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OBITUARY

Kiyoshi Itô (1915–2008)

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In his speech on receipt of the Kyoto Prize, Kiyoshi Itô told us:[1]

> Ever since I was a student, I have been attracted to the fact that statistical laws reside in seemingly random phenomena. Although I knew that probability theory was a means of describing such phenomena, I was not satisfied with contemporary papers or works on probability theory, since they did not clearly define the random variable, the basic element of probability theory. At that time, few mathematicians regarded probability theory as an authentic mathematical field, in the same strict sense that they regarded differential and integral calculus. With clear definition of real numbers formulated at the end of the nineteenth century, differential and integral calculus had developed into an authentic mathematical system. When I was a student, there were few researchers in probability; among the few were Kolmogorov of Russia, and Paul Lévy of France.
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> In 1938, upon graduation from university, I joined the Cabinet Statistics Bureau, where, until I became an associate professor at Nagoya University, I worked for five years. During those five years I had much free time, thanks to the special consideration given me by then Director Kawashima (grandfather of Princess Akishino). Accordingly, I was able to continue studying probability theory, by reading Kolmogorov's Basic Concept of Probability Theory (1933) and Paul Lévy's Theory of Sum of Independent Random Variables (1937). At that time, it was commonly believed that Lévy's works were extremely difficult, since Lévy, a pioneer in the new mathematical field, explained probability theory based on his intuition. I attempted to describe Lévy's ideas, using precise logic that Kolmogorov might use. Introducing the concept of regularization, developed by Doob of

[1] http://www.inamori-f.or.jp/laureates/k14_b_kiyoshi/lct_e.html.

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the U.S., I finally devised stochastic differential equations, after painstaking solitary endeavors. My first paper was thus developed; today, it is common practice for mathematicians to use my method to describe Lévy's theory.

Only a few mathematicians alive today can claim to have made contributions so fundamental and innovative as to create a substantial and deep new area of mathematics. I believe that, until his death on November 10th 2008, Itô was a member of this select group; Itô was probably unique among them in having created a field with such a substantial impact outside of mathematics per se, in science, engineering and economics.

The theory of differential equations, introduced by Isaac Newton, is a hugely successful tool for expressing and studying relationships between evolving systems. Differential equations such as

$$dy(t) = f(t, y(t))dt$$

and

$$dy(t) = f(t, y(t))dt + \sum_{i=1}^{n} f^i(t, y(t))dx^i(t)$$

are basic tools in applied mathematics and control theory and are used to express the behaviour of systems as they evolve and interact. Such equations are also key to understanding parts of pure mathematics. The development of a path into a manifold using a gauge, or from a manifold into a fibre bundle using a connection, or from the Lie algebra of a Lie group into that Lie group all provide examples where one evolving system interacts with and implies the evolution of a second. It is a key concept in any discussion of curvature or holonomy.

However, these models begin to fail at a practical level when applied in high dimensional systems. The superposition of billions of low frequency smooth nonlinear vector fields $f^i(t, y(t))dx^i(t)$ all acting on $y(t)$ quickly becomes, on the original computable scales, a highly oscillatory and unpredictable field. Classical numerical techniques are not able to capture these fluctuations. To understand the evolution of $y(t)$ on the original scale under circumstances where there is a huge degree of fine structure leads one, almost inevitably, to use stochastic models.

Today mathematical models combining systematic and stochastic features are widely used and are essential for effective modelling of key aspects of human endeavour. In climate modelling and weather prediction, in signal processing, in modelling epidemics, in space flight, in banking, in finance, in engineering, in chemistry, … adequate models have to take account of randomness. For an economic example, one can prove easily that any model for the evolution of prices in a liquid and deterministic market would also have to be static or it must admit arbitrage opportunities. A market without free lunches either has unpredictable "corrections" or volatility or both.

