Central European Journal of Operations Research (2020) 28:539–559
https://doi.org/10.1007/s10100-020-00677-7

How relieving is public debt relief? Monetary and fiscal policies in a monetary union during a debt crisis

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Published online: 7 March 2020

Abstract

We use a dynamic game model of a two-country monetary union to study the consequences of sovereign debt reliefs for a member country or bloc of countries of the union after an exogenous fall in aggregate demand and the resulting increase in public debt. The debt reliefs are assumed to occur endogenously, being enacted after an increase of government debt beyond a certain threshold. We assume that the governments of the countries participating in the union pursue national goals when deciding on fiscal policies whereas the common central bank's monetary policy aims at union-wide objective variables. The union considered is asymmetric, consisting of a "core" with lower initial public debt, and a "periphery" with higher initial public debt. The "periphery" may experience debt reliefs due to the high level of its sovereign debt. We calculate numerical solutions of the dynamic game between the governments and the central bank using the OPTGAME algorithm. We show that a debt relief as modeled in our study is disadvantageous for both the "core" and the "periphery" of the monetary union, and that after an initial haircut further debt reliefs will be required to an extent that threatens the existence of the entire union.

Keywords Dynamic game · Numerical solutions · Feedback Nash equilibrium · Pareto solution · Economic dynamics · Monetary union · Macroeconomics · Public debt

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1 Introduction

Dynamic game theory has been shown to be a valuable analytical tool for economic policy analysis (see, e.g., Petit 1990; Dockner et al. 2000). Dynamic games have been used as models for conflicts between monetary and fiscal policies by several authors (e.g. Pohjola 1986). There is also a large body of literature on dynamic conflicts between policy makers from different countries on issues of international stabilization policy (e.g. Hughes Hallett et al. 1986; Miller and Salmon 1985). Both types of conflict are present in a monetary union because a supranational central bank interacts strategically with sovereign governments as national fiscal policy makers in the member states. Such conflicts have previously been analyzed using either large empirical macroeconomic models (e.g. Haber et al. 2002) or small stylized models (e.g. van Aarle et al. 2002; Engwerda et al. 2013; Michalak et al. 2014; Neck and Blueschke 2014). More recently, Canofari et al. 2015, 2017, 2018 developed similar strategic models of a monetary union resembling the Euro Area (EA) and used it to analyse effects of the debt crisis and policies to preserve the stability of the European Monetary Union.

In Neck and Blueschke (2014), we analyzed the consequences of a one-time debt relief ("haircut") for a country or a bloc of countries for macroeconomic variables and showed that such a measure can be disadvantageous for both the indebted and the donor country. One reason for this is the fact that financial markets do not quickly "forget" that a country has failed to fulfil its obligations and add a risk premium to the interest rate on such a country's government bonds over an extended period of time. For instance, the bail-out package for Greece proposed by the troika of the IMF, European Central Bank and European Commission included a debt relief of 50% by the banks. There is a long-standing discussion about the costs of such a haircut for an economy (e.g., Bulow and Rogoff 1989; Panizza et al. 2009). A more recent study (Cruces and Trebesch 2013) has shown that financial markets punish the haircut by introducing a higher risk premium. For a 40% haircut for the entire "periphery" of a two-country monetary union, of which three quarters are paid by the public sector of the "core", we showed that the macroeconomic consequences of such a measure are inferior to a scenario without such a debt relief.

In the present paper, we aim at analyzing the question of how debt reliefs for a highly indebted member of a monetary union affect the policy strategies of fiscal and monetary policy makers and their results under alternative assumptions about the amount, timing and circumstances of the haircut.

In particular, we conduct an analysis of the consequences of endogenously triggered haircuts. This has some importance within the context of the Euro Area's recent (and possibly future) policy problems as some countries may consider demanding another "haircut" after the first one, especially if the first did not really succeed in reducing their public debt substantially. One may ask whether a sequence of debt reliefs may improve the public finances of such countries which are struggling with the consequences of many years of irresponsible fiscal policy in the medium and long run. For this purpose, we assume that more than one haircut can occur. In particular, we consider "endogenous" haircuts, meaning that they are triggered if a certain threshold of public debt in the "periphery" country or bloc of countries is crossed.

As in the previous paper, we use a dynamic game analysis of the strategic interactions between fiscal and monetary policy makers in a stylized monetary union to answer the question about the macroeconomic consequences of such endogenous debt reliefs. As the dynamic game model is too complex to allow for an analytical solution, numerical solutions or approximations are the only tool available. Here we use the OPTGAME algorithm (Blueschke et al. 2013) to analyze such a macroeconomic policy problem for a two-country asymmetric monetary union. The OPTGAME algorithm delivers approximate solutions for discrete-time nonlinear-quadratic difference games, i.e. games with quadratic objective functions and a nonlinear dynamic system. Dynamic games with a finite planning horizon are considered. We apply OPTGAME to calculate the feedback Nash equilibrium solution and a cooperative Pareto-optimal solution for our model of an asymmetric monetary union. The feedback Nash equilibrium solution is strongly time consistent (Markov perfect) and serves as model for uncoordinated policy making in the monetary union. The Pareto solution assumes a cooperative agreement, in our case by the member states and the central bank, and reflects a compromise between these agents (assumed to have equal weights in our model). It can be regarded as a formal pact between them, which reflects the preferences and the power of the policy makers but is not necessarily a socially optimal outcome of the game. In spite of the simple character of the model, we can shed some light on current sovereign debt problems in Europe by comparing and interpreting the results from this debt relief modeling exercise.

2 MUMOD1: a dynamic model of a monetary union

In order to analyze the dynamic effects of a haircut in a monetary union we use a dynamic macroeconomic model consisting of two countries (or two blocs of countries) with a common central bank. This model is called MUMOD1 and slightly improves on the one introduced in Blueschke and Neck (2011). For a similar framework in continuous time, see van Aarle et al. (2002). The model is calibrated so as to deal with the problem of public debt targeting in a situation that resembles the one currently prevailing in the European Union.

We consider a monetary union which itself is a closed economy, without exports to and imports from other countries. This assumption is, of course, not fully realistic but seems acceptable as a first approximation in view of the European Union's relatively small amount of trade with the rest of the world. Financial markets within the monetary union are integrated except interest rates depending on country-specific risk premiums. There are no differences between interest rates on private and public debt. This implies that each government and each private sector can lend and borrow at domestic interest rates at home and at foreign interest rates abroad. The assumption of private debt being subject to the same risk premium as public debt can be justified by the fact that in case of high private default risk the government will have to bail out private firms (especially banks) and households, and this is expected by foreign and domestic bond holders.

The model is formulated in terms of deviations from a long-run growth path and includes three decision-makers. The common central bank decides on the prime rate

R_{Et}, a nominal rate of interest under its direct control. The national governments decide on fiscal policy. g_{it} denotes country i's ($i = 1, 2$) real fiscal surplus (or, if negative, its fiscal deficit), measured in relation to real GDP.

