Rock Mech Rock Eng (2012) 45:981–988
DOI 10.1007/s00603-012-0276-4

ISRM SUGGESTED METHOD

The Hoek–Brown Failure Criterion

Erik Eberhardt

Published online: 3 July 2012

List of Symbols

σ_1	Major principal stress
σ_3	Minor principal stress
C_o	Uniaxial compressive strength
m_i	Hoek–Brown material constant (intact rock)
m_b	Hoek–Brown material constant (rock mass)
s	Hoek–Brown material constant
a	Hoek–Brown material constant
GSI	Geological Strength Index
D	Disturbance factor
T_o	Uniaxial tensile strength
σ'_{3max}	Upper limit of confining stress
r^2	Coefficient of determination

1 Description

The Hoek–Brown failure criterion is an empirically derived relationship used to describe a non-linear increase in peak strength of isotropic rock with increasing confining stress. Hoek–Brown follow a non-linear, parabolic form that distinguishes it from the linear Mohr–Coulomb failure criterion. The criterion includes companion procedures developed to provide a practical means to estimate rock mass strength from laboratory test values and field observations. Hoek–Brown assumes independence of the intermediate principal stress.

E. Eberhardt (✉)
Geological Engineering, EOAS, University of British Columbia, Vancouver, Canada
e-mail: erik@eos.ubc.ca

2 Background and Formulation

The Hoek–Brown criterion was developed as a means to estimate rock mass strength by scaling the relationship derived according to the geological conditions present. The criterion was conceived based on Hoek's (1968) experiences with brittle rock failure and his use of a parabolic Mohr envelope derived from Griffith's crack theory (Griffith 1920, 1924) to define the relationship between shear and normal stress at fracture initiation. By associating fracture initiation with fracture propagation and rock failure, Hoek and Brown (1980) proceeded through trial and error to fit a variety of parabolic curves to triaxial test data to derive their criterion. Accordingly, the Hoek–Brown criterion is empirical with no fundamental relationship between the constants included in the criterion and any physical characteristics of the rock (Hoek 1983).

The original non-linear Hoek–Brown failure criterion for intact rock (Hoek and Brown 1980) was introduced as:

$$\sigma_1 = \sigma_3 + \sqrt{m\, C_o \sigma_3 + s\, C_o^2} \tag{1}$$

where σ_1 and σ_3 are the major and minor principal stresses at failure, C_o is the uniaxial compressive strength of the intact rock, and m and s are dimensionless empirical constants. The criterion is non-linear in the meridian plane (defined as the plane which passes through the hydrostatic axis and cuts the failure envelope) and linear in appearance in the π-plane (defined as the plane perpendicular to hydrostatic axis and cuts the failure envelope; see Fig. 1). The criterion is also linear in the biaxial ($\sigma_1 - \sigma_2$) plane (e.g., see Fig. 6).

The non-linear form of the Hoek–Brown criterion distinguishes it from the linear Mohr–Coulomb failure criterion (Fig. 1a). In terms of equivalencies, the parameter m is

