




Seat assignment problem with the payable up-grade as an ancillary service of airlines

Moongil Yoon¹ · Habin Lee² 

Accepted: 30 September 2021 / Published online: 24 October 2021
© The Author(s) 2021

Abstract

The de-bundling and re-bundling of service products offered by low cost carriers from service innovation have become 'à-la-carte' services and chargeable ancillary services to generate additional revenues in the airline industry. In this paper, we focus on the payable upgrade option in airlines to increase revenue when customers' demands are uncertain. This study is as an extension of planned upgrade, which guarantees the use of low-quality services at a low price, but requires customers to pay to use upgraded benefits. In this paper, we focus on the seat assignment problem with the payable upgrade option to maximize revenue. Also, we set the condition under which payable upgrade options can generate more revenue. Furthermore, applying Belobaba's EMSR approach, we suggest an effective seat assignment method for multiple fare classes with a payable upgrade option to increase the total revenue. With a simple numerical example, we find that introducing a payable upgrade option can increase the revenue. Our method for allocating seats for multiple fare classes with a payable upgrade option can contribute effectively to revenue increases in airlines' branded fare products, as well as in other service industries.

Keywords Option products · Payable upgrade · Seat assignment · Revenue management · Ancillary revenue

1 Introduction

Deregulation and the advent of low-cost carriers (LCCs) during the past decades have accelerated the serious competition among airlines. Airlines are offering their customers innovative services to improve their competitiveness and profits. Traditionally, airlines increase their revenue by applying complicated revenue management (RM) models and systems. RM in airlines is aimed at effective sales controls and pricing to maximize total revenues for airline

✉ Habin Lee
habin.lee@brunel.ac.uk

Moongil Yoon
mgyoon@kau.ac.kr

¹ Korea Aerospace University, Goyang-si, Gyeonggi-do, South Korea

² Brunel University London, Uxbridge, Middlesex, UK

tickets, which are perishable products whose values become zero if they are not consumed within a certain time period (Chiu et al., 2017; Feng et al., 2021; Qin et al., 2020). Recently, the de-bundling and re-bundling of service products offered by LCCs have become 'à-la-carte' services and chargeable ancillary services to generate additional revenues. Ancillary revenues are recognized as an important revenue source for both LCCs and traditional full-service carriers (FSCs).

It is very important to respond effectively to the uncertainty of future demands for a perishable product in order to maximise revenues. Gallego et al. (2008) suggested several mechanisms to overcome the future demand uncertainties in the airline industry: standbys, bumping, the re-plane concept, flexible products, last-minute discounts, and auctions. However, none of them is perfect to maximize the revenues of airlines. Some (standbys and bumped passengers) add operational complexities, some require additional processes from customers, and some may reduce customer convenience.

Numerous RM models have been widely used in the airline industry to improve financial performance in recent decades, but most of them have low forecasting accuracy due to uncertain future demands (Gallego et al., 2008). In particular, seats protected from the low fare demand for high fare products frequently become empty or insufficient and airlines lose the opportunity to get more revenue. There are many studies on RM, and comprehensive reviews on revenue management problems can be found in Feng et al. (2015), Talluri and van Ryzin (2004) and McGill and van Ryzin (1999). Although airlines have been able to increase revenues by using those RM techniques, they have struggled to make profit for decades and the development of new services to increase their revenues is pivotal for their sustainability.

In this paper, we suggest an innovative product, called the payable upgrade option. This product allows customers to buy an upgrade option at additional cost when they buy one of multiple products with different prices. The upgrade option guarantees the use of higher fare products depending on their availability at the time. If there is no availability, the airlines recall the upgrade option and customers are paid compensation. In this case, they can simply use the lower fare products that they originally bought. The new product creates value for both customers and airlines, as customers can buy high fare products at a lower price, while airlines can create additional revenue if the demand for the high fare products is lower than expected.

The payable upgrade option is similar to the call option or callable product (Gallego et al., 2008) that is widely used in the airline industry. Airlines sell some flight tickets with call options to respond to uncertain future demands. A callable flight ticket can be recalled by an airline at some point in the future before the departure time if the demand for higher fare tickets is high: therefore, the airline can increase its revenue. Customers whose tickets are recalled by the airline need to give up these tickets, with compensation that is higher than the original price. Callable products are attractive to customers whose flight schedules are flexible, as they can use higher fare seats with lower prices, while airlines can resell these tickets at higher prices if demand for seats on a flight suddenly increases. Similar to callable productions, the payable upgrade option product requires customers to purchase a lower fare product and the upgrade option as well. Once the upgrade option is recalled by an airline, customers receive compensation for this option and use the original product. Otherwise, customers can use the higher fare product at the lower fare and pay a fee to upgrade. Our study is an extension of planned upgrade that guarantees the use of services with a low quality at a low price; however, customers are required to pay to use the upgrade benefit in this case. In this paper, we concentrate on the seat assignment problem with the payable upgrade option to maximize airlines' revenues.

The organization of the paper is as follows. We review existing ancillary services in the airline industry in the next section. The main focus is to highlight the novelty of the new service proposed in this paper. Then, we define a capacity allocation model with a payable upgrade option that is illustrated using numerical examples. We discuss the implications of the findings of the study and conclude.

2 Revenue management models in the airline industry

There have been two types of development with regard to chargeable ancillary services for RM in the airline industry. One is the de-bundling of products, which is developed via decomposing existing full services. The other is developing new products by re-designing the business processes.

