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ORIGINAL PAPER

On the Ramsey-Goodness of Paths

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Abstract For a graph G, we denote by $\nu(G)$ the order of G, by $\chi(G)$ the chromatic number of G and by $\sigma(G)$ the minimum size of a color class over all proper $\chi(G)$-colorings of G. For two graphs G_1 and G_2, the Ramsey number $R(G_1, G_2)$ is the least integer r such that for every graph G on r vertices, either G contains a G_1 or $\overline{G}$ contains a G_2. Suppose that G_1 is connected. We say that G_1 is G_2-good if $R(G_1, G_2) = (\chi(G_2) - 1)(\nu(G_1) - 1) + \sigma(G_2)$. In this note, we obtain a condition for graphs H such that a path is H-good.

Keywords Ramsey number · Goodness · Path

Mathematics Subject Classification 05C55 · 05D10

1 Introduction

Throughout this paper, all graphs are finite and simple. Let G_1 and G_2 be two graphs. The *Ramsey number* $R(G_1, G_2)$, is defined as the least integer r such that for every

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graph G on r vertices, either G contains a G_1 or $\overline{G}$ contains a G_2, where $\overline{G}$ is the complement of G.

We denote by $\nu(G)$ the order of G, by $\delta(G)$ the minimum degree of G, by $\omega(G)$ the component number of G, by $\chi(G)$ the chromatic number of G and by $\sigma(G)$ the minimum size of a color class over all proper $\chi(G)$-colorings of G. For two disjoint graphs G_1 and G_2, the *union* of G_1 and G_2 is defined as $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$; and the *join* of G_1 and G_2 is defined as $V(G_1 \vee G_2) = V(G_1) \cup V(G_2)$, and $E(G_1 \vee G_2) = E(G_1 + G_2) \cup \{xy : x \in V(G_1), y \in V(G_2)\}$. The union of k disjoint copies of the same graph G is denoted by kG.

Theorem 1 **(Burr** [5]**)** *For all graphs G and H, with G connected and $\nu(G) \geq \sigma(H)$,*

$$R(G, H) \geq (\chi(H) - 1)(\nu(G) - 1) + \sigma(H).$$

We say G is *H-good* if $R(G, H) = (\chi(H) - 1)(\nu(G) - 1) + \sigma(H)$. Lin et al. proved the following theorem on Ramsey-goodness of trees.

Theorem 2 **(Lin et al.** [11]**)** *Let T be a tree and H be a graph. If T is H-good and $\sigma(H) = 1$, then T is also $K_1 \vee H$-good.*

By P_n and C_n we denote the *path* and *cycle* on n vertices, respectively. For the case of T being a path, we get an extension of Theorem 2.

Theorem 3 *Let $n \geq 2$, and H be a subgraph of $K_{a_1,a_2,\ldots,a_k}$, $k = \chi(H)$, such that*

$$a_i \leq \left\lceil \frac{k(n-1)+1}{k+i} \right\rceil, 1 \leq i \leq k,$$

If P_n is H-good, then P_n is also $K_1 \vee H$-good.

Note that H is a subgraph of $K_{a_1,a_2,\ldots,a_k}$ if and only if there is a proper coloring of H such that the size of the ith color class is at most a_i, $1 \leq i \leq k$.

We prove Theorem 3 in Sect. 3. In Sect. 2, we will apply Theorem 3 to show some results of Ramsey values involving paths.

Note that $\sigma(K_1 \vee H) = 1$. By Theorem 2, we can see that under the conditions of Theorems 2 and 3, P_n is $K_t \vee H$-good for all $t \geq 1$.

Theorem 4 *Let $n \geq 3$ and H be a graph with $\sigma(H) \leq 2$. If P_n is H-good, then P_n is also $(2K_1 \vee H)$-good.*

Proof Since P_n is H-good, $R(P_n, H) = (\chi(H) - 1)(n - 1) + \sigma(H)$. Set

$$r = (\chi(2K_1 \vee H) - 1)(n - 1) + \sigma(2K_1 \vee H) = \chi(H)(n - 1) + \sigma(H).$$

Then $r = R(P_n, H) + n - 1$.