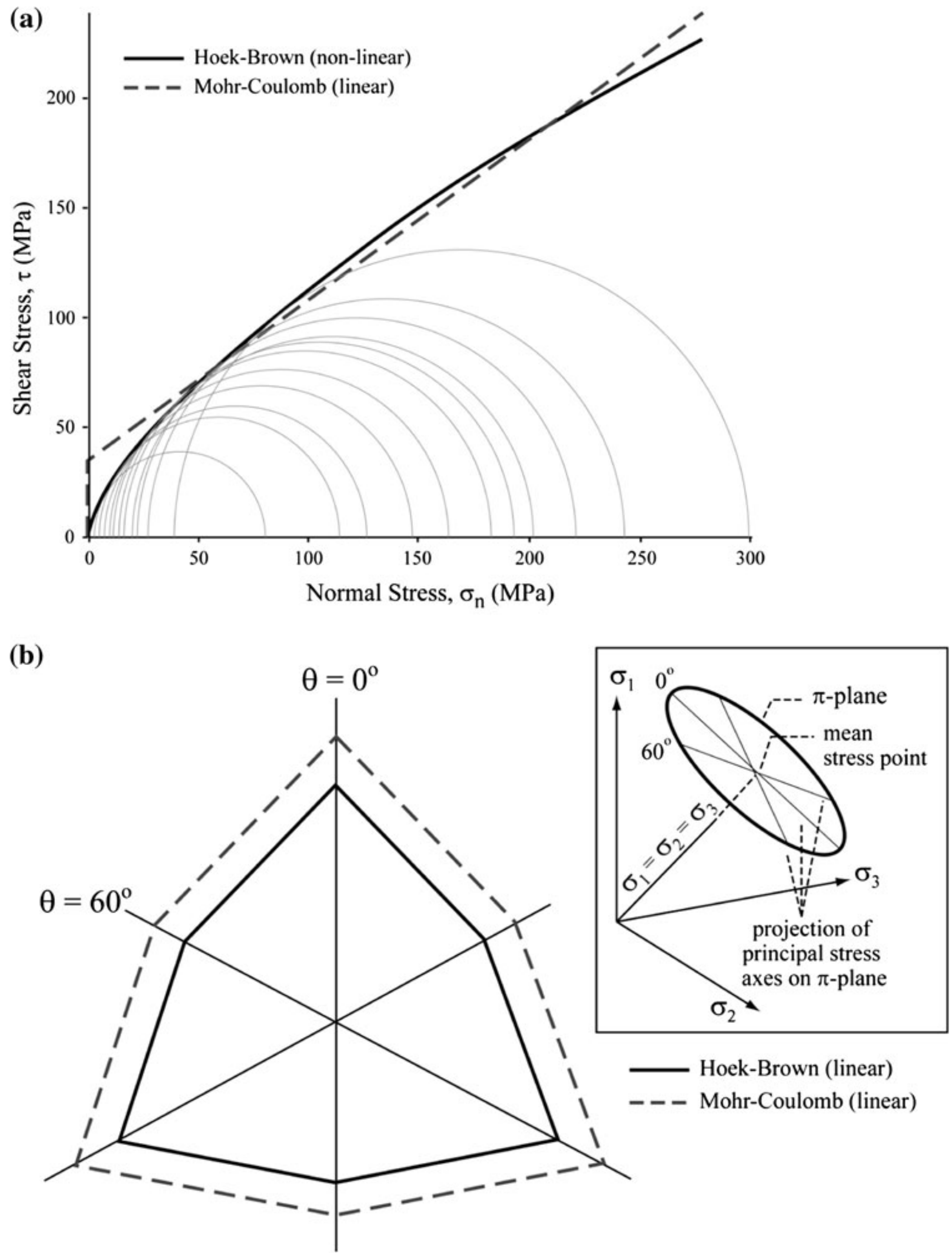

Fig. 1 **a** Comparison of the linear Mohr–Coulomb and non-linear Hoek–Brown failure envelopes plotted against triaxial test data for intact rock and **b** similar comparison but projected onto the π-plane. *Inset* shows definition of π-plane (i.e., plane perpendicular to hydrostatic stress axis)

analogous to the frictional strength of the rock and *s*, which is a measure of how fractured the rock is, is related to the rock mass cohesion. Large values of *m* give steeply inclined Mohr envelopes and high instantaneous friction angles at low effective normal stresses, as is generally found for strong brittle rocks; lower *m* values give lower instantaneous friction angles as observed for more ductile rocks (Hoek 1983). This is demonstrated in Fig. 2. The constant *s* varies as a function of how fractured the rock is from a maximum value of 1 for intact rock to zero for heavily fractured rock where the tensile strength has been reduced to zero.

As can be seen in Eq. (1), the Hoek–Brown criterion assumes that rock failure is controlled by the major and minor principal stress, σ_1 and σ_3; the intermediate principal stress, σ_2, does not appear in the equations except insofar as $\sigma_2 = \sigma_3$ (i.e., conventional triaxial compression test) or $\sigma_2 = \sigma_1$. This assumption is later discussed in more detail in the treatment of the advantages and limitations of the criterion.

3 Rock Mass Properties

As the primary focus of this Working Group report is failure criterion for intact rock, the application of Hoek–Brown to rock mass strength is only briefly discussed here.