The model consists of the following equations:

$$y_{it} = \delta_i(\pi_{jt} - \pi_{it}) - \gamma(r_{it} - \theta) + \rho_i y_{jt} - \beta_i \pi_{it} + \kappa_i y_{i,t-1} - \eta_i g_{it} + zd_{it}, \quad (1)$$
$$r_{it} = I_{it} - \pi^e_{it}, \quad (2)$$
$$I_{it} = R_{Et} - \lambda_i g_{it} + \chi_i D_{it} + zhp_{it}, \quad (3)$$
$$\pi_{it} = \pi^e_{it} + \xi_i y_{it}, \quad (4)$$
$$\pi^e_{it} = \varepsilon_i \pi_{i,t-1} + (1 - \varepsilon_i)\pi^e_{i,t-1}, \ \ \varepsilon \in [0, 1], \quad (5)$$
$$y_{Et} = \omega y_{1t} + (1 - \omega) y_{2t}, \ \ \omega \in [0, 1], \quad (6)$$
$$\pi_{Et} = \omega \pi_{1t} + (1 - \omega)\pi_{2t}, \ \ \omega \in [0, 1], \quad (7)$$
$$D_{it} = (1 + BI_{i,t-1} - \pi^e_{i,t-1}) D_{i,t-1} - g_{it} + zh_{it}, \quad (8)$$
$$BI_{it} = \frac{1}{6} \sum_{\tau=t-5}^{t} I_{it}. \quad (9)$$

The goods markets are modelled for each country i by the short-run income-expenditure equilibrium relation (IS curve) (1) for real output y_{it} (the deviation of short-run output from a long-run growth path) at time t ($t = 1, \ldots, T$). The natural real rate of output growth, $\theta \in [0, 1]$, is assumed to be equal to the natural real rate of interest.

The current real rate of interest r_{it} is given by Eq. (2). The nominal rate of interest I_{it} is given by Eq. (3), where $-\lambda_i$ and χ_i (assumed to be positive) are risk premiums for country i's fiscal deficit and public debt level. This allows for different nominal rates of interest in the union in spite of a common monetary policy.

The inflation rates for each country π_{it} are determined in Eq. (4) according to an expectations-augmented Phillips curve. π^e_{it} denotes the rate of inflation expected to prevail during time period t, which is formed according to the hypothesis of adaptive expectations at (the end of) time period $t - 1$ [(Eq. (5)]. $\varepsilon_i \in [0, 1]$ are positive parameters determining the speed of adjustment of expected to actual inflation.

The average values of output and inflation in the monetary union are given by Eqs. (6) and (7), where parameter ω expresses the weight of country 1 in the economy of the whole monetary union as defined by its output level. The same weight ω is used for calculating union-wide inflation.

The government budget constraint is given as an equation for real government debt D_{it} (measured in relation to (real) GDP) and is defined in Eq. (8). The interest rate on public debt (on bonds) is denoted by BI_{it}, which assumes an average government bond maturity of six years, as estimated in Krause and Moyen (2016).

The MUMOD1 model allows to include several exogenous shocks and to analyze the dynamic behavior of the whole system as a result of these shocks. The variables $zd_{it} (i = 1, 2)$ represent demand-side shocks in the goods markets, zh_{it} allows us to model an exogenous shock on public debt, zhp_{it} allows for exogenous shocks on the nominal rate of interest.

Table 1 Parameter values for an asymmetric monetary union, $i = 1, 2$

T	θ	ω	$\delta_i, \eta_i, \varepsilon_i$	$\beta_i, \gamma_i, \rho_i, \kappa_i, \lambda_i$	ξ_i	χ_i	μ_i, μ_E
30	3	0.6	0.5	0.25	0.1	0.0125	0.333

The parameters of the model are specified for a slightly asymmetric monetary union. Here an attempt has been made to calibrate the model parameters so as to fit for the euro area (EA). The data used for calibration include average economic indicators for the 17 EA countries from EUROSTAT up to the year 2007 (pre-crisis state). Mainly based on the public finance situation, the EA is divided into two blocs: a "core" (country or bloc 1) and a "periphery" (country or bloc 2). The first bloc includes ten EA countries (Austria, Belgium, Estonia, Finland, France, Germany, Luxembourg, Malta, Netherlands, and Slovakia) with a more solid fiscal situation and inflation performance. This bloc has a weight of 60% in the entire economy of the monetary union. The second bloc has a weight of 40% in the economy of the union; in the EA, it consists of seven countries with higher public debt and/or deficits and higher interest and inflation rates on average (Cyprus, Greece, Ireland, Italy, Portugal, Slovenia, and Spain). The weights correspond to the respective shares in EA real GDP. For the other parameters of the model, we use values in accordance with econometric studies and plausibility considerations (see Table 1).

Using the MUMOD1 model we consider an intertemporal nonlinear game which is given in tracking form. The players aim at minimizing quadratic deviations of the objective (state) variables from given "ideal" (desired) values. The individual objective functions of the national governments ($i = 1, 2$) and of the common central bank (E) are given by

$$J_i = \frac{1}{2}\sum_{t=1}^{T}\left(\frac{1}{1+\frac{\theta}{100}}\right)^t \Big\{\alpha_{\pi i}\left(\pi_{it} - \tilde{\pi}_{it}\right)^2 + \alpha_{yi}\left(y_{it} - \tilde{y}_{it}\right)^2 + \alpha_{Di}\left(D_{it} - \tilde{D}_{it}\right)^2 + \alpha_{gi} g_{it}^2\Big\} \quad (10)$$

$$J_E = \frac{1}{2}\sum_{t=1}^{T}\left(\frac{1}{1+\frac{\theta}{100}}\right)^t \Big\{\alpha_{\pi E}\left(\pi_{Et} - \tilde{\pi}_{Et}\right)^2 + \alpha_{yE}\left(y_{Et} - \tilde{y}_{Et}\right)^2 + \alpha_{E}\left(R_{Et} - \tilde{R}_{Et}\right)^2\Big\} \quad (11)$$

where all α are weights of state variables representing their relative importance to the respective policy maker. A tilde denotes the desired ("ideal") values of the respective variable.

The joint objective function for calculating the cooperative Pareto-optimal solution is given by the weighted sum of the three objective functions:

$$J = \mu_1 J_1 + \mu_2 J_2 + \mu_E J_E, \qquad (\mu_1 + \mu_2 + \mu_E = 1). \quad (12)$$

Table 2 Negative demand shocks in the asymmetric monetary union

t	1	2	3	4	5	6	7	8	9	...
zd_{1t}	−6	−1	−6	0	0	0	0	0	0	0
zd_{2t}	−6	−1	−6	−6	−8	−6	−4	−2	0	0

The dynamic system, which constrains the choices of the decision makers, is given in state-space form by the MUMOD1 model as presented in Eqs. (1)–(9). Equations (10), (11) and the dynamic system (1)–(9) define a nonlinear dynamic tracking game problem which can be solved for different solution concepts. Using the OPTGAME3 algorithm (see Blueschke et al. 2013) we are able to solve this dynamic tracking game and to analyze the dynamic effects of different shocks acting on the system. In this paper, two alternative game strategies are considered, a cooperative Pareto solution and a non-cooperative Nash equilibrium solution with feedback information pattern.

3 Public debt relief

In a previous paper (Neck and Blueschke 2014) we already investigated the impact of a haircut in a monetary union modelled by MUMOD1. The present study extends this research and uses a similar experiment setting.