From Theorem 1, we have $R(P_n, 2K_1 \vee H) \geq r$. Now let G be an arbitrary graph of order r without P_n as a subgraph. We will prove that $\overline{G}$ contains $2K_1 \vee H$. Let $P = v_1 v_2 \ldots v_k$ be a longest path of G. Thus $k \leq n - 1$.

If $k = 1$ then G is an empty graph and $\overline{G}$ is complete. Note that $\nu(G) = r \geq R(P_n, H) + 2 \geq \nu(H) + 2$. So $\overline{G}$ contains $2K_1 \vee H$.

Now we assume that $2 \leq k \leq n - 1$. Let G' be a subgraph of G induced by $V(G) - V(P)$. Then $\nu(G') = r - k \geq R(P_n, H)$. So $\overline{G'}$ contains H. Note that v_1 and v_k are nonadjacent to every vertex of G'. Thus $\overline{G}$ contains $2K_1 \vee H$. □

From Theorem 4 we get the following result.

Corollary 1 *Let $n \geq 3$ and H be a graph with $\sigma(H) \leq 2$. If P_n is H-good, then P_n is also $(\overline{tK_2} \vee H)$-good.*

2 Some Corollaries

In this section, we will list some known results for the Ramsey numbers involving paths. After each result, we apply Theorems 2, 3 and 4 to get a new Ramsey numbers involving paths. We denote by L_s^t $(s \geq t + 1)$ the graph obtained from K_{s+t} by removing the edges of a matching of size t, i.e., $L_s^t = \overline{tK_2} \vee K_{s-t}$. We use $\text{par}(m)$ to denote the parity of m. In the following corollaries, we always assume that $n \geq 3$.

Theorem 5 **(Gerencsér and Gyárfás [10])** *If $2 \leq m \leq n$, then*

$$R(P_n, P_m) = n + \left\lfloor \frac{m}{2} \right\rfloor - 1.$$

Corollary 2 *Let $t \geq 0$ be an integer. If $2 \leq m \leq n$, then*

$$R(P_n, L_s^t \vee P_m) = (s + 1)(n - 1) + 1.$$

Proof By Theorem 5, P_n is P_m-good. Take $a_1 = \lceil m/2 \rceil$ and $a_2 = \lfloor m/2 \rfloor$. By Theorem 3, P_n is $(K_1 \vee P_m)$-good. So by Theorems 2 and 4 we get the assertion. □

The *kipas* $\widehat{K}_m$ is the graph obtained by joining a K_1 and a path P_m. For the case $(s, t) = (1, 0)$ in Corollary 2, we can get the values of path-kipas Ramsey numbers $R(P_n, \widehat{K}_m)$ for $3 \leq m \leq n$, which was already obtained by Saleman and Broersma [16].

Theorem 6 **(Faudree et al. [9])** *If $n \geq 2$ and $m \geq 3$, then*

$$R(P_n, C_m) = \begin{cases} 2n - 1, & n \geq m \text{ and } m \text{ is odd;} \\ n + m/2 - 1, & n \geq m \text{ and } m \text{ is even;} \\ \max\{m + \lfloor n/2 \rfloor - 1, 2n - 1\}, & m > n \text{ and } m \text{ is odd;} \\ m + \lfloor n/2 \rfloor - 1, & m > n \text{ and } m \text{ is even.} \end{cases}$$

Corollary 3 *Let $t \geq 0$ be an integer. If m is even, $4 \leq m \leq n$; or m is odd and $3 \leq m \leq \lceil 3n/2 \rceil$, then*

$$R(P_n, L_s^t \vee C_m) = (s + \text{par}(m) + 1)(n - 1) + 1.$$

Proof From Theorem 6, one can check that P_n is C_m-good. For the case m is even, take $a_1 = a_2 = m/2$ and apply Theorems 3, 2 and 4; for the case m is odd, apply Theorems 2 and 4. In both case, we have the assertion. □

The *wheel* W_m is the graph obtained by joining K_1 and a cycle C_m. For the case $(s, t) = (1, 0)$ in Corollary 3, we can get the values of path-wheel Ramsey numbers $R(P_n, C_m)$ under the condition of Corollary 3, which was already obtained by Chen et al. [6].

