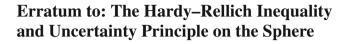
ERRATUM



Feng Dai · Yuan Xu

Published online: 12 March 2015 © Springer Science+Business Media New York 2015

Erratum to: Constr Approx (2014) 40:141–171 DOI 10.1007/s00365-014-9235-5

Several forms of uncertainty principles on the unit sphere are established in [1]. When stated in term of the vector

$$\tau(f) := \frac{1}{\omega_d} \int_{\mathbb{S}^{d-1}} x |f(x)|^2 \, d\sigma(x)$$

of \mathbb{R}^d (normalization constant $1/\omega_d$ was missing in [1]), our main result is in:

Corollary 4.4 Let $f \in W_2^1(\mathbb{S}^{d-1})$ be such that $\int_{\mathbb{S}^{d-1}} f(y) d\sigma(y) = 0$ and $||f||_2 = 1$. If $d \ge 2$, then

$$(1 - \|\tau(f)\|)\|\nabla_0 f\|_2^2 \ge C_d^{-1}.$$
(4.11)

CONSTRUCTIVE

PPROXIMATION

Here C_d is a constant given in Theorem 4.1. We next attempted to remove the condition that $\int_{\mathbb{S}^{d-1}} f(y) d\sigma(y) = 0$ and stated:

F. Dai

Y. Xu (⊠) Department of Mathematics, University of Oregon, Eugene, OR 97403-1222, USA e-mail: yuan@uoregon.edu



The online version of the original article can be found under doi:10.1007/s00365-014-9235-5.

Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB T6G 2G1, Canada e-mail: dfeng@math.ualberta.ca

Theorem 4.5 Assume that $d \ge 2$, and let $f \in W_2^1(\mathbb{S}^{d-1})$ be such that $||f||_2 = 1$. Then

$$(1 - \|\tau(f)\|)\|\nabla_0 f\|_2^2 \ge c_d \|\tau(f)\|.$$
(4.14)

This theorem, however, is incorrect. This was pointed out to us by Stefan Steinerberger, who showed that the inequality (4.14) does not hold for the function $f(\cos\theta, \sin\theta) = 1 + \varepsilon \sin\theta$ for small enough ε when d = 2. The mistake in the proof appeared on the line 6 of page 166, which states that $\|\tau(f)\| \le (2|m_f|+1)\|g\|_2^2$, but it should have been $\|\tau(f)\| \le \|g\|_2^2 + 2|m_f|\|g\|_2$. As a consequence, the right-hand side of (4.14) has to be replaced by $c_d \|\tau(f)\|^2$. Since $\|\tau(f)\| \le \|f\|_2^2$, the resulted inequality is then equivalent to

$$(1 - \|\tau(f)\|^2) \|\nabla_0 f\|_2^2 \ge c_d \|\tau(f)\|^2, \tag{1}$$

which was already known in the literature; see the discussion in [1] and references therein.

Since (4.14) no longer holds, an immediate question is whether the uncertainty principle in (4.11) and that in (1) are equivalent, assuming $\int_{\mathbb{S}^{d-1}} f(y) d\sigma(y) = 0$. That they are not equivalent is shown in the following proposition.

Proposition 1 For $n \ge 3$, let $Y \in \mathcal{H}_n^d$, a spherical harmonic of degree n on \mathbb{S}^{d-1} , and let Q be a polynomial of degree at most n - 2 such that $\int_{\mathbb{S}^{d-1}} Q(x) d\sigma = 0$. Assume that both $[Y(x)]^2$ and $[Q(x)]^2$ are even in every coordinate. Let

$$f = b(Y + Q)$$
, where $b^{-1} := ||Y + Q||_2 > 0$.

Then $\tau(f) = 0$. In particular, (1) becomes the trivial inequality $\|\nabla_0 f\|_2^2 \ge 0$, whereas (4.11) shows that $\|\nabla_0 f\|_2^2 \ge c > 0$.

Proof Since the degree of Q is at most n - 2, it follows from the orthogonality of Y and the even parity of Y^2 and Q^2 that

$$\int_{\mathbb{S}^{d-1}} x_i |f(x)|^2 d\sigma = \int_{\mathbb{S}^{d-1}} x_i \left(Y(x)^2 + 2Y(x)Q(x) + Q(x)^2 \right) d\sigma(x) = 0$$

for $1 \le k \le d$. Hence, $\tau(f) = 0$. By its definition, $||f||_2 = 1$, and, by the orthogonality of *Y* and the zero mean of *Q*, we see that $\int_{\mathbb{S}^{d-1}} f(x) d\sigma = 0$ so that (4.11) is applicable on *f*.

As a simple example of the function f, we can choose $Q(x) = x_1^k$ and $Y(x) = C_n^{\lambda}(x_1)$ for $x = (x_1, \dots, x_d) \in \mathbb{S}^{d-1}$, where $\lambda = (d-2)/2$ and $1 \le k \le n-2$.

Acknowledgments The authors thank Stefan Steinerberger for pointing out the mistake in Theorem 4.5. of [1].

Reference

 Dai, F., Xu, Y.: The Hardy–Rellich inequality and uncertainty principle on the sphere. Constr. Approx. 40, 141–171 (2014)