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ERRATUM

Erratum to: Optimal adaptive sampling recovery

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We correct the definitions of the quantities of optimal sampling recovery $e_n(W)_q$ and $r_n(W)_q$ which have been introduced in [1]. All the results and proofs of [1] are unchanged and hold true for the new corrected definitions given below.

Recall that the definition of sampling recovery method S_n^B was given by (1.4) in [1]. The worst case error of the recovery by S_n^B for the function class W, is measured by

$$\sup_{f \in W} \| f - S_n^B(f) \|_q.$$

Given a family $\mathcal{B}$ of subsets in L_q, we consider optimal sampling recoveries by B from $\mathcal{B}$ in terms of the quantity

$$R_n(W, \mathcal{B})_q := \inf_{B \in \mathcal{B}} \inf_{S_n^B} \sup_{f \in W} \| f - S_n^B(f) \|_q. \tag{1}$$

We assume a restriction on the sets $B \in \mathcal{B}$, requiring that they should have, in some sense, a finite capacity. The capacity of B is measured by its cardinality or pseudo-dimension. This reasonable restriction would provide nontrivial lower bounds of asymptotic order of $R_n(W, \mathcal{B})_q$ for well known function classes W.

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Denote $R_n(W, \mathcal{B})_q$ by $e_n(W)_q$ if $\mathcal{B}$ in (1) is the family of all subsets B in L_q such that $|B| \le 2^n$, where $|B|$ denotes the cardinality of B, and by $r_n(W)_q$ if $\mathcal{B}$ in (1) is the family of all subsets B in L_q of pseudo-dimension at most n.

Let $\Phi = \{\varphi_k\}_{k \in J}$ be a family of elements in L_q. Denote by $\Sigma_n(\Phi)$ the non-linear set of linear combinations of n free terms from Φ, that is $\Sigma_n(\Phi) := \{\varphi = \sum_{j=1}^n a_j \varphi_{k_j} : \ k_j \in J\}$. The quantity $s_n(W, \Phi)_q$ which has been introduced in [1, 2] (denoted by $\nu_n(W, \Phi)_q$ in [2]), can be equivalently redefined as

$$s_n(W, \Phi)_q \ := \inf_{S_n^B :\, B = \Sigma_n(\Phi)} \ \sup_{f \in W} \ \| f - S_n^B(f) \|_q .$$

A different definition of $s_n(W, \Phi)_q$ is as follows. For each function $f \in W$, we choose a sequence $\{x^s\}_{s=1}^n$ of n points in $\mathbb{I}^d$. This choice defines n sampled values $\{f(x^s)\}_{s=1}^n$. Then we choose a sequence $a = \{a_s\}_{s=1}^n$ of n numbers and a sequence $\{\varphi_{k_s}\}_{s=1}^n$ of n functions from Φ, depending on the sampling information of $\{x^s\}_{s=1}^n$ and $\{f(x^s)\}_{s=1}^n$. This choice defines a sampling recovery method given by

$$A_n^\Phi(f) \ := \sum_{s=1}^n a_s \varphi_{k_s}. \tag{2}$$

Notice that the set of all sampling recovery methods A_n^Φ coincides with the set of all S_n^B such that $B = \Sigma_n(\Phi)$. Therefore, there holds true the equality

$$s_n(W, \Phi)_q = \inf_{A_n^\Phi} \ \sup_{f \in W} \ \| f - A_n^\Phi(f) \|_q, \tag{3}$$

where the infimum is taken over all sampling recovery methods A_n^Φ of the form (2). Hence, we can take (3) as an alternative definition of $s_n(W, \Phi)_q$.

It is easy to verify that for the above corrected and new definitions, there hold true the inequalities $e_n(W)_q \ge \varepsilon_n(W)_q$, $r_n(W)_q \ge \rho_n(W)_q$ and $s_n(W, \Phi)_q \ge \sigma_n(W, \Phi)_q$ which were used in [1, 2] for the proofs the lower bounds of $e_n(U^\alpha_{p,\theta})_q$, $r_n(U^\alpha_{p,\theta})_q$ and $s_n(U^\alpha_{p,\theta}, \mathbf{M})_q$.

References

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