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ERRATUM

Branching-coalescing particle systems

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It has been pointed out by Martin Hutzenthaler that Theorem 7 in [1] as stated there is incorrect. The purpose of this note is to state a weaker version of the theorem that is still sufficient for our applications in the mentioned paper.

We start by recalling some definitions. If E is a metrizable space, then we denote by $M(E)$ the space of real Borel measurable functions on E and set $B(E) := \{f \in M(E) : f \text{ is bounded}\}$. If A is a linear operator from a domain $\mathcal{D}(A) \subset M(E)$ into $M(E)$ and X is an E-valued process, then we say that X solves the martingale problem for A if X has cadlag sample paths, for each $f \in \mathcal{D}(A)$ one has $E\big[|f(X_t)|\big] < \infty$ and $\int_0^t E\big[|Af(X_s)|\big]\mathrm{d}s < \infty$ for all $t \geq 0$, and the process $(M_t)_{t\geq 0}$ defined by $M_t := f(X_t) - \int_0^t Af(X_s)\mathrm{d}s$ $(t \geq 0)$ is a martingale with respect to the filtration generated by X.

The following theorem corrects [1, Theorem 7].

Theorem 1 (Duality with error term) *Assume that E_1, E_2 are metrizable spaces and that for $i = 1, 2$, A_i is a linear operator from a domain $\mathcal{D}(A_i) \subset B(E_i)$ into $M(E_i)$. Assume that $\Psi \in B(E_1 \times E_2)$ satisfies $\Psi(\cdot, x_2) \in \mathcal{D}(A_1)$ and $\Psi(x_1, \cdot) \in \mathcal{D}(A_2)$ for each $x_1 \in E_1$ and $x_2 \in E_2$, and that*

$$\Phi_1(x_1, x_2) := A_1\Psi(\,\cdot\,, x_2)(x_1) \quad \textit{and} \quad \Phi_2(x_1, x_2) := A_2\Psi(x_1,\,\cdot\,)(x_2) \tag{1}$$

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$(x_1 \in E_1,\ x_2 \in E_2)$ *are jointly measurable in* x_1 *and* x_2. *Assume that* X^1 *and* X^2 *are independent solutions to the martingale problems for* A_1 *and* A_2*, respectively, and that*

$$\int_0^T \mathrm{d}s \int_0^T \mathrm{d}t\ E\left[\left|\Phi_i\left(X_s^1, X_t^2\right)\right|\right] < \infty \quad (T \geq 0,\ i = 1, 2). \tag{2}$$

Then

$$E\left[\Psi\left(X_T^1, X_0^2\right)\right] - E\left[\Psi\left(X_0^1, X_T^2\right)\right] = \int_0^T \mathrm{d}t\ \left(\Phi_1\left(X_t^1, X_{T-t}^2\right) - \Phi_2\left(X_t^1, X_{T-t}^2\right)\right) \tag{3}$$

holds for a.e. T *with respect to Lebesgue measure. Moreover, the left-hand side of* (3) *is continuous in* T.

This theorem differs in two ways from [1, Theorem 7]. First, the latter does not contain the statement about the continuity of the left-hand side of (3), and second, it is claimed there that (3) holds for every $T \geq 0$ (instead of a.e. T). This latter claim is wrong in general. For a counterexample demonstrating this as well as a proof of the corrected theorem we refer to [2].

Theorem 7 in [1] is applied at two places in that article: in the proof of Theorem 1, pages 401–403, and in the proof of Proposition 23, pages 404–405. Luckily, in both instances, all that is needed is the following corollary, which still holds.

Corollary 1 (Everywhere inequality) *Under the assumptions of Theorem 1, if*

$$A_1\Psi(\,\cdot\,, x_2)(x_1) \geq A_2\Psi(x_1,\,\cdot\,)(x_2) \quad (x_1 \in E_1,\ x_2 \in E_2), \tag{4}$$

then

$$E\left[\Psi\left(X_T^1, X_0^2\right)\right] \geq E\left[\Psi\left(X_0^1, X_T^2\right)\right] \quad (T \geq 0). \tag{5}$$

The same statement holds with both inequality signs reversed.

Proof Set $g(T) := E[\Psi(X_T^1, X_0^2)] - E[\Psi(X_0^1, X_T^2)]$. Then Theorem 1 shows that g is a continuous function satisfying $g \geq 0$ a.e., hence $g(T) \geq 0$ for every $T \geq 0$. □

References

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