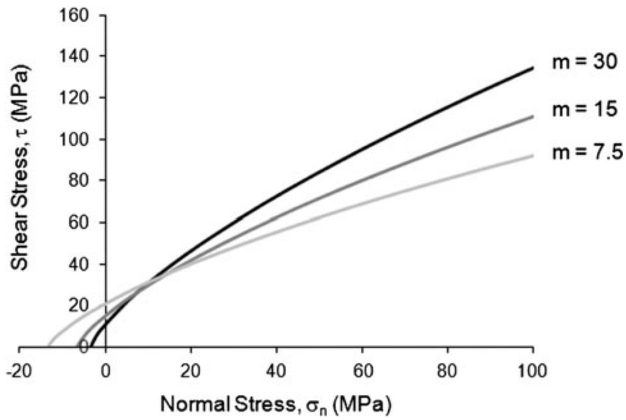

Fig. 2 Change in Hoek–Brown failure envelope as a function of m plotted in shear versus normal stress space. Note how larger values of m give more steeply inclined Mohr envelopes and higher equivalent friction angles than lower m values

By adjusting the m and s parameters according to the rock mass conditions, the criterion can be applied to the estimation of rock mass strength properties. This requires the assumption that any fractures present are numerous enough that the overall strength behavior has no preferred failure direction; i.e., the rock mass responds as an isotropic, equivalent continuum.

As an empirical criterion, the Hoek–Brown criterion has been updated several times in response to experience gained with its use and to address certain practical limitations (Hoek and Brown 1988; Hoek et al. 1992, 1995, 2002). These primarily involve adjustments to improve the estimate of rock mass strength. One key update was the reporting of the 'generalised' form of the criterion (Hoek et al. 1995):

$$\sigma_1' = \sigma_3' + C_o\left(m_b\frac{\sigma_3'}{C_o} + s\right)^a. \tag{2}$$

The term m_b was introduced for broken rock. The original m_i value had been reassessed and found to depend upon the mineralogy, composition and grain size of the intact rock (Hoek et al. 1992). The exponential term a was added to address the system's bias towards hard rock and to better account for poorer quality rock masses by enabling the curvature of the failure envelope to be adjusted, particularly under very low normal stresses (Hoek et al. 1992). The Geological Strength Index (GSI) was subsequently introduced together with several relationships relating m_b, s and a, with the overall structure of the rock mass (or blockiness) and surface conditions of the discontinuities (Hoek et al. 1995). The principal stress terms in the original equation had been replaced earlier with effective principal stress terms as it was assumed that criterion was valid for effective stress conditions (Hoek 1983).

In 2002, Hoek et al. (2002) re-examined the relationships between the GSI and m_b, s and a, and introduced a new factor D to account for near surface blast damage and stress relaxation. This edition of the criterion represents the last major revision of the Hoek–Brown system. The rock mass scaling relationships for m_b, s and a were reported as:

$$m_b = m_i \exp\left(\frac{\mathrm{GSI} - 100}{28 - 14\mathrm{D}}\right) \tag{3}$$

$$s = \exp\left(\frac{\mathrm{GSI} - 100}{9 - 3\mathrm{D}}\right) \tag{4}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-\frac{GSI}{15}} + e^{-\frac{20}{3}}\right). \tag{5}$$

From above, m_i is a curve fitting parameter derived from triaxial testing of intact rock. The parameter m_b is a reduced value of m_i, which accounts for the strength reducing effects of the rock mass conditions defined by GSI (Fig. 3). Adjustments of s and a are also done according to the GSI value. GSI is estimated from the chart of Marinos et al. (2005); Sönmez and Ulusay (2002) discuss the sensitivity of the Hoek–Brown strength envelope to GSI. Although relationships exist to convert RMR_{89} and Q to GSI (see Hoek et al. 1995), Hoek (2007) recommends that GSI be estimated directly by means of the charts published on its use.

For practicing engineers, the Hoek–Brown and GSI procedures (see Hoek et al. 2002) provide a straight forward means to scale laboratory test values to obtain isotropic rock mass properties. However, it must first be decided whether the representation of the engineered rock mass as an equivalent continuum is appropriate or not. The criterion should not be used where discontinuities have a significant influence on the mobilization of failure and failure kinematics, for example where the discontinuity spacing is large compared with the dimensions of the underground opening or when a rock slope is being analyzed and stability is more governed by the shear strength of individual discontinuities. Where the rock mass is more moderately to heavily jointed and the rock mass strength is approximately isotropic, then the GSI and Hoek–Brown treatment of the rock mass as an equivalent continuum are applicable.