First, we assume that the monetary union is confronted with a symmetric negative demand shock (representing the economic crisis 2008–2010), which is followed by another demand shock on the second country only (representing the sovereign debt crisis in Europe affecting mainly the "periphery" bloc). Table 2 summarizes these shocks.

Such a negative demand shock requires a countercyclical (expansionary) fiscal policy. However, this leads to rapidly growing public debts, especially for the periphery bloc. Greece is the most prominent example of the European sovereign debt crisis with its government bonds rated close to default. One bailing-out package for Greece, which included a 53.5% haircut by non-institutional foreign creditors, was implemented in 2011. In 2013, a bail-out (or rather bail-in) deal was put into effect for the Cypriot economy, which included a haircut of approximately 47.5% for bank deposits above EUR 100,000.

In order to simulate an event of this kind for the entire "periphery", Neck and Blueschke (2014) introduce a 40 percentage points haircut for the public debt of country 2 ("periphery" bloc) at time 11. Three quarters of this haircut are assumed to be paid by the governments of the "core" bloc. This results in an increase in public debt of 20 percentage points for country 1 (the "core" bloc). According to the study by Cruces and Trebesch (2013), larger haircuts are not forgotten by the markets in the short run; instead, the country which has experienced such a haircut has to pay a higher risk premium for several years to come. Neck and Blueschke (2014) use the average values from the results of that study to calibrate the exogenous variable zhp_{2t} which denotes the additional risk premium after the haircut. Table 3 summarizes the haircut shock and the additional after-shocks which are triggered by such a haircut.

Table 3 Haircut and its after-effect shocks in the asymmetric monetary union

t	11	12	13	14	15	16	17	18	19	20	21	...
zh_{2t}	−40	0	0	0	0	0	0	0	0	0	0	0
zh_{1t}	20	0	0	0	0	0	0	0	0	0	0	0
zhp_{2t}	10	6	5.5	5	4.5	4	3.5	3	2	1	0	0

The occurrence of the haircut shock (and the other after-shocks) is analyzed in Neck and Blueschke (2014) in two different ways: as an expected event (the players know already at the beginning of the game that in time period 11 a haircut will occur) and as an unexpected event (the players are informed in time period 11 about this shock). Neck and Blueschke (2014) find that the proposal of a haircut can be counterproductive. A haircut creates different incentives and, as a consequence, different policies for the countries in the monetary union. In anticipation of a haircut, especially if it is foreseen, the best strategy for the “periphery” (given its policy makers’ preferences) is to produce even higher budget deficits before this event. This result occurs for both the cooperative Pareto solution and the noncooperative feedback Nash equilibrium solution. Taking the higher risk premium into account, which is usually paid after a haircut, results in a situation where all the players in the monetary union perform worse than in the respective scenario without a haircut.

4 Endogenous haircuts

In the present paper we extend the work by Neck and Blueschke (2014) by introducing endogenously triggered haircut shocks. We use the MUMOD1 model and the nonlinear tracking game framework as presented in Sect. 2. We apply the same negative demand shock as presented in Table 2. We also start with the same haircut shock as in Table 3 but do not define a certain time period when the haircut occurs. Instead, we define a threshold for the level of public debt in the periphery (denoted by th_{D2}). Whenever the threshold is reached the haircut and the after-effects (as given in Table 3) are triggered. In our first experiment we start with $th_{D2} = 150$ which means that once the public debt in the periphery is above 150% of GDP, a haircut will be triggered. The chosen value of 150% is slightly below the situation in Greece in 2011 (170% of GDP) but above the public debt level in Cyprus in 2013 (112% of GDP) and hence seems to be a plausible value for a haircut threshold in the EA.

Before we can start with our experiments we need to define the initial values for the state variables, the target values for state and control variables considered in the objective function and the weights of the state variables as presented by the αs in Eqs. 10 and 11 as inputs in the tracking game framework. The MUMOD1 model represents a monetary union consisting of three players, two national governments (or blocs) and a central bank. Both national fiscal authorities are assumed to care about stabilizing inflation (π), output (y), debt (D), and fiscal deficits of their own countries (g) at each time t. This is a policy setting which seems plausible for the

Table 4 Weights of the variables in the objective functions

α_{yi}, α_{gi}	$\alpha_{\pi E}$	$\alpha_{yE}, \alpha_{\pi i}$	α_{D1}	α_{D2}	α_{RE}
1	2	0.5	0.01	0.0001	3

Table 5 Initial values of the two-country monetary union

$y_{i,0}$	$\pi_{i,0}$	$\pi^e_{i,0}$	$I_{i,0}$	$D_{1,0}$	$D_{2,0}$	$R_{E,0}$	$g_{1,0}$	$g_{2,0}$
0	2.5	2.5	3	60	80	3	-2	-4

Table 6 Target values for the asymmetric monetary union

$\tilde{D}_{1t}$	$\tilde{D}_{2t}$	$\tilde{\pi}_{it}$	$\tilde{\pi}_{Et}$	$\tilde{y}_{it}$	$\tilde{y}_{Et}$	$\tilde{g}_{it}$	$\tilde{R}_{Et}$
60	80 ↘ 60	2	2	0	0	0	3

real EA as well, with full employment (output at its potential level) and price level stability relating to country (or bloc) i's primary domestic goals, and government debt and deficit relating to its obligations according to the Maastricht Treaty of the European Union. The common central bank is interested in stabilizing inflation and output in the entire monetary union, also taking into account a goal of low and stable interest rates in the union. It attaches a weight of 2 to the inflation target and 0.5 to the output gap target. Several studies report quite different weights of a central bank (see, e.g., Dennis 2004; Mandler 2012; Assenmacher-Wesche 2006). However, in all studies the inflation target was found to be considerably more important than the output target, and we think that a weight of 0.25 for the output target relative to the inflation target underlines this difference quite well. The weights of all objective variables (state and control) are given in Table 4.

The initial values of the state variables of the dynamic game model are presented in Table 5. The ideal or target values assumed for the objective variables of the players are given in Table 6. Country 1 (the core bloc) has an initial debt level of 60% of GDP and wants to hold this level over time. Country 2 (the periphery bloc) has an initial debt level of 80% of GDP and aims at decreasing its level to 60% by the end of the planning horizon, which means that it is going to fulfil the Maastricht criterion for this economic indicator. The ideal rate of inflation is calibrated at 2%, which corresponds to the Eurosystem's aim of keeping inflation below, but close to, 2%. The initial values of the two blocs' government debts and budget deficits correspond to those at the beginning of the Great Recession, the recent financial and economic crisis. Otherwise, the initial situation is assumed to be close to equilibrium, with parameter values calibrated accordingly.

