirical CDF versus standard normal CDF for Gasoline log-returns. The dotted black lines represent the DKW upper and lower bounds. From the top left to the right, second row: **a** Gasoline log-returns Q–Q plot from Gaussian distribution, **b** Gasoline log-returns Q–Q plot from skew-normal distribution

theoretical distribution chosen, the Q–Q plot thickens across the line that connects the first and third quantiles of the data. The Q–Q plots show that the skew-normal performs better than the Gaussian (with the exception of some outliers).

The second picture of first row of Figs. 4 and 5 reveals that the empirical CDF of returns is far from the theoretical normal CDF and widely exceeds from the DKW lower and upper bounds.

The last check on the normality is performed by the KS test (see Table 2), which demonstrates that the most indicated distributions are the skew-normal and the generalized hyperbolic while the Gaussian and the generalized Pareto distributions do not seem to fit well.

2.4 Calibration and numerical simulation

The parameters of a skew-geometric Brownian motion are calibrated through the maximum log-likelihood (MLE) estimation.

Let R_t^F be the returns of the corn price process defined in Eq. (18), then $R_t^F = \mu t + \sigma Y_t^F \sim SN(\mu t, \sigma^2 t, \beta)$. Hence, if r_t^C denote the observations of R_t^C over n periods, the likelihood

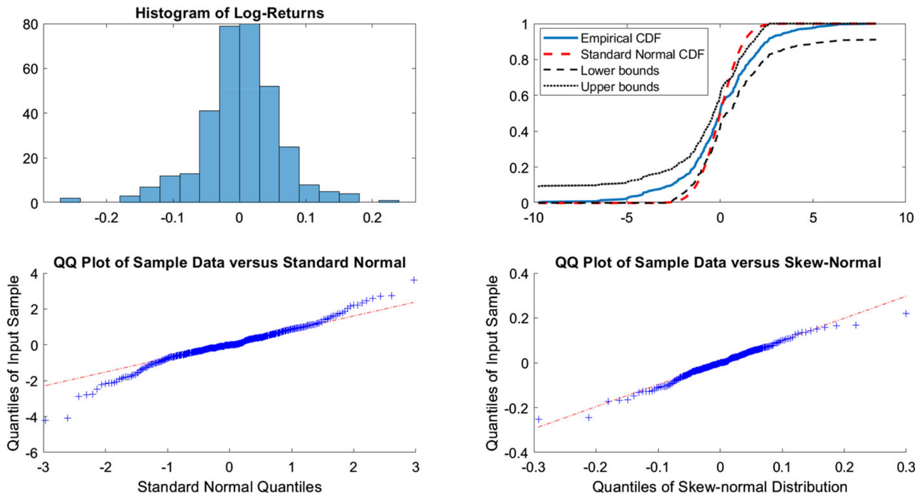

Fig. 5 From the top left to the right, first row: **a** Corn log-returns histogram, **b** Empirical CDF versus standard normal CDF for Corn log-returns. The dotted black lines represent the DKW upper and lower bounds. From the top left to the right, second row: **a** Corn log-returns Q–Q plot from Gaussian distribution, **b** Corn log-returns Q–Q plot from skew-normal distribution

Table 2 K–S test to detect the original distribution for the log-return time series

Time series	Normal			Skew-normal		Gen. Hyperbolic		Gen. Pareto	
	Resp	*p*-value	DKW exceeds (%)	Resp	*p*-value	Resp	*p*-value	Resp	*p*-value
Gasoline	1	0.0035	48.04	0	0.9641	0	0.4451	1	0
Corn	0	0.0316	54.65	0	0.9023	0	0.4877	1	0

The response is a boolean where 0 indicates that there is no evidence to reject the null hypothesis, and the value 1 is the opposite case

function of R_t^C is

$$\mathcal{L}^C(\mu, \sigma, \beta) = \frac{2^n}{(\sigma\sqrt{t})^n} \prod_{t=1}^{n} \varphi\left(\frac{r_t^C - \mu t}{\sigma\sqrt{t}}\right) \Phi\left(\beta \frac{r_t^C - \mu t}{\sigma\sqrt{t}}\right),$$

and the estimated parameters can be found as

$$(\hat{\mu}, \hat{\sigma}, \hat{\beta}) = \arg\left(\max_{(\mu,\sigma,\beta)} \ln \mathcal{L}^C(\mu, \sigma, \beta)\right).$$

The above procedure is implemented in the helpful R package of Azzalini (2021).