The à-la-carte service and branded fare products (Vinod & Moore, 2009) in airlines are examples of the former. Air New Zealand introduced the first brand fare products in 2004 and since then numerous airlines have offered similar products. In this scheme, airlines bundle elements, such as pre-reserved seats, the amount of frequent flyer miles, lounge access, baggage limits and so on, into the base fares. For example, British Airways offers four different products including Economy, Premium Economy, Business/Club, and First Class tickets that have different compositions of ancillary services depending on their privileges.

A flexible service is a product developed by redesigning business processes. There are many types of flexible services that are proposed in various forms depending on the scope of rights in the transaction processes between airlines and customers. For example, the purchase delay option allows a delay of payment after a reservation. Aydin et al. (2016) developed an RM model with commitment options, which allow a time delay to make payments after ticket reservations. The callable or call-back option, the buy-back option, and the upgrade option are examples of flexible services in the airline industry and play a significant role in increasing revenues.

Callable products have been developed to reduce inventories due to market uncertainty in the airline industry (Sheffi, 2005). Call-back is a simple concept in airlines. Callable flight tickets can be recalled by airlines at some point in the future before the departure time. Customers whose tickets are recalled need to give up these tickets and are compensated by the airline with pre-specified prices (which are of course higher than the original prices). If the airline does not recall the tickets, customers can use them as normal. Customers whose flight schedules are flexible would choose callable tickets for potential profit, while airlines can resell these tickets at higher prices if demand suddenly increases. Gallego et al. (2008) reveal that callable products are more effective for high-fare customers. Also, they show that airlines can increase their revenues from selling any or all flight tickets in combination with callable products. This is also called the "call-back" option (Li et al., 2016).

A similar idea to callable products is "buy back", which is an option by airlines to re-purchase already sold passenger tickets (Lardeux et al., 2018). Once airlines re-purchase tickets from their customers, their reservations are cancelled, and the seats can be sold to customers who are willing to pay the higher prices. In the buy-back option, customers have the choice to accept or reject re-call offers. The buy-back option differs from the callable product in that customers can reject the recall offer. IT solutions supporting buy-back options have been launched recently in practice (Avisell, 2018; Caravelo, 2017; TransferTravel, 2017).

Elmaghraby et al. (2009) address a case in which a service provider offers both callable and non-callable products with different prices at different times. The provider announces

that the product will be sold at the discounted price if there are any left after the sale at the regular price by the specific time. In that case, two operating schemes are considered: "No Reservation" and "With Reservation". They show that the reservation option enables the service provider to have a "back-up" demand if the demand at the regular price is not enough.

Upgrading allows customers who have purchased lower fare products to use higher fare products without extra costs. This is expected to improve companies' profitability in the presence of demand uncertainty by increasing customer loyalty. There are two types of upgrading scheme: full cascading and limited cascading. Full cascading allows upgrades to any higher fare products, while limited cascading restricts upgrades only to the next highest band product. Alstrup et al. (1986) and Karaesmen and van Ryzin addressed overbooking problems with upgrades and downgrades. Shumsky and Zhang (2009) developed an integrated approach of capacity control with planned upgrades based on protection limits. Steinhart and Gösch (2012) and Gösch et al. (2013) focused on a capacity control problem with upgrades in the rental car industry.

The payable upgrade option proposed in this paper is different from the call option, as customers who have bought the upgrade option can still use their original lower band products even if the upgrade option is recalled by airlines. On the other hand, customers who have bought callable options are not able to travel if their tickets are recalled. The upgrade option is also different from airlines' upgrading practice, as customers are paying smaller fees for the option to ensure a certain level of guarantee of upgrading. On the other hand, upgrading is random and customers do not have any control over receiving the service. From the airlines' perspective, a payable upgrade option is more attractive than upgrading, as they can increase revenues from the fee attached to the service when demands for higher band services are low.

3 Capacity allocation models with recall options

Airlines offer several booking classes to respond to uncertain demands. Each booking class has a different constraint on conditions for cancellation, changes, and service levels. Customers prefer to buy a flight ticket with few restrictions at a low price. However, airlines sell flight tickets with fewer restrictions at higher prices. The lower-class tickets, which are sold for the lower prices, usually have stricter restrictions on the ability to make changes or cancellations, as well as lower service quality. The payable upgrade products meet the needs of customers who wish to buy a low band ticket and then consider other options for upgrading the ticket with additional cost.

We define the following notations to model the airlines' decision problem for the payable upgrade option.

N, Q	The number of fare types and the total capacity respectively,
r_i	Fare of fare class i ($r_1 > r_2 > \dots > r_N$),
D_i	Demand for fare class i having the mean \overline{D}_i , and the variance σ_i^2 ,
z_i	Sale amount of upgrade option for fare class i ,
Δ_i	Number of upgrade options recalled for fare class i ,
g_i	Upgrade option price,
h_i	Compensation for the recall for fare class i ,
x_i	Number of seats assigned for fare class i ,
$S_i(x_i)$	Number of seats sold for fare class i , once the number of seats is assigned at x_i ,
$R_i(x_i)$	Revenue for fare class i from the assigned seats x_i ,
$R(Q)$	The total revenue with the capacity Q .