Theorem 7 *If* $n \geq 2$, *then*

$$R(P_n, mK_1) = m.$$

This theorem is trivial and the following corollary can be get immediately. We omit the proof.

Corollary 4 *Let* $t \geq 0$ *be an integer. If* $m \leq \lceil n/2 \rceil$, *then*

$$R(P_n, L_s^t \vee mK_1) = s(n-1) + 1.$$

For $m \geq 2$, the graph $K_{1,m}$ is called a *star*; the graph $K_2 \vee mK_1$ is called a *book*; and the graph $K_t \vee mK_1$, $t \geq 3$, is called a *generalized book*. We remark here that the Ramsey numbers of paths versus stars and paths versus (generalized) books under the condition of Corollary 4 was already obtained by Parsons [12], and Rousseau and Sheehan [14], respectively.

Theorem 8 **(Faudree and Schelp** [8]**)** *If* $n, m_i \geq 2$, $1 \leq i \leq k$, *then*

$$R\left(P_n, \bigcup_{i=1}^{k} P_{m_i}\right) = \max\left\{n + \sum_{i=1}^{k} \left\lfloor \frac{m_i}{2} \right\rfloor - 1, \sum_{i=1}^{k} m_i + \left\lfloor \frac{n}{2} \right\rfloor - 1\right\}.$$

Corollary 5 *Let* $t \geq 0$ *be an integer. If* $m_i \geq 2$, $1 \leq i \leq k$ *and* $\sum_{i=1}^{k} \lceil m_i/2 \rceil \leq \lceil n/2 \rceil$, *then*

$$R\left(P_n, L_s^t \vee \bigcup_{i=1}^{k} P_{m_i}\right) = (s+1)(n-1) + 1.$$

Proof By Theorem 8, P_n is $(\bigcup_{i=1}^{k} P_{m_i})$-good. Take

$$a_1 = \sum_{i=1}^{k} \left\lceil \frac{m_i}{2} \right\rceil \text{ and } a_2 = \sum_{i=1}^{k} \left\lfloor \frac{m_i}{2} \right\rfloor.$$

By Theorem 3, P_n is $(K_1 \vee \bigcup_{i=1}^{k} P_{m_i})$-good. By Theorems 2 and 4 we get the assertion. □

The graph $F_m = K_1 \vee mK_2$ is called a *fan*. From the above corollary, we can see that if $m \leq \lceil n/2 \rceil$, then $R(P_n, F_m) = 2n - 1$. This result was already obtained by Saleman and Broersma [15].

Let P_n^k be the k-th power of P_n, i.e., the graph with vertex set $\{v_1, \ldots, v_n\}$ and edge set $\{v_i v_j : |i - j| \leq k\}$.

Theorem 9 **(Pokrovskiy** [13]**)** *If* $n \geq k + 1$, *then*

$$R(P_n, P_n^k) = k(n-1) + \left\lfloor \frac{n}{k+1} \right\rfloor.$$

Corollary 6 *Let* $t \geq 0$ *be an integer. If* $n \geq k + 1$, *then*

$$R(P_n, L_s^t \vee P_n^k) = (t + k)(n - 1) + 1.$$

Proof Note that $\chi(P_n^k) = k + 1$ and $\sigma(P_n^k) = \lfloor n/(k+1) \rfloor$. By Theorem 9, P_n is P_n^k-good. Take

$$a_i = \left\lfloor \frac{n + i - 1}{k + 1} \right\rfloor, 1 \leq i \leq k + 1.$$

