

The divergence of Banach space valued random variables on Wiener space

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The original version of this article unfortunately contained two mistakes.

Corollary 3.5 (as well as Corollary 3.17a whose proof relies on it) should be ignored since the inequality $|F_n|_{p,1} \leq \|F_n\|_{p,1}$ in its proof is false, and we have been unable to find a simple alternative argument.

More importantly, in Propositions 3.14 and 3.18 one needs to add the assumption

(A) Y^{**} has the Radon Nykodim property (RNP) with respect to μ (cf. [1])

on the Banach space Y , as we shall now explain. Note (Sect. III.3 in [1]) that Y^{**} has the RNP with respect to any measure if, for example, Y^{**} is separable or Y is reflexive.

Unfortunately, in the proof of Proposition 3.14, the natural imbedding of $L^p(\mu; Y^{**})$ in the operator space $L(Y^*, L^p(\mu))$ was erroneously claimed to be surjective. In addition, the observation associated with (3.12) was also incorrect as stated (although not used in the rest of the paper). We restate this observation in (1) below, and prove it under the additional assumption (A); it will then be used in (2) to replace the incorrect proof of Proposition 3.14.

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- (1) Assume (A) and let $\mathbf{K} \in L^p(\mu; L(W^*, Y^{**}))$. If (3.12) holds for some $\gamma > 0$ and all $F \in \mathcal{S}(Y^*)$ then $\mathbf{K} \in \mathbf{dom}_{p, Y^{**}} \delta$ (the converse is obvious).

Proof The bound (3.12) implies the existence of a $\Lambda_{\mathbf{K}} \in L^q(\mu, Y^*)^*$ such that

$$E \operatorname{tr} \left(\mathbf{K}^T \nabla^{W^*} F \right) = \Lambda_{\mathbf{K}}(F) \quad \forall F \in \mathcal{S}(Y^*).$$

Due to assumption (A), $L^q(\mu, Y^*)^*$ can be identified with $L^p(\mu, Y^{**})$ (cf. Theorem IV.1 in [1]) in the sense that there exists a $\delta \mathbf{K} \in L^p(\mu, Y^{**})$ such that $\Lambda_{\mathbf{K}}(F)$ is given by $E_{Y^*} \langle F, \delta \mathbf{K} \rangle_{Y^{**}}$. Thus, for any $F \in \mathcal{S}(Y^*)$

$$E \operatorname{tr} \left(\mathbf{K}^T \nabla^{W^*} F \right) = E_{Y^{**}} \langle \delta \mathbf{K}, F \rangle_{Y^{**}}. \tag{†}$$

For $F = \phi(\delta(e_1), \dots, \delta(e_m)) \otimes l$, $(\{e_1, \dots, e_m\} \subset W^*$, orthonormal in H , and $l \in Y^*$), (†) amounts to $E_{Y^{**}} \langle E(\sum_{i=1}^m \partial_i \phi K e_i), l \rangle_{Y^{**}} = E_{Y^{**}} \langle E \phi \delta \mathbf{K}, l \rangle_{Y^{**}}$ which, if true $\forall l \in Y^*$, is true $\forall l \in Y^{**}$ as well. Thus (†) holds for all $F \in \mathcal{S}(Y^{**})$, which means that $\mathbf{K} \in \mathbf{dom}_{p, Y^{**}} \delta$.

- (2) We now present a modified proof of the “if” implication in the first statement of Proposition 3.14, using the characterization provided by (1) instead of the erroneous identification of $L^p(\mu; Y^{**})$ and $L(Y^*, L^p(\mu))$ mentioned above:

It follows from (3.13) that there exists a $\Delta_{\mathbf{K}} \in L(Y^*, L^p(\mu))$ such that for all $l \in Y^*$

$$\delta(\mathbf{K}^T l) = \Delta_{\mathbf{K}}(l) \tag{3.17}$$

so that, for any $F = \sum_{j=1}^m \Phi_j l_j \in \mathcal{S}(Y^*)$

$$\begin{aligned} E \operatorname{tr} (\mathbf{K}^T \nabla F) &= E \sum_{j=1}^m \operatorname{tr} \mathbf{K}^T \nabla (\Phi_j l_j) = \sum_{j=1}^m E_{W^*} \langle \nabla \Phi_j, \mathbf{K}^T l_j \rangle_{W^{**}} \\ &= \sum_{j=1}^m E \delta(\mathbf{K}^T l_j) \Phi_j = \sum_{j=1}^m E \Delta_{\mathbf{K}}(l_j) \Phi_j \\ &= E \Delta_{\mathbf{K}} \left(\sum_{j=1}^m \Phi_j l_j \right) = E \Delta_{\mathbf{K}}(F) \end{aligned}$$

and thus for any $q \geq 1$, and with $\|\Delta_{\mathbf{K}}\|$ denoting the operator norm,

$$\left| E \operatorname{tr} (\mathbf{K}^T \nabla F) \right| \leq E \|\Delta_{\mathbf{K}}\| \|F\|_{Y^*} \leq \|\Delta_{\mathbf{K}}\| \left(E \|F\|_{Y^*}^q \right)^{1/q}$$

so that from (1) it follows that $\mathbf{K} \in \mathbf{dom}_{p, Y^{**}} \delta$.

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Reference

1. Diestel, J., Uhl, J.J., Jr: Vector measures. *AMS Math. Surv.* 15, (1977)