Using the OPTGAME algorithm we calculate three different solutions for each experiment: a non-controlled simulation (a strategy without policy intervention), the cooperative Pareto solution and the non-cooperative Nash feedback solution. As a baseline scenario we use the experiment with the demand side shocks (Table 2) but without a haircut. We do not analyze this scenario in detail as it can be found in Neck and Blueschke (2014). Instead, we use this scenario for comparison and a better understanding of the dynamics in the endogenous haircut scenarios. The alternative

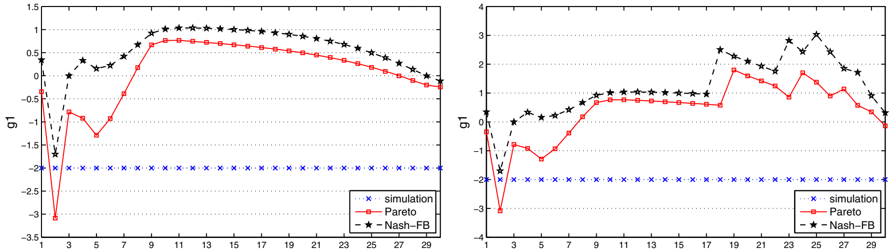


Fig. 1 Country 1's fiscal surplus g_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

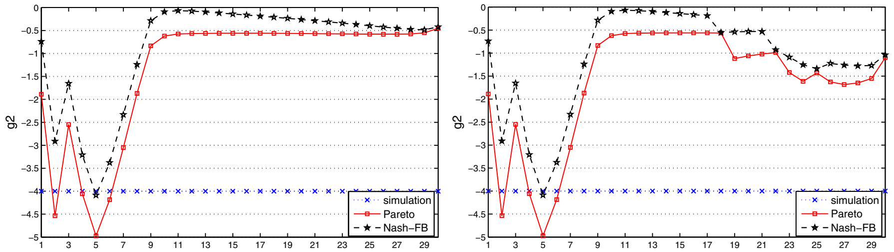


Fig. 2 Country 2's fiscal surplus g_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

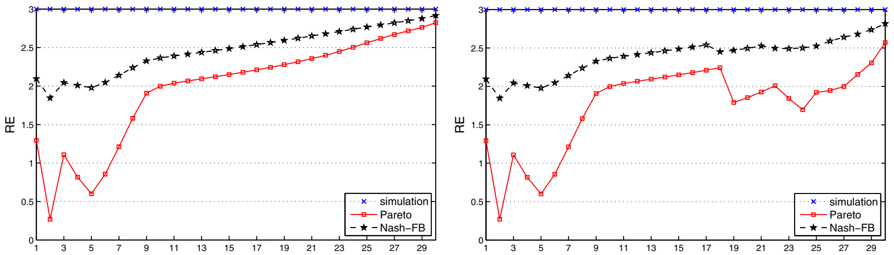


Fig. 3 Prime rate R_{Et} controlled by the central bank (left: scenario without haircut; right: scenario with haircuts) (color figure online)

scenario is a haircut shock (and after-shocks as detailed in Table 3) triggered if the public debt of the periphery reaches 150% of GDP.

Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 show the simulation and optimization results of our experiment. Figures 1, 2 and 3 show the results for the control variables of the players and Figs. 4, 5, 6 7, 8, 9, 10, 11, 12 and 13 show the results of selected state variables.

In the baseline scenario, both countries suffer dramatically (output drops by more than 6%) from the economic downturn modelled by the demand side shock in the first three periods. The periphery suffers even more in the periods 4–8 due to the second negative demand shock, hitting the second bloc only. The calculated solutions of the baseline scenario imply that the optimal policies of both the governments and the common central bank are countercyclical during the immediate influence of the demand shock but not afterwards; instead, if governments want (or are obliged by

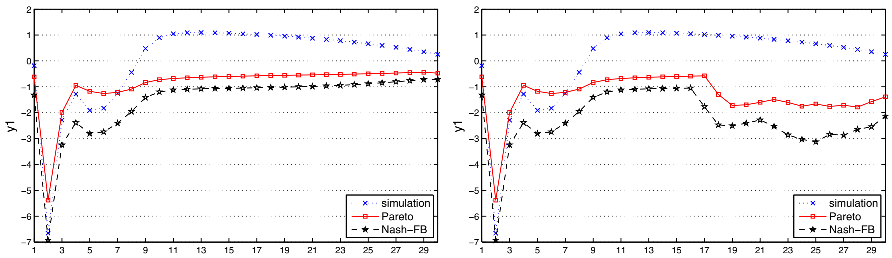


Fig. 4 Country 1's output y_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

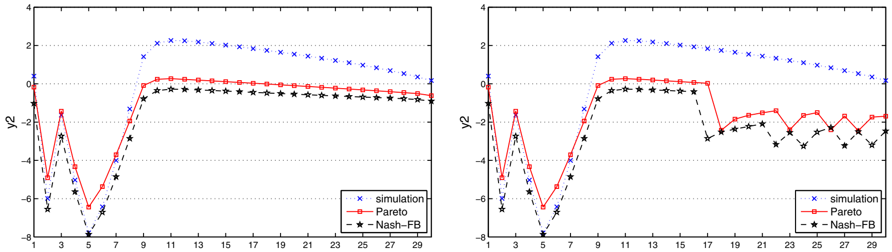


Fig. 5 Country 2's output y_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

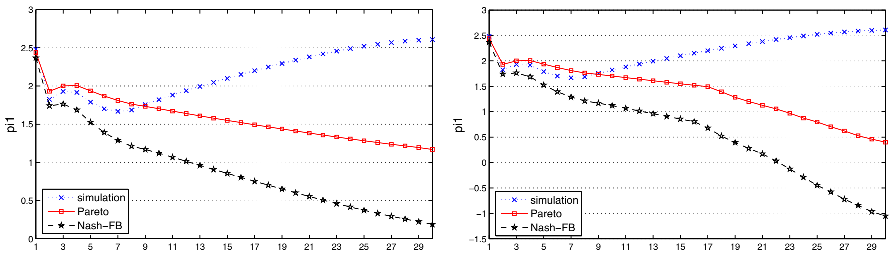


Fig. 6 Country 1's inflation rate π_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

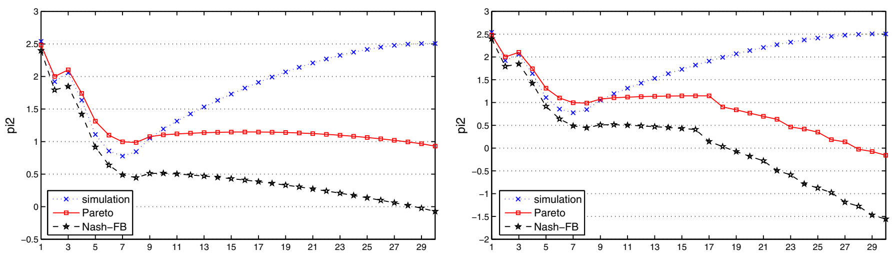


Fig. 7 Country 2's inflation rate π_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

the union's rules) to keep their public debt under control and avoid state bankruptcy, they have to implement prudent fiscal policies as soon as the crisis is over. The core bloc, which gives higher importance to the public debt target, follows this strategy

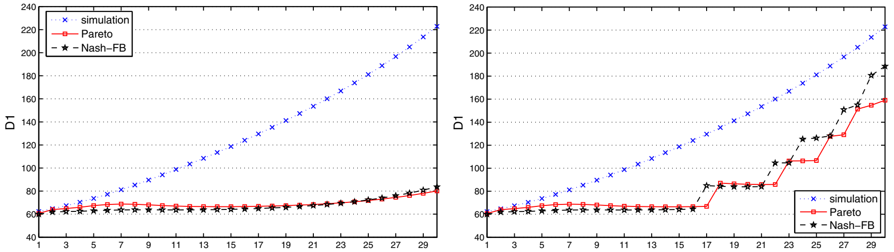


Fig. 8 Country 1's debt level D_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

