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ERRATUM

Erratum to: On the weak L^p Hodge decomposition and Beurling–Ahlfors transforms on complete Riemannian manifolds

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In this Erratum, we correct an error in the representation formulas of the Beurling–Ahlfors transforms and a gap in the original proof of the L^p-norm estimates of the Beurling–Ahlfors transforms obtained in original article. The original estimates in the main theorems proved in original article remain valid.

As pointed out by Bañuelos and Baudoin [1], various formulas in original article of the form $\int_0^T e^{a(t-T)} M_T M_t^{-1} \alpha_t dX_t$ need to be rewritten as $e^{-aT} M_T \int_0^T e^{at} M_t^{-1} \alpha_t dX_t$ since M_T is not $\mathcal{F}_t = \sigma(X_s, s \le t)$-measurable. In view of this, the correct representation formula for $S^T_{A_i}$ in the Beurling–Ahlfors transforms on k-forms over complete Riemannian manifolds should be given by

$$\langle d^* d(a + \Box)^{-1}\omega, \eta\rangle = 2 \lim_{T\to\infty} \int\limits_M \langle S^T_{A_1}\omega, \eta\rangle\, dx, \tag{1}$$

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where, for a.s. $x \in M$,

$$S_{A_i}^T \omega(x) = E\left[M_T e^{-aT} \int_0^T e^{at} M_t^{-1} A_i \nabla \omega_a(X_t, T-t) dX_t \middle| X_T = x \right], \quad i = 1, 2. \tag{2}$$

The proof is an easy modification of the original proof of Theorem 3.4 in original article.

In [1], Bañuelos and Baudoin also pointed out that, since $M_T M_t^{-1}$ is not adapted with respect to the filtration $\mathcal{F}_t = \sigma(X_s, s \leq t)$, there is a gap in the original proof of Theorem 1.2 in original article where the Burkholder–Davies–Gundy inequality was used. They proved a new martingale inequality (Theorem 2.6 in [1]) which can be used to correct the gap in original article. Indeed, based on (1) and (2), and using Theorem 2.6 in [1], we can correct the gap in the proof of Theorem 1.2 in original article as follows.

Correction of the proof of Theorem 1.2. By (2) and using Theorem 2.6 in [1], we have

$$\|S_{A_i}\omega\|_p \leq C_p \|A_i\|_{\mathrm{op}} \|J\|_p. \tag{3}$$

where $C_p = 3\sqrt{p(2p-1)}$, and $J = \{\int_0^T \overline{\nabla}\omega_a(X_t, T-t)dt\}^{1/2}$. This corrects the gap in the original proof of Theorem 1.2 in original article (see line 7–line 13 in p. 135 in original article). Then, using (44) in original article, we can derived the original estimates in original article

$$\|S_{A_i}\omega\|_p \leq C(p^* - 1)^{3/2} \|A_i\|_{\mathrm{op}} \|\omega\|_p.$$

Note that there is a misprint in line 12 in p. 137 in original article: "$p > 1$" should be "$p > 2$". □

Correction of the proof of Theorem 5.1. The original proof remains valid except that we should correct the representation formula of S_{A_i} by (2). Indeed, when $W_k = -a$, we have

$$S_{A_i}^T \omega(x) = E\left[U_T \int_0^T U_t^{-1} A_i \nabla \omega_a(X_t, T-t) dX_t \middle| X_T = x \right], \quad i = 1, 2.$$

By the Burkholder martingale subordination inequality and the same argument as in original article, we have

$$\begin{aligned} \|S_{A_i}\omega\|_p &\leq (p^* - 1) \sup_{0 \leq t \leq T} \|U_t^{-1} A_i U_t\| \left\| \int_0^T U_t^{-1} \nabla \omega_a(X_t, T-t) dX_t \right\|_p \\ &\leq 2(p^* - 1) \|A_i\|_{\mathrm{op}} \|\omega\|_p. \end{aligned}$$

□

For the details of the above corrections, see [2]. Moreover, we can check that the original estimates stated in Theorem 1.2, Theorem 1.3 and Theorem 1.4, Theorem 5.1 and Corollary 5.2 in original article remain valid. As a consequence, the main theorems proved in original article remain valid with the correct L^p-norm estimates $C(p^* - 1)^{3/2}$. In particular, see Theorem 1.3 in original article, on complete and stochastically complete Riemannian manifolds non-negative Weitzenböck curvature operator $W_k \geq 0$, where $1 \leq k \leq n = \dim M$, the Weak L^p-Hodge decomposition theorem holds for k-forms, the De Rham projection $\mathrm{P}_1 = dd^*\square^{-1}$, the Leray projection $\mathrm{P}_2 = d^*d\square^{-1}$ and the Beurling–Ahlfors transform $B_k = (d^*d - dd^*)\square^{-1}$ on k-form is bounded in L^p for all $1 < p < \infty$.

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