To estimate the correlation coefficient ρ we use the Spearman's rank correlation (see, e.g., Wayne, 1990) between r_t^c and the observations r_t^F of R_t^F. Then, Proposition 1 soon implies that

$$\hat{\alpha} = \frac{\hat{\beta}}{\sqrt{1 + (1 + \hat{\beta}^2)\left(1 - \frac{2\hat{\delta}^2}{\pi}\right)\left(\frac{1}{\hat{\rho}^2} - 1\right)}},$$

Table 3 Estimated parameters for the processes C_t and F_t

Parameters	μ	σ	β	δ	ρ	θ	η	α
Estimates	0.4669	0.3675	12.0651	0.9966	0.2265	0.9942	0.1633	0.3705

where $\hat{\delta} = \frac{\hat{\beta}}{\sqrt{1+\hat{\beta}^2}}$ (see also Appendix A).

Finally, following Bufalo et al. (2022, Section 5.1), the likelihood function of the correlated process $R_t^F = \theta t + \eta Y_t^F \sim SN(\theta t, \eta^2 t, \alpha)$ (conditioned to $(\hat{\alpha}, \hat{\delta}, \hat{\rho})$) is

$$\mathcal{L}^C(\theta, \eta) = \frac{2^n}{\left(\eta\sqrt{\hat{\rho}^2 t + \left(1 - \frac{2\hat{\delta}^2}{\pi}\right)(1-\hat{\rho}^2)t}\right)^n} \cdot \prod_{t=1}^{n} \varphi\left(\frac{r_t^F - \theta t}{\eta\sqrt{\hat{\rho}^2 t + \left(1 - \frac{2\hat{\delta}^2}{\pi}\right)(1-\hat{\rho}^2)t}}\right) \Phi\left(\hat{\alpha}\frac{r_t^F - \theta t}{\eta\sqrt{\hat{\rho}^2 t + \left(1 - \frac{2\hat{\delta}^2}{\pi}\right)(1-\hat{\rho}^2)t}}\right),$$

and the remaining estimations can be found as

$$(\hat{\theta}, \hat{\eta}) = \arg\left(\max_{(\theta,\eta)} \ln \mathcal{L}^F(\theta, \eta)\right).$$

In particular, in order to simulate the processes F_t, C_t under $\mathbb{Q}$, we use the risk-free rate r in their drift instead of the estimates $\hat{\mu}$, $\hat{\theta}$, which are involved into the skew-Brownian motions $\bar{Y}_t^F$, $\bar{Y}_t^C$ (according to Eq. (16)).

The above parameters are estimated in the time horizon $[0, t_0]$, while the out of sample simulations regards the remaining period $(t_0, T]$. The simulations are taken through the mean of $N = 10^5$ trajectories of a discretized scheme for the processes C_t, F_t.

Figure 6 shows the corn and gas prices (the solid black line and the dotted black one, in the horizon $[0, t_0]$ and $(t_0, T]$, respectively) versus the their future expected values (the solid blue line in $(t_0, T]$). We set $t_0 = 14\,Y$, $T = 13\,Y$, according to the case study analyzed in Sect. 3. The dotted red lines represent the 95% confidence interval of our predictions, which are calculated according to Qi et al. (2022). Moreover, Table 3 lists the estimated parameters of our model for any $t \in [0, t_0]$.

Observe that the correlation ρ is 0.2265, while Fig. 7 displays the Spearman's correlation between C_t and F_t from t_0 on, through a rolling window of (fixed) size equal to 60, i.e., 5 years. With a minimum value of 0.18, we can assess that the corn and gas time series are always correlated both before and after the starting date of project t_0.

3 Case study

3.1 The project valuation with compound options approach

In this section we propose an ideal case study by using plausible parameters that involves all the steps to start a corn ethanol plant project with 20 million gallons total production capacity of fuel ethanol. With refer to Sect. 2.1, we set the construction-engineering phases equal to $n = 2$. The investor expects to pursues the engineering costs I_0 at time $t_0 = 0$,

