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ERRATUM

Erratum to: Almost automorphic mild solutions to some classes of nonautonomous higher-order differential equations

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In this Note we correct some errors that occurred in our paper [T. Diagana, Almost automorphic mild solutions to some classes of nonautonomous higher-order differential equations. *Semigroup Forum*. **82**(3) (2011), 455–477].

I. It was recently found that Lemma 3.7 in Diagana [1], which is taken from the paper by Goldstein and N'Guérékata [2], contained an error. Indeed, the injection $BC^{1-\beta}(\mathbf{R}, \mathbf{X}_\alpha) \hookrightarrow BC(\mathbf{R}, \mathbf{X})$ as stated in both [1] and [2], is in fact not compact and that is crucial for the use of the Schauder fixed point to prove the existence of an almost automorphic solution to Eq. (3.1) appearing in [1]. The above-mentioned issue has been fixed in a recent Note by Goldstein and N'Guérékata [3]. The main objective of this paper is to correct Lemma 3.7 and slightly modify assumptions (H.4)–(H.5) to adapt Theorem 3.8 of [1] to this new setting.

A function $f \in BC(\mathbb{R}, \mathbb{X})$ is said to belong to $AA(\mathbb{X})$ [resp., $AA_u(\mathbb{X})$] if for every sequence of real numbers $(s'_n)_{n\in\mathbb{N}}$ there exists a subsequence $(s_n)_{n\in\mathbb{N}}$ such that

$$\lim_{n\to\infty} f(t+s_n) = g(t), \qquad \lim_{n\to\infty} g(t-s_n) = f(t)$$

pointwise on $\mathbb{R}$ [resp., uniformly on compacts of $\mathbb{R}$].

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In contrast with [1], here the space $BC^{\gamma}(\mathbf{R}, \mathbf{X}_{\alpha})$ will be viewed as a locally convex Fréchet space equipped with the following metric (see [3])

$$\Delta(f,g) = \sum_{n=1}^{\infty} 2^{-n} \frac{\rho_n(f,g)}{1+\rho_n(f,g)}$$

where, for $h = f - g$,

$$\Delta_n(f,g) = \Delta_n(h,0) = \|h\|_{C[-n,n]} + \gamma . \sup\left\{ \frac{\|h(t)-h(s)\|_{\alpha}}{|t-s|^{\gamma}} : t, s \in [-n,n], t \neq s \right\}.$$

Let $\mathrm{GN}_{\gamma}(\mathbb{R}, \mathbb{X}_{\alpha})$ be the locally convex Fréchet space $(BC^{\gamma}(\mathbb{R}, \mathbb{X}_{\alpha}), \Delta)$.

Lemma 3.7 in [1] should be replaced with

Lemma 0.1 *The set* $\mathrm{GN}_{1-\beta}(\mathbb{R}, \mathbb{X}_{\alpha})$ *is compactly contained in* $\mathrm{GN}_0(\mathbb{R}, \mathbb{X}_{\alpha})$, *that is, the canonical injection* $id : \mathrm{GN}_{1-\beta}(\mathbb{R}, \mathbb{X}_{\alpha}) \hookrightarrow \mathrm{GN}_0(\mathbb{R}, \mathbb{X}_{\alpha})$ *is compact, which yields*

$$id : \mathrm{GN}_{1-\beta}(\mathbb{R}, \mathbb{X}_{\alpha}) \cap AA_u(\mathbb{X}_{\alpha}) \hookrightarrow AA_u(\mathbb{X}_{\alpha})$$

is compact, too.

II. For the matrix $A_l(t)$ in Eq. (4.1) to be decomposed as $A_l(t) = K_l^{-1} J_l(t) K_l(t)$ as stated in [1], one has to suppose that each root ρ_k^l $(k = 1, 2, \ldots, n)$ of the polynomial $Q_t^l(\cdot)$ in page 457 is of multiplicity one.

III. Consider the following assumptions:

(h.4) $R(\omega, A(\cdot)) \in AA_u(B(\mathbb{X}_{\alpha}))$.

(h.5) The function $F : \mathbb{R} \times \mathbb{X}_{\alpha} \mapsto \mathbb{X}$ is such that $t \mapsto F(t,u)$ belongs to $AA_u(\mathbb{X})$ for all $u \in \mathbb{X}_{\alpha}$. The function $u \mapsto F(t,u)$ is uniformly continuous on any bounded subset K of $\mathbb{X}$ for each $t \in \mathbb{R}$. Finally,

$$\|F(t,u)\|_{\infty} \le \mathcal{M}\big(\|u\|_{\alpha,\infty}\big),$$

where $\mathcal{M} : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is a continuous, monotone increasing function satisfying

$$\lim_{r\to\infty} \frac{\mathcal{M}(r)}{r} = 0.$$

In view of the above, Theorem 3.8 in [1] should be replaced with

Theorem 0.2 *Suppose assumptions* (H.1)–(H.2)–(H.3)–(h.4)–(h.5) *hold, then the nonautonomous differential equation* (3.1) *has a mild solution which belongs to* $AA_u(\mathbb{X}_{\alpha})$.

References

1. Diagana, T.: Almost automorphic mild solutions to some classes of nonautonomous higher-order differential equations. Semigroup Forum **82**(3), 455–477 (2011)
2. Goldstein, J.A., N'Guérékata, G.M.: Almost automorphic solutions of semilinear evolution equations. Proc. Am. Math. Soc. **133**(8), 2401–2408 (2005)
3. Goldstein, J.A., N'Guérékata, G.M.: Corrigendum on "Almost automorphic solutions of semilinear evolution equations". Proc. Am. Math. Soc. **140**, 1111–1112 (2012)