It is easy to see that P_n^k is a subgraph of $K_{a_1, a_2, \ldots, a_{k+1}}$. By Theorems 3, 2 and 4, we have the assertion. □

Theorem 10 **(Sudarsana et al.** [18]**)** *If* $m \geq 2$, *then*

$$R(P_n, 2K_m) = (m - 1)(n - 1) + 2.$$

Corollary 7 *Let* $t \geq 0$ *be an integer. If* $m \geq 2$ *and* $n \geq 3$, *then*

$$R(P_n, L_s^t \vee 2K_m) = (s + m - 1)(n - 1) + 1.$$

Proof By Theorem 10, P_n is $2K_m$-good. Take $a_i = 2$, $1 \leq i \leq m$. Note that $2K_m$ is a subgraph of $K_{a_1, a_2, \ldots, a_m}$. By Theorems 3, 2 and 4, we have the assertion. □

Theorem 11 **(Sudarsana** [17]**)** *If* $m, k \geq 2$ *and* $n \geq (k-2)((km-2)(m-1)+1)+3$, *then*

$$R(P_n, kK_m) = (m - 1)(n - 1) + k.$$

Corollary 8 *Let* $t \geq 0$ *be an integer. If* $m, k \geq 2$ *and* $n \geq (k - 2)((km - 2)(m - 1) + 1) + 3$, *then*

$$R(P_n, L_s^t \vee kK_m) = (s + m - 1)(n - 1) + 1.$$

Proof By Theorem 11, P_n is kK_m-good. Take $a_i = k$, $1 \leq i \leq m$. Note that kK_m is a subgraph of $K_{a_1, a_2, \ldots, a_m}$. By Theorems 3, 2 and 4, we have the assertion. □

The *cocktail party graph* (or *hyperoctahedral graph*) H_m is the graph obtained by removing a perfect matching from a complete graph K_{2m} (i.e., $H_m = \overline{mK_2}$).

Theorem 12 **(Ali et al.** [1]**)** *If* $n, m \geq 3$, *then*

$$R(P_n, H_m) = (n-1)(m-1) + 2.$$

Corollary 9 *Let* $t \geq 0$ *be an integer. If* $n, m \geq 3$, *then*

$$R(P_n, L_s^t \vee H_m) = (s+m-1)(n-1) + 1.$$

Proof By Theorem 12, P_n is H_m-good. Take $a_i = 2$, $1 \leq i \leq m$. Note that $H_m = K_{a_1, a_2, \ldots, a_m}$. By Theorems 3, 2 and 4, we have the assertion. □

The *sunflower graph* SF_m is the graph on $2m+1$ vertices obtained by taking a wheel W_m with hub x, an m-cycle $v_1 v_2 \cdots v_m v_1$, and additional m vertices $w_1, w_2, \ldots, w_m$, where w_i is joined by edges to v_i, v_{i+1}, $1 \leq i \leq m$, where $v_{m+1} = v_1$.

Theorem 13 **(Ali et al.** [4]**)** *If* $m \geq 3$, *then*

$$R(P_n, SF_m) = \begin{cases} 2n + m/2 - 2, & m \text{ is even and } n \geq 4m^2 - 7m + 4; \\ 3n - 2, & m \text{ is odd and } n \geq 2m^2 - 9m + 11. \end{cases}$$

Corollary 10 *Let* $t \geq 0$ *be an integer. If* $m \geq 4$ *is even and* $n \geq 4m^2 - 7m + 4$, *or* $m \geq 3$ *is odd and* $n \geq 2m^2 - 9m + 11$, *then*

$$R(P_n, L_s^t \vee SF_m) = (s + 2 + \text{par}(m))(n-1) + 1.$$

Proof By Theorem 13, P_n is SF_m-good. If m is even, then take $a_1 = m+1$ and $a_2 = a_3 = m/2$; if m is odd, then $\sigma(SF_m) = 1$. By Theorems 3, 2 and 4, we have the assertion. □

The *Beaded wheel* BW_m is a graph on $2m+1$ vertices which is obtained by inserting one vertex in each spoke of the wheel W_m.