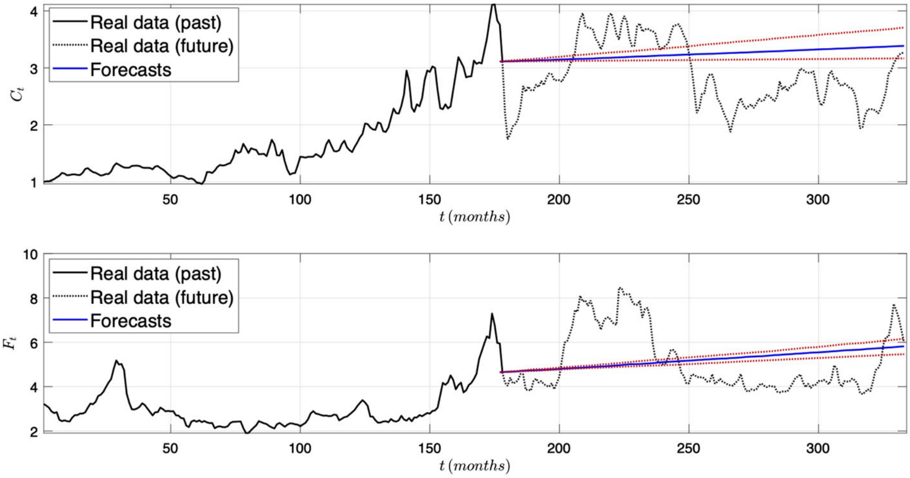

Fig. 6 Upper figure. Real corn prices (past, i.e., $t \in [0, t_0]$—solid black line, and future, i.e., $t \in (t_0, T]$—dotted black line) versus forecasted corn prices ($t \in (t_0, T]$—solid blue line). Lower figure. Real gas prices (past, i.e, $t \in [0, t_0]$—solid black line, and future, i.e., $t \in (t_0, T]$—dotted black line) versus forecasted gas prices ($t \in (t_0, T]$—solid blue line). The dotted red line represent the 95% confidence interval. (Color figure online)

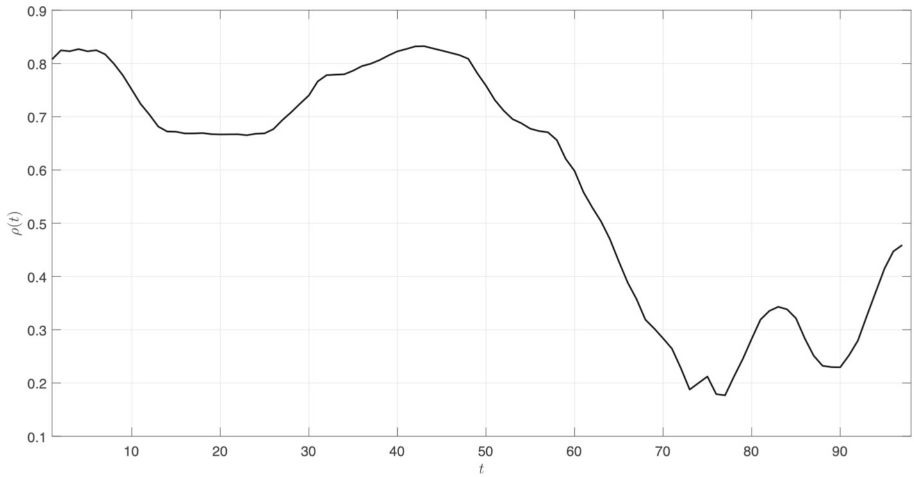

Fig. 7 Spearman's correlation between C_t and F_t computed for any $t \in [t_0, T]$ through a rolling window of size 60 months

the construction investment I_1 at time $t_1 = 1\,Y$, the operating costs I_2 at time $t_2 = 3\,Y$ that lasts up to maturity $T = 13\,Y$.[4] We expect that in the operating period the plant is able to produce an expected quantity of fuel to be sold 1.818 million gallons per year (20 million gallons/11 Y) in order to obtain a total production capacity of 20 million gallons.

[4] Notice that we have used likely time-frames for different stages that characterized a corn ethanol plant project. However, these period can change according unpredictable variable that can affect these project (public administration behavior, investor performance, environment, etc.).

This corn ethanol plant involves a construction cost of 2.25 \$/gallons and the plant expects to produce 20 million gallons of fuel ethanol in order to obtain $I_1 = \$\,45$ million (2.25×20 million gallons).[5] Once the construction investment is calculated, it is easy to calculate the engineering investment I_0 as a percentage of I_1 that in this case is equal to 2%, by obtaining $I_0 = \$\,0.90$ million. As above, to proceed in the operating period requires another investment I_2 that includes production expenses such as labor and maintenance costs, administrative and insurance costs, fees. This cost is equal to 0.15 \$/gallons $\times$ 1.818 million gallons of fuel ethanol per year (our expected production).[6] The value of $I_2 = \$\,2.103$ million is calculated as the discounted sum of operating costs at time $t_2 = 3\,Y$ that is the starting year of operating period by using a discount rate equal to $i = 8\%$.[7] Moreover, we use a conversion factor $\bar{\gamma} = 3.09$ that has been chosen by using the average value of the range studied by Kirby and Davison (2010) that goes from 2.87 to 3.31. We set the parameters b_1, b_2 of Eq. (7) equal to 1 and 2, respectively.[8] Before proceeding to apply our compound options model with stochastic parameters, we pursue a static Net Present Value (NPV) based on the simple Discounted Cash Flows (DCF) approach. This allows to make a comparison between the two approaches by discussing them. To make this analysis we consider the annual average of global prices of corn in terms of U.S. Dollars per Bushel[9] and the U.S. all grades all formulations annual average retail gasoline prices in terms of Dollars per Gallon from 2011 to 2021.[10] By using a discount rate $i = 8\%$, a static DCF analysis is shown in Table 4.

