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ERRATUM

Erratum to: Polyhedral approximation of ellipsoidal uncertainty sets via extended formulations: a computational case study

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The purpose of this erratum is to correct a signing error in the statement of the inner approximation of the second-order cone $\mathbb{L}^n$ presented in Bärmann et al. (2016).

In Bärmann et al. (2016), we developed a construction for the inner approximation of $\mathbb{L}^n$ based on the ideas of Ben-Tal and Nemirovski (2001) and Glineur (2000). We showed—using the same decomposition as in the aforementioned papers—that it suffices to find an inner approximation of $\mathbb{L}^2$, which in turn can be obtained from an inner approximation of the unit ball $\mathbb{B}^2 \subset \mathbb{R}^2$. However, in the statement of the latter two approximations, there was a signing error which we would like to correct here.

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Our inner approximation of $\mathbb{B}^2$ is a regular m-gon $\bar{P}_m$ inscribed into it. Via an extended formulation, we can state this m-gon using a number of variables and constraints logarithmic in m:

Theorem 2.9 *The polyhedron*

$$\bar{D}_k = \left\{ (p_0, \dots, p_{k-1}, d_0, \dots, d_{k-1}) \in \mathbb{R}^{2k} \;\middle|\; \begin{aligned} p_{i-1} &= \gamma_i p_i + \sigma_i d_i, & (\forall i = 1, \dots, k-1) \\ -d_{i-1} &\leq \sigma_i p_i - \gamma_i d_i, & (\forall i = 1, \dots, k-1) \\ d_{i-1} &\leq \sigma_i p_i - \gamma_i d_i, & (\forall i = 1, \dots, k-1) \\ p_{k-1} &= \gamma_k, & \\ -d_{k-1} &\leq \sigma_k, & \\ d_{k-1} &\leq \sigma_k & \end{aligned} \right\}$$

for $k \geq 2$ is an extended formulation for $\bar{P}_{2^k}$ with $proj_{p_0,d_0}(\bar{D}_k) = \bar{P}_{2^k}$.

Proof In the following, we describe the construction of the inner approximation as an iterative procedure. We start by defining the polytope

$$P_{k-1} := \{(p_{k-1}, d_{k-1}) \mid p_{k-1} = \gamma_k, -\sigma_k \leq d_{k-1} \leq \sigma_k\}.$$

Now, we construct a sequence of polytopes $P_{k-1}, P_{k-2}, \dots, P_0$. Assume that polytope P_i has already been constructed. In order to obtain polytope P_{i-1} from polytope P_i, we perform the following actions which we will translate into mathematical operations below:

1. Rotate P_i counterclockwise by an angle of $\theta_i = \frac{\pi}{2^i}$ around the origin to obtain a polytope P_i^1,
2. Reflect P_i^1 at the x-axis to obtain a polytope P_i^2,
3. Form the convex hull of P_i^1 and P_i^2 to obtain polytope P_{i-1}.

The first step is a simple rotation and can be represented by the linear map

$$\mathcal{R}_\theta : \mathbb{R}^2 \mapsto \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

The reflection at the x-axis corresponds to the linear map

$$\mathcal{M} : \mathbb{R}^2 \mapsto \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Thus, the composition $\mathcal{M}\mathcal{R}_{\theta_i}$ which first applies $\mathcal{R}_{\theta_i}$ and then $\mathcal{M}$, is given by

$$\mathcal{M}\mathcal{R}_{\theta_i} : \mathbb{R}^2 \mapsto \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

With this, we obtain $P_i^1 = \mathcal{R}_{\theta_i}(P_i)$ and $P_i^2 = (\mathcal{M}\mathcal{R}_{\theta_i})(P_i)$. Finally, adding the two constraints

$$-d_{i-1} \leq \sigma_i p_i - \gamma_i d_i$$

and

$$d_{i-1} \leq \sigma_i p_i - \gamma_i d_i$$

yields a polyhedron whose projection onto the variables (d_{i-1}, p_{i-1}) is $P_{i-1} = conv(P_i^1, P_i^2)$. Keeping this correspondence in mind, we show that $P_0 = \bar{P}_{2^k}$.

