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CORRECTION

Correction to: On the strong universal consistency of local averaging regression estimates

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There is a gap at the end of the proof of Theorem 1, since there the application of the conditional McDiarmid inequality yields

$$J_n - \mathbf{E}\{J_n|X_1,\ldots,X_n\} \to 0 \quad a.s.,$$

where $J_n = \int \left|\sum_{i=1}^n W_{n,i}(x) \cdot (Y_i - m(X_i))\right| \mu(\mathrm{d}x)$, and not yet the assertion

$$J_n \to 0 \quad a.s.$$

in the last step of the proof of Theorem 1.
This gap can be filled by adding into assumption $(A3)$ the second condition

$$\sum_{i=1}^{n} \int |W_{n,i}(x)|^2 \mu(\mathrm{d}x) \to 0 \quad a.s. \tag{29}$$

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Using this condition together with $|Y| \leq L$ a.s., it is easy to see that one has

$$\mathbf{E}\{J_n|X_1,\ldots,X_n\} \to 0 \quad a.s.,$$

which is still needed to obtain the assertion.
In order to verify (29) in the applications of Theorem 1, for kernel estimation in the context of Lemma 6 one notices that, up to some constant factor, the left-hand side of (29) is majorized by

$$\int \frac{1}{1+\sum_{i=1}^n I_{S_{r_1}}\left(\frac{x-X_i}{h_n}\right)}\mu(\mathrm{d}x),$$

which can be treated similarly to the verification of $(A4)$ in Lemma 6. The verification of (29) for partitioning estimation in the context of Lemma 9 is analogous.

Details

Last part of the proof of Theorem 1. It remains to show

$$J_n \cdot I_{B_n} \to 0 \quad a.s.$$

Application of the conditional McDiarmid inequality as in the proof of Theorem 1 yields

$$J_n \cdot I_{B_n} - \mathbf{E}\{J_n \cdot I_{B_n}|X_1,\ldots,X_n\} \to 0 \quad a.s.$$

Hence, it suffices to show

$$\mathbf{E}\{J_n|X_1,\ldots,X_n\} \to 0 \quad a.s. \tag{30}$$

By the inequality of Jensen, the independence of the data and $|Y| \leq L\ a.s.$, we get

$$\begin{aligned}
&(\mathbf{E}\{J_n|X_1,\ldots,X_n\})^2 \\
&\quad\leq \mathbf{E}\{J_n^2|X_1,\ldots,X_n\} \\
&\quad\leq \mathbf{E}\left\{\int\left|\sum_{i=1}^n W_{n,i}(x)\cdot(Y_i-m(X_i))\right|^2\mu(\mathrm{d}x)\middle|X_1,\ldots,X_n\right\} \\
&\quad= \mathbf{E}\left\{\left|\sum_{i=1}^n W_{n,i}(X)\cdot(Y_i-m(X_i))\right|^2\middle|X_1,\ldots,X_n\right\} \\
&\quad= \mathbf{E}\left\{\mathbf{E}\left\{\left|\sum_{i=1}^n W_{n,i}(X)\cdot(Y_i-m(X_i))\right|^2\middle|X,X_1,\ldots,X_n\right\}\middle|X_1,\ldots,X_n\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \mathbf{E}\left\{\sum_{i=1}^{n} W_{n,i}(X)^2 \cdot \mathbf{E}\left\{(Y_i - m(X_i))^2 \middle| X, X_1, \ldots, X_n\right\} \middle| X_1, \ldots, X_n\right\} \\
&\leq 4L^2 \cdot \mathbf{E}\left\{\sum_{i=1}^{n} W_{n,i}(X)^2 \middle| X_1, \ldots, X_n\right\} \\
&= 4L^2 \cdot \sum_{i=1}^{n} \int |W_{n,i}(x)|^2 \mu(\mathrm{d}x).
\end{aligned}
$$

Thus, (30) follows from (29).

Proof of (29) **in the context of Lemma 6.** On the one hand, we have

$$
\sum_{i=1}^{n} W_{n,i}(x)^2 = \frac{\sum_{i=1}^{n} K\left(\frac{x-X_i}{h_n}\right)^2}{\left(\sum_{j=1}^{n} K\left(\frac{x-X_j}{h_n}\right)\right)^2} \leq 1.
$$