Following the DCF analysis, the investor should reject the corn ethanol plant project since it gives a negative result (NPV = − \$ 1.096 million). At this point, we can apply the compound options approach by considering the stochastic nature of revenues and costs.

The analysis starts by calculating the project value E_{t_2} where the gasoline (F_t) and corn price (C_t) follows a skew-geometric Brownian motion (as described in Sect. 2.2). To obtain E_{t_2} we consider a simulation derived from the historical series of monthly global price of corn in terms of U.S. Dollars per Bushel[11] and the U.S. all grades all formulations monthly retail gasoline prices in terms of Dollars per Gallon[12] from January 1994 to September 2021. Since we consider monthly time series, we use a monthly expected quantity of fuel to be sold $q = 151\,515.15$ gallons (1 818 181/12).

The first step gives a project value E_{t_2} equal to \$ 53.6 million after using a risk-free rate $r = 4\%$.[13] Then, we proceed via backward induction according the logic described in

[5] The construction cost of 2.25 \$/gallons has been extrapolated from the study of Pederson and Zou (2009).

[6] The operating cost of 0.15 \$/gallons has been extrapolated from the study of Pederson and Zou (2009).

[7] The discount rate value has been taken considering the study of Schmit et al. (2009).

[8] Based on Report CTI -Biomass Plant for the energy production by Riva et al., it is plausible that on the basis of a technical-economic analysis, the profitability index of this type of plant, taking into account the humidity accumulated of the vegetable fuel and the weight of the ashes, can be estimated assuming $b_2 = 2$.

[9] Note that the yearly global prices of corn from 2011 to 2021 in terms of U.S. Dollars per Bushel have been extrapolated from FRED economic data (https://fred.stlouisfed.org/series/PMAIZMTUSDM) after converting the metric ton in bushels and making an average of monthly prices.

[10] Note that yearly retail gasoline prices in terms of dollars per gallon from 2011 to 2021 have been extrapolated from "eia" website (https://www.eia.gov/dnav/pet/pet_pri_gnd_a_epm0_pte_dpgal_m.htm), Indipendent Statistics & Analysis U. S. Energy Information Administration after making an average of monthly prices.

[11] Note that the global price of corn in terms of U.S. Dollars per Bushel have been extrapolated from FRED economic data (https://fred.stlouisfed.org/series/PMAIZMTUSDM) after converting the metric ton in bushels.

[12] Note that retail gasoline prices in terms of dollars per gallon have been extrapolated from "eia" website (https://www.eia.gov/dnav/pet/pet_pri_gnd_a_epm0_pte_dpgal_m.htm), Indipendent Statistics & Analysis U. S. Energy Information Administration.

[13] The risk-free rate $r = 4\%$ has been extrapolated from the study of Pederson and Zou (2009).

Table 4 Discounted cash flows method

Years	Gasoline prices ($)	Corn prices ($)	Margin after conversion ($)	Cash flows (million $)
0	0	0	0	− 0.90
1	0	0	0	− 45.00
2	0	0	0	0
3	3.58	7.41	3.6378	6.341
4	3.69	7.58	3.8097	6.654
5	3.58	6.58	4.4729	7.860
6	3.44	4.90	5.7350	10.154
7	2.51	4.31	3.4510	6.002
8	2.25	4.04	2.9159	5.029
9	2.53	3.93	3.8919	6.803
10	2.82	4.18	4.5269	7.958
11	2.69	4.32	3.9765	6.957
12	2.26	4.21	2.7768	4.776
13	2.98	6.66	2.5564	4.375
NPV = − $ 1.096 million				

Eqs. 8 and 1 by using monthly instant times, and we obtain a real option (RO) value skew-geometric Brownian motion equal to $ 0.8 million. Moreover, we extrapolated the RO value under the geometric Brownian motion in order to make a comparison between it and our skew-geometric Brownian motion assumption.

All the values used for the NPV and compound options valuation are summarized in Table 5.

3.2 Sensitivity analysis

This section provides a sensitivity analysis of how the RO value under skew-geometric Brownian motion varies by changing t_1, t_2, $\bar{\gamma}$ and b_2. Figure 8 shows that by delaying the time period of construction investment t_1 or time period of operating investment t_2, the RO value tends to increase. This is because the extension of the time period of project investment makes the project riskier about its future performance and the operational flexibility captured by RO value becomes more valuable. Figure 8 also shows that if the value of $\bar{\gamma}$ goes up, the RO value tends to increase. This is a quite intuitive aspect since by improving the conversion factor allows to makes the project more attractive. Moreover, we can also state that an increase in the plant efficiency intensity b_2 increases average plant efficiency and the RO value.

