Calc. Var. (2013) 47:433–435
DOI 10.1007/s00526-012-0565-5

Calculus of Variations

ERRATUM

Erratum to: Partial regularity of stable solutions to the Emden equation

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Published online: 17 November 2012

Erratum to: Calc. Var.
DOI 10.1007/s00526-011-0446-3

Recently the authors of [1] pointed out that the proof of Lemma 3.2 in [2] is incorrect.[1] The following is the corrected formulation and proof.

Lemma 0.1 $\exists \varepsilon_0 > 0, \theta \in (0, 1)$, *which depend only on the dimension n, such that for a stable solution u of* $-\Delta u = e^u$ *in* $B_2(0)$, *if*

$$2^{2-n} \int\limits_{B_2(0)} e^u = \varepsilon, \tag{0.1}$$

where $\varepsilon \le \varepsilon_0$, *then*

$$\theta^{2-n} \int\limits_{B_\theta(0)} e^u \le \frac{1}{2}\varepsilon. \tag{0.2}$$

We can iterate (0.2) to get a bound of $\|e^u\|_{M^{n/2+\delta}(B_\theta(0))}$ for some $\delta > 0$ (depending only on θ). Then we get the smoothness of u in $B_{\theta/2}(0)$ as before. Note that, by the Hölder inequality, the assumption of Theorem 3.1 in [2] implies that u satisfies (0.1).

[1] I would like to thank Louis Dupaigne for explaining their results to me. Their method can be used to give another proof of the partial regularity result in [2].

The online version of the original article can be found under doi:10.1007/s00526-011-0446-3.

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Proof Take a function $\eta \in C_0^\infty(B_2(0))$, such that $0 \le \eta \le 1$, and $\eta \equiv 1$ in $B_1(0)$. By Farina's estimate (i.e. taking $\alpha = 1/2$ in the estimate Eq. (2.1) in [2]), we have

$$\int e^{2u}\eta^2 \le C \int e^u(|\nabla \eta|^2 + |\Delta \eta^2|).$$

Hence $\|e^u\|_{L^2(B_1(0))} \le C\varepsilon^{1/2}$.

Take the decomposition $u = v + w$ in $B_1(0)$, where v satisfies

$$\begin{cases} -\Delta v = e^u, & \text{in } B_1(0), \\ v = 0, & \text{on } \partial B_1(0). \end{cases} \tag{0.3}$$

Since $\|v\|_{L^1(B_1(0))} \le C\varepsilon$ (cf. Lemma 3.4 in [2]) and $\|e^u\|_{L^2(B_1(0))} \le C\varepsilon^{1/2}$, by the global $W^{2,2}$ estimate, $\|v\|_{W^{2,2}(B_1(0))} \le C\varepsilon^{1/2}$. Then by the Sobolev embedding theorem, $\|v\|_{L^{\frac{2n}{n-4}}(B_1(0))} \le C\varepsilon^{1/2}$. By interpolation between L^q spaces, we obtain

$$\|v\|_{L^2(B_1(0))} \le \|v\|_{L^1(B_1(0))}^{\frac{4}{n+4}} \|v\|_{L^{\frac{2n}{n-4}}(B_1(0))}^{\frac{n}{n+4}} \le C\varepsilon^\alpha,$$

where $\alpha = \frac{n+8}{2n+8} > 1/2$. Then by interpolation between Sobolev spaces, we get

$$\|\nabla v\|_{L^2(B_1(0))} \le \varepsilon^{\frac{1}{4}(\alpha - \frac{1}{2})} \|\nabla^2 v\|_{L^2(B_1(0))} + C\varepsilon^{-\frac{1}{4}(\alpha-\frac{1}{2})}\|v\|_{L^2(B_1(0))} \le C\varepsilon^\beta, \tag{0.4}$$

where $\beta > 1/2$ depends only on n. From this we get

$$\int_{B_1(0)} ve^u = \int_{B_1(0)} -v\Delta v = \int_{B_1(0)} |\nabla v|^2 \le C\varepsilon^{2\beta}. \tag{0.5}$$

We decompose the estimate of $r^{2-n}\int_{B_r(0)} e^u$ into two parts: $\{v \le \varepsilon^\gamma\}$ and $\{v > \varepsilon^\gamma\}$, where $\gamma = \frac{1}{2}(2\beta - 1) > 0$.

The first part can be estimated as in [2] (by using the fact that e^w is subharmonic and $w < u$ in $B_1(0)$, see Lemma 3.3 in [2])

$$\begin{aligned} r^{2-n} \int_{B_r(0)\cap\{v\le\varepsilon^\gamma\}} e^u \le r^{2-n} \int_{B_r(0)\cap\{v\le\varepsilon^\gamma\}} e^{\varepsilon^\gamma} e^w &\le r^2 e^{\varepsilon^\gamma} r^{-n} \int_{B_r(0)} e^w \\ &\le r^2 e^{\varepsilon^\gamma} \int_{B_1(0)} e^u \\ &\le Cr^2\varepsilon. \end{aligned}$$

The second part can be estimated using (0.5)

$$r^{2-n} \int_{B_r(0)\cap\{v>\varepsilon^\gamma\}} e^u \le \varepsilon^{-\gamma} r^{2-n} \int_{B_r(0)} ve^u \le Cr^{2-n}\varepsilon^{2\beta-\gamma}.$$

Putting these together we get

$$r^{2-n} \int_{B_r(0)} e^u \le Cr^2\varepsilon + Cr^{2-n}\varepsilon^{2\beta-\gamma}.$$

Note that $2\beta - \gamma > 1$. We can first choose $r = \theta$ small enough, then ε_0 small enough, such that for any $\varepsilon < \varepsilon_0$

$$C\theta^2\varepsilon + C\theta^{2-n}\varepsilon^{2\beta-\gamma} \leq \frac{1}{2}\varepsilon.$$

By this choice we get (0.2). □

References

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2. Wang, K.: Partial regularity of stable solutions to the Emden equation. Calc. Var. **44**, 601–610 (2012)