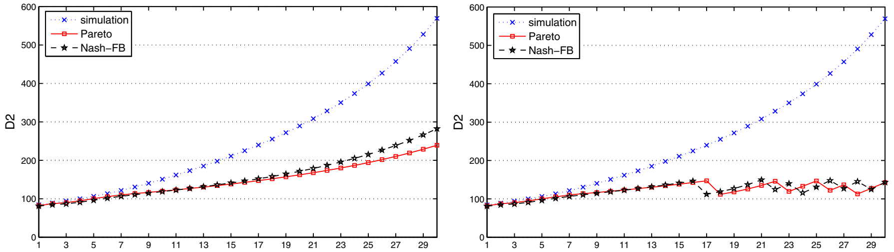


Fig. 9 Country 2's debt level D_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

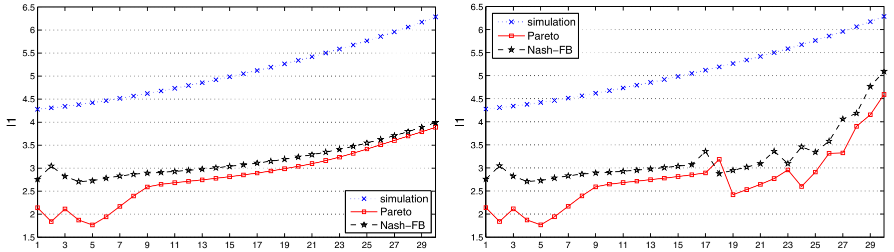


Fig. 10 Country 1's nominal interest rate I_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

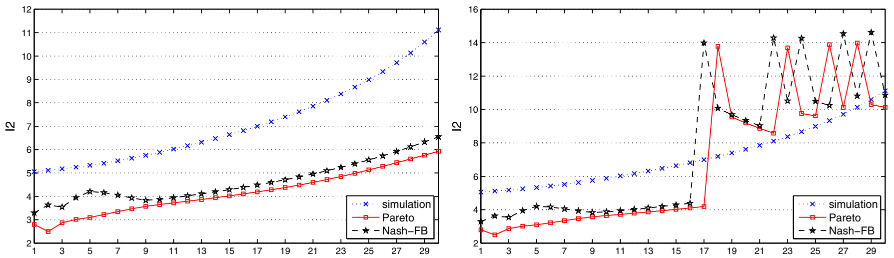


Fig. 11 Country 2's nominal interest rate I_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

and creates budget surpluses. In contrast, the periphery bloc runs a less prudent fiscal policy. As a result, the public debt of the periphery bloc goes up to 240% of GDP in the

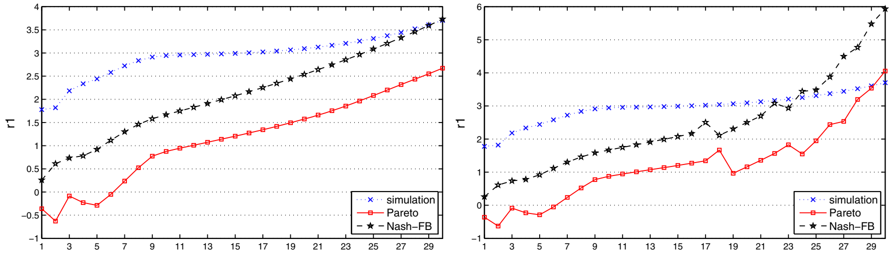


Fig. 12 Country 1's real interest rate r_{1t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

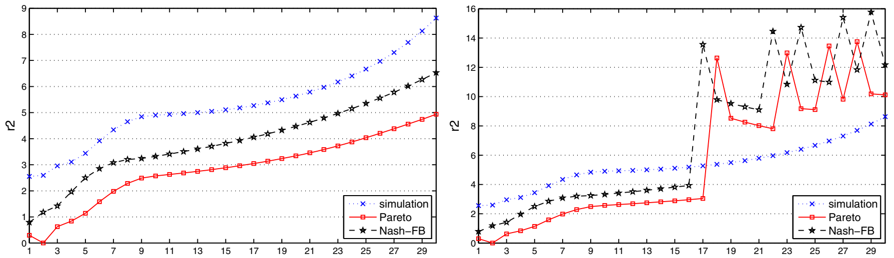


Fig. 13 Country 2's real interest rate r_{2t} (left: scenario without haircut; right: scenario with haircuts) (color figure online)

case of the Pareto solution and up to 290% of GDP in the case of the Nash feedback solution.

One may wonder why the periphery follows a policy of budget deficits after a haircut, even in the Pareto solution. The basic reason is the same as in the noncooperative solution: it considers such a policy to be expansionary (and it is so in the short run in our model), and the periphery gives a strong weight to increasing output even at the expense of further increasing debt. In the Pareto solution, the periphery has a weight of 1/3 in the joint objective function, and it uses its power to enforce the "permit" to further increase its debt. In the noncooperative solution, the core and the central bank accommodate such a policy to a lower extent, preventing the periphery from following an even more expansionary budgetary policy. It should also be noted that in the model the timing of the haircut depends only on the threshold (is not part of the policy strategies of the players but of a regulation in the monetary union), hence the incentive of the periphery to restrain its debt is relatively weak given the additional money it has at its disposal after the debt relief. This can also be interpreted as a case of the "voracity effect" (see Lane and Tornell 1996; Tornell and Lane 1999): the weak institutional structure of the union (in the model because of the separation of decisions about haircuts and budgetary policies, in the EU because of the separation of decisions about weak regulations and budgetary policies) induce policy makers to implement policies which seem to be advantageous in the short run but are detrimental to overall goals like stabilization of output and debt.

If we run the scenario with endogenous haircuts at $th_{D2} = 150$ (and assumed to be unexpected by the players) we see identical results to the baseline scenario until the first haircut occurs. After it the players are confronted with an unexpected shock and have to adjust their game strategies. The most remarkable result is that this scenario produces several haircuts. In the case of the Pareto solution these haircuts occur in time periods 18, 23, 26, and 28. In the case of the Nash feedback solution we even see five haircuts, namely in time periods 17, 22, 24, 27, and 29. The two blocs (or countries) react to these shocks very differently. The core bloc runs an even more prudent fiscal policy. In contrast, the periphery bloc creates additional deficits. Monetary policy is required to be more expansionary than without the haircuts.

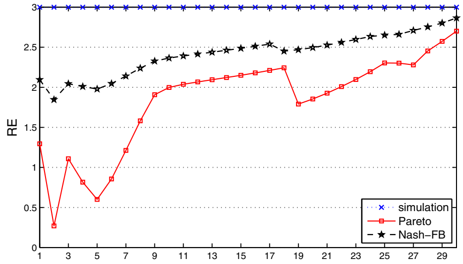
These haircut shocks lead to additional losses in output of around 1 percentage point for the core and 2 percentage points for the periphery. This results in decreasing inflation for both countries. In the case of the Nash feedback solution an ongoing deflation is expected. Due to the skyrocketing interest rate in the periphery the initially positive effect of a haircut on public debt disappears very quickly, and in many cases just two periods afterwards the next haircut is required. But each time a haircut for the public debt in the periphery bloc is executed, the core bloc has to pay for it with an increase in its own public debt. A relatively large number of such haircuts leads to the absurd situation that the public debt of the core becomes higher than the public debt of the periphery at the end of the planning horizon (160% of GDP in the Pareto case and 190% of GDP in the Nash feedback case). Altogether, the results show the uselessness and non-sustainability of such haircuts. In addition, such a scenario is not very realistic due to political reasons, as the core will not agree to such a solution of the periphery's sovereign debt problem.

