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ERRATUM

Erratum to: Multivariate spectral multipliers for tensor product orthogonal expansions

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In order for the main theorem of the original paper to be true one needs the additional assumption on L^p contractivity of the heat semigroups of the investigated operators. We need to assume that for each $n = 1, \ldots, d$ and all $p \in [1, \infty]$,

$$\|e^{-t_n \mathcal{L}_n} f\|_{L^p(A_n,\nu_n)} \leq \|f\|_{L^p(A_n,\nu_n)}, \quad t_n > 0, \quad f \in L^p(A_n, \nu_n) \cap L^2(A_n, \nu_n). \tag{1}$$

Since (1) implies item (ii) of Theorem 2.3, to draw the conclusion of the main theorem (Theorem 2.3) of the original paper (i.e. the L^p boundedness of the operator $m(\mathcal{L})$), it now suffices to assume only item (i),

$$\int_{\mathbb{R}^d} \sup_{T \in (0,\infty)^d} |\mathcal{M}(m_{N,T})(u_1, \ldots, u_d)| \, \|\mathcal{L}^{iu_1,\ldots,iu_d}\|_{p \to p} \, du < \infty.$$

We kindly refer the reader to the original paper for the definitions of the quantities considered above. Condition (1) is not a serious restriction and it is satisfied in case of all applications presented in the original paper. We need to include (1) in order to obtain Theorem 2.4 for $1 < p < 2$, i.e. the L^p boundedness of the g-function g_N

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for $1 < p < 2$. The proof of Theorem 2.4 from the original paper, is incorrect for $1 < p < 2$. The correction we present here is a slight modification of the proof of [2, Theorem 1.5 ii)].

For the sake of simplicity we focus on $d = 2$. The notations we use are from the original paper. Let $N \in \mathbb{N}_+$ be fixed. Take a smooth function h on $\mathbb{R}$, supported in $[-1, 1]$, and such that

$$\sum_{l\in\mathbb{Z}} h(x-l) = 1, \quad x \in \mathbb{R}.$$

Then we set $h_k(x) = h_{k_1}(x_1)h_{k_2}(x_2) = h(x_1 - k_1)h(x_2 - k_2)$. Next, for each $j \in \mathbb{Z}^2$ we define the functions

$$\begin{aligned} b_{j,k}(\xi) &= \int_{\mathbb{R}^2} h_k(u)\Gamma(N - iu_1)\Gamma(N - iu_2)e^{-ij_1u_1}e^{-ij_2u_2}\xi_1^{iu_1}\xi_2^{iu_2}\,du \\ &= \int_{\mathbb{R}} h_{k_1}(u_1)\Gamma(N - iu_1)e^{-ij_1u_1}\xi_1^{iu_1}\,du_1 \\ &\quad \times \int_{\mathbb{R}} h_{k_2}(u_2)\Gamma(N - iu_2)e^{-ij_2u_2}\xi_2^{iu_2}\,du_2, \quad \xi_1, \xi_2 \in \Sigma_{\pi/2}, \end{aligned}$$

where $\Sigma_{\pi/2} = \{z \in \mathbb{C} : \mathrm{Re}(z) > 0\}$, is the right complex half plane. Proceeding as in [2, p. 2207] we easily see that

$$\|g_N(f)\|_p \le \sum_{k\in\mathbb{Z}^2} \left\| \left(\sum_{j\in\mathbb{Z}^2} |b_{j,k}(L_1, L_2)f|^2 \right)^{1/2} \right\|_p .$$

Then from a two-dimensional variant of [2, Lemma 1.3] (which is easily proved by using a two-dimensional Khinchine inequality) and the product structure of $b_{j,k}$ it follows that

$$\|g_N(f)\|_p \le \sum_{k\in\mathbb{Z}^2} \sup_{|a_{j_1}|\le 1, |a_{j_2}|\le 1} \left\| \left(\sum_{j_1\in\mathbb{Z}} a_{j_1} b_{j_1,k_1}(L_1) \right) \left(\sum_{j_2\in\mathbb{Z}} a_{j_2} b_{j_2,k_2}(L_2) \right) f \right\|_p .$$

Using the latter inequality and [2, Lemma 1.4] we easily adjust the final steps of [2, Theorem 1.5 ii)] to our situation, obtaining the desired bound

$$\|g_N(f)\|_p \le C_p \|f\|_p, \quad 1 < p < 2.$$

Note that assumption (1) is needed to justify the use of the crucial transference result of [1, Theorem 1 and Lemma 1.4].

References

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