3.3 Discussion of results

According the analysis made in the previous section, the corn ethanol plant investment is profitable for potential investor since the RO value under skew-geometric Brownian motion, equal to $ 0.8 million, is positive and quite high. In fact, we note that the ratio between the compound option value (c_2^m) and the investment to initiate the project (I_0) is higher than 1 to demonstrate the positive revenue-generating capacity of the plant ($\frac{c_2^m}{I_0} = \frac{1.7}{0.9} = 1.89 > 1$).

Table 5 Results of compound ROA

Item	Description	Value
E_{t_2}	Project value at time t_2	\$ 53.6 million
I_0	Investment for engineering	\$ 0.90 million
I_1	Investment for construction	\$ 45 million
I_2	Investment for operation	\$ 2.103 million
$\bar{\gamma}$	Conversion factor	3.09
r	Risk free rate	4%
i	Discount rate	8%
q	Expected quantity of fuel to be sold per month	0.1515 million gallons
t_0	Year of engineering investment	0 (starting point)
t_1	Year of construction investment	1
t_2	Year of operating investment	3
T	Year of maturity	13
NPV	Static DCF approach	− \$ 1.096 million
RO value with geometric Brownian motion	Value of project under uncertainty and sequential logic	\$ 0.194 million
c_2^m	Compound option value	\$ 1.7 million
RO value with skew-geometric Brownian motion	Value of project under uncertainty and sequential logic	\$ 0.8 million

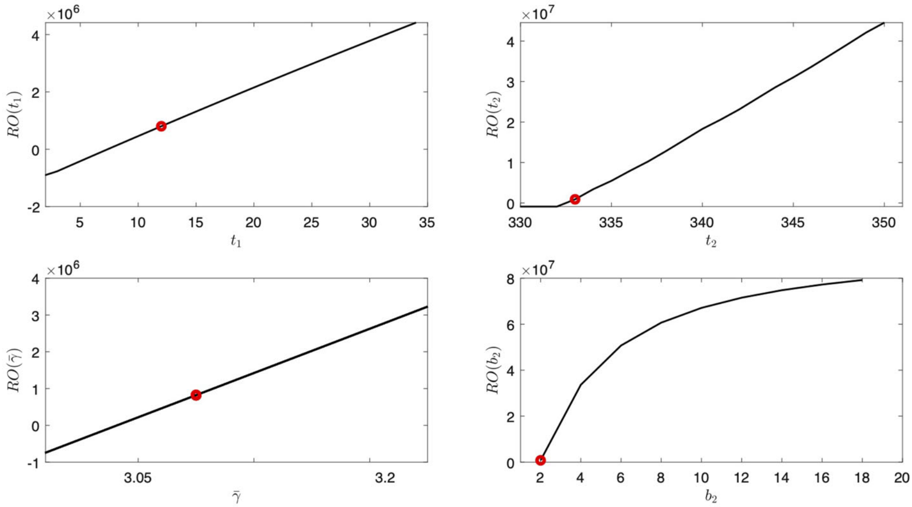

Fig. 8 From the top left to the right, first row: **a** RO values when $t_1 \in [1, 36]$ (months), **b** RO values when $t_2 \in [330, 350]$ (months). From the top left to the right, second row: **a** RO values when $\bar{\gamma} \in [3, 3.2]$, **b** RO values when $b_2 \in [2, 18]$. The red circle point denotes the RO value reported in Table 5, i.e., \$ 0.8 million. (Color figure online)

The profitability of the corn-ethanol plant is confirmed also by the RO value under Geometric-Brownian motion since its positive value is equal to \$ 0.194 million, even if it is lower than RO value under skew-geometric Brownian motion. In fact, by making a comparison between these two RO values, we note that the compound options model based on geometric Brownian motion tends to underestimates the project value. This is because the skew-Brownian motion allows to price more adequately the irregularity of corn and gasoline prices in comparison to Geometric-Brownian motion. The compound options valuation under skew-Brownian motion encourages the corn-ethanol plant projects. These findings are in line with the studies of Pederson and Zou (2009) and Schmit et al. (2009). The RO value represents an ex-ante project valuation that allows to makes aware the investor about the future opportunities of the project. Differently from the classical Net Present Value (NPV), the RO value embeds the "optionality" to change investment decision during the lifetime of the project if a certain stage turns out to be unprofitable. The literature terms this as 'managerial flexibility'. We have also shown a valuation comparison between NPV and RO approaches. Following the NPV method the investor should reject the corn-ethanol plant project since it is not able to value the managerial flexibility. So, we can state that, there is a huge difference between the discounted cash flows approach and our innovative approach. The standard indicator based on discounted cash flows method leads to a rejection of this corn-ethanol plant investments since it gives a negative results (NPV = − \$ 1.096 million). This would represents a wrong support to decision making. Differently from the NPV, the corn-ethanol plant valuation based on our compound options approach increases in value (RO = + \$ 0.8 million) since the model includes the managerial flexibility aspects and an adequate stochastic approach of prices evolution based on skew-Brownian motion. In this sense, the operational flexibility to activate the investment options at each stage—acting as compound options—is valuable. These results are in line with the study of Ross (1995) that explained that the NPV can lead to reject a project that should be accepted.