The economic intuition behind the repeated haircuts and the increasing velocity of their occurrence can be explained by three effects. First, immediately after each haircut, the public debt of the periphery is, of course, lower than before the haircut but not necessarily lower than in $t = 0$, while the public debt of the core is higher due to the transfer to the periphery. Second, and most important, the risk premium for the periphery's interest rate is only gradually decreasing, which implies increasing interest costs for the periphery. Our model may even underestimate this effect, as it is very well possible that repeated haircuts lead to ever-increasing risk premiums, and a risk premium for the core may arise from the increase in its public debt. Finally, the government of the periphery has a moral hazard problem: it knows that it will get some debt relief if its public debt is rising beyond the threshold and has hence no strong incentive to avoid ever rising sovereign debt after each haircut (cf. also Canofari et al. 2017, p. 861f). Altogether, the impact of the apparent help to the periphery leads to an increase in the monetary union's overall public debt beyond limits and hence to a breakdown of the union.

5 Some extensions

In the experiment with the haircut threshold given by 150% of GDP, the system leads to a situation with several haircuts (4 in the Pareto case and 5 in the Nash feedback case). This development becomes obviously unsustainable, "too expensive" for the core and politically non-affordable. Next, we intend to design the endogenous haircut scenario in a more realistic way and adjust it by making additional assumptions. This

Fig. 14 Prime rate R_{Et} controlled by the central bank (color figure online)



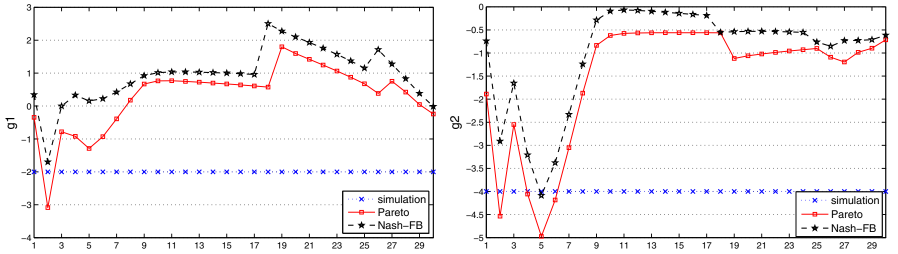


Fig. 15 Country i's fiscal surplus g_{it} (control variable) for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

can be also interpreted as a sensitivity analysis for the scenarios presented in Sect. 4. In particular, we look at a scenario where only two haircuts take place and the second debt relief is triggered at a higher value of government 2's debt (Sect. 5.1). Then we examine consequences of a nonlinear risk premium for high public debt, which has been proposed in the theoretical and empirical literature on the sovereign debt problem in the European Monetary Union (Sect. 5.2).

5.1 Scenario with two different thresholds

First, we examine a scenario with two thresholds for a haircut and no further haircut after the second. We assume that the threshold changes after the first occurrence of the haircut (using a threshold $th_{D2} = 150$). The next haircut will be triggered by a threshold $th'_{D2} = 200$, which means that now the public debt in the periphery bloc is allowed to grow up to 200% of GDP. Furthermore, we assume that only two haircuts are allowed to occur.

Figures 14, 15, 16, 17 and 18 show the simulation and optimization results of this experiment. Figures 1, 2 and 3 show the results for the control variables of the players and Figs. 16, 17 and 18 show the results of selected state variables.

In this scenario the first haircut occurs in the same period as in the previous scenario with $th_{D2} = 150$, namely in $t = 18$ (the Pareto solution) and $t = 17$ (the Nash feedback solution). The second haircut occurs significantly later, namely in $t = 26$ (the Pareto solution) and $t = 25$ (the Nash feedback solution). The optimal strategies are similar to the previous scenario with $th_{D2} = 150$ but with some changes: monetary policy is required to be more expansionary and fiscal policy depends on the player.

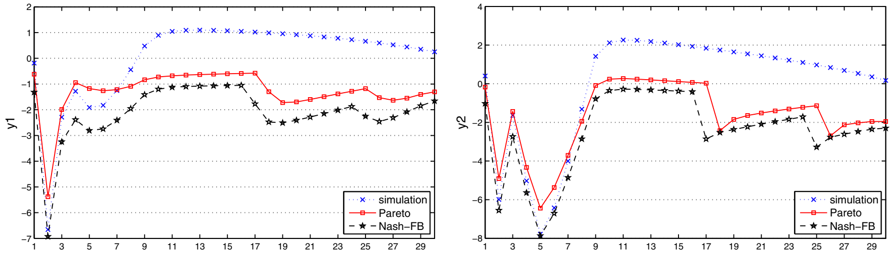


Fig. 16 Country i's output y_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

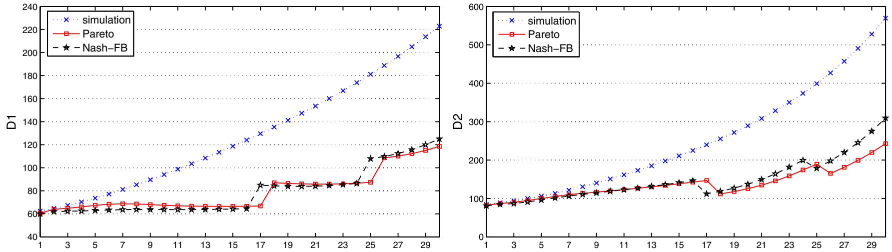


Fig. 17 Country i's debt level D_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

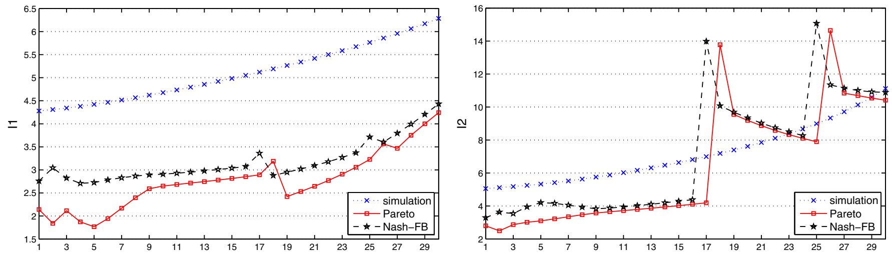


Fig. 18 Country i's nominal interest rate I_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

The core bloc runs a more prudent fiscal policy and the periphery bloc creates budget deficits.

This experiment leads to smaller losses in output for the core and higher losses in output for the periphery. Realizing just two haircuts instead of four (Pareto) or five (Nash feedback) haircuts in the previous experiment has a significant impact on public debt. The public debt of the periphery rises to 240% of GDP in the case of the Pareto solution and to 300% of GDP in the case of the Nash feedback solution. The public debt of the core stays within a more or less reasonable range, namely about 120% of GDP. However, we see again that the interest rate dynamics dominate the development of the public debt and lead to the result that the haircuts are counterproductive as long as the targets and preferences of the players remain unchanged.

