

Erratum to: Rough Burgers-like equations with multiplicative noise

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Unfortunately, the proof of Proposition 4.8 in the original article is not correct. In fact, equation (4.60), where R^θ is rewritten as a stochastic integral, has no meaning because the integrand is not adapted. Hence in (4.61) we are not allowed to apply the Burkholder–Davies–Gundy inequality.

The argument was corrected in [1, Lemma 3.6 and Corollary 3.7] in a more complicated situation. In order to transfer these statement to the situation in the original article some small changes are needed. First of all, the regularity of the linearised process X should be measured in a Hölder norm with slightly bigger index α_\star than the solution u . This can be done without further problems. With this change, the definition of the stopping time τ_K^X in (4.53) should be replaced by the following.

For $K > 0$ and for an $\alpha_\star \in (\alpha, 1/2)$ we introduce the stopping time

$$\tau_K^X = \inf \left\{ t \in [0, T] : \sup_{\substack{x_1 \neq x_2 \\ 0 \leq s_1 < s_2 \leq t}} \frac{|X(s_1, x_1) - X(s_2, x_2)|}{|s_1 - s_2|^{\alpha_\star/2} + |x_1 - x_2|^{\alpha_\star}} > K \right\}. \quad (1)$$

The online version of the original article can be found under doi:[10.1007/s00440-011-0392-1](https://doi.org/10.1007/s00440-011-0392-1).

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With this changed definition, Lemma 3.6 of [1] implies, in the notation of the original article, the following result.

Lemma 1 *Suppose that $0 < \alpha < \alpha_* < \frac{1}{2}$ and let τ be a stopping time that almost surely satisfies*

$$0 \leq \tau \leq \tau_K^X \wedge T.$$

For every $0 \leq t \leq T$ we set

$$\begin{aligned} \tilde{\theta}(t) &:= \theta(t \wedge \tau), \\ \tilde{\Psi}^\theta(t) &:= \int_0^{t \wedge \tau} S(t-r) \tilde{\theta}(r) dW(r), \\ \tilde{X}(t) &:= \int_0^{t \wedge \tau} S(t-r) dW(r), \\ \tilde{R}^\theta(t; x, y) &:= \delta \tilde{\Psi}^\theta(t; x, y) - \tilde{\theta}(t, x) \delta \tilde{X}(t; x, y). \end{aligned}$$

Then, for any p large enough and for any $\gamma > 0$ such that

$$\gamma < \alpha_* + \alpha - \frac{1}{p} - \sqrt{\frac{1}{2p}(1 + \alpha - \alpha_*)},$$

the following bound holds true:

$$\sup_{0 < t \leq T} \mathbb{E} |\tilde{R}^\theta(t)|_{\Omega C^\gamma}^p \lesssim \|\theta\|_{p, \alpha}^p. \tag{2}$$

The statement given here is actually slightly stronger than the bound stated in [1] because the norm appearing on the left hand side of (2) is bounded uniformly in t instead of allowing a blow up near 0. In [1] we had to introduce this blowup due to a slightly modified definition of the Gaussian process X : the process used in [1] does not start at 0, but with stationary initial condition, which was convenient for other reasons. When going through the proof given in [1], one realises that when considering the process with zero initial condition, one can apply bound (3.74) for all times t and there is no need to use (3.75) for small times.

Based on this version of Lemma 1, it is then straightforward to use the a priori information on the time regularity of R^θ , combined with the fact that the “tilde” processes coincide with the “non-tilde” processes before time τ , to obtain the bound

$$\mathbb{E} \left[\|R^\theta\|_{C^\kappa([0, \tau]; \Omega C^{2\alpha})}^p \right] \leq C(K, T) \|\theta\|_{p, \alpha}^p,$$

for sufficiently small values of κ and sufficiently large values of p , as required.

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Reference

1. Hairer, M., Maas, J., Weber, H.: Approximating rough stochastic PDEs (arXiv e-prints)