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Preface

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During the last two decades, the mathematical study and numerical treatment of inverse problems have become one of the fastest growing areas in applied mathematics. Most of mathematical problems from applications are inverse problems, which are ill-posed in the sense that they may not have a unique solution or their solutions are not continuously dependent on the given data. Mathematical treatment of these problems requires regularization methods. To capture the fruitful development of this area we edit this special issue that highlights new approaches used in mathematical development of inverse problems, aiming at identifying new research directions of the area.

This special issue consists of twelve original research papers in the area of inverse problems. These papers were submitted by invitations of the editors and went through rigorous refereeing processing. The scope of this special issue covers general theory and numerical solutions of the operator equations of the first kind, inverse scattering problems, numerical solutions for electrical impedance tomography and image restoration problems. We briefly introduce below the papers included in this special issue.

An inverse scattering problem is to recover an unknown object from measurements of waves or far-field patterns of scatterer. The work "The inverse scattering problem for cavities with impedance boundary condition" by Hai-Hua Qin and David Colton considers the inverse scattering problem of determining the shape of a cavity with a impedance boundary condition from sources and measurements placed on a curve inside the cavity. They showed

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that both the shape ∂D of the cavity and the surface impedance λ are uniquely determined by the measured data. The work "Recovering the Dirichlet-to-Neumann map in inverse scattering problems using integral equation methods" by Haibing Wang and Jijun Liu studies the reconstruction of Dirichlet-to-Neumann map from the far-field patterns of the scattered waves in inverse scattering problems, and proposed schemes which give possible ways to realize the probe methods using practical far-field data, with the advantage of no numerical differentiation for scattered wave in their implementations.

The general theory and numerical solutions of ill-posed operator equations of the first kind are a main topic of this special issue. The paper "Application of high order numerical quadratures to numerical inversion of the Laplace transform" by Fu-Rong Lin and Fen Liang presents a new algorithm for solving the Fredholm integral equation of the first kind which corresponds to finding the inverse Laplace transform. The work "The regularizing properties of the composite trapezoidal method for weakly singular Volterra integral equations of the first kind" by Robert Plato investigates the regularizing properties of the product trapezoidal method for solving weakly singular first kind Volterra integral equations with perturbed right-hand sides. The paper "On the quasi-optimal rules for the choice of the regularization parameter in case of a noisy operator" by Toomas Raus · Uno Hämarik generalizes the concept of the quasi-optimality for the case of a noisy operator. The paper "Quadrature based collocation methods for integral equations of the first kind" by M. Thamban Nair studies the problem of solving integral equations of the first kind by a collocation method when the nodes are associated with a convergent quadrature rule. The paper by Xingjun Luo, Fanchun Li and Suhua Yang entitled *"A posteriori* parameter choice strategy for fast multi-scale methods solving ill-posed integral equations" proposes a fast multiscale method for solving ill-posed integral equations via the Tikhonov regularization and gives a modified *a posteriori* parameter choice strategy which leads to optimal convergence rates. G. H. Zheng and T. Wei in their work "A new regularization method for a Cauchy problem of the time fractional diffusion equation" consider a Cauchy problem of the time fractional diffusion equation (TFDE), and show that the Cauchy problem is severely ill-posed and further develop a new regularization method to solve it.

The core problem in seismic exploration is to invert the subsurface reflectivity from the surface recorded seismic data. Most of the seismic inverse problems are ill-posed by natural. To overcome the ill-posedness, Y. F. Wang in his paper "Preconditioning non-monotone gradient methods for retrieval of seismic reflection signals" develops a preconditioned non-monotone gradient method and applied it to seismic deconvolution and imaging.

Electrical impedance tomography (EIT) is a non-destructive imaging technique which has various applications in medical imaging, geophysics and many other fields. Its purpose is to reconstruct the electric conductivity and permittivity of hidden objects inside a medium with the help of boundary field measurements. The work "Second-order topological expansion for electrical impedance tomography" by M. Hintermüller, A. Laurain and A. A. Novotny

suggests second-order topological expansions in EIT problems with piecewise constant conductivities. The study aims at determining the relevance of the non-local and interaction terms from a numerical point of view. A level set based shape algorithm is proposed and initialized by using topological sensitivity analysis.

Recently, graph cuts algorithms were used to solve variational image restoration problems, especially for noise removal and segmentation. The work "Domain decomposition methods with graph cuts algorithms for total variation minimization" by Yuping Duan and Xue-Cheng Tai combines the domain decomposition method and the graph cuts algorithm for solving the total variation minimizations with L^1 and L^2 fidelity terms. Numerous numerical experiments on large-scale data demonstrate the proposed algorithm yields good results in terms of computational time and memory usage.

The work "Recovery of an interface from boundary measurement in an elliptic differential equation" by Weifu Fang and Suxing Zeng studies the inverse problem of recovering an interior interface from a boundary measurement in an elliptic boundary value problem arising from a semiconductor transistor model. They set up a nonlinear least-squares formulation for solving the inverse problem, and establish the necessary derivatives with respect to the interface, then propose numerical methods by using integral equations.

The successful publication of this special issue is the combined effort of the authors who contribute their excellent papers and the referees who provide constructive comments which improve the quality of the papers included in this special issue. We greatly appreciate the efforts of all authors and all referees who make this special issue a successful one. We also thanks the editor-in-chief, Dr. Charles Micchelli, for his encouragement and guidance for editing this special issue.

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