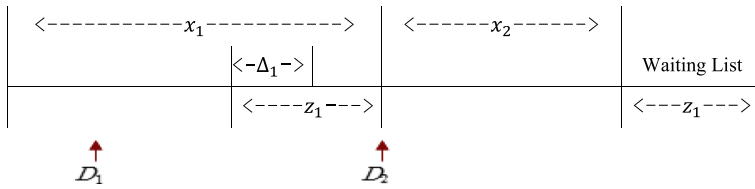


Fig. 1 The concept of seat assignment with a payable upgrade option for two fare classes

In this paper, we have the following assumptions;

1. Given upgrade option sales (z_1, z_2, \dots, z_{N-1}), option price and recall compensation are g_i, h_i ($g_i < h_i$), respectively.
2. Demands are independent of each other, and upgrades will be available only for the next class up.
3. The number of recalls for fare class i (Δ_i) is a random variable depending on the demand for class (D_i).
4. Once the upgrade option is available, it will be possible to re-sell the seat by using a waiting list for the lower fare tickets. The maximum number on the waiting list will be charged by z_i .
5. Payable upgrade options will be sold out.

Considering 2 two class products, Fig. 1 illustrates the concept of seat assignment with a payable upgrade option. The total revenue without the payable upgrade option is defined as follows:

$$\begin{aligned}
 R(Q) &= R_1(x_1) + R_2(x_2), \quad (x_1 + x_2 = Q), \\
 R_1(x_1) &= r_1 S_1(x_1), \quad (S_1(x_1) = \text{Min}(D_1, x_1)), \\
 R_2(x_2) &= r_2 S_2(x_2), \quad (S_2(x_2) = \text{Min}(D_2, Q - x_1)).
 \end{aligned}$$

However, once the payable upgrade option is offered to customers, the total revenue can be defined depending on the amount of sales of the upgrade option as well as the recalled option. Then the total revenue can be presented as follows:

$$\begin{aligned}
 \hat{R}(Q) &= \hat{R}_1(x_1) + \hat{R}_2(x_2), \quad (x_1 + x_2 = Q), \\
 \hat{R}_1(x_1) &= r_1 S_1(x_1) + g_1 z_1 - h_1 \Delta_1 + r_2(z_1 - \Delta_1), \quad (S_1(x_1) = \text{Min}(D_1, x_1)), \\
 \hat{R}_2(x_2) &= r_2 S_2(x_2), \quad (S_2(x_2) = \text{Min}(D_2, Q - x_1)).
 \end{aligned}$$

Then $\hat{R}(Q)$ can be represented as the sum of $R(Q)$ and the additional revenue $\nabla R(x_1)$.

$$\hat{R}(Q) = R_1(x_1) + R_2(x_2) + \nabla R(x_1), \quad (\nabla R(x_1) = g_1 z_1 - h_1 \Delta_1 + r_2(z_1 - \Delta_1)).$$

Hence, if $\nabla R(x_1) > 0$, the expected revenue with the payable upgrade option is greater than that without this option.

Proposition 1 *Let us consider two fare-class products and let x_1, x_2 be the number of assigned seats. For any given values of z_1, x_1, g_1, h_1 , the actual revenue with a payable upgrade option product is at least as large as the corresponding revenue without a recall option product if $\frac{s_1}{z_1} \leq \frac{(r_2 + g_1)}{(r_2 + h_1)}$ is satisfied for all s_1 and r_2 . (If a re-sale is not permitted, $r_2 = 0$).*

Proof $\nabla R(x_1)$ which means that the revenue gain obtained from the payable upgrade option is represented as the following equation.

$$\nabla R(x_1) = g_1 z_1 - h_1 \Delta_1 + (z_1 - \Delta_1)r_2 = (g_1 + r_2)z_1 - (h_1 + r_2)\Delta_1.$$

We can see that if the revenue gain is positive, the revenue with a payable upgrade option product is greater than that without the option.

Therefore, if $\nabla R(x_1) = (g_1 + r_2)z_1 - (h_1 + r_2)\Delta_1 \geq 0$, we can see $\frac{z_1}{\Delta_1} \leq \frac{(r_2+g_1)}{(r_2+h_1)}$.

Since $\nabla R(x_1)$ denotes the revenue gain from the sale of payable upgrade option products, it represents the providers’ surplus. That is, when airlines offer payable upgrade option products, they will have revenue gains $\nabla R(x_1)$ in the case of $\frac{z_1}{\Delta_1} \leq \frac{(r_2+g_1)}{(r_2+h_1)}$. Customer surplus will be calculated by defining customers’ benefits from purchasing payable upgrade option products. Customers’ benefits can be defined as three types: upgrade, compensation, and saving. Upgrade benefit is defined as the fare difference between two fare classes with the payable upgrade option price. The benefit will occur when it is possible for customers who have already bought the payable upgrade option without recall to use a higher fare class. Compensation benefit is defined as the difference between the compensation when recalled and the price of the payable upgrade option product. Saving benefit is dependent on the number of payable upgrades, and is defined by the fare difference between two fare classes without an upgrade option price.

If airlines do not recall the upgrade options, customers paying $(r_2 + g_1)$ including the upgrade option can use the higher band service that is worth r_1 . Then, customers get the upgrade benefit $r_1 - (r_2 + g_1)$. In this case, customers who were rejected for the r_2 fare service and registered on the waiting list for this service can now use the r_2 fare service. Therefore, a customer can save $(r_1 - r_2)$, because if s/he cannot use the r_2 fare, s/he has to pay the r_1 fare to use the service, and this is defined as the saving benefit. If airlines re-call the option, a customer having this option receives compensation h_1 from airlines, and the compensation benefit is defined as $(h_1 - g_1)$.