In this case study, a positive RO value means that the project appears attractive in financial terms in addition to pursue sustainability goal considering that it is viewed as a renewable investment. As the results of Sect. 3.1 show, despite the presence of uncertainty and sequential stage decisions, using compound ROA with skew-geometric Brownian motion driven price uncertainties and incorporating the value of managerial flexibility allows the investor to appropriately price corn ethanol plant projects.

4 Conclusions

This article proposes a methodology to value corn ethanol plant projects characterized by corn and gasoline price uncertainty. The staged nature of the decisions available to the project owner arises because the investor proceeds with the following investment only if it is on average profitable given the information obtained at the end of the previous stage. To consider these valuation characteristics, we adopt compound ROA modelling corn and gasoline prices to follow, in accordance with historical data, skew-geometric Brownian motions. We also propose a case study to apply our valuation methodology to likely data. The results show that the corn ethanol plant project appears attractive and financially profitable by using compound ROA with skew-geometric Brownian motions for corn and gasoline prices. By adding these results to the sustainability goal of the plant allows to have a wider vision of the benefits of the renewable investments like this. In this paper we model a corn ethanol plant valuation as a compound options approach with stochastic revenues and costs parameters. Further research can embed in this approach the option to abandon the corn ethanol plant project for a salvage value. This would allow to provide a broader view of the operational flexibility insight in the project valuation. In addition, it could be interesting to determine, as another subsequent contribution, an optimal time in which, before reaching the lifetime ends in which efficiency approaches zero, it is advisable to replace the plant with a new one, modelled as the real option to switch.

Acknowledgements This paper is dedicated to the memory of Peter Carr, for his contribution in the field of mathematical finance. Moreover, we thank the anonymous reviewers for their many insightful comments and suggestions.

Funding Open access funding provided by Università degli Studi di Bari Aldo Moro within the CRUI-CARE Agreement.

A Skew-normal density

In this appendix we recall some well-known definitions and properties of the skew-normal density, described by Azzalini (see e.g., Azzalini, 1985, 2013).

Definition A1 A random variable X is said to be (standard) skew-normal if it has the following density

$$p(x) = 2\varphi(x)\Phi(\beta x) \quad (x \in \mathbb{R}), \tag{23}$$

where φ and Φ are the normal density and the normal cumulative distribution functions, respectively. The real number β is called shape parameter.

More generally, if X is a standard skew-normal random variable, the random variable

$$Y = \xi + \omega X \quad (\xi \in \mathbb{R},\ \omega \in R_+),$$

is a skew-normal with location parameter ξ, scale parameter ω and shape parameter β. Its density function is

$$p(y) = \frac{2}{\omega}\varphi\left(\frac{y-\xi}{\omega}\right)\Phi\left(\beta\,\frac{y-\xi}{\omega}\right) \quad (y \in \mathbb{R}), \tag{24}$$

and it is named $Y \sim SN(\xi, \omega^2, \beta)$.

The principal moments of a skew-normal random variable are given by the following proposition.

Proposition A2 *If $Y \sim SN(\xi, \omega^2, \beta)$, its moment generating function is given by*

$$\mathbb{E}\big[e^{kY}\big] = 2e^{k\xi + \frac{k^2\omega^2}{2}}\Phi(k\delta\omega), \tag{25}$$

