Math. Z. (2015) 279:1225–1226
DOI 10.1007/s00209-015-1427-6

Mathematische Zeitschrift

ERRATUM

Erratum to: Pseudo-Anosov extensions and degree one maps between hyperbolic surface bundles

Michel Boileau · Yi Ni · Shicheng Wang

Published online: 15 January 2015

**Erratum to: Math. Z. (2007) 256:913–923
DOI 10.1007/s00209-007-0113-8**

In the original publication, the proof of Lemma 4 was incorrect and it is now corrected.

In [1, Lemma 4], we constructed two homotopic pinches between closed surfaces, such that the boundary circles of the two regions we pinched are not homotopic. However, as pointed out by Tao Li, our construction in Case 1 in the original proof is not correct. In this note, we will provide a new proof of this lemma.

We start by recalling some notations from [1]. If C is a subpolyhedron of a manifold M, let $N(C)$ be a regular neighborhood of C in M. If M is a compact manifold, let $\mathrm{int}(M)$ be the interior of M. Suppose that $g_s > g_t \geq 1$ are two integers. Let F_s, F_t be two closed oriented surfaces with genus g_s, g_t, respectively. Fix a disk $D \subset F_t$, let $V = F_t - \mathrm{int}(D)$. Suppose that $p_0, p_1 : F_s \to F_t$ are two pinches. Namely, there exist two compact subsurfaces $V_0, V_1 \subset F_s$, such that p_j maps V_j homeomorphically to V, and p_j maps $W_j = F_s - \mathrm{int}(V_j)$ into D, $j = 0, 1$. Let $e_j : V \hookrightarrow F_s$ be the inverse of p_j.

Lemma 4 *With the notation above, there exist two pinches $p_0, p_1 : F_s \to F_t$ such that*

(i) *p_0 and p_1 are homotopic;*
(ii) *$e_0(\partial D)$ is not homotopic to $e_1(\partial D)$.*

The online version of the original article can be found under doi:10.1007/s00209-007-0113-8.

M. Boileau
Institut de Mathématiques de Marseille, Aix Marseille Université, Marseille, France
e-mail: michel.boileau@cmi.univ-mrs.fr

Y. Ni (✉)
Department of Mathematics, Caltech, Pasadena, CA 91125, USA
e-mail: yini@caltech.edu

S. Wang
LMAM, Department of Mathematics, Peking University, Beijing 100871, China
e-mail: wangsc@math.pku.edu.cn

Proof Let $h = g_s - g_t$, S be a closed oriented surface of genus $h+1$. As in [1, Figure 2], we can choose three simple closed curves α, β_0, $\beta_1 \subset S$, such that α intersects β_j transversely at exactly one point, $j = 0, 1$, and β_0 is homologous to β_1. Moreover, let $\gamma_j = \partial N(\alpha \cup \beta_j)$, then γ_0 is not freely homotopic to γ_1. We may further assume that $\alpha \cap \beta_0 = \alpha \cap \beta_1$, and a singular two-chain Φ connecting β_0 and β_1 is disjoint from α except in a small neighborhood of $\alpha \cap \beta_0$.

Let S' be the surface obtained by removing an open disk about $\alpha \cap \beta_0$ from S, and let $\alpha' = \alpha \cap S'$, $\beta'_j = \beta_j \cap S'$. Clearly, β'_0 and β'_1 are homologous in S' relative to $\partial S'$. So we can find a properly embedded surface $B \subset S' \times [0, 1]$, such that $B \cap (S' \times \{j\}) = \beta'_j \times \{j\}$, $j = 0, 1$. We may assume that $B \cap (\alpha' \times [0, 1]) = \emptyset$, since B can be chosen to be a lift of $\Phi \cap S'$, and we have assumed that Φ is disjoint from α except in a small neighborhood of $\alpha \cap \beta_0$.

Let T be a torus, $\xi, \eta \subset T$ be two simple closed curves which intersect transversely at exactly one point. Let T' be the surface obtained by removing an open disk about $\xi \cap \eta$ from T, and let $\xi' = \xi \cap T'$, $\eta' = \eta \cap T'$.

Now we construct a map $Q : S' \times [0, 1] \to T'$ in three steps:

Step 1. Construct

$$Q_1 : \big(\partial S' \times [0, 1]\big) \cup \big(\alpha' \times [0, 1]\big) \to \partial T' \cup \xi'$$

by first projecting to $\partial S' \cup \alpha'$, then map $\partial S' \cup \alpha'$ homeomorphically to $\partial T' \cup \xi'$.

Step 2. Extend Q_1 to a map

$$Q_2 : \big(\partial S' \times [0, 1]\big) \cup \big(\alpha' \times [0, 1]\big) \cup B \to \partial T' \cup \xi' \cup \eta'.$$

This can be achieved by first mapping $\beta'_j \times \{j\}$ homeomorphically to η', $j = 0, 1$, then send the interior of B to η' by using the contractibility of η'.

Step 3. Extend Q_2 to a map

$$Q_3 : N\left(\big(\partial S' \times [0, 1]\big) \cup \big(\alpha' \times [0, 1]\big) \cup B\right) \to N\left(\partial T' \cup \xi' \cup \eta'\right),$$

then send the closure of

$$\big(S' \times [0, 1]\big) \setminus N\left(\big(\partial S' \times [0, 1]\big) \cup \big(\alpha' \times [0, 1]\big) \cup B\right)$$

to the closure of

$$T' \setminus N\left(\partial T' \cup \xi' \cup \eta'\right)$$

using the contractibility of the target, and we can get the map Q we want.

Let Σ be a compact surface of genus $g_t - 1$ with exactly one boundary component. Let P_0 be the projection from $\Sigma \times [0, 1] \to \Sigma$. Gluing the two maps P_0 and Q together, we get a map $P : F_s \times [0, 1] \to F_t$, which is a homotopy connecting two pinches $p_0, p_1 : F_s \to F_t$. Clearly, Condition(ii) in the statement of this lemma is also satisfied.

Acknowledgments We are grateful to Tao Li for pointing out the mistake. The second author was partially supported by NSF Grant No. DMS-1252992 and an Alfred P. Sloan Research Fellowship.

Reference

1. Boileau, M., Ni, Y., Wang, S.: Pseudo-Anosov extensions and degree one maps between hyperbolic surface bundles. Math. Z. **256**, 913–923 (2007)