Given the number of the payable upgrade option z_1 and the number of options re-called Δ_1 , the customer benefit $CV(z_1, \Delta_1)$ denoting the customer surplus is defined as follows:

$$CV(z_1, \Delta_1) = (r_1 - r_2 - g_1)(z_1 - \Delta_1) + (h_1 - g_1)\Delta_1 + (r_1 - r_2)(z_1 - \Delta_1).$$

□

Proposition 2 *In case of purchasing an upgrade option, there always exists a customer surplus.*

Proof In the customer benefit, we already know that the compensation (h_1) for the recalled product should be greater than the re-call option price (g_1). The customers’ benefits can be represented as follows:

$$\begin{aligned} CV(z_1, \Delta_1) &= (r_1 - r_2 - g_1)(z_1 - \Delta_1) + (h_1 - g_1)\Delta_1 + (r_1 - r_2)(z_1 - \Delta_1) \\ &= \{2(r_1 - r_2) - g_1\}z_1 - \{2(r_1 - r_2) - h_1\}\Delta_1 \end{aligned}$$

Noting that the compensation and the option price should be less than or equal to $(r_1 - r_2)$; $0 \leq g_1 \leq h_1 \leq (r_1 - r_2)$. □

Then the customer surplus has a positive value, so the customer surplus exists when airlines offer customers a payable upgrade option.

$$\begin{aligned} CV(z_1, \Delta_1) &= \{2(r_1 - r_2) - g_1\}z_1 - \{2(r_1 - r_2) - h_1\}\Delta_1 \\ &\geq \{2(r_1 - r_2) - g_1\}\Delta_1 - \{2(r_1 - r_2) - h_1\}\Delta_1 \\ &= (h_1 - g_1)\Delta_1 \geq 0 \end{aligned}$$

Given a capacity Q , the total expected revenue ($TR(Q)$) will be calculated as follows:

$$TR(Q) = E[\hat{R}(Q)] = E[R_1(x_1) + R_2(Q - x_1) + \nabla R(x_1)], (x_1 + x_2 = Q).$$

The maximum revenue will occur at the point where the expected marginal revenue is 0.

$$\begin{aligned} \frac{dTR(Q)}{dx_1} &= \frac{dE[R_1(x_1)]}{dx_1} + \frac{dE[R_2(Q - x_1)]}{dx_1} + \frac{dE[\nabla R(x_1)]}{dx_1} \\ &= r_1\{1 - F_1(x_1)\} + \frac{dE[\nabla R(x_1)]}{dx_1} - r_2\{1 - F_2(Q - x_1)\}. \end{aligned}$$

$(F_i(x_1) : \text{cumulative probability at } x_1 \text{ for fare class } i)$

Let $EMR_1(x_1) = r_1\{1 - F_1(x_1)\} + \frac{dE[\nabla R(x_1)]}{dx_1}$ and $EMR_2(Q - x_1) = r_2\{1 - F_2(Q - x_1)\}$. Then we can get the maximum revenue at point x_1 with $EMR_1(x_1) = EMR_2(Q - x_1)$.

The number of products recalled is defined as the different value depending on the demand (D_1) of fare r_1 , and we can obtain the value $\frac{dE[\nabla R(x_1)]}{dx_1}$.

$$\begin{aligned} \Delta_1 &= \begin{cases} 0, & D_1 \leq x_1, \\ D_1 - (x_1 - z_1), & x_1 - z_1 \leq D_1 \leq x_1, \\ D_1 \geq x_1. & D_1 \geq x_1. \end{cases} \\ \frac{dE[\nabla R(x_1)]}{dx_1} &= -(h_1 + r_2) \frac{d[\int_{x_1-z_1}^{x_1} (D_1 - (x_1 - z_1)) f_1(x_1) dD_1 + \int_{x_1}^{\infty} z_1 f_1(x_1) dD_1]}{dx_1} \\ &= -(h_1 + r_2)\{-(F_1(x_1) - F_1(x_1 - z_1))\} = (h_1 + r_2)\{F_1(x_1) - F_1(x_1 - z_1)\}. \end{aligned}$$

$(f_i(x_i) : \text{the probability density function of the demand for fare class } i)$

The expected marginal revenues $EMR_1(x_1)$ and $EMR_2(Q - x_1)$ can be defined as follows:

$$\begin{aligned} EMR_1(x_1) &= r_1\{1 - F_1(x_1)\} + (h_1 + r_2)\{F_1(x_1) - F_1(x_1 - z_1)\}, \\ EMR_2(Q - x_1) &= r_2\{1 - F_2(Q - x_1)\}. \end{aligned}$$

Assuming that the low fare demand comes first, the probability that the demand for r_2 fare is greater than the number of available seats ($Q - x_1$) is equal to 1: i.e. $F_2(Q - x_1) = 0$. The number of seats to be assigned for fare r_1 can be obtained from the following equation:

$$EMR_1(x_1) = r_1\{1 - F_1(x_1)\} + (h_1 + r_2)\{F_1(x_1) - F_1(x_1 - z_1)\} = r_2.$$

In our model, we can find that the expected marginal revenue of fare r_2 with the payable upgrade option is relatively low compared to the value without the option. Since the expected marginal value is decreased with the increase of seats assigned, the more seats are assigned, the lower the marginal value. Therefore, with the payable upgrade option, more seats will be assigned to higher fares compared to existing methods, and more revenue will be available by inducing up-selling to higher fares with the payable upgrade option.