Hoek (2007) recommends, where possible, the Hoek–Brown criterion be applied directly. However, given that many geotechnical design calculations are written for the Mohr–Coulomb failure criterion, it is often necessary to calculate equivalent rock mass cohesion, c, and friction angle, ϕ, values from the Hoek–Brown parameters. Moreover, most practitioners have an intuitive feel for the physical meanings of cohesion and friction, which is not the case for m_b, s and a. The quantitative conversion of Hoek–Brown to Mohr–Coulomb parameters is done by

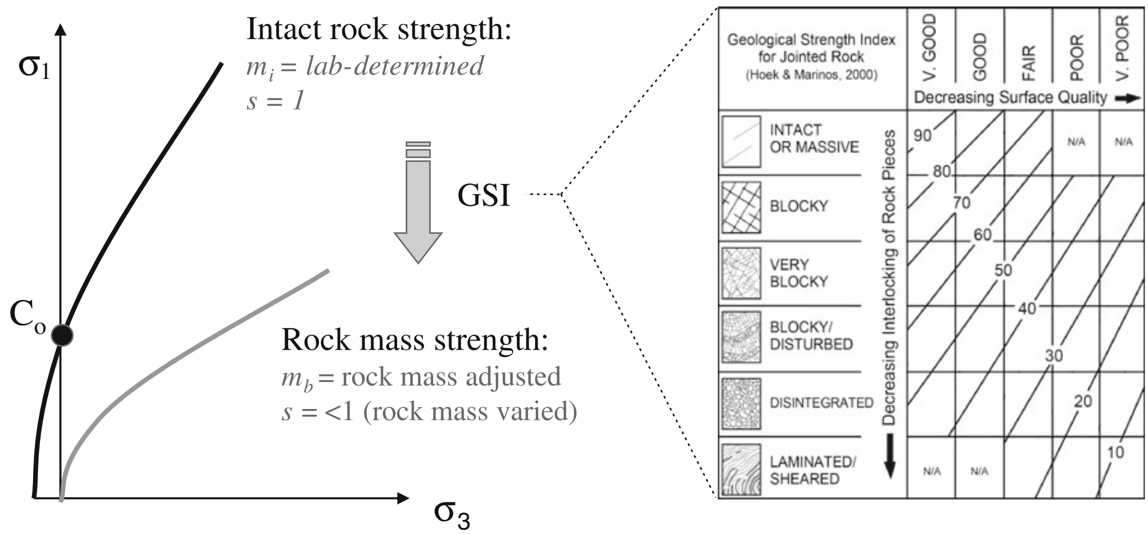

Fig. 3 Scaling of Hoek–Brown failure envelope for intact rock to that for rock mass strength. See Marinos et al. (2005) for full details on use of the GSI chart

fitting an average linear relationship to the non-linear Hoek–Brown envelope for a range of minor principal stress values defined by $T_o < \sigma_3 < \sigma'_{3\max}$ (Hoek et al. 2002). Note that the value of $\sigma'_{3\max}$, the upper limit of confining stress over which the relationship between the Hoek–Brown and Mohr–Coulomb criteria is considered, has to be determined for each individual case (Fig. 4). Brown (2008) warns against applying programs that calculate equivalent Mohr–Coulomb parameters too automatically without thinking clearly about the range of effective normal stress that applies to the case being considered. If high values of $\sigma'_{3\max}$ are used, then the equivalent effective cohesion value may be too high and the equivalent effective friction angle too low.

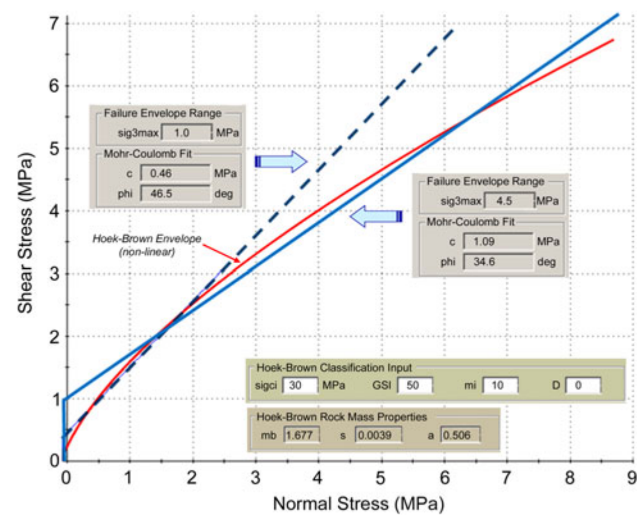

Fig. 4 Fitting of linear Mohr–Coulomb failure envelopes (*blue solid* and *dashed lines*) along two different stress ranges of a non-linear Hoek–Brown failure envelope (*red curve*). Note the change in equivalent cohesion and friction angle values for the two different stress ranges specified. Failure envelopes plotted using Rocscience's (2007) RocLab (color figure online)

