EDITORIAL

Supermodularity and Complementarity in Economic Theory

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Abstract

This special issue brings together eight separate contributions reflecting recent advances in the methodology of supermodular optimization and games. Three of the papers fall in the main area of supermodular games. Balbus et al. (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-017-1075-7) study supermodular games with a continuum of players. Jimenez-Martinez (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-018-1107-y) develops a model of versioning in social networks. Barthel and Hoffman (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-017-1092-6) deal with a class of games with mixed-monotonic best responses (in both directions). The second group of papers falls in the related areas of mechanism design, principal agent, and matching. Johnson (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-018-1127-7) considers synchronized Becker-style matching with incomplete information. Kushnir and Liu (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-018-1124-x) generalizes the equivalence between Bayesian and dominant strategy implementation to the case of nonlinear utilities. Zambrano (Econ Theory 2019, https://doi.org/10.1007/s00199-017-1087-3) studies a principal-agent model with choice between a risky and a safe projects. Christensen (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-018-1116-x) investigates the stability, existence, and uniqueness of equilibria, as solutions to systems of equations. Finally, Drugeon et al. (Econ Theory 67, 2019, https://doi.org/10.1007/s00199-018-1166-0) consider a class of dynamic programming problems with endogenous discount factor.

Keywords Complementarity · Supermodular games · Submodular games · Becker matching · Monotone comparative statics · Correspondence principle

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1 Introduction

The novel methodology of supermodular optimization and games now permeates several different strands of the literature in pure and applied economic theory.[1] The main characteristic of this methodology is that it relies essentially on critical assumptions on a model's primitives to establish the desired monotone comparative statics (henceforth, MCS). In so doing, it dispenses with superfluous assumptions that are often imposed only to facilitate the use of the classical method, based on the implicit function theorem. The latter typically requires smoothness, interiority, and concavity conditions. The main insight of this new methodology is indeed quite elementary. If, in a maximization problem, the objective function reflects complementarity between an endogenous variable and an exogenous parameter, in the sense that having more of one increases the marginal returns to having more of the other, then the optimal value of the former will be increasing in the latter. In the case of multiple endogenous variables, then all of them must also be complements in order to guarantee that their increases are mutually reinforcing. This conclusion follows directly from the underlying complementarity relationship and is thus independent of the other, superfluous, assumptions. It thus holds even if there are multiple optimal values of the endogenous variables.

The most widely applicable advance due to this framework of analysis is the theory of supermodular games, better known in economics as games with strategic complementarities, introduced by Topkis (1979). The main characteristic of these games is that they possess increasing reaction correspondences (that need not be continuous), reflecting a complementarity relationship between own actions and rivals' actions. As a consequence of Tarski's fixed-point theorem, the latter property guarantees the existence of pure-strategy Nash equilibrium points. Because the latter type of equilibrium is most often the desired concept in economic models, the scope of game-theoretic modeling in economics is thereby substantially enlarged. Another key observation in this respect is that supermodularity is again the relevant notion behind the monotone comparative statics of Nash equilibrium points. Furthermore, it will be argued that supermodular games are more conducive to predictable comparative statics properties than games with continuous best responses, the latter being the other class of games that always admit pure-strategy Nash equilibrium points.

This special issue brings together eight separate contributions reflecting recent advances in the methodology of supermodular optimization and games. While three of the papers fall in the main area of supermodular games, they deal with a variety of novel features in this literature.[2] Balbus et al. (2019) extend some of the basic insights of supermodular games to the case of a continuum of players. Jimenez-Martinez (2019) develops a model of versioning with advertising in social networks. Finally, Barthel and Hoffman (2019) deal with a class of games with mixed-monotonic best responses (in both directions).

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[1] For comprehensive treatments of the general theory and its applications, see Topkis (1998) and Vives (1999).

[2] Early applications are described in Vives (1990, 1999), Milgrom and Roberts (1990) and Amir (1996a, c), among others.

The second group of papers falls in the related areas of mechanism design, principal agent, and matching. Johnson (2019) considers synchronized Becker-style matching with incomplete information. Kushnir and Liu (2019) generalizes the equivalence between Bayesian and dominant strategy implementation to the case of nonlinear utilities. Zambrano (2019) studies a principal-agent model with choice between a risky and a safe projects.

Going back to some aspects of the methodology itself, Christensen (2019) investigates the stability, existence, and uniqueness of equilibria, as solutions to systems of equations. Finally, Drugeon et al. (2019) consider a class of dynamic programming problems with endogenous choice of discount factor.