3.1 Example for two fares

We consider the example with two fare classes. Input parameters, including fares, option price, compensation, demands and the number of options to be sold, are as follows:

$$r_1 = 100, r_2 = 60, g_1 = 10, h_1 = 15, z_1 = 5, Q = 100, D_1 \sim N(50, 10^2), D_2 \sim N(80, 10^2).$$

Then, the expected marginal revenue for fare class 1 can be obtained as follows:

$$EMR_1(x_1) = 100(1 - F_1(x_1)) + (60 + 15)(F_1(x_1) - F_1(x_1 - 5)).$$

Since the expected marginal revenue for fare class 2 is $EMR_2(Q - x_1) = 60$, the number of assigned seats for fare class 1 is determined at 51, which is the closest between $EMR_1(x_1)$ and $EMR_2(Q - x_1)$.

x_1	$EMR_1(x_1)$	$EMR_2(Q - x_1)$	r_2	Remark
45	80.39	69.15	60	– Since the expected marginal values for fare r_1 and fare r_2 without the payable upgrade option are $EMR_1(x_1) = 61.79$ and $EMR_2(Q - x_1) = 60$, respectively, the seat assignment is determined as $x_1 = 47$ and $x_2 = 53$
46	77.58	65.54	60	
47	74.56	61.79	60	– Since the marginal expected values for fare r_1 and fare r_2 with the payable upgrade option are $EMR_1(x_1) = 60.66$ and $EMR_2(Q - x_1) = 60$, the numbers of assigned seats are $x_1 = 51$ and $x_2 = 49$ respectively
48	71.33	57.93	60	
49	67.93	53.98	60	
50	64.36	50.00	60	
51	60.66	46.02	60	
52	56.86	42.07	60	
53	53.00	38.21	60	

Without the payable upgrade option, we assign 47 seats for fare r_1 and 53 seats for fare r_2 , and expect a total revenue of 7612.9. While, once we offer the payable upgrade option, the assigned numbers of seats are 51 and 49 for fares r_1 and r_2 respectively, and the total expected revenue will be increased to 7729.4. Therefore, we can get a revenue gain of 116.7 (1.53% gain) from offering the payable upgrade option.

	Revenues					Total Revenue
	Fare r_1 Product	Fare r_2 Product	Option Sales	Compensation for Re-called tickets	Extra sales from upgrade	
Option	4649.4	2939.8	50.0	– 41.85	132.0	7729.4
No-option	4433.5	3179.4	–	–	–	7612.9
Revenue gain	215.9	– 239.6	50.0	– 41.85	132.0	116.7
% Gain	4.87	– 7.54	–	–	–	1.53

For multiple fare-classes with more than three classes, we will apply Belobaba’s (1989) EMSR approach to assign the number of seats for each fare class. Given M fare classes ($M > 3$), the total expected revenue (TR) can be represented as follows:

$$TR = E \left[\sum_{i=1}^M \{R_i(x_i) + \nabla R_i(x_i)\} \right]$$

$$= E \left[\sum_{i=1}^M \{r_i \text{Min}(D_i, x_i) + g_i z_i - h_i \Delta_i + (z_i - \Delta_i)r_{i+1}\} \right], \nabla R_M(x_M) = 0.$$

As in the EMSR approach, we adjust the fare and the demand for higher fare classes to determine the protection level for higher fare classes. For all fare classes greater than or equal to fare class i , the adjusted demand D_{1i} for the adjusted fare \bar{r}_{1i} has the distribution with an average of \bar{D}_{1i} and a standard deviation of $\bar{\sigma}_{1i}$.

$$\bar{r}_{1i} = \frac{\sum_{k=1}^i r_k \bar{D}_k}{\sum_{k=1}^i \bar{D}_k}, \quad \bar{D}_{1i} = \sum_{k=1}^i \bar{D}_{1k}, \quad \bar{\sigma}_{1i} = \sqrt{\sum_{k=1}^i \sigma_k^2}.$$

Let \hat{K} be the set of all fare classes greater than or equal to fare class K , and \bar{x}_{1K} be the number of seats assigned for the set \hat{K} . Let $EMR_{1K}(\bar{x}_{1K})$ be the expected marginal revenue with \bar{x}_{1K} for the set \hat{K} . Then \bar{x}_{1K} is obtained at the value with $EMR_{1K}(\bar{x}_{1K}) = EMR_{1K+1}(Q - \bar{x}_{1K})$ which depends on the adjusted demand for the set \hat{K} .

$$EMR_{1K}(\bar{x}_{1K}) = \frac{dT R_{1K}}{d\bar{x}_{1K}} = \frac{d(R_{1K}(\bar{x}_{1K}) + \nabla R_{1K}(\bar{x}_{1K}))}{d\bar{x}_{1K}} = \frac{d\{\bar{r}_{1K} \text{Min}(D_{1K}, \bar{x}_{1K})\}}{d\bar{x}_{1K}},$$

$$EMR_{1K+1}(Q - \bar{x}_{1K}) = r_{K+1}, \quad (\nabla R_{1K}(\bar{x}_{1K}) = g_1 z_1 - h_1 \Delta_1 + (z_1 - \Delta_1) r_2).$$

Since \bar{x}_{1K} means the number of seats assigned for fare class set \hat{K} , it denotes the protection level for fare class set \hat{K} . Customers who bought the product with fare class $K + 1$ and the payable upgrade option are permitted to upgraded only to fare class K . The seat assignment process for the multiple fare classes with the payable upgrade option is almost the same as EMSR and is represented as follows:

[Step 0] The protection level for the highest service level: x_1 .

$$EMR_1(x_1) = EMR_2(Q - x_1) (= r_2, \text{ as the demand for the lower service comes first}).$$

[Step 1] For fare class i products, adjust the demand and the price for products with a higher fare than product i ; \bar{D}_{1i} , $\bar{\sigma}_{1i}$, and \bar{r}_{1i} .