4 Experimental Data on Intact Rock

There are several laboratory testing procedures from which the peak strength of intact rock can be measured. These include uniaxial compression, conventional triaxial compression ($\sigma_2 = \sigma_3$) and true triaxial compression. Empirical strength criteria have been developed based on fitting the best line or curve to these data. The accuracy of a criterion's fit to the data is generally evaluated based on the biaxial plane-stress condition (σ_1–σ_2 plane) and any meridian cross section $\left(\sqrt{2J_2} - I_1/\sqrt{3}\right.$ plane for $0° \leq \theta \leq 60°$), including the $\sigma_1 - \sqrt{2}\sigma_3$ plane (conventional triaxial test condition, where $\sigma_2 = \sigma_3$ or $\theta = 0°$).

In developing the Hoek–Brown criterion, Hoek and Brown (1980) analyzed published conventional triaxial test data for more than 14 intact rock types covering a range of igneous, sedimentary and metamorphic rocks, with peak strengths ranging from 40 MPa for a sandstone to 580 MPa for a chert. This analysis included multiple tests for the same rock type carried out in different laboratories and only considered data sets containing a minimum of five tests covering a range of confining stresses. The choice of a non-linear criterion was based on this review and the m_i parameter was derived from best-fit linear regression to these data. The coefficient of determination, r^2, for these fits ranged from 0.68 to 0.99, with most being >0.9.

Zhao (2000) compared Mohr–Coulomb and Hoek–Brown fits to experimental data from a series of dynamic

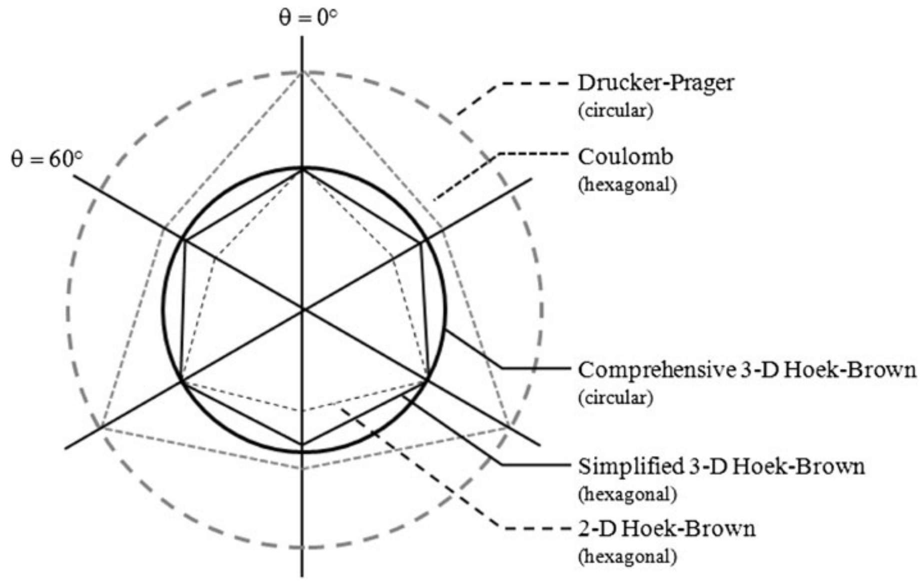

Fig. 5 π-Plane plot comparing Priest's (2005) comprehensive and simplified 3-D Hoek–Brown criteria relative to other commonly used criteria. See Fig. 1b inset for definition of the π-plane projection (modified after Priest 2005)

uniaxial and triaxial compression, uniaxial tension and unconfined shear tests performed on Bukit Timah granite from Singapore (average UCS approximately 190 MPa). This comparison showed that the intact rock strength under dynamic loads, at both low and high confining pressures, was better represented by the non-linear Hoek–Brown criterion. Similarly, Ghazvinian et al. (2008) found that the non-linear form of the Hoek–Brown criterion gave a better fit to their experimental data than the linear Mohr–Coulomb, in this case for weak marlstones (average UCS approximately 12 MPa).