Theorem 14 **(Ali et al.** [3]**)** *If* $m \geq 3$, *then*

$$R(P_n, BW_m) = \begin{cases} 2n - 1 & m \text{ is even and } n \geq 2m^2 - 5m + 4; \\ 2n & m \text{ is odd and } n \geq 2m^2 - 5m + 3. \end{cases}$$

Corollary 11 *Let* $t \geq 0$ *be an integer. If* $m \geq 4$ *is even and* $n \geq 2m^2 - 5m + 4$, *or* $m \geq 3$ *is odd and* $n \geq 2m^2 - 5m + 3$, *then*

$$R(P_n, L_s^t \vee BW_m) = (s+2)(n-1) + 1.$$

Proof By Theorem 14, P_n is BW_m-good. If m is even, then $\sigma(BW_m) = 1$; if m is odd, then take $a_1 = m$ and $a_2 = a_3 = (m+1)/2$. By Theorems 3, 2 and 4, we have the assertion. □

The *Jahangir graph* J_{2m} is a graph on $2m + 1$ vertices consisting of a cycle C_{2m} with one additional vertex which is adjacent alternatively to m vertices of C_{2m}.

Theorem 15 (Surahmat and Tomescu [19]) *If $m \geq 2$ and $n \geq (4m-1)(m-1)+1$, then*

$$R(P_n, J_{2m}) = n + m - 1.$$

Corollary 12 *Let $t \geq 0$ be an integer. If $m \geq 2$ and $n \geq (4m-1)(m-1)+1$, then*

$$R(P_n, L_s^t \vee J_{2m}) = (t+1)(n-1)+1.$$

Proof By Theorem 15, P_n is J_{2m}-good. Take $a_1 = m$ and $a_2 = m + 1$. By Theorems 3, 2 and 4, we have the assertion. □

The *generalized Jahangir graph* $J_{k,m}$ is a graph on $km + 1$ vertices consisting of a cycle C_{km} with one additional vertex which is adjacent to m vertices of the C_{km} each of which is at distance k to the next one on C_{km}.

Theorem 16 (Ali et al. [2]) *If $m, k \geq 2$, then*

$$R(P_n, J_{k,m}) = \begin{cases} n + km/2 - 1, & k \text{ is even and } n \geq (2km-1)(km/2-1)+1; \\ 2n - 1 & k \text{ is odd, } m \text{ is even and } n \geq km(km-2)/2; \\ 2n & k, m \text{ are odd and } n \geq (km-1)^2/2. \end{cases}$$

Corollary 13 *Let $t \geq 0$ be an integer. If $n, m, k \geq 2$, and if k is even and $n \geq (2km-1)(km/2-1)+1$, or k is odd, m is even and $n \geq km(km-2)/2$, or k, m are odd and $n \geq (km-1)^2/2$, then*

$$R(P_n, L_s^t \vee J_{k,m}) = (s + 1 + \mathrm{par}(k))(n-1)+1.$$

Proof By Theorem 16, P_n is $J_{k,m}$-good. If k is even, then take $a_1 = km/2 + 1$ and $a_2 = km/2$; if k is odd, then take

$$a_1 = m \cdot \left\lfloor \frac{k+2}{3} \right\rfloor + 1, a_2 = m \cdot \left\lfloor \frac{k+1}{3} \right\rfloor \text{ and } a_3 = m \cdot \left\lfloor \frac{k}{3} \right\rfloor.$$

By Theorems 3, 2 and 4, we have the assertion. □

3 Proof of Theorem 3

From Theorem 1, it is sufficient to prove that $R(P_n, K_1 \vee H) \leq k(n-1)+1$. Let G be a graph of order $k(n-1)+1$. Suppose that G contains no P_n and $\overline{G}$ contains no $K_1 \vee H$.