Calculate $EMR_{1i}(\bar{x}_{1i})$ for the demand distribution with the mean \bar{D}_{1i} , the variance $\bar{\sigma}_{1i}^2$, and the price \bar{r}_{1i} . The protection level \bar{x}_{1i} is determined as the following condition:

$$EMR_{1i}(\bar{x}_{1i}) = EMR_{i+1}(Q - \bar{x}_{1i})(= r_{i+1}).$$

When the seat assignment process is completed, we obtain the protection level for the higher fare products. Using the protection level, the booking level BL_i of fare class i is defined as follows:

$$BL_i = Q - \bar{x}_{1i-1}, \quad (\bar{x}_{11} = 0).$$

4 Numerical examples

There are many changes to the airline fare systems due to fierce competition in the airline industry. A notable one is the branded fare, which is adopted in most LCCs. LCCs sell their products with three to four branded fares instead of the traditional booking classes. In this paper, we consider four classes to test the performance of the seat assignment for the multiple fare classes. For convenience, we assume the input parameters that are given in Table 1. The total capacity Q is assumed as 200, and we assume that demand is normally distributed.

For the performance evaluation, we measure the revenue gain obtained from the seat assignment with the payable upgrade option by comparing it to the revenue for the seat assignment without the option. We calculate the expected revenues from three different

Table 1 Input parameters

Fare class	Fare	Payable upgrade option			Demand	
		Number of seats assigned	Price	Compensation for re-call	Average	SD
1	100	5	5	10	30	5
2	80	5	10	15	50	10
3	60	5	15	20	80	10
4	30	5	–	–	100	20

models: Model 1 for seat assignment with the payable upgrade option, Model 2 for the payable upgrade option sale after the seat assignment without the payable upgrade option, and Model 3 for seat assignment without the payable upgrade option.

The seat assignments for fare classes in our example are given in Table 2.

Given the total capacity of 200 and applying protective nesting, the booking limit and the protection level for each fare class are shown in Table 3. It shows that more seats are assigned to the higher-class fares. The expected revenue for each model is given in Table 4.

In Model 1, for the given 200 seats, the average number of seats sold is 192.2. Among the total 15 recall options, we can see that 8.1 seats are recalled and 6.9 seats are upgraded to

Table 2 Protection levels

	Assigned number of seats*	Expected marginal revenues
x_1	30 (26)	$EMR_1(x_1)=100(1-F_1(x_1)) + (80 + 10)(F_1(x_1)-F_1(x_1 - 5)), EMR_2(Q - x_1)= 80$
\bar{x}_{12}	79 (75)	$\bar{D}_{12}=80, \bar{\sigma}_{12}=\sqrt{125}, \text{ and } \bar{r}_{12}=(100 \times 30 + 80 \times 50)/80 = 87.5$ $EMR_{12}(\bar{x}_{12})= 87.5(1-F_{12}(\bar{x}_{12})) + (60 + 15)(F_{12}(\bar{x}_{12})-F_{12}(\bar{x}_{12} - 5)), EMR_3(Q - \bar{x}_{12})=60$
\bar{x}_{13}	167 (164)	$\bar{D}_{13}=160, \bar{\sigma}_{13}=\sqrt{225}, \bar{r}_{13}=(100 \times 30 + 80 \times 50 + 60 \times 80)/160 = 73.75$ $EMR_{13}(\bar{x}_{13})=73.75(1-F_{13}(\bar{x}_{13})) + (30 + 20)(F_{13}(\bar{x}_{13})-F_{13}(\bar{x}_{13} - 5)), EMR_4(Q - \bar{x}_{13}) = 30$

*($\bar{\cdot}$): the number of seats assigned without the payable upgrade option

Table 3 Seat assignment and booking limit

Fare class (Fare)	Assigned seats*	Protection level*	Booking limit*
1 (100)	30 (26)	30 (26)	200 (200)
2 (80)	49 (49)	79 (75)	170 (174)
3 (60)	88 (89)	167 (164)	121 (125)
4 (30)	33 (36)	0 (0)	33 (36)

*($\bar{\cdot}$): the number of seats assigned without the payable upgrade option

Table 4 Summary of expected values

	Fare r_1			Fare r_2			Fare r_3			Fare r_4		
	Seat	Re-call	Up grade	Seat	Re-call	Up grade	Seat	Re-call	Up grade	Fare	Re-call	Up grade
No. of assigned seats	30			49			88			33		
	Model1											200
	Model2			49			89			36		200
	Model3			49			89			36		200
No. of option sales	5.0			5.0			5.0			-		-
	Model2			5.0			5.0			-		-
	Model3			-			-			-		-
Average no. of sales	28.0	3.4	1.6	45.5	3.2	1.8	78.8	1.5	3.5	33.0		192.2/6.9
Average no. of sales	Model2			45.5			79.0			36.0		191.9/6.0
	Model3			45.5			79.0			36.0		185.9/0.0
Average no. of sales	25.4	-	-	45.5	-	-	79.0	-	-	36.0		185.9/0.0
	Model3			45.5			79.0			36.0		185.9/0.0
Average expected revenue	2801.2	116.8		3639.5	111.7		4728.0	151.8		989.9		12,538.9