Pariseau (2007) compared Mohr–Coulomb, Hoek–Brown and Drucker–Prager fits to triaxial experimental data of several intact rock types using the unconfined compressive and tensile strength intercepts as common reference points between the different criteria (it was assumed that the criteria are independent of the intermediate principal stress). Based on data from a sandstone, a high-strength norite, an Indiana limestone and a Dunham dolomite, the non-linear Hoek–Brown envelope provided a significantly better fit over the entire data range (i.e., low to high confining pressures) than Mohr–Coulomb and Drucker–Prager. Pariseau (2007) concluded, based on added comparisons involving other non-linear criteria, that a non-linear failure criterion is required to address the short comings of linear failure criteria.

A similar comparison was reported by Benz and Schwab (2008), assessing six different criteria: Mohr–Coulomb, Lade–Duncan, an approximation to Wiebols–Cook, Mogi, Hoek–Brown and a combined Hoek–Brown Matsuoka–Nakai criterion proposed by Benz et al. (2008), which accounts for the influence of the intermediate principal stress, σ_2. These criteria were fitted to true triaxial test data for eight different intact rocks taken from previously published studies: Dunham dolomite, Solnhofen limestone, Shirahama sandstone, Yuubari shale, KTB amphibolite, Mizuho trachyte, a dense marble and Westerly granite. Again, in each case, the non-linear Hoek–Brown envelope gave either an equal or better fit than the linear Mohr–Coulomb criterion. Comparisons between Hoek–Brown and the other criteria were variable, though in six out of the eight cases, a clear reduction in the misfit between criteria and data was found when the intermediate principal stress was considered in the failure criterion.

It should be emphasized that the relevance of these comparisons and the level of fit achieved are dependent, in part, on the confining stress range (i.e., regression range) and the coordinate system in which the data and criterion are compared (e.g., σ_1–σ_3 plane). Fitting of criteria near the origin of a normal stress–shear stress plot, including tensile strength, is typically more important for engineering excavations in rock; the closeness of fit in this region may thus be of more concern than that at high confining pressures.

5 Advantages and Limitations

The main advantages of the Hoek–Brown criterion are:

(a) It is non-linear in form (in the meridian plane), which agrees with experimental data over a range of confining stresses;
(b) It was developed through an extensive evaluation of laboratory test data covering a wide range of intact rock types;
(c) It provides a straight forward empirical means to estimate rock mass properties;