Since H is a subgraph of $K_{a_1,a_2,\ldots,a_k}$, we have

$$\sigma(H) \leq a_k \leq \left\lceil \frac{k(n-1)+1}{2k} \right\rceil = \left\lceil \frac{n}{2} - \frac{k-1}{2k} \right\rceil = \left\lceil \frac{n}{2} \right\rceil.$$

Since P_n is H-good,

$$R(P_n, H) = (\chi(H) - 1)(n - 1) + \sigma(H) \leq (k - 1)(n - 1) + \left\lceil \frac{n}{2} \right\rceil.$$

If there is a vertex v in G with $d(v) \leq \lfloor n/2 \rfloor - 1$, then let G' be a subgraph of G induced by $V(G) - \{v\} - N(v)$, where $N(v)$ is the set of vertices adjacent to v in G. Note that

$$\begin{aligned} \nu(G') &= \nu(G) - 1 - d(v) \geq k(n-1) + 1 - \left\lfloor \frac{n}{2} \right\rfloor \\ &= (k-1)(n-1) + \left\lceil \frac{n}{2} \right\rceil \geq R(P_n, H). \end{aligned}$$

This implies that G' contains a path P_n or $\overline{G'}$ contains a subgraph isomorphic to H. Note that v is nonadjacent to every vertex of G'. G contains a P_n or $\overline{G}$ contains a $K_1 \vee H$, a contradiction. Thus we assume that $\delta(G) \geq \lfloor n/2 \rfloor$.

If there is a component B of G with $\nu(B) \geq n$, then by Dirac's Theorem (see [7]), B contains a P_n, a contradiction. Thus we assume that every component of G has order at most $n - 1$. Note that the minimum degree of G is at least $\lfloor n/2 \rfloor$. Every component of G has order between $\lfloor n/2 \rfloor + 1$ and $n - 1$.

If $\omega(G) \leq k$, then $\nu(G) \leq k(n-1)$; and if $\omega(G) \geq 2k$, then $\nu(G) \geq k(n+1)$, both a contradiction. This implies that

$$k + 1 \leq \omega(G) \leq 2k - 1.$$

Let $\mathcal{B} = \{B_1, B_2, \ldots, B_\omega\}$, $\omega = \omega(G)$, be the set of the components of G. We assume without loss of generality that $\nu(B_1) \geq \nu(B_2) \geq \cdots \geq \nu(B_\omega)$. Thus we have

$$\nu(B_i) \geq \left\lceil \frac{\nu(G) - (i-1)(n-1)}{\omega - i + 1} \right\rceil = \left\lceil \frac{(k-i+1)(n-1)+1}{\omega - i + 1} \right\rceil, 1 \leq i \leq k < \omega.$$

Now we partition $\mathcal{B}$ into $k + 1$ parts such that the order sum of the components in the ith part is at least a_i, $1 \leq i \leq k$.

Let $t = \omega - k - 1$. For $1 \leq i \leq t$, let $\mathcal{B}_i = \{B_{\omega-2i+1}, B_{\omega-2i}\}$; for $t + 1 \leq i \leq k$, let $\mathcal{B}_i = \{B_{i-t}\}$; and let $\mathcal{B}_{k+1} = \{B_\omega\}$.

If $1 \leq i \leq t$, then $\mathcal{B}_i$ contains two components each of which has order at least $\lfloor n/2 \rfloor + 1$. Thus $\sum\{\nu(B_j) : B_j \in \mathcal{B}_i\} \geq n + 1$. On the other hand,

$$a_i \leq \left\lceil \frac{k(n-1)+1}{k+i} \right\rceil \leq \left\lceil \frac{k(n-1)+1}{k} \right\rceil = n < \sum_{B_j \in \mathcal{B}_i} \nu(B_j).$$

If $t + 1 \leq i \leq k$, then $\mathcal{B}_i = \{B_{i-t}\}$. Note that

$$\nu(B_{i-t}) \geq \left\lceil \frac{(k-i+t+1)(n-1)+1}{\omega - i + t + 1} \right\rceil = \left\lceil \frac{(\omega - i)(n-1)+1}{2\omega - k - i} \right\rceil.$$