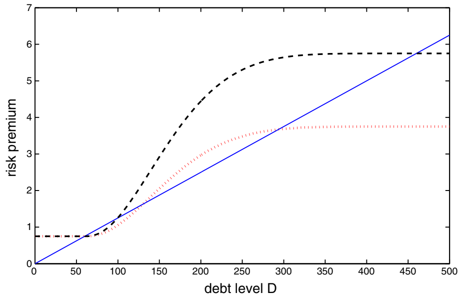


Fig. 19 Risk premium calculation; Beta distribution (14) versus linear (color figure online)

5.2 Nonlinear risk premiums

So far we assumed that the interest rates depend on the risk premiums in a linear way. This implies that an increase in public debt from, say, 30–40% of GDP has the same effect on the interest rate as an increase from 130–140%. This assumption does not seem to be realistic; in particular, there exists empirical evidence (e.g. Bi 2012; Ghosh et al. 2013) that when the fiscal space becomes limited the reaction of interest rates becomes strongly nonlinear. This implies that at high levels of government debt, further increases result in much stronger increases in the risk premium and hence in the interest rates. In order to capture this effect, we assumed that the interest rates depend on public debt in a nonlinear way via a function derived from the cumulative distribution function of the beta distribution:

$$I_{it} = R_{Et} - \lambda_i g_{it} + F^*(D_{it}) + zhp_{it}. \tag{13}$$

$F^*(.)$ is the cumulative beta distribution function:

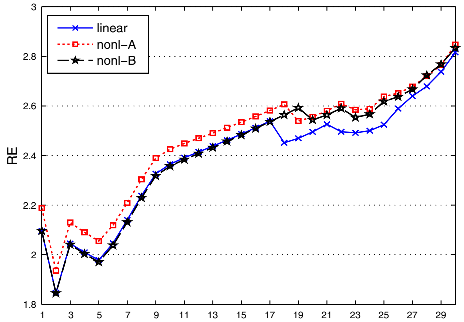
$$F^* = F^*\left(\frac{D_{it} - 60}{450}\right), 2.5, 8) = a \cdot \frac{1}{B(2.5, 8)} \int_0^{\frac{D_{it}-60}{450}} t^{1.5}(1-t)^7 dt + 0.75, \tag{14}$$

where $B(.)$ is the Beta function.

The functions are shown in Fig. 19 above, with the blue solid line showing the linear risk premium, the red dotted line and the black dashed line showing Beta distribution functions with parameter $a = 3$ (A) and $a = 5$ (B), respectively. The actual risk premium function may be argued to increase even further at higher debt levels instead of the flat portion of the function, but this would cause instabilities in the simulations; that part of the function is irrelevant anyway as we assumed a haircut to occur at a debt level of 150%.

Figures 20, 21, 22, 23 and 24 show the simulation and optimization results of these experiments for the noncooperative (Feedback Nash) solutions. The scenario from Sect. 4 serves as reference scenario (blue solid curves); scenarios A (red dotted curves) and B (black dashed curves) correspond to the parameter values of A and B

Fig. 20 Nonlinear risk premium scenarios; prime rate R_{Et} (color figure online)



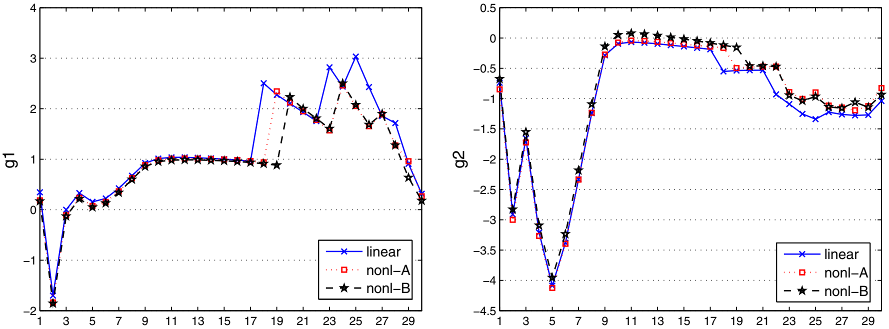


Fig. 21 Nonlinear risk premium scenarios; fiscal surplus g_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

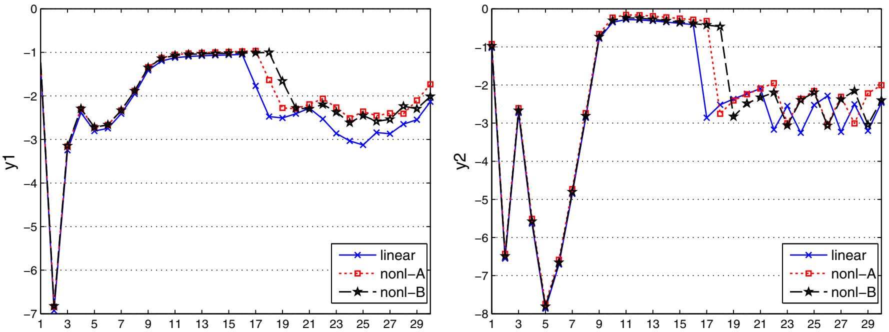


Fig. 22 Nonlinear risk premium scenarios; y_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

above, respectively. Figures 20, 21 show the results for the control variables of the players and Figs. 22, 23 and 24 show the results of selected state variables.

As can be seen from Figs. 20, 21, 22, 23 and 24 the dynamics of the public debt depends on the risk premium curve. The assumed nonlinear cases A and B yield lower risk premiums for the public debt below 130–100% of GDP and higher risk premiums for the higher debt levels. Thus, it is logical that the public debt in the nonlinear cases stays below the linear case in the first half of the planning horizon. Interestingly, we can not observe a clear reverse of this dynamics in the second half of

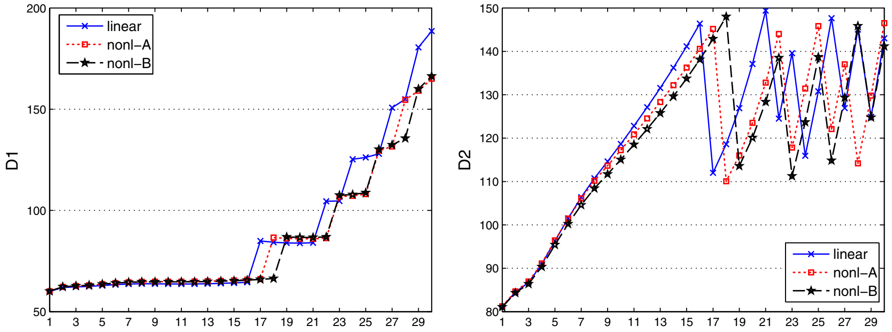


Fig. 23 Nonlinear risk premium scenarios; debt level D_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