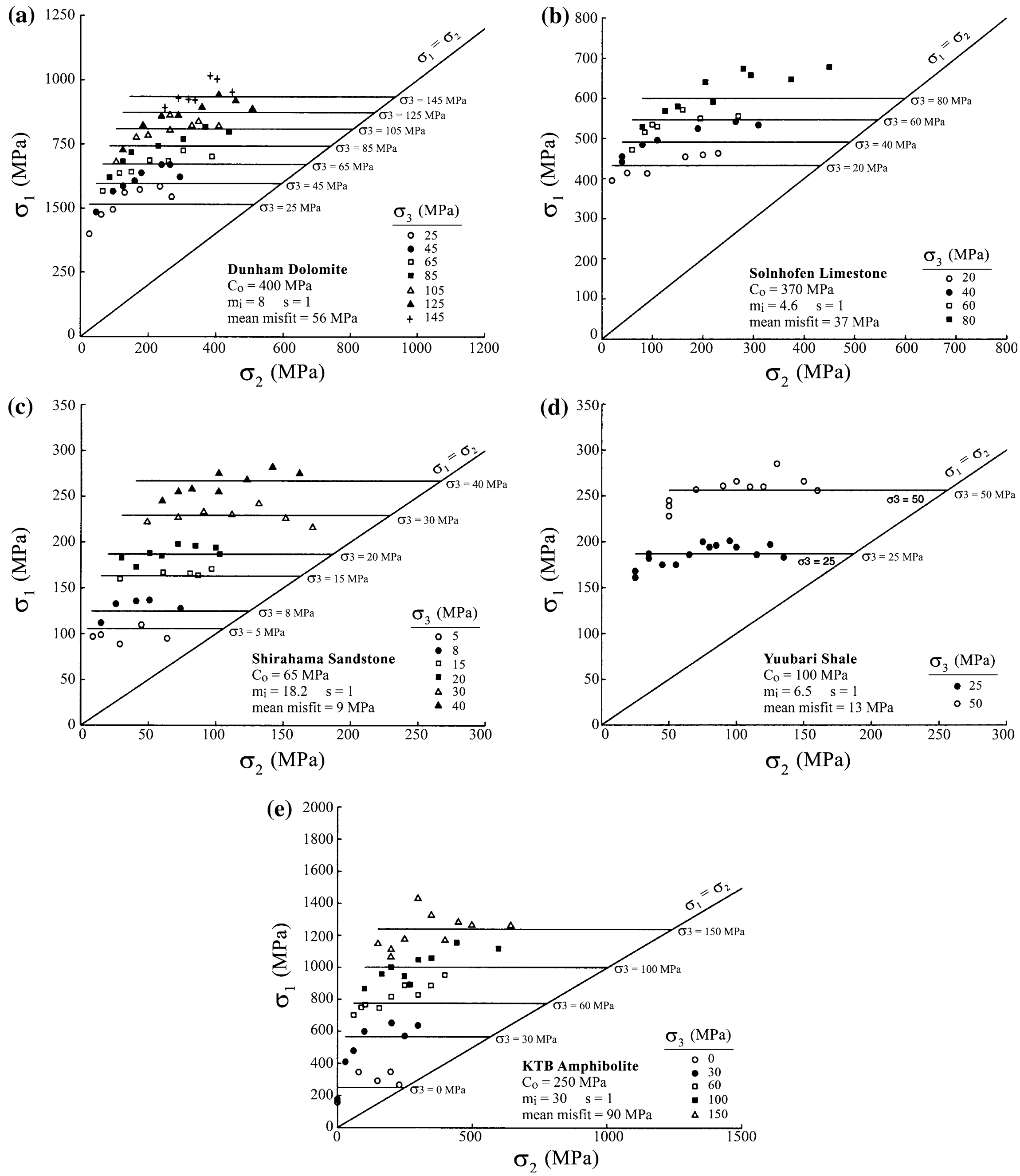

Fig. 6 Best-fit comparison of the Hoek–Brown criterion to true triaxial ($\sigma_1 > \sigma_2 > \sigma_3$) tests of intact rock for: **a** Dunham dolomite, **b** Solnhofen limestone, **c** Shirahama sandstone, **d** Yuubari shale, and **e** KTB amphibolite. The Hoek–Brown criterion is represented by *straight lines* in σ_1 versus σ_2 space, extending laterally from each σ_3 value (after Colmenares and Zoback 2002)

(d) There is almost three decades worth of experience with its use by practitioners on a variety of rock engineering projects.

Considerable progress has also been made in applying the Hoek–Brown criterion to the assessment and prediction of brittle fracture damage in overstressed massive rock. Martin et al. (1999) provide an empirical depth of spalling failure relationship using the Hoek–Brown criterion, setting $m = 0$ and $s = 0.11$. The fundamental assumption made by the authors is that the stress-controlled failure process around the tunnel is dominated by cohesion loss. Hence the m_b parameter, which can be equated to frictional strength, is set to zero. It should be emphasized that this treatment (i.e., $m = 0$) differs from that which would be used for an elasto-plastic yielding failure mechanism where the frictional strength component mobilizes and dominates

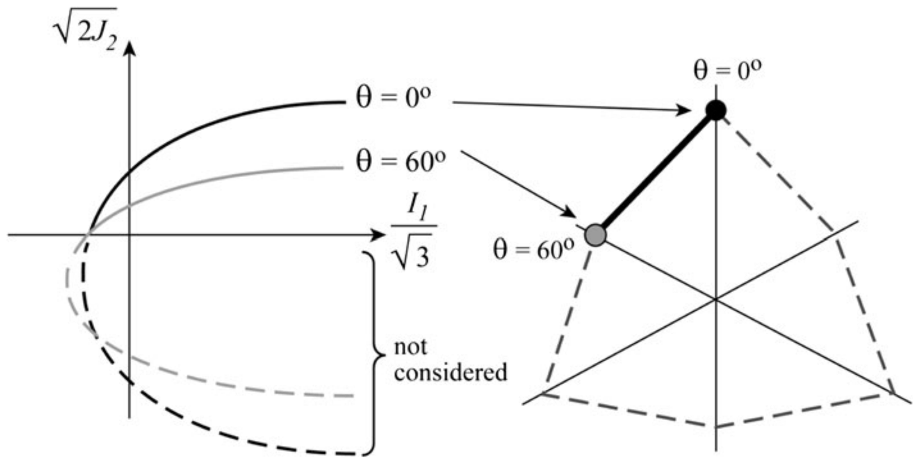

Fig. 7 Illustration of underlying assumption in the development of the Hoek–Brown criterion in which $\sigma_1 \geq \sigma_2 \geq \sigma_3$ (or $0° \leq \theta \leq 60°$ in the π-plane). This forces a positive mean shear stress. The Hoek–Brown criterion is a segment of the parabola which starts from the hydrostatic stress axis

