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ERRATUM

Erratum to: Stable periodicity and negative circuits in differential systems

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1 Erratum to: J. Math. Biol. (2011) 63:593–600 DOI 10.1007/s00285-010-0388-y

In a private communication, Frederic Beck (University of Mainz) pointed out that, in the originally published article, when proving that Ω contains a stable periodic solution we use a wrong argument, the following: $(\partial f_i/\partial x_i)(x) < 0$ for all $x \in \Omega$, $i = 1, 2$. Indeed, for instance, if $x_1 \in]2-\varepsilon, 2+\varepsilon[$ and $x_2 \in \times[1+\varepsilon, 3-\varepsilon]\backslash]2-\varepsilon, 2+\varepsilon[$ then $(\partial f_i/\partial x_i)(x) = 4\varphi_2'(x_1) - 1$, and this term can not be less than zero for all $x_1 \in]2-\varepsilon, 2+\varepsilon[$ because φ_2' has to be greater than $1/2\varepsilon > 1$ somewhere in this region. So there are $x \in \Omega$ with $(\partial f_i/\partial x_i)(x) > 0$.

However, using slightly more involved arguments, we proved here that the system is still a counter-example of Conjecture $2'$. More precisely, we prove that if $\varepsilon \leq 1/8$ then there is a stable periodic solution in the domain $\Omega' = [0, 4]^2 \setminus \Gamma$, where Γ is the interior of the convex hull of the set containing the points $A = (1-\varepsilon, 3-\varepsilon)$, $B = (2-\varepsilon, 3+\varepsilon)$, $C = (3-\varepsilon, 3+\varepsilon)$, $D = (3+\varepsilon, 2+\varepsilon)$, $E = (3+\varepsilon, 1+\varepsilon)$, $F = (2+\varepsilon, 1-\varepsilon)$, $G = (1+\varepsilon, 1-\varepsilon)$, and $H = (1-\varepsilon, 2-\varepsilon)$; see Fig. 1 for an illustration.

First, since $\Omega' \subseteq \Omega$, there is no equilibrium point in Ω'. Suppose now that $\varepsilon \leq 1/8$, and let us prove that all the solutions starting in Ω' remain in Ω'. As showed in the originally published article, no solution starting in $[0, 4]^2$ leaves $[0, 4]^2$, thus it is

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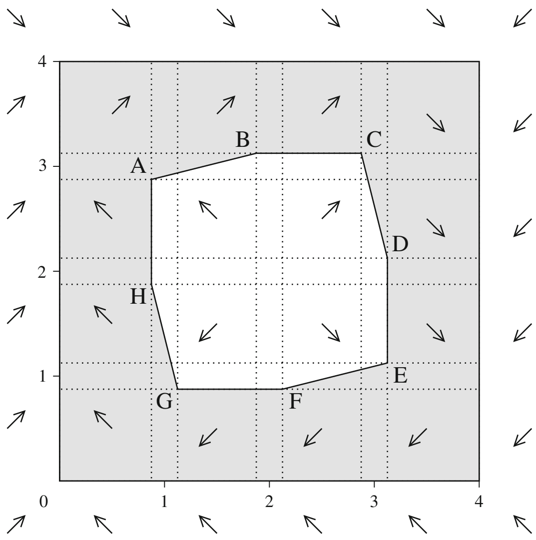

Fig. 1 The *gray region* is an illustration of Ω'

sufficient to prove that no solution starting in Ω' reaches the interior of the convex hull Γ. Consider first the line segment L with endpoints A and B. For all $x \in L$ we have

$$\begin{aligned} f_1(x) &= 4\varphi_3(x_2) - x_1 \leq 4 - x_1 \leq 3 + \varepsilon \\ f_2(x) &= 4 - x_2 \geq 1 - \varepsilon. \end{aligned}$$

Thus for all $x \in L$ the scalar product between $f(x)$ and the vector $v = (-2\varepsilon, 1)$ is at least $-2\varepsilon(3+\varepsilon) + (1-\varepsilon) = 1 - 7\varepsilon - 2\varepsilon^2$, and this term is positive since $\varepsilon \leq 1/8$. Since v is orthogonal to L and is pointing outside Γ, this means that if a solution starts in Ω', then it cannot reach Γ by crossing the line segment $L = AB$. Also, for all x that lies in the line segment BC we have $f_2(x) = 1 - \varepsilon > 0$. Thus if a solution starts in Ω', then it cannot reach Γ by crossing BC. Reasoning similarly with the segments CD, DE, EF, FG, GH, and HA, we deduce that all the solutions starting in Ω' remains in Ω'. Thus, following the Poincaré–Bendixon theorem, there exists a periodic solution ψ of period $T > 0$ starting in Ω'.

Finally, let us prove that ψ is stable. For all $x \in \mathbb{R}^2$ we have $(\partial f_1(x)/\partial x_1)(x) = 4\varphi_2'(x_1)(\varphi_1(x_2) - \varphi_3(x_2)) - 1$. Thus $(\partial f_1(x)/\partial x_1)(x) \geq 0$ implies $4\varphi_2'(x_1)(\varphi_1(x_2) - \varphi_3(x_2)) > 0$ which implies that x belongs to the domain $]2-\varepsilon, 2+\varepsilon[\times]1-\varepsilon, 3+\varepsilon[$ which is disjoint from Ω'. Thus $(\partial f_1/\partial x_1)(x) < 0$ for all $x \in \Omega'$, and we prove with similar arguments that $(\partial f_2/\partial x_2)(x) < 0$ for all $x \in \Omega'$. Thus

$$\int_0^T \frac{\partial f_1}{\partial x_1}(\psi(t)) + \frac{\partial f_2}{\partial x_2}(\psi(t))\mathrm{d}t < 0$$

and we deduce that ψ is stable.