2 A qualitative theory of large games with strategic complementarities

Balbus et al. (2019) study equilibria in large games with strategic complementarities, in which the payoff of an individual agent depends inherently on the entire distribution of actions and characteristics of other players. This paper brings together the well-known literature on supermodular games started with the seminal works of Topkis (1978, 1979), and the more specialized literature dealing with non-cooperative games with a continuum of players (see e.g., Mas-Colell 1984 and many others). The authors exploit a minimal supermodularity structure to extend the existence and equilibrium characterization results, earlier known only for games with a small number of players, to their large counterparts. From a technical perspective, in their key construction, they show that the space of distributions on players' actions and characteristics, ordered with respect to first-order stochastic dominance, is a partially ordered set. The authors provide conditions on payoffs that imply that the appropriate best response operator is isotone. These prerequisites allow them to prove existence of a greatest and least distributional Nash equilibrium (see Mas-Colell 1984), under a different set of sufficient conditions than those in the existing literature. In addition, in line with the lattice-theoretic approach used, the authors provide constructive methods for computing the extremal equilibria by their successive approximation starting from the extremal elements of the space of all feasible distributions of player actions and "traits".

The authors' approach and methods are general. Their framework allows not only for the study of semi-anonymous games with traits (Khan et al. 2013), but also games that are have no aggregative structure (Guesnerie and Jara-Moroni 2011). What should be also highlighted, in addition to the absence of the customary aggregation assumption, is that the proposed novel methods are applicable to economic problems in which strategies are non-monotone with respect to players' characteristics. When doing so, the authors are also able to extend some recent monotone comparative statics results, for large games proposed by Acemoglu and Jensen (2010, 2013), among others. Last but not least, the reader may appreciate the section of their paper dealing with economic applications, which illustrates the wide scope of application of the underlying framework to quite a few interesting examples from the broader economic literature. The novel applications include games of social distance, large stopping games, or keeping-up-with-the-Joneses models.

3 Discrimination through versioning with advertising in social networks

Jimenez-Martinez ([2019](#)) explores second-degree price discrimination over a service (say, online video streaming) provided by a monopolistic platform to consumers connected by a social network. The utility that each consumer receives from the service increases with the number of direct neighbors in the network. The platform is able to increase the network (positive) externalities by offering a premium version of the service. Consumers must then choose between the free version, which includes ads about some unrelated good (say, a new car model), and the premium version, which is costly to the consumer, includes no ads, and augments the utility increase that the consumer receives from each neighbor. The trade-off of the platform is then whether to rely on its profits from the advertising activity or from its sales of the premium version of the service. The trade-off of each consumer is whether to incur a cost in terms of ads exposure and lower network externalities, or to pay a price and enjoy higher externalities. Whether the platform has incentives to offer only the free version, only the premium version, or both versions, as well as its optimal pricing policy, can be related to the shape of the hazard rate distribution of the social network. The welfare implications of the second-degree discriminatory optimal policy are characterized by first-order stochastic dominance over networks.

The proposed model is closely related, and owes much, to the influential theoretical benchmark of supermodularity and complementarity. On the one hand, the premium version of the service increases, in a complementary way, the utility increase per neighbor in the network. On the other hand, advertising is assumed to inform consumers about their own valuations for the good and thus rotate clockwise consumers' demand. Building upon the developments of Lewis and Sappington ([1994](#)) and Johnson and Myatt ([2006](#)), this informative role of advertising reflects complementarities between the consumers' choices over the service and their endogenous valuations for the good. Furthermore, under the perspective of Bayesian supermodular games proposed by Van-Zandt and Vives ([2007](#)), the role of advertising considered in this paper parallels the convexity assumption for information structures proposed by Amir and Lazatti ([2016](#)). Finally, another connection with supermodularity insights comes from the persuasion literature (Kamenica and Gentzkow [2011](#)). The informative role of advertising can be viewed as a choice about committed information disclosure. Thus, the analysis of second-degree discrimination relates crucially to the key convexity condition on the value of information for an information designer to have incentives to provide an uninformed party with additional information.

4 Rationalizability and learning in games with strategic heterogeneity

Games in which players exhibit monotone best responses have been the focus of extensive study and describe two common interactions between players. Games of strategic complements or GSC (resp. strategic substitutes or GSS) describe situations where players exhibit monotone increasing (resp. decreasing) best responses. Grounded in

order-theoretic tools developed by Topkis (1998), exploiting these two seemingly simple properties offers numerous advantages in the analysis of many economic settings. For example, in the context of GSC, Milgrom and Roberts (1990) show a surprising connection between learning in games and rationalizability: As long as players learn to play adaptively, resulting play will eventually fall within the highest and lowest Nash equilibria, which themselves bound all rationalizable strategies. Roy and Sabarwal (2012) show that the same relationship holds in GSS with the caveat that the bounding strategies, although serially undominated, need not be Nash equilibria.