the behavior of the rock mass, requiring the m value to be set to a typical value for the rock type in question. These findings and the empirical relationship suggested by Martin et al. (1999) have since been repeated and confirmed in other studies on tunnel stability in highly stressed rock (e.g., Kaiser et al. 2000; Diederichs et al. 2004). Diederichs (2007) also uses the Hoek–Brown relationship to develop a reliable procedure for modelling the depth and extent of brittle spalling for deep tunnels in blocky to massive rock (GSI > 65). His procedure introduces a bi-linear failure criterion that accounts for different stress thresholds under which brittle fractures initiate and propagate during spalling. Considering the influence of confinement on self-stabilization of the spalling process at some distance into the rock mass, this criterion captures the dependence of fracture propagation on confinement and can be incorporated into a non-elastic numerical model using modified Hoek–Brown parameters.

Limitations in the Hoek–Brown criterion have been documented through detailed discussions on the simplifying assumptions made in deriving the criterion (Hoek and Brown 1980; Hoek 1983; Brown 2008). One of the most important of these is the independency of the criterion from the intermediate principal stress, σ_2. Hoek and Brown (1980) justified this by pointing to triaxial extension and compression tests by Brace (1964) that showed no significant variation between results when $\sigma_2 = \sigma_3$ and $\sigma_2 = \sigma_1$. Brace concluded that σ_2 had a negligible influence on failure. True triaxial testing by others (for e.g., Mogi 1971) shows that a more pronounced influence of σ_2 was discounted as involving brittle/ductile transitions in the failure process.

Subsequent experimental studies have since suggested that the intermediate principal stress has a substantial influence on rock strength (e.g., Takahashi and Koide 1989; Colmenares and Zoback 2002; Haimson 2006). This has led to the development of several 3-D versions of the Hoek–Brown failure criterion (Pan and Hudson 1988; Priest 2005; Zhang and Zhu 2007; Zhang 2008; Melkoumian et al. 2009). Figure 5 compares the comprehensive and simplified 3-D Hoek–Brown envelopes developed by Priest (2005) to other commonly used criteria for a given hydrostatic stress. Melkoumian et al. (2009) explain that despite the capacity of the Hoek–Brown criterion for modelling a wide range of intact and fractured rock types, its use has not been widely adopted in the petroleum industry, partly because it does not take into account the intermediate principal stress. A stress state where the intermediate principal stress is substantially larger than the minor principal stress can occur adjacent to boreholes drilled for petroleum and gas extraction and thus the strength of the rock is higher than what the criterion predicts. Figure 6 compares the fit of the Hoek–Brown criterion to true triaxial test data for five different intact rock types as reported by Colmenares and Zoback (2002).

Another limitation of the Hoek–Brown criterion, as discussed by Pariseau (2007), is with respect to its mathematical characteristics. He noted that the parabola form of the criterion is not centered on the hydrostatic stress axis. However, this does not have any influence on the practical application of Hoek–Brown. The underlying assumption in the development of the Hoek–Brown criterion is $\sigma_1 \geq \sigma_2 \geq \sigma_3$ (or $0° \leq \theta \leq 60°$ in the π-plane), which implies a positive mean shear stress $\tau_m \geq 0$. Therefore, Hoek–Brown is actually a segment of the parabola in the meridian plane which starts from the hydrostatic stress axis, I_1 (Fig. 7).

6 Recommendations

As a peak strength criterion for intact rock, the Hoek–Brown criterion has the advantage of describing a nonlinear increase in strength with increasing confinement that agrees with extensive laboratory triaxial test data covering a wide range of intact rock types. Its use can be recommended for most rock types (igneous, sedimentary, metamorphic) under both low and high confining pressures. Similarly, its use can be recommended for problems involving a varying range of confining stress magnitudes (from low to very high confinement). Where rock mass strength is more appropriate, empirical procedures, which provide an important and straight forward means to estimate rock mass properties, are also available. These are not discussed in detail here as the scope of the Working Group's report is dedicated to reporting on failure criteria for isotropic intact rock (see WG Introduction).

Acknowledgments The author would like to thank Dr. Evert Hoek, Masoud Rahjoo and Prof. Bill Pariseau, together with the reviewers of the Working Group's report for their constructive review comments and suggestions.

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