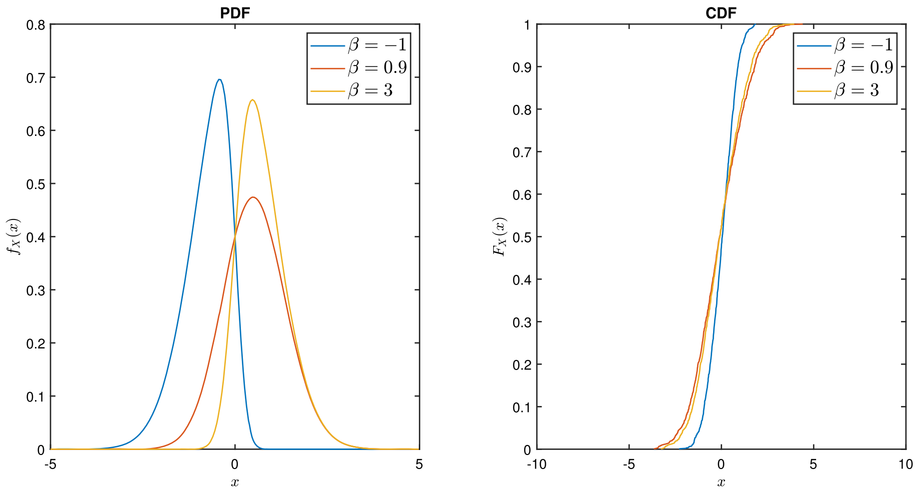

Fig. 9 PDF and CDF of a skew-normal random variable X for different values of β

where

$$\delta = \frac{\beta}{\sqrt{1+\beta^2}}, \tag{26}$$

is the rescaled shape parameter. In particular, one has

$$\mathbb{E}[Y] = \xi + \omega\delta\sqrt{\frac{2}{\pi}}, \tag{27}$$

and

$$Var(Y) = \left(1 - \frac{2\delta^2}{\pi}\right)\omega^2. \tag{28}$$

From the above result comes an interesting property that will be helpful in Sect. 2.2.

Proposition A3 *If Y_1, Y_2 are two independent random variables, with $Y_1 \sim SN(\xi, \omega^2, \beta)$ and $Y_2 \sim N(\mu, \sigma^2)$, then*

$$Y_1 + Y_2 \sim SN(\xi + \mu, \omega^2 + \sigma^2, \widetilde{\beta}), \tag{29}$$

where

$$\widetilde{\beta} = \frac{\beta}{\sqrt{1 + (1+\beta^2)\frac{\sigma^2}{\omega^2}}}.$$

Proof See Azzalini (2013, Proposition 2.3). □

Remark A4 Observe that the sum of two independent skew-normal random variables is not skew-normally distributed. The exact distribution of the sum of independent is given by Nadarajah and Li (2017, Theorem 2.1), and it involves the so-called Kampé de Fériet function de Fériet (1937).

B Proofs of Subsection 2.2

B.1 Proof of Proposition 1

Proof Definition 1 states that $Y_0^C = 0$, Y_t^C has continuous sample paths and $Y_t^C \sim SN(0, t, \alpha)$, with shape parameter α given by Eq. (13) due to Proposition 1. In particular,

$$Var(Y_t^C) = \rho^2\, Var(Y_t^F) + \left(1 - \frac{2\delta^2}{\pi}\right)(1-\rho^2)Var(B_t) = \left(1 - \frac{2\delta^2}{\pi}\right)t.$$

Moreover, due to the independence between Y_t^F and B_t, we can write

$$\begin{aligned} Cov(Y_t^F, Y_t^C) &= \mathbb{E}[Y_t^F \cdot Y_t^C] - \mathbb{E}[Y_t^F] \cdot \mathbb{E}[Y_t^C] \\ &= \rho\mathbb{E}[(Y_t^F)^2] + \sqrt{\left(1 - \frac{2\delta^2}{\pi}\right)(1-\rho^2)} \cdot \mathbb{E}[Y_t^F] \cdot \mathbb{E}[B_t] - \rho(\mathbb{E}[Y_t^F])^2 \\ &= \rho Var(Y_t) \\ &= \rho\left(1 - \frac{2\delta^2}{\pi}\right)t. \end{aligned}$$

which soon implies

$$Corr\left(Y_t^F, Y_t^C\right) = \frac{Cov\left(Y_t^F, Y_t^C\right)}{\sqrt{Var\left(Y_t^F\right) \cdot Var\left(Y_t^C\right)}} = \rho.$$

□

B.2 Proof of Proposition 2

Proof As suggested in Zhu and He (2018) the skew-Brownian motion does not have stationary increments, and generally, for any $0 \le s < t$, the conditioning density of the skew-Brownian motion Y_t defined in Eq. (10) is given by

$$f_{Y_t|Y_s} = f_{\sqrt{1-\delta^2}\cdot W_t\left|\sqrt{1-\delta^2}\cdot W_s\right.} * f_{\delta|U_t|\left|\delta|U_s|\right.}, \tag{30}$$

where

$$\begin{aligned} f_{\sqrt{1-\delta^2}\cdot W_t\left|\sqrt{1-\delta^2}\cdot W_s\right.}(u_1|w_1) &= \frac{1}{\sqrt{2\pi(t-s)(1-\delta^2)}} e^{-\frac{(u_1-w_1)^2}{2(t-s)(1-\delta^2)}} \quad (u_1 \in \mathbb{R}), \\ f_{\delta|U_t|\left|\delta|U_s|\right.}(u_2|w_2) &= \frac{1}{\delta\sqrt{2\pi(t-s)}}\left[e^{-\frac{(u_2-w_2)^2}{2(t-s)\delta^2}} + e^{-\frac{(u_2+w_2)^2}{2(t-s)\delta^2}}\right] \quad (u_2 \in \mathbb{R}_+), \end{aligned}$$

and $*$ denotes the usual convolution product (being W_t and U_t independent). In particular, we have set $w_1 = \sqrt{1-\delta^2} \cdot W_s,\ w_2 = \delta|U_s|$, while $f_{\delta|U_t|\left|\delta|U_s|\right.}$ represents a (conditioning) folded-normal density (see e.g., Tsagris et al., 2014).