On the other hand, it holds

$$
\begin{aligned}
\sum_{i=1}^{n} W_{n,i}(x)^2 &\leq c_2 \cdot \frac{\sum_{i=1}^{n} K\left(\frac{x-X_i}{h_n}\right)}{\left(\sum_{j=1}^{n} K\left(\frac{x-X_j}{h_n}\right)\right)^2} \cdot I_{\left\{\sum_{j=1}^{n} K\left(\frac{x-X_j}{h_n}\right)>0\right\}} \\
&\leq c_2 \cdot \frac{1}{\sum_{j=1}^{n} K\left(\frac{x-X_j}{h_n}\right)}.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\sum_{i=1}^{n} W_{n,i}(x)^2 &\leq \min\left\{1, c_2 \cdot \frac{1}{\sum_{j=1}^{n} K\left(\frac{x-X_j}{h_n}\right)}\right\} \\
&\leq \min\left\{1, \frac{c_2}{c_1} \cdot \frac{1}{\sum_{j=1}^{n} I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\right\} \\
&\leq \max\left\{1, \frac{c_2}{c_1}\right\} \cdot \min\left\{1, \frac{1}{\sum_{j=1}^{n} I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\right\} \\
&\leq \max\left\{1, \frac{c_2}{c_1}\right\} \cdot \frac{2}{1 + \sum_{j=1}^{n} I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}.
\end{aligned}
$$

Hence, it suffices to show

$$
W_n := \int \frac{1}{1 + \sum_{j=1}^{n} I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)} \mu(\mathrm{d}x) \to 0 \quad a.s. \tag{31}
$$

For any bounded sphere S around 0, by Lemma 2a and by assumption (9), we get

$$\begin{aligned}
&\mathbf{E}\left\{\int_S \frac{1}{1+\sum_{j=1}^n I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\mu(\mathrm{d}x)\right\}\\
&= \int_S \mathbf{E}\left\{\frac{1}{1+\sum_{j=1}^n I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\right\}\mu(\mathrm{d}x)\\
&\leq \int_S \frac{1}{n\cdot\mu(x+h_n\cdot S_{r_1})}\mu(\mathrm{d}x)\\
&\leq \frac{const}{n\cdot h_n^d} \to 0 \quad (n\to\infty),
\end{aligned}$$

where the last inequality holds because of equation (5.1) in Györfi et al. (2002). Thus, it suffices to show

$$W_n - \mathbf{E}\{W_n\} \to 0 \quad a.s. \tag{32}$$

Analogously to the proof of (A4), with X_1', X_1, …, X_n independent and identically distributed and

$$W_n' := \int \frac{1}{1+I_{S_{r_1}}\left(\frac{x-X_1'}{h_n}\right)+\sum_{j=2}^n I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\mu(\mathrm{d}x),$$

by Lemma 4.2 in Kohler et al. (2003), one has

$$\mathbf{E}\{|W_n-\mathbf{E}\{W_n\}|^4\} \leq c_{11}\cdot n^2\cdot \mathbf{E}\{(W_n-W_n')^4\} \quad (n\in\mathbb{N}).$$

Furthermore, by the second part of Lemma 5 one gets

$$\begin{aligned}
&\mathbf{E}\{|W_n-W_n'|^4\}\\
&\leq 16\cdot\mathbf{E}\left\{\left(\int \frac{I_{S_{r_1}}\left(\frac{x-X_1}{h_n}\right)}{\left(1+\sum_{j=2}^n I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)\right)^2}\mu(\mathrm{d}x)\right)^4\right\}\\
&\leq 16\cdot\mathbf{E}\left\{\left(\int \frac{I_{S_{r_1}}\left(\frac{x-X_1}{h_n}\right)}{1+\sum_{j=2}^n I_{S_{r_1}}\left(\frac{x-X_j}{h_n}\right)}\mu(\mathrm{d}x)\right)^4\right\}\\
&\leq \frac{const}{n^4}.
\end{aligned}$$

From these relations, one obtains (32) by the Borel–Cantelli lemma and the Markov inequality.

Proof of (29) in the context of Lemma 9. Analogously to above it suffices to show

$$V_n := \int \frac{1}{1 + \sum_{j=1}^{n} I_{A_{\mathcal{P}_n}(x)}\left(X_j\right)} \mu(\mathrm{d}x) \to 0 \quad a.s.$$

For any bounded sphere S around zero, by assumption (12) we get

$$\int_S \frac{1}{n \cdot \mu(A_{\mathcal{P}_n}(x))} \mu(\mathrm{d}x) \to 0 \quad (n \to \infty),$$

from which by Lemma 2a we can conclude analogously to above

$$\mathbf{E} V_n \to 0 \quad (n \to \infty).$$

Hence, it suffices to show

$$V_n - \mathbf{E}\{V_n\} \to 0 \quad a.s.,$$

which follows analogously to above from the second part of Lemma 7.