In each iteration, P_i is rotated counterclockwise by an angle of θ_i around the origin, such that the vertex of P_i with minimal vertical coordinate is rotated to (γ_k, σ_k), therefore $P_i^1 = \mathcal{R}(P_i)$. It is $|\mathcal{V}(P_i^1)| = |\mathcal{V}(P_i)|$ and P_i^1 lies strictly above the horizontal axis. Applying $\mathcal{M}$, we obtain $P_i^2 = \mathcal{M}(P_i^1)$, which satisfies $|\mathcal{V}(P_i^2)| = |\mathcal{V}(P_i^1)|$ and lies strictly below the horizontal axis. Then $P_{i-1} = conv(P_i^1, P_i^2)$ satisfies $|\mathcal{V}(P_{i-1})| = 2|\mathcal{V}(P_i)|$ because all vertices $v \in \mathcal{V}(P_i^1) \cup \mathcal{V}(P_i^2)$ remain extreme points of P_i. We obtain polytope P_0 after $k-1$ iterations of the above procedure, which has $|\mathcal{V}(P_0)| = 2^k$ vertices. As the interior angles at each vertex of P_0 are of equal size, it follows $P_0 = \bar{P}_{2^k}$. This proves the correctness of our construction. □

The intermediate steps of the construction are depicted in Fig. 1 for the case $k = 3$, which leads to an octagon-approximation. The upper left picture shows the initial polytope P_2, which is an interval on the line $x = \gamma_k$. The upper middle and upper right picture show its rotation by 45° counterclockwise and subsequent reflection at the x-axis, thus representing P_2^1 and P_2^2, respectively. The lower left picture shows P_1 as the convex hull of P_2^1 and P_2^2. The lower middle picture contains both P_1^1 and P_1^2 as a rotation of P_1 by 90° counterclockwise and subsequent reflection at the x-axis, respectively. Finally, the lower right picture shows $P_0 = \bar{P}_{2^3}$ as the convex hull of P_1^1 and P_1^2.

By homogenization, we can obtain an inner ϵ-approximation of $\mathbb{L}^2$, i.e., a set $\bar{\mathcal{L}}_\epsilon^2$ with $\{(r, x) \in \mathbb{R} \times \mathbb{R}^2 \mid \|x\| \leq \frac{1}{1+\epsilon} r\} \subseteq \bar{\mathcal{L}}^2 \subseteq \mathbb{L}^2$:

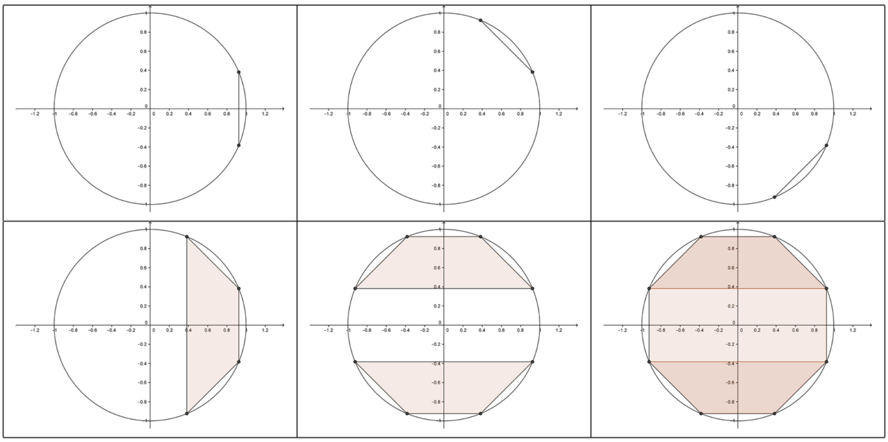

Fig. 1 Construction of the inner approximation of the unit disc $\mathbb{B}^2$ for $k = 3$

Corollary 2.10 *The projection of the set*

$$\bar{\mathcal{L}}^2_\epsilon = \left\{ (s, p_0, \ldots, p_{k-1}, d_0, \ldots, d_{k-1}) \in \mathbb{R}^{2k} \;\middle|\; \begin{array}{rll} p_{i-1} &= \gamma_i p_i + \sigma_i d_i, & (\forall i = 1, \ldots, k-1) \\ -d_{i-1} &\leq \sigma_i p_i - \gamma_i d_i, & (\forall i = 1, \ldots, k-1) \\ d_{i-1} &\leq \sigma_i p_i - \gamma_i d_i, & (\forall i = 1, \ldots, k-1) \\ p_{k-1} &= \gamma_k s, & \\ -d_{k-1} &\leq \sigma_k s, & \\ d_{k-1} &\leq \sigma_k s & \end{array} \right\}$$

with $\epsilon > 0$ *and* $k = \lceil \log(\pi \arccos(\frac{1}{\epsilon+1})^{-1}) \rceil$ *onto the variables* (s, p_0, d_0) *is an inner* ϵ*-approximation of* $\mathbb{L}^2$.

We apologize for the incorrect statements of the two approximations in the initial paper.

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