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Correction

Correction to: Uniqueness of the Representation in Homogeneous Isotropic LQC

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We correct some oversights contained in [1].

(1) "Campiglia" was accidentally misspelled at several points in the paper. The authors apologize for this.

(2) The authors forgot to provide the definition of the characters [first used in equation (9)]

$$\chi_\lambda \colon \mathbb{R} \mapsto \mathbb{C}, \quad t \mapsto \mathrm{e}^{\mathrm{i}\lambda t} \qquad \forall\, \lambda \in \mathbb{R};$$

which are the generators of the almost periodic functions $C_{\mathrm{AP}}(\mathbb{R})$ on $\mathbb{R}$.

(3) As noticed in [2], in the continuity part of Sect. 4 it had been overseen that, instead of $\nu \circ \varphi \in C_0(\mathbb{R})$, we have $\nu \circ \varphi \in \mathrm{span}_{\mathbb{C}}(\chi_0) + C_0(\mathbb{R})$ for $\varphi \in C_0(\mathbb{R})$ and $\nu \colon (-1, 1) \ni t \mapsto \sqrt{1+t}$. This issue can be fixed, of course, by adding "-1" in the definition of ν. We then have $\sqrt{1+\varphi} \in \mathfrak{D}$ (instead of $\sqrt{1+\varphi} \in \mathfrak{d}$, as wrongly stated in the paper) – This, however, makes no difference for our conclusion that $1 + \omega_{\mathfrak{d}}(\varphi) \geq 0$ holds, because ω is defined on full $\mathfrak{D}$. The revised argumentation—starting after Eq. (14) on page 238, and ending after the mentioned conclusion—then reads as follows.

Revised argumentation:
To see this, first observe that the smooth function

$$\nu \colon (-1, 1) \ni t \mapsto \sqrt{1+t} - 1$$

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(and each of its derivatives) can be represented by a power series. Thus, by completeness of $\overline{\mathfrak{d}}$, for $\varphi \in \overline{\mathfrak{d}}$ real valued with $\|\varphi\|_\infty < 1$, we have $\nu \circ \varphi \in \overline{\mathfrak{d}}$ as well as

$$\nu^{(n)} \circ \varphi \in \mathrm{span}_{\mathbb{C}}(\chi_0) + \overline{\mathfrak{d}} \qquad \forall\, n \geq 1.$$

Then, for $\varphi \in \mathfrak{d}$, the chain rule gives

$$\partial_t(\nu \circ \varphi) = (\dot{\nu} \circ \varphi) \cdot \dot{\varphi} \in \overline{\mathfrak{d}}.$$

Now, the right hand side is differentiable; and, applying the same arguments inductively, we find that $(\nu \circ \varphi)^{(n)} \in \overline{\mathfrak{d}}$ holds for each $n \in \mathbb{N}$; hence,

$$\nu \circ \varphi = \sqrt{1 + \varphi} - \chi_0 \in \mathfrak{d}.$$

We thus have $\varphi_\nu := \sqrt{1+\varphi} \in \mathfrak{D}$ with

$$1 + \omega_{\mathfrak{d}}(\varphi) = \omega(\mathbb{1} + \widehat{\varphi}) = \omega(\widehat{\varphi}_\nu^* \widehat{\varphi}_\nu) \geq 0.$$

(4) In the equations (24) and (25), the γ-factor was misplaced. More specifically,

- Equation (24) must read

$$\sum_{i=1}^{3} \frac{V_0[\mathcal{C}]}{k\gamma} \frac{|v_1 v_2 v_3|}{v_i} \frac{\mathrm{d}}{\mathrm{d}x^{\mathbf{0}}} \left(\gamma\, \omega_{\mathbf{i}}^{i0} - \frac{1}{2} \epsilon^{ijk} \omega_{\mathbf{i}jk} \right). \tag{24}$$

- Equation (25) must read

$$c^i := \gamma\, \omega_{\mathbf{i}}^{i0} - \frac{1}{2}\epsilon^{ijk}\omega_{\mathbf{i}jk}, \quad p_i := \frac{V_0[\mathcal{C}]}{k\gamma} \frac{|v_1 v_2 v_3|}{v_i}. \tag{25}$$

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References

1. Engle, J., Hanusch, M., Thiemann, Th.: Uniqueness of the representation in homogeneous isotropic LQC. Commun. Math. Phys. **354**, 231–246 (2017)
2. Fleischhack, Ch.: Continuity of States on Non-Unital Differential Algebras in Loop Quantum Cosmology. e-print: arXiv:1803.08944 [math-ph]

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