Itô created the mathematical tools for modelling jumpy and diffusive stochastic systems that allowed a calculus to be built to express interactions and relationships between these systems and build new processes from the old ones. Most of the natural models for noisy evolving systems are far from differentiable and the classical approach to differential equations does not address this situation. As Itô explained, starting with his 1942 paper, one can extend the ideas of Newton to deal with equations that have the form

$$dy(t) = f(t, y(t))dt + g(t, y(t))db(t)$$

where a deterministic evolution is perturbed by a Brownian noise $b(t)$ where the effect depends on the current state of the system. Based on developing an isometry between two L^2 spaces it was a truly remarkable achievement and introduced a completely new concept and unexpected notion of integration. It gave these differential equations meaning even though as Brownian motion is no-where differentiable and the functional connecting the controlling process $b(t)$ and the response process $y(t)$ is, in general, a profoundly nonlinear and discontinuous map.

Without doubt Itô is best known to non-experts for his introduction of Itô calculus, with its ability to give meaning to stochastic differential equations, with its "novel" change of variable formula compensating for the volatility and nowhere differentiability of Brownian paths. His fame in this area is built on a sequence of 13 or more papers, starting with his 1942 paper, in which he steadily built up the theory to something close to that we recognise today. He created a concept, and the core analytic tools needed to sustain stochastic analysis for 60 years: the connections between diffusions, PDEs, and geometry (through contact transformations and Cartan Development) can all be traced back to these early papers.

Even in the early days, this work attracted a great deal of attention and his main book was quickly translated from Japanese into Russian (by Wentzel, edited by Dynkin) and into Chinese. In the west, the seminal book "Stochastic Integrals" by H.P. McKean appeared in 1969 and has been followed by many other books. Some, such as the book by Ikeda and Watanabe, aimed at development of stochastic analysis itself, and others, such as A. Friedman's, aimed at using this mathematics in applications. But few (except perhaps Itô himself who was motivated, by dissatisfaction with the famous paper of Bachelier, to try and construct geometric Brownian motion and so to discover stochastic differential equations) could have foreseen its impact in applied science, engineering and economics. Stochastic models are everywhere today. Frequently, stochastic differential equations are used directly as models for phenomena but—perhaps even more importantly—they also appear as "diffusion limits" of rescaled discrete time models. Techniques of this sort have had a huge impact in fields such as population genetics and telecommunications engineering where the large scale nature of the problem makes aggregation and rescaling one of the most viable approaches yielding computable results. In the mid 1960s, Paul Samuelson and Robert Merton began "continuous time finance" using stochastic differential equations as models for asset prices and this led quickly to the appearance in 1973 of the Black–Scholes option pricing formula, for which Scholes and Merton were eventually awarded the economics Nobel Prize. These developments formed the basis for what has now become a huge industry in the global financial markets. The vanilla pricing of options using the celebrated Black and Scholes Paradigm can be viewed as a simple re-statement of Itô's stochastic integral in practical terms. It is a part of the financial modelling sector that was severely tested and not found wanting in the recent turbulence. Itô calculus definitely enhances our ability to describe the world we live in.

Although the applications to economics are the most widely known, as we have alluded above, Itô's contributions to mathematics and to engineering are much wider, and go well beyond the application of his calculus. His work on infinitely divisible random variables, stationary processes and excursions are all deep highly influential contributions that would have justified his position as a leading twentieth century mathematician had he never introduced Stochastic Calculus. A look on MathSciNet will show a huge number of individually starred items. His wider contribution to mathematics has also been substantial. One particular achievement has been the wonderful Encyclopaedic Dictionary of Mathematics, of which he edited the second edition, which allowed mathematicians to access to much more of their subject than would otherwise have been possible. He also set up the Kyoto Research Institute for Mathematics.

Itô has been mathematically active throughout his life. The author last witnessed him give a speech (in English) after a mathematics conference for his 88th Birthday which he also attended!

Itô created stochastic differential equations, and a mathematical framework of great substance in which to study them.

The impact of his creation goes forward.[2]

[2] The author would like to acknowledge that while he takes responsibility for this note, he drew information in it from an earlier letters that had been written concerning Professor Itô that were a joint effort with M.H.A.Davis, P.Malliavin, H.P.McKean, B.Oksendal, S.R.S.Varadhan.