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*Erratum*

# Erratum to: Multidimensional Potential Burgers Turbulence

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In the original article we overlooked that the space $L_\infty/\mathbb{R}$ on which we consider the dual-Lipschitz metric is not separable: thus we are not in the standard setting in which stationary measures for stochastic PDEs are studied (cf. [1]). Moreover, since $C^\infty$ is not dense in $L_\infty$, the semigroup $S_t^\omega$ is not well-defined on the space of functions in $L_\infty$ defined modulo an additive constant $L_\infty/\mathbb{R}$. Thus Lemma 8.4. does not hold.

However, we can consider the separable space of continuous functions modulo an additive constant $C^0/\mathbb{R}$. In this setting, the semigroups $S_t^\omega$ and $S_t^*$ are well-defined. The proof of the following result is almost word-for-word the same as the proof of the corresponding 1d $L_1$-nonexpansion result for the stochastic Burgers equation in [2].

**Lemma.** *There exist positive constants $C'$, $\delta$ such that for $\psi_1^0, \psi_2^0 \in C^0$ we have*

$$\mathbf{E}|S_t^\omega \psi_1^0 - S_t^\omega \psi_2^0|_{C^0/\mathbb{R}} \leq C' t^{-\delta}, \quad t \geq 1. \tag{1}$$

*In particular, these constants do not depend on $\psi_1^0, \psi_2^0$.*

For all $\omega$ the solution of the stochastic Burgers equation is $C^\infty$-smooth in space for $t > 0$: this is proved in Appendix 1. This allows us to define the semigroups $\tilde{S}_t^\omega$ and $\tilde{S}_t^*$, acting respectively on $L_1$ and on the space of probability measures on $L_1$. Indeed, first we consider two solutions $\psi_1, \psi_2$ to the stochastic Hamilton–Jacobi equation with the same noise and different smooth initial conditions, as well as the corresponding solutions

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$\mathbf{u}_1, \mathbf{u}_2$ to the stochastic Burgers equation. By the Gagliardo–Nirenberg inequality we get

$$|\mathbf{u}_1 - \mathbf{u}_2|_1 \leq C|\psi_1 - \psi_2 - \int_{\mathbf{T}^d} (\psi_1 - \psi_2)|_1 |\nabla(\psi_1 - \psi_2)|_{1,1}$$
$$\leq C|\psi_1 - \psi_2 - \int_{\mathbf{T}^d} (\psi_1 - \psi_2)|_\infty |\mathbf{u}_1 - \mathbf{u}_2|_{1,1}.$$

Thus, using Theorem 6.2. and the lemma stated above we obtain the existence of a $u_1^0, u_2^0$-independent constant $C'$ such that:

$$\mathbf{E}|\tilde{S}_t^\omega \psi_1^0 - \tilde{S}_t^\omega \psi_2^0|_{L_1} \leq C' t^{-\delta/2}, \quad t \geq 1, \tag{2}$$

with the same $\delta$ as above. These inequalities allows us first to prove by density of $C^\infty$ in $L_1$ that $\tilde{S}_t^\omega$ and $\tilde{S}_t^*$ are well-defined respectively on $L_1$ and on the space of probability measures on $L_1$, and then to obtain Theorem 8.5.

As a conclusion, while Lemma 8.4. does not hold, all other results remain valid.

## References

1. Kuksin, S., Shirikyan, A.: Mathematics of two-dimensional turbulence. In: Cambridge Tracts in Mathematics, vol. 194. Cambridge University Press, Cambridge (2012)
2. Boritchev, A.: Sharp estimates for turbulence in white-forced generalised Burgers equation. GAFA **23**(6), 1730–1771 (2013)

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