Since $\omega - k \leq i \leq k$, one can check that

$$a_i \leq \left\lceil \frac{k(n-1)+1}{k+i} \right\rceil \leq \left\lceil \frac{(\omega - i)(n-1)+1}{2\omega - k - i} \right\rceil \leq \nu(B_{i-t}).$$

Clearly $\nu(B_\omega) \geq 1$. Thus $\overline{G}$ contains a $K_{a_1,a_2,\ldots,a_k,1}$, which is a supergraph of $K_1 \vee H$, our final contradiction.

The proof is complete.

References

1. Ali, K., Baig, A.Q., Baskoro, E.T.: On the Ramsey number for a linear forest versus a cocktail party graph. J. Comb. Math. Comb. Comput. **71**, 173–177 (2009)
2. Ali, K., Baskoro, E.T., Tomescu, I.: On the Ramsey numbers for paths and generalized Jahangir graphs $J_{s,m}$. Bull. Math. Soc. Sci. Math. Roumanie Tome **51**(3), 177–182 (2008)
3. Ali, K., Baskoro, E.T., Tomescu, I.: On the Ramsey number for paths and beaded wheels. J. Prime Res. Math. **5**, 133–138 (2009)
4. Ali, K., Tomescu, I., Javaid, I.: On path-sunflower Ramsey numbers. Math. Rep. **17**(4), 385–390 (2015)
5. Burr, S.A.: Ramsey numbers involving graphs with long suspended paths. J. Lond. Math. Soc. **24**, 405–413 (1981)
6. Chen, Y., Zhang, Y., Zhang, K.: The Ramsey numbers of paths versus wheels. Discret. Math. **290**(1), 85–87 (2005)
7. Dirac, G.A.: Some theorems on abstract graphs. Proc. Lond. Math. Soc. **2**, 69–81 (1952)
8. Faudree, R.J., Schelp, R.H.: Ramsey numbers for all linear forests. Discret. Math. **16**(2), 149–155 (1976)
9. Faudree, R.J., Lawrence, S.L., Parsons, T.D., Schelp, R.H.: Path-cycle Ramsey numbers. Discret. Math. **10**(2), 269–277 (1974)
10. Gerencsér, L., Gyárfás, A.: On Ramsey-type problems. Ann. Univ. Sci. Budapest Eötvös Sec. Math. **10**, 167–170 (1967)
11. Lin, Q., Li, Y., Dong, L.: Ramsey goodness and generalized stars. Eur. J. Comb. **31**(5), 1228–1234 (2010)
12. Parsons, T.D.: Path-star Ramsey numbers. J. Combin. Theory Ser. B **17**(1), 51–58 (1974)
13. Pokrovskiy, A.: Calculating Ramsey numbers by partitioning coloured graphs, eprint arXiv:1309.3952
14. Rousseau, C.C., Sheehan, J.: A class of Ramsey problems involving trees. J. Lond. Math. Soc. **18**, 392–396 (1978)
15. Salman, A.N.M., Broersma, H.J.: Path-fan Ramsey numbers. Discret. Appl. Math. **154**(9), 1429–1436 (2006)
16. Salman, A.N.M., Broersma, H.J.: Path-kipas Ramsey numbers. Discret. Appl. Math. **155**(14), 1878–1884 (2007)
17. Sudarsana, I.W.: The goodness of long path with respect to multiple copies of complete graphs. J. Indones. Math. Soc. **20**(1), 31–35 (2014)
18. Sudarsana, I.W., Assiyatun, H., Adiwijaya, Musdalifah, S.: The Ramsey number for a linear forest versus two identical copies of complete graphs. Discret. Math. Alg. Appl. **2**(4), 437–444 (2010)
19. Surahmat, Tomescu, I.: On path-Jahangir Ramsey numbers. Appl. Math. Sci. **8**(99), 4899–4904 (2014)