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ERRATUM

Erratum to: Dynamics and spectrum of the Cesàro operator on $C^\infty(\mathbb{R}_+)$

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The statement of Theorem 1.2 in the original article is incorrect and should read as follows. The notation is from the original article.

Theorem 1.2 *The spectra of the Cesàro operator* C *acting on the Fréchet space* $C^\infty(\mathbb{R}_+)$ *are given by*

$$\sigma_{pt}(\mathsf{C}) = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$$

and

$$\sigma(\mathsf{C}) = \sigma^*(\mathsf{C}) = \sigma_{pt}(\mathsf{C}).$$

The proof of Theorem 1.2 is based on Proposition 3.2 in the original article, which is where the error occurs. The correct statement of this result is the following one.

The online version of the original article can be found under doi:10.1007/s00605-015-0863-z.

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With this new version of Proposition 3.2 (note that only (0.1) changes) the proof of Theorem 1.2 above proceeds as in the original article.

Proposition 3.2 *The family $(T(t))_{t\in\mathbb{R}}$ is a uniformly continuous, locally equicontinuous C_0-group on $C^\infty(\mathbb{R}_+)$. The infinitesimal generator A of $(T(t))_{t\in\mathbb{R}}$ is the continuous, everywhere defined linear operator*

$$(Af)(x) := -xf'(x), \quad x \in \mathbb{R}_+,\ f \in C^\infty(\mathbb{R}_+).$$

Moreover, $(T(t))_{t\geq 0}$ is an equicontinuous C_0-semigroup on $C^\infty(\mathbb{R}_+)$. Concerning the spectra of A, it is the case that

$$\sigma_{pt}(A) = \{-n : n \in \mathbb{N}_0\}$$

and that

$$\sigma(A) = \sigma^*(A) = \sigma_{pt}(A). \tag{0.1}$$

Concerning the proof of Proposition 3.2 above, the spectrum of A can be calculated directly as follows. Define $T_m g(x) := \sum_{j=0}^m \frac{g^{(j)}(0)}{j!}x^j$, for $m \in \mathbb{N}_0 := \{0\} \cup \mathbb{N}$, i.e., the mth Taylor polynomial, for any $g \in C^\infty(\mathbb{R}_+)$. Fix $\lambda \in \mathbb{C}\backslash\{0, -1, -2, \ldots\}$ and select $n \in \mathbb{N}$ such that $\mathrm{Re}\lambda > -n$. The resolvent operator of A at λ is then given by

$$\begin{aligned} R(\lambda, A)f(x) &= x^n \int_0^1 \frac{f(xy) - T_{n-1}f(xy)}{(xy)^n} y^{n+\lambda-1}\, dy + \sum_{j=0}^{n-1} \frac{f^{(j)}(0)}{j!} \frac{x^j}{(\lambda + j)} \\ &=: H_n(\lambda)f(x) + K_n(\lambda)f(x), \end{aligned} \tag{0.2}$$

for every $f \in C^\infty(\mathbb{R}_+)$ and $x \in \mathbb{R}_+$. Indeed, direct calculation shows that $R(\lambda, A)(\lambda I - A)f = If = (\lambda I - A)R(\lambda, A)f$ for each $f(x) = x^k$, $k \in \mathbb{N}_0$, from which the claim follows provided that the continuity of $R(\lambda, A)$ is established. For this we proceed as follows.

Select $\varepsilon > 0$ such that if $\mu \in \mathbb{C}$ satisfies $|\mu - \lambda| < \varepsilon$, then $\mathrm{Re}\mu > (\delta - n)$ and $|\mu + j| > \delta$ for each $j \in \mathbb{N}_0$ and some $\delta > 0$. Since $(K_n(\mu)f)^{(m)}(x) = 0$, for $m \geq n$ and $x \in \mathbb{R}_+$, it follows that

$$(K_n(\mu)f)^{(m)}(x) = \sum_{r=m}^{n-1} \frac{f^{(r)}(0)x^{r-m}}{(r-m)!(\mu + r)}, \quad f \in C^\infty(\mathbb{R}_+), \quad 0 \leq m < n.$$

It is then routine to show that $\{K_n(\mu)f : |\mu-\lambda| < \varepsilon\}$ is a bounded subset of $C^\infty(\mathbb{R}_+)$. Accordingly, $\{K_n(\mu) : |\mu - \lambda| < \varepsilon\}$ is an equicontinuous set in $\mathcal{L}(C^\infty(\mathbb{R}_+))$.

The equicontinuity of $\{H_n(\mu) : |\mu - \lambda| < \varepsilon\}$ is more involved. One first treats $n = 1$, in which case $(H_1(\mu)f)^{(m)}(x)$ equals

$$m\int_0^1 y^{m+\mu-1} \left(\mathsf{C}(f')\right)^{(m-1)}(xy)\, dy + x\int_0^1 y^{m+\mu} \left(\mathsf{C}(f')\right)^{(m)}(xy)\, dy$$

for $x \in \mathbb{R}_+$ and $m \in \mathbb{N}_0$. This formula leads to the estimates needed to show that $\{H_1(\mu)f : |\mu - \lambda| < \varepsilon\}$ is a bounded set in $C^\infty(\mathbb{R}_+)$. Now fix $n \geq 2$. It follows from (0.2) that

$$(H_n(\mu)f)^{(m)}(x) = \int_0^1 \left(f^{(m)}(xy) - \sum_{j=m}^{n-1} \frac{f^{(j)}(0)}{(j-m)!}(xy)^{j-m} \right) y^{m+\mu-1}\, dy,$$

for $x \in \mathbb{R}_+$ and $m = 1, \ldots, (n-2)$, and that

$$(H_n(\mu)f)^{(n-1)}(x) = H_1(n+\mu-1) f^{(n-1)}(x), \quad x \in \mathbb{R}_+,$$

with $(n + \mu - 1) \in (-1, 0]$. Via these identities one can deduce the estimates needed to verify that $\{H_n(\mu)f : |\mu - \lambda| < \varepsilon\}$ is a bounded subset of $C^\infty(\mathbb{R}_+)$ for each $f \in C^\infty(\mathbb{R}_+)$.

The authors thank Prof. P. Domanski for pointing out the error. He also suggested a formula needed to correctly calculate $R(\lambda, A)$, as above. Further information concerning the operator $P(xd/dx)$ acting in the space $C^\infty(\mathbb{R}_+)$, for $P(z)$ a polynomial, can be found in [1].

Reference

1. Domański, P., Langenbruch, M.: Surjectivity of Euler type differential operators on spaces of smooth functions. Poznań (2016, Preprint)