

Preface

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This special issue consists of 13 original research papers in an emerging important field—mathematical methods for image processing. All papers included in this issue were submitted by invitations of the editors and went through rigorously refereeing processing.

In the last two decades, mathematics methods, such as variational partial differential equation (PDE) methods, stochastic methods, and multiscale methods, have played a key role in developing powerful algorithms in the area of digital image processing, an interdisciplinary field of imaging science. The demand in digital image processing also leads to development of many sophisticated mathematical theories and new algorithms. This special issue highlights new approaches and new research directions for the digital image processing by collecting together thirteen technical papers in areas including image denoising, restoration, decomposition, and segmentation as well as mathematical theory related to image analysis such as sampling and filter design.

Images are often modeled as real-valued functions on \mathbb{R}^2 . Typically, a class of images is a subspace of $L^2(\mathbb{R}^2)$ with certain smoothness property. Modern image processing schemes proceed in three steps. The first step consists in transforming the image (a function on \mathbb{R}^2) into a sequence (a function on \mathbb{Z}^2) with the requirement of the image being fully characterized by the sequence. This process is called sampling. The inverse process that

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transforms the sequence back into the image is called the reconstruction and is often of the third step in the processing. Image processing algorithms, such as denoising, compression/decompression, etc., are performed between these two steps using digital devices (e.g., computers). Therefore, the steps of sampling and reconstruction are fundamental in any image processing scheme. The main focus of the article by Acosta-Reyes, Aldroubi, and Krishtal is on describing and quantifying “admissible” perturbations of the sampling model. The dual frame method for recovering the original signal from its samples is studied by using the theory of localized frames.

Recently, hexagonal image processing has attracted attention and has been used in many fields such as medical imaging, computer vision, and geoscience. The hexagonal lattice has several advantages in comparison with the conventionally used rectangular lattice. For example, a hexagonal lattice needs fewer sampling points; it has better consistent connectivity; it has higher symmetry; and its structure is plausible to human vision systems. Hence, there is a need to design hexagonal wavelet frame filter banks in order that the multiresolution analysis method can be used for hexagonal image processing. In the article by Jiang, hexagonal wavelet frame filter banks having three “idealized” high-pass filters are constructed. The symmetry of the constructed filters are discussed via several examples.

Image denoising is a central topic in digital image processing. Digital images are often degraded by noise in the acquisition and/or transmission phase. The goal of image denoising is to recover the true/original image from a distorted/noisy copy. This special issue contains three papers on image denoising. Bilateral filter and PDE models are two popular methods for image denoising. In the article by Chui and Wang, the theoretical aspects of the bilateral filter are developed by deriving a class of nonlinear diffusion PDE associated with the bilateral filtering process. In the article by Bollt, Chartrand, Esedoglu, Schultz, and Vixie, a variational image denoising model, which interpolates total variation denoising and isotropic diffusion denoising, is proposed. They prove existence and uniqueness of minimizers for the proposed models. In the article by Chen, Song, and Tai, the dual algorithm of Chambolle is applied to the minimization of a high-order PDE image denoising model. The convergence of the proposed algorithm is analyzed.

Recovering missing data from an incomplete samples is a central issue in applied mathematics and it has a wide range of applications in image analysis and processing. In the article authored by Cai, Chan, Shen, and Shen, an iterative algorithm based on tight framelets for image recovery from incomplete observed data is proposed. The convergence and optimal property of the proposed algorithm are given. In particular, the performance of the proposed algorithm for impulsive noise removal and image inpainting are illustrated. In the article by Fornasier, Ramlau, and Teschke, mathematical inpainting and

recolorization are reviewed and discussed. The authors propose a wavelet-based joint sparsity model for recolorization. Novel numerical experiments in the real-life problem of the A. Mantegna's frescoes are performed.

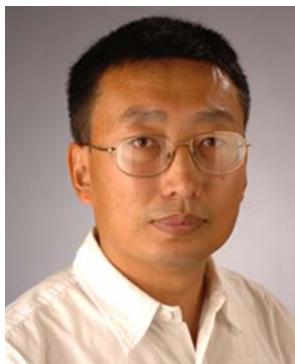
In the paper by Jian, a cryptographic watermarking technique is proposed for digital rights management and copyright protection. The author develops an algorithm to randomly generate the watermark indices based on the discrete logarithm problem and the Fermat's little theorem. The robustness and security analysis of the proposed algorithm are demonstrated under common signal degradations such as Gaussian noise and random multiplicative noise.

Image deblurring and image segmentation are two important fields in image processing. The aim of image deblurring is to recover a sharp image from a blurred and noisy observed image. The aim of image segmentation is to partition the image into homogeneous regions so that pixels within a region belong to the same object. This special issue includes two papers on image deblurring and two papers on image segmentation. In the article by Bardsley and Luttman, an image deblurring model with Poisson type noise is discussed. The authors propose to minimize a total variation-penalized Poisson likelihood functional. A nonnegatively constrained, projected quasi-Newton method is introduced to find the minimizer of the functional. In the article by Krishnan, Pham, and Yip, a primal-dual active-set algorithm is proposed for bilaterally constrained total variation deblurring and piecewise constant Mumford-Shah segmentation problems. In the article by Hong, Soatto, and Vese, a variational framework is presented to compute statistics of an ensemble of shapes. The obtained shape statistics are integrated into segmentation as a prior knowledge. The effectiveness of the method is demonstrated through experimental results with synthetic and real images.

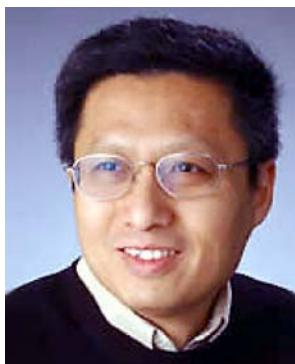
Image decomposition is another active research topic. In the article by Shen, a connection is established for two seemingly uncorrelated realms in mathematical and computational image processing: beamlet and texture modeling using Sobolev distributions. The author shows that beamlets are densely embedded in and can completely describe H^{-1} textures. In the article by Romero, Alexander, Baid, Jain, and Papadakis, an isotropic multiresolution analysis is characterized in terms of the Lax-Wiener theorem. The authors show that the designed isotropic wavelets are suitable for 2D and 3D texture segmentation.

We greatly appreciate the efforts of all authors of this special issue and the conscientious reviews of all referees. We especially thanks the editor-in-chief, Dr. Charles Micchelli, for his encouragement and guidance for editing this special issue.

Guest Editors: Lixin Shen and Yuesheng Xu



Lixin Shen received the B.Sc. and M.Sc. degrees from Peking University, Beijing China, in 1987 and 1990, respectively, and the Ph.D. degree from Sun Yat-sen University of China, in 1996, all in mathematics. From September 1996 to July 2001, he was a Research Fellow at the Center for Wavelets, Approximation, and Information Processing, National University of Singapore, Singapore. From August 2001 to July 2002, he was a Post-Doctoral Fellow at Virtual Environments Research Institute, University of Houston. From August 2002 to July 2004, he was a Research Assistant Professor, supported by a NSF grant, in the Department of Mathematics at West Virginia University. From August 2004 to July 2006, he was an Assistant Professor in the Department of Mathematics, Western Michigan University. He is currently an Assistant Professor in the Department of Mathematics, Syracuse University, Syracuse, NY. His current research interests are wavelets and their application in image processing. His research has been supported by the US National Science Foundation.



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