Table 4 (continued)

	Fare r_1			Fare r_2			Fare r_3			Fare r_4		
	Seat	Re-call	Up grade	Seat	Re-call	Up grade	Seat	Re-call	Up grade	Seat	Re-call	Up grade
Average expected revenue	Model2 2540.4	25.4	111.7	3639.5	111.7	160.0	4739.9	160.0	1079.9	12,293.7		
Average expected revenue	Gain1 260.8	91.4	0.0	0.0	0.0	- 8.2	- 11.9	- 8.2	- 90.0	245.2 (2.0%)		
Average expected revenue	Model3 2540.4	-	-	3639.5	-	-	4739.9	-	1079.9	11,999.7		
Average expected revenue	Gain2 260.8	116.8	111.7	0.0	111.7	151.8	- 11.9	151.8	- 90.0	539.2 (4.5%)		

Gain1 = Expected Revenue of Model 1—Expected Revenue of Model 2

Gain2 = Expected Revenue of Model 1—Expected Revenue of Model 3

higher class fare products. The total expected revenue is 12,538.9. In Model 3, without the payable upgrade option, we can expect the average number of seats sold to be 185.9 and the expected revenue is 11,999.7, which is less than that of Model 1. This means that Model 1 can achieve a 4.5% revenue gain compared to Model 3. Even in Model 2, we find a revenue gain compared to Model 3. It is expected from the proposition 1. Compared to Model 2, the revenue gain of our model (Model 1) is expected to be 2.0% in the total revenue. Consequently, in our simple model, it is proven that once airlines offer their customers payable upgrade options, these options contribute to the increase in revenue. Furthermore, we can see that managing seat control by considering the recall process will lead to a significant revenue gain for airlines.

5 Conclusion

Airlines operate complex RM systems to increase revenues with various fare classes. However, with the advent of ancillary services, airlines are paying more attention to increasing revenues from developing additional services instead of traditional sales of full packaged services. Callable options or similar ones have been popular for increasing revenues. The payable upgrade option suggested in this paper is an innovative and novel service. Unlike the callable options, customers are guaranteed to be able to use the services even if their upgrade options are recalled by the airlines. In this paper, we showed that the payable upgrade options increase airlines' revenue to manage demand uncertainty.

We showed that the payable upgrade option will generate more revenue if $\frac{s_i}{z_i} \leq \frac{(r_{i+1}+g_i)}{(r_{i+1}+h_i)}$ is guaranteed. Furthermore, applying Belobaba's EMSR approach (1989), we suggested an effective seat assignment method for multiple fare classes with the payable upgrade option to increase total revenue. With a numerical example, we found that the revenue can be increased by introducing a payable upgrade option. If airlines sell payable upgrade options as an ancillary service, they can achieve a 2.0% revenue gain. That is, if we consider a payable upgrade option sale, it is effective to allocate more seats to the higher fare classes and to induce service upgrades via payable upgrade options for the customers whose first priority is for the lower fare classes.

The findings of this study make the following contributions. Firstly, the paper contributes to the RM literature by proposing an analytical model for a payable upgrade option service. We identified the properties of the upgrade option model and proved its positive performance in the presence of demand uncertainties. Secondly, the findings of the paper provide a clear practical implication for airlines to introduce a payable upgrade option product in their portfolios. Our method for allocating seats for multiple fare classes with a payable upgrade option will contribute effectively to the revenue increase in airlines' branded fare products and can also be applied by companies in other service industries, including hotels, car rentals and restaurants.

However, before applying our model to real-world practice, it needs further studies on consumers' behavioural analysis regarding waiting time to be recalled, the number of payable upgrade options to sell, and the prices for the various options. Furthermore, it is important to analyse customers' emotional response to payable upgrade options. The analysis should be extended to examine the effect of cancellation and no-show behaviour. This paper pioneers a stream of such studies in the area.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give

appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Alstrup, J., Boas, S., Madsen, O. B. G., & Valqui, R. V. (1986). Booking policy for flights with two types of passengers. *European Journal of Operational Research*, 27, 274–288.
- Anderson, C., Rasmussen, H., & Davison, M. (2004). Revenue management: A real options approach. *Naval Research Logistics*, 51(5), 686–703.
- Avisell. (2018). <https://www.avisell.com/>.
- Aydin, N., Birbil, S. I., & Topaloglu, H. (2016). Delayed purchase options in single-eg revenue management. *Transportation Science*, 51(4), 1031–1045. <https://doi.org/10.1287/trsc.2015.0643>
- Bell, P. C. (2008). Short selling and replanning as tools to enhance revenues. *Journal of the Operational Research Society*, 59(3), 313–321.
- Belobaba, P. P. (1989). Application of a probabilistic decision model to airline seat inventory control. *Operations Research*, 37, 183–197.
- Biyalogorsky, E., Carmon, Z., Fruchter, G., & Gerstner, E. (1999). Overselling with opportunistic cancellations. *Marketing Science*, 18, 605–610.
- Biyalogorsky, E., & Gerstner, E. (2004). Note: Contingent pricing to reduce price risks. *Marketing Science*, 23(1), 146–155.
- Caravelo (2017). <https://www.caravelo.com/solutions/seat-resale>.
- Chiu, C.-H., Hou, S.-H., & Liu, W. (2017). Real options approach for fashionable and perishable products using stock loan with regime switching. *Annals of Operations Research*, 257, 357–377.
- Ekstein, N. (2017). United wants to sell your seat to someone else for more money. *Bloomberg News*. Retrieved from: <https://www.bloomberg.com/news/articles/2017-07-12/united-wants-to-sell-your-seat-to-someone-else-for-more-money>.
- Elmaghraby, W., Lippman, S. A., Tang, C. S., & Yin, R. (2009). Will more purchasing options benefit customers? *Production and Operations Management*, 18(4), 381–401.
- Feng, B., Li, Y., & Shen, Z. M. (2015). Air cargo operations: Literature review and comparison with practices. *Transportation Research Part C*, 56, 263–280.
- Feng, B., Zhao, J., & Jiang, Z. (2021). Robust pricing for airlines with partial information. *Annals of Operations Research*. <https://doi.org/10.1007/s10479-020-03926-9>
- Gallego, G., & Lee, H. (2018). Callable products with early exercise and overbooking. In *Proceedings of the AGIFORS Revenue Management Study Group*.
- Gallego, G., Kou, S. G., & Phillips, R. (2008). Revenue management of callable products. *Management Science*, 54(3), 550–564.
- Gallego, G., & Phillips, R. (2004). Revenue management of flexible products. *Manufacturing and Service Operations Management*, 6, 1–17.
- Gönsch, J., Koch, S., & Steinhardt, C. (2013). An EMSR-based approach for revenue management with integrated upgrade decisions. *Computers and Operations Research*, 40(2013), 2532–2542.
- Graf, M., & Kimms, A. (2011). An option-based revenue management procedure for strategic airline alliances. *European Journal of Operational Research*, 215(2), 459–469.
- Graf, M., & Kimms, A. (2013). Transfer price optimization for option-based airline alliance revenue management. *International Journal of Production Economics*, 145, 281–293.
- Iliescu, D. C., & Garrow, L. A. (2008). A hazard model of US airline passengers' refund and exchange behavior. *Transportation Research Part b*, 42, 229–242.
- Karaesmen, I., & van Ryzin, G. (2004). Overbooking with substitutable inventory classes. *Operations Research*, 52(1), 83–104.
- Lardeux, B., Sabatier, G., Delahaye, T., Boudia, M., Tonnet, O., & Mathieu, P. (2018). Yield optimization for airlines from ticket resell. *Journal of Revenue and Pricing Management*, 18, 213–217. <https://doi.org/10.1057/s41272-018-00167-1>
- Li, T., Xie, J., Lu, S., & Tang, J. (2016). Duopoly game of callable products in airline revenue management. *European Journal of Operational Research*, 254(2016), 925–934.

- Lim, W. S. (2009). Overselling in a competitive environment: Boon or bane? *Marketing Science*, 28(6), 1129–1143.
- Littlewood, K. (1972). Forecasting and control of passenger bookings. In *12th AGIFORS Symposium Proceedings, Nathanya, Israel*, (Vol. 12, pp. 95–117).
- Marcus, B., & Anderson, C. K. (2006). Online low-price guarantees—A real options analysis. *Operations Research*, 54(6), 1041–1050.
- Marcus, B., & Anderson, C. K. (2008). Revenue management for low-cost providers. *European Journal of Operations Research*, 188, 258–272.
- McGill, J. I., & Van Ryzin, G. J. (1999). Revenue management: Research overview and prospects. *Transportation Science*, 33(2), 233–256.
- Phillips, R. L. (2005). *Pricing and revenue optimization*. Stanford University Press.
- Qin, J., Wang, K., Wang, Z., & Xia, L. (2020). Revenue sharing contracts for horizontal capacity sharing under competition. *Annals of Operations Research*, 291, 731–760.
- Sheffi, Y. (2005). *The resilient enterprise: Overcoming vulnerability for competitive advantage*. MIT Press.
- Shumsky, R. A., & Zhang, F. (2009). Dynamic capacity management with substitution. *Operations Research*, 57, 671–684.
- Steinhardt, C., & Gösch, J. (2012). Integrated revenue management approaches for capacity control with planned upgrades. *European Journal of Operational Research*, 223(2), 380–391.
- Talluri, K. T., & Van Ryzin, G. J. (2004). *The theory and practice of revenue management*. Springer.
- TransferTravel. (2017). <https://www.transfertravel.com/>.
- Vinod, B., & Moore, K. (2009). Promoting branded fare families and ancillary services: Merchandising and its impacts on the travel value chain. *Journal of Revenue and Pricing Management*, 8(2/3), 174–186.
- Walker, J. S., Case, T. S., Jorasch, J. A., & Sparico, T. M. (1998). Method, apparatus, and program for pricing, selling, and exercising options to purchase airline tickets. United States Patent 5,797,127.
- Zhong, Z.-Y., Li, J.-L., & Lun, R. (2010). Airline inventory control of callable tickets based on real option. In *Proceedings from: 17th international conference on management science and engineering*. November 24–26, 2010, Melbourne, Australia.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.