Now, if

$$X_t^F := \sigma\mathcal{Y}_t^F,$$

is the stochastic part of the return process R_t^F, by virtue of Eq. (30), we get

$$f_{X_t^F|X_s^F}(x_1|y,z) = \frac{1}{2\pi(t-s)\sigma^2\delta\sqrt{1-\delta^2}} \int_0^{+\infty} e^{-\frac{(x_1-\zeta-y)^2}{2(t-s)(1-\delta^2)\sigma^2}} \left[e^{-\frac{(\zeta-z)^2}{2(t-s)\delta^2\sigma^2}} + e^{-\frac{(\zeta+z)^2}{2(t-s)\delta^2\sigma^2}} \right] d\zeta,$$

which easily leads to formula (19).

With analogous arguments, we may conclude that the conditioning density of

$$X_t^C := \eta\left[\rho\sqrt{1-\delta^2}\cdot W_t + \sqrt{\left(1-\frac{2\delta^2}{\pi}\right)(1-\rho^2)}\cdot \mathcal{B}_t\right] + \rho\delta|U_t|,$$

is given by Eq. (20). □

C Stochasticity of costs

As a further generalization, we may assume that also the costs I_h ($h \in [1, n]$) are stochastic processes, whose dynamics are given by

$$I_{h,t} = k_h(\bar{I}_h - I_{h,t}) + \beta_h dZ_{h,t}, \tag{31}$$

where $Z_{h,t}$ are independent Brownian motions, $\bar{I}_h$ represent the long-run mean with speed rate $k_h \in \mathbb{R}_+$ and volatility $\beta_h \in \mathbb{R}_+$. For simplicity, one can set the starting value of such processes equal to $\bar{I}_h$ so that any expectation coincide with $\bar{I}_h$ itself (see e.g., the values taken in Sect. 3.1). Otherwise, if real data were available for I_h, one could calibrate the parameters of the process (31) like in Orlando et al. (2019, Section 4.3).

In the general case of random costs, the boundary conditions (9) become

$$\begin{cases} s\left(\mathcal{V}_{t_n}^m, I_n, 0\right) = \mathbb{E}^{\mathbb{P}}\left[\max\{E_{t_n} - I_{h,n}; 0\}\right] \\ c\left(c_{h-1}^m, I_{n-h+1}, 0\right) = \mathbb{E}^{\mathbb{P}}\left[\max\{\mathbb{E}_{t_0}^{\mathbb{Q}}[c_{h-1}^m] - I_{h,n-h+1}; 0\}\right]. \end{cases} \tag{32}$$

It is clear the measure used to compute each payoff is just the objective probability measure ($\mathbb{P}$), as distinct from the risk-neutral ($\mathbb{Q}$) measure used for option pricing.

D Generating skew-Brownian motions

As described in Lejay (2006) a natural construction of a (standard) skew-Brownian motion is given by solving the stochastic equation

$$Y_t = Y_t^0 + \delta L_t^Y, \tag{33}$$

where Y_t^0 is a standard Brownian motion, $|\delta| < 1$, and L_t^Y is the symmetric local tine at 0 of Y_t, i.e.,

$$L_t^Y = \lim_{\varepsilon \searrow 0} \frac{1}{2\varepsilon} \int_0^t \mathbf{1}_{[-\varepsilon,\varepsilon]}(Y_u)\, du.$$

As observed in Corns and Satchell (2007), an alternative construction of a (standard) skew-Brownian motion Y_t consists of the sum of a standard Brownian motion and a reflected

Brownian motion, i.e., $Y_t = \sqrt{1-\delta^2} \cdot W_t + \delta|U_t|$, as in Eq. (10). By virtue of Tanaka's formula, Eq. (10) may be rewritten as

$$\sqrt{1-\delta^2} \cdot W_t + \delta \int_0^t \operatorname{sign}(U_u)\, dU_u + \delta L_t^U, \tag{34}$$

which is equivalent to Eq. (33) (see Corns & Satchell, 2007, Proposition 2.1).

Lejay and Martinez (2006) develops a numerical comparison about several methods, including an Euler and a deterministic scheme, used to simulate skew processes. It is clear that the main difficulty in simulating the skew-Brownian motion in Eq. (10) is the presence of a reflected Brownian motion. However, as showed in Asmussen et al. (1995), the discretization error associated with the Euler scheme for simulating such process has an order of convergence of $1/2$. In Lejay (2006), Lejay and Martinez (2006) many sophisticated techniques are considered in order to approximate a skew-Brownian motion by some classical SDEs (without a reflected Brownian motion) but they are more expensive, having an order of convergence greater than 1. Therefore, it appears that the representation (10) is the most convenient from a numerical point of view.

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