Mathematical Biology

CORRECTION

Correction to: Adaptive dynamics of saturated polymorphisms

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In the original publication, Proposition 4 is mistaken. The proposition considers models with two environmental feedback variables, E_1 and E_2, and two resident strategies with trait values x_1 and x_2. The original proposition claimed that if the invasion fitness $r(x, E_1, E_2)$ is uniformly monotonic in E_i ($i = 1$ or 2), then the equilibrium value of this same feedback as a function of the trait values, $\hat{E}_i(x_1, x_2)$, has a saddle point at a generic evolutionarily stable singularity. In fact, it is not $\hat{E}_i(x_1, x_2)$, but the *other* feedback that has a saddle point under the conditions stated in Proposition 4 of the original publication. The corrected proposition and its proof appear below. The error does not affect the rest of the article.

In most models that we encounter, the invasion fitness is uniformly monotonic in all feedback variables (this also pertains to the example presented in Section 2 of the original publication). In this case, both feedbacks have a saddle point and the correction of the proposition has no practical consequence; yet the proof needs to be corrected.

Proposition 4 *Suppose there are two environmental feedback variables and two coexisting resident strategies ($n = 2$). If the invasion fitness is uniformly monotonic in the environmental feedback variable E_1, then $\hat{E}_2(x_1, x_2)$ has a saddle point at a generic evolutionarily stable dimorphic singularity, and vice versa, if the invasion fitness is uniformly monotonic in E_2, then $\hat{E}_1(x_1, x_2)$ has a saddle point at a generic evolutionarily stable dimorphic singularity. Specifically, the environmental feedback at the saddle is maximized as a function of one trait value and minimized as a function of the other.*

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Proof For $n = 2$, equation (16) of the original article reads

$$\mathbf{A} \begin{bmatrix} \frac{\partial^2 \hat{E}_1}{\partial x_j \partial x_k} \\ \frac{\partial^2 \hat{E}_2}{\partial x_j \partial x_k} \end{bmatrix} = \begin{cases} -\partial_{11} r(x_j, \hat{E}_1, \hat{E}_2) \mathbf{e}_j & \text{if } k = j \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where $j, k \in \{1, 2\}$ and the matrix $\mathbf{A}$ has elements $A_{ij} = \partial_{j+1} r(x_i, \hat{E}_1, \hat{E}_2)$. In particular, with $k = j$, we have

$$\mathbf{A} \begin{bmatrix} \frac{\partial^2 \hat{E}_1}{\partial x_1^2} \\ \frac{\partial^2 \hat{E}_2}{\partial x_1^2} \end{bmatrix} = \begin{bmatrix} -\partial_{11} r(x_1, \hat{E}_1, \hat{E}_2) \\ 0 \end{bmatrix}$$

and

$$\mathbf{A} \begin{bmatrix} \frac{\partial^2 \hat{E}_1}{\partial x_2^2} \\ \frac{\partial^2 \hat{E}_2}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 0 \\ -\partial_{11} r(x_2, \hat{E}_1, \hat{E}_2) \end{bmatrix}$$

for $j = 1$ and for $j = 2$, respectively. Using Cramer's rule, we thus obtain

$$\frac{\partial^2 \hat{E}_1}{\partial x_1^2} = \frac{-\partial_{11} r(x_1, \hat{E}_1, \hat{E}_2) \cdot \partial_3 r(x_2, \hat{E}_1, \hat{E}_2)}{\det \mathbf{A}} \tag{1a}$$

$$\frac{\partial^2 \hat{E}_1}{\partial x_2^2} = \frac{\partial_{11} r(x_2, \hat{E}_1, \hat{E}_2) \cdot \partial_3 r(x_1, \hat{E}_1, \hat{E}_2)}{\det \mathbf{A}} \tag{1b}$$

and

$$\frac{\partial^2 \hat{E}_2}{\partial x_1^2} = \frac{\partial_{11} r(x_1, \hat{E}_1, \hat{E}_2) \cdot \partial_2 r(x_2, \hat{E}_1, \hat{E}_2)}{\det \mathbf{A}} \tag{2a}$$

$$\frac{\partial^2 \hat{E}_2}{\partial x_2^2} = \frac{-\partial_{11} r(x_2, \hat{E}_1, \hat{E}_2) \cdot \partial_2 r(x_1, \hat{E}_1, \hat{E}_2)}{\det \mathbf{A}} \tag{2b}$$

The second derivatives $\partial_{11} r(x_1, \hat{E}_1, \hat{E}_2)$ and $\partial_{11} r(x_2, \hat{E}_1, \hat{E}_2)$ are both negative at a generic evolutionarily stable singularity. If the invasion fitness is uniformly monotonic in E_2, then the factors $\partial_3 r(x_2, \hat{E}_1, \hat{E}_2)$ and $\partial_3 r(x_1, \hat{E}_1, \hat{E}_2)$ in Eqs. (1a) and (1b) have the same sign. Due to the minus sign that appears in (1a) but not in (1b), $\frac{\partial^2 \hat{E}_1}{\partial x_1^2}$ and $\frac{\partial^2 \hat{E}_1}{\partial x_2^2}$ have opposite signs. Together with Proposition 2 of the original article, this shows that $\hat{E}_1(x_1, x_2)$ is maximized as a function of one trait value and minimized as a function of the other, i.e., it has a saddle point at a generic evolutionarily stable dimorphic singularity. Similarly, if the invasion fitness is uniformly monotonic in E_1, then the factors $\partial_2 r(x_2, \hat{E}_1, \hat{E}_2)$ and $\partial_2 r(x_1, \hat{E}_1, \hat{E}_2)$ in Eqs. (2a) and (2b) have the same sign,

so that $\frac{\partial^2 \hat{E}_2}{\partial x_1^2}$ and $\frac{\partial^2 \hat{E}_2}{\partial x_2^2}$ have opposite signs; $\hat{E}_2(x_1, x_2)$ is therefore maximized as a function of one trait value and minimized as a function of the other. $\qquad \square$