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ERRATUM

Erratum to: Semigroups of max-plus linear operators

Boris Andreianov[1] · Marjeta Kramar Fijavž[2,3] · Aljoša Peperko[3,4] · Eszter Sikolya[5]

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Boris Andreianov determined that the examples of max-additive and max-plus linear semigroups in the last section of the article [6] are given inaccurately, i.e., [6, Proposition 4.1] is not true as stated and [6, Proposition 4.2] does not hold without some additional assumptions. Jointly we are able to correct the issues as follows.

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✉ Marjeta Kramar Fijavž
marjeta.kramar@fgg.uni-lj.si

Boris Andreianov
boris.andreianov@lmpt.univ-tours.fr

Aljoša Peperko
aljosa.peperko@fs.uni-lj.si

Eszter Sikolya
seszter@cs.elte.hu

1 LMPT CNRS UMR7350, Université de Tours, Parc Grandmont, 37200 Tours, France

2 Faculty of Civil and Geodetic Engineering, University of Ljubljana, Jamova 2, 1000 Ljubljana, Slovenia

3 Institute of Mathematics, Physics, and Mechanics, Jadranska 19, 1000 Ljubljana, Slovenia

4 Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, 1000 Ljubljana, Slovenia

5 Department of Applied Analysis and Computational Mathematics, ELTE TTK, Pázmány Péter sétány 1/C, 1117 Budapest, Hungary

In the case of conservation law (CL) studied in [6, Subsec. 4.1], a smaller set of weak solution should be considered. A function $v\colon [0,\infty) \to \mathrm{L}^1(\mathbb{R})$ is called an *isentropic solution* to (CL), if it satisfies the initial condition $v(0) = h$ and the Kružkov condition (14) from [6] with the equality sign.

We now give the proper formulation of [6, Proposition 4.1] and its proof. Let

$$\mathcal{C} := \{h \in \mathrm{L}^1(\mathbb{R}) \mid (CL) \text{ admits an isentropic solution } v \text{ with } v(0) = h\} \\ \cup \{-\infty\}.$$

Proposition ([6, Proposition 4.1]) *The semigroup* $\mathcal{T}^{\mathrm{CL}} := (T(t))_{t\geq 0}$, *where* $u(t,x) = T(t)h(x)$ *is the unique isentropic solution to* (CL) *and where* $T(t)(-\infty) := -\infty$, *is a max-additive strongly continuous semigroup on* $\mathcal{C}$.

Proof By [5, Theorem 1], $T(t)h_1 \oplus T(t)h_2$ is an isentropic solution to (CL) for any $h_1, h_2 \in \mathcal{C}$. Moreover, $T(t)h_1 \oplus T(t)h_2|_{t=0} = h_1 \oplus h_2$, hence $T(t)h_1 \oplus T(t)h_2$ is the isentropic solution to (CL) with initial condition $h = h_1 \oplus h_2 \in \mathcal{C}$. By uniqueness of the isentropic solutions, we obtain $T(t)h_1 \oplus T(t)h_2 = T(t)(h_1 \oplus h_2)$. □

Remark 1 The isentropic solutions to (CL) often exist only on some limited time interval. However, our results also hold in this case considering the local time flows $(T(t))_{t\in[0,T]}$ defined on

$$\mathcal{C}_T := \{h \in \mathrm{L}^1(\mathbb{R}) \mid (CL) \text{ admits an isentropic solution } v \text{ on } [0,T] \\ \text{with } v(0) = h\} \cup \{-\infty\}.$$

In the case of Hamilton–Jacobi equation (HJ) considered in [6, Subsec. 4.2], we should specify the choice of the generalized solutions and regularity assumptions. We assume the following for the function f appearing in the problem (HJ).

1. For every $x \in \mathbb{R}^n$, $\|x\| = 1$, there exists the limit $\lim_{r\downarrow 0} rf(x/r)$.
2. For any $(x,r), (x',r') \in \mathbb{R}^n \times \mathbb{R}_+$ with $\|x\|^2 + r^2 \leq 1$ and $\|x'\|^2 + r'^2 \leq 1$,

$$\left|rf(x/r) - r'f(x'/r')\right| \leq K\left(\|x - x'\|^2 + (r - r')^2\right)^{1/2},$$

   where $K > 0$ is some constant.
3. The function f is convex.

In the literature, there are many notions for generalized weak solutions to (HJ): Crandall–Lions' viscosity solutions, minimax solutions by Subbotin, Maslov idempotent weak solutions, Kružkov generalized solutions. However, under our assumptions they all agree, see [7, Sec. 5], and [6, Proposition 4.2] holds as stated in the paper. For the proof, we refer directly to [7, Theorem 3.2] (note that there the min-plus terminology is used; therefore, the concavity instead of convexity of f is assumed).

Remark 2 Stability of solutions of (HJ) (understood in the viscosity sense of Crandall–Lions) under the $\oplus$-operation is closely related to the properties of liminf of a sequence of viscosity solutions of first order Hamilton–Jacobi equations with convex Hamiltonians, see [2] and [1, Th. 2.1 and 2.3]. The fact that the maximum of two viscosity solutions is a viscosity sub-solution is classical, see [3]. Heuristically, the fact that the maximum of two viscosity solutions is also a viscosity super-solution stems from the semi-concavity property of viscosity solutions, see [1, p.1125].

Remark 3 We add a reference for yet another proof of the max-linearity of Hamilton–Jacobi–Bellman semigroup presented in Subsection 4.3. In [4], this fact is proved using a probabilistic approach.

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