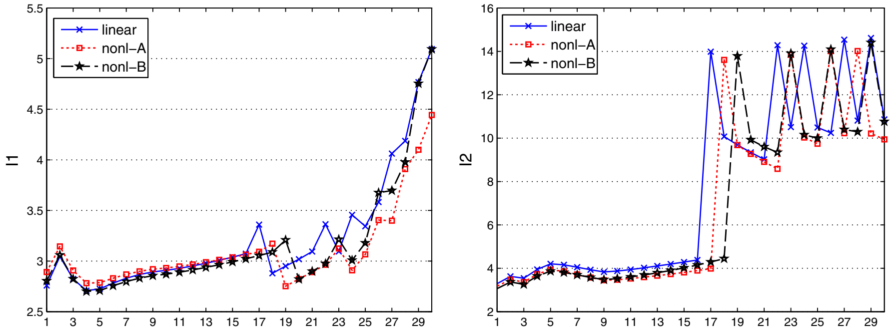


Fig. 24 Nonlinear risk premium scenarios; nominal interest rate I_{it} for $i = 1$ (core; left) and $i = 2$ (periphery; right) (color figure online)

the planning horizon, with higher public debts of the countries in the monetary union. This is partially explained by the positive effect of the haircut reducing the debt level of the periphery. The number of haircuts is reduced to four cases as compared to five haircuts in the linear case. This holds true for both nonlinear scenarios A and B. The first haircut occurs in the linear case at the end of period 17. In the nonlinear scenarios the first haircut is slightly delayed and occurs in period 18 in the nonlinear scenario A and 19 in scenario B.

The control variables are used in the nonlinear scenarios in a similar way to the linear case. Accordingly, the drop in GDP and the increase in public debt are slower in the nonlinear case. However, introducing the nonlinear risk premiums does not change the overall negative effects of the endogenously triggered haircuts.

6 Sensitivity analysis

In this section we run a sensitivity analysis regarding different weights of the objective variables. We consider all target state variables, namely inflation (π), output gap (y) and public debt (D). For each variable we run two separate experiments by increasing (+) or decreasing (−) the weight of the respective variable in order to study the shift

Table 7 Robustness experiments

Experiment	Target variable	Factor	New weights
'π_+'	π_i, π_E	2	$\alpha_{\pi_i} = 1, \alpha_{\pi_E} = 4$
'π_-'	π_i, π_E	0.5	$\alpha_{\pi_i} = 0.25, \alpha_{\pi_E} = 1$
'y_+'	y_i, y_E	2	$\alpha_{y_i} = 2, \alpha_{y_E} = 1$
'y_-'	y_i, y_E	0.5	$\alpha_{y_i} = 0.5, \alpha_{y_E} = 0.25$
'D_+'	D_i	10	$\alpha_{D_1} = 10^{-1}, \alpha_{D_2} = 10^{-3}$
'D_-'	D_i	0.1	$\alpha_{D_1} = 10^{-3}, \alpha_{D_2} = 10^{-5}$

Table 8 Objective function values

	CB			C1			
	y_E	π_E	RE_E	y_1	π_1	D_1	g_1
'base'	40.3	50.0	14.5	64.8	9.9	107.4	15.2
'π_+'	33.7	70.0	26.8	52.0	13.0	86.1	9.8
'π_-'	45.4	30.4	8.1	74.5	6.2	131.9	19.6
'y_+'	63.1	41.4	18.8	107.9	8.6	194.7	24.7
'y_-'	23.5	55.6	13.2	36.6	10.8	68.5	12.5
'D_+'	41.3	47.7	27.4	64.9	9.1	54.7	19.3
'D_-'	33.0	36.0	7.3	47.5	5.8	41.9	4.1

	C2				$\sum$			
	y_2	π_2	D_2	g_2	CB	$C1$	$C2$	$CB + C1 + C2$
'base'	121.1	17.3	2.7	28.0	104.9	197.3	169.1	471.3
'π_+'	107.4	25.9	2.5	30.2	130.6	161.0	166.0	457.5
'π_-'	131.7	10.1	2.6	26.7	83.9	232.1	171,1	487.1
'y_+'	177.0	13.5	2.7	73.5	123.3	335.9	266.8	725.9
'y_-'	72.8	19.4	2.3	10.0	92.3	128.5	104.5	325.3
'D_+'	128.0	17.2	19.3	14.1	116.5	148.0	178.5	442.9
'D_-'	112.1	15.3	0.3	31.5	76.3	99.3	159.2	334.9

caused by a stronger/weaker focus on these target values. In addition we examine the consequences of the fiscal policy maker emphasizing/understating the reduction of the debt ratio. Table 7 contains the detailed descriptions of the conducted experiments as compared to the baseline scenario given by the weights in Table 4.

In the Table 8 we present the detailed results for the six experiments and the baseline scenario. We calculate the individual part for each target variable which it adds to the accumulated objective function as given by Eqs. 10 and 11 . All results are given for the Nash equilibrium solutions.

Table 8 helps to understand how the shocks on the weights influence the objective function values and indicates the sensitivity of the system in this regard. As an example we compare the first two rows of the first column, namely values 40.3 and 33.7. The value 40.3 is the part of the accumulated objective function (loss function) of the central bank (CB) triggered by the variable union-wide output gap (y_E). If we increase the weight (importance) of the target variable inflation (π_+) then the loss associated with the union-wide output gap drops down to 33.7.

The last part of the Table 8 (the lower right corner) gives summarized effects of the sensitivity shock.

7 Concluding remarks

We apply a dynamic game analysis to the MUMOD1 model, a simple macroeconomic model of a monetary union consisting of two countries (two blocs) and a central bank. The monetary union is assumed to be asymmetric in the sense of consisting of a "core" with less initial public debt and a "periphery" with higher initial public debt. We analyze the effects of multiple endogenously realized haircuts, which can be regarded as consequences of a negative demand shock and resulting increases in public debt. We run two experiments with different thresholds at which a haircut is triggered and compare their results among others to a baseline solution without haircuts.

In the main scenario with endogenous haircuts we assume as threshold the public debt in the periphery to be above 150% of GDP. This scenario results in four haircuts in the case of the Pareto solution and in five haircuts in the case of the Nash feedback solution. The frequency of the haircuts is increasing and the public debt of the core is growing even above the one of the periphery, which indicates that such a scenario is unsustainable and politically non-affordable. The main reason for this development is the increase in the risk premium for the periphery's interest rate after each haircut, giving rise to a positive (adverse) feedback to its public debt, which is aggravated by the moral hazard problem for the periphery's government.

In a few alternative scenarios we first allow for two endogenous haircuts triggered first at 150% of GDP and later at 200% of GDP level of the public debt in the periphery bloc; for nonlinear risk premiums for high public debt; and for higher/lower weights of the target variables in the objective function. In all of these simulation experiments, the resulting scenarios with the haircuts are unsustainable. Summarizing we conclude that the suggested alternative of haircuts in periods of high debt can be counterproductive for both the core and the periphery under our assumptions.

Acknowledgements Open access funding provided by University of Klagenfurt. Earlier versions of this paper were presented at the 16th International Symposium on Dynamic Games and Applications, Amsterdam, 9-12 July 2014, the CESifo Area Conference 2016 on Macro, Money and International Finance, Munich, 26-27 February 2016, and the 21st International Conference on Macroeconomic Analysis and International Finance, Rethymno, Greece, 25-27 May 2017. We are grateful to discussants at these conferences and two anonymous referees for helpful suggestions for improvement. The usual disclaimer applies.

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