However, little attention has been given to a more general class Barthel and Hoffman (2019) call games of strategic heterogeneity (GSH). In a GSH, players are allowed to best respond in an either increasing or decreasing way to the actions of others, and hence encompass both GSC and GSS as special cases. Instances of such settings include oligopolistic markets containing both price and quantity competitors, models of crime and punishment, resource conservation, and games on networks, among others. Well-known versions of these models are developed as special cases of the framework in the paper.

However, allowing for both increasing and decreasing best responses leads to a non-monotone joint best response correspondence, which complicates analysis. Nevertheless, results analogous to those for GSC extend to GSH as well. In particular, the existence of highest and lowest serially undominated strategies which bound the limit of all adaptive learning processes is proved, which leads to the main result: In a GSH, a game is dominance solvable if and only if all adaptive learning processes converge to the unique Nash equilibrium. Lastly, a game without the monotonicity requirements of a GSH may still be analyzed as such as long as it can be "monotonically embedded" into a GSH, thus extending the scope of the monotonicity analysis.[3]

5 Synchronized matching with incomplete information

Since Becker (1973), supermodularity assumptions have played a key role in deriving the structure of efficient allocations and stable equilibria in one-to-one matching models. When the match surplus is supermodular in each partner's quality, the efficient outcome is positive assortative matching, where the best agents are matched together, the second best together, and so on. Hoppe et al. (2009) extend Becker's insight in a setting where agents have private information about their quality, but can invest in a costly and public signal. Supermodularity of the match surplus guarantees that higher types have higher returns to signaling in any Bayesian equilibrium, and positive assortative matching is then implementable. In many cases, however, paying for a mate might be considered immoral, and ostentatious displays of money burning might be considered uncouth.

Johnson (2019) asks, if agents discount future payoffs but their types are private information, can they signal their quality by delaying the time at which they agree to match, and what patterns of matching are implementable? When only one side has

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[3] It can also be shown that other oligopoly models (e.g., Vives 1990; Amir and Lambson 2000; Hoernig 2003; Cosandier et al. 2018; Reynolds and Rietzke 2018) may fit this setting under suitably modified assumptions.

private information, the surplus for that side must satisfy the stronger condition of log supermodularity for positive assortative matching to be implementable, similar to Shimer and Smith (2000). When information is private on both sides, all types must agree on exactly the same matching schedule, which is satisfied only when the match surpluses on the two sides exhibit the same growth rate from partner quality for those types matched in equilibrium. These are much stronger sufficient conditions than those of similar static models and seem unlikely to hold in general. One might think that coarse matching (McAfee 2002; Damiano and Li 2007), in which the market is divided into assortative sub-markets in which matching occurs randomly, might create positive correlation between matched types. However, Johnson provides conditions for a coarse matching to be incentive compatible and shows that under reasonable conditions, the only implementable coarse matching is uniformly random. This illustrates how incorporating dynamics and match timing can complicate or even reverse classical results in one-to-one matching models.

6 On the equivalence of Bayesian and dominant strategy implementation for environments with nonlinear utilities

A recent important result in the mechanism design literature establishes that one can often insist on the robustness of the mechanisms, i.e., restrict attention to more robust dominant strategy incentive-compatible mechanisms, without sacrificing the objectives of the designer. In particular, in linear environments with private and one-dimensional types, for any Bayesian incentive-compatible mechanism, one could always find an equivalent dominant strategy incentive-compatible mechanism that yields the same expected utilities to all agents and generates the same social surplus (Manelli and Vincent 2010; Gershkov et al. 2013).

Kushnir and Liu 2019 significantly expands the scope of this equivalence result to include nonlinear agent utilities. To accomplish this, an extension of the standard property of increasing differences (or supermodularity condition; see Topkis 1998) to the space of lotteries is proposed: *increasing differences over distributions*. This is shown to be a necessary and sufficient condition on agents' utilities for Bayesian incentive-compatible mechanisms to be conveniently characterized by a monotonicity condition and an envelope formula. The functions that satisfy this novel property are characterized. Roughly, a function of two arguments satisfies the property of increasing differences over distributions if and only if it can be decomposed into the multiplication of single-argument functions. Thus, this novel property is closely related to the utility independence assumption originally considered in decision theory (Fishburn 1974; Keeney and Raiffa 1976).

The authors demonstrate the value added of their general results by providing a diverse set of possible economic applications to recent models, including the principal-agent problem with allocative externalities, environmental mechanism design, and the provision of public goods. These applications were carefully selected in the sense that existing results would be of limited usefulness in tackling these models at a high level of generality. This paper reinforces in interesting ways the existing links between

incentive theory and mechanism design on the one hand, and the methodology of supermodularity and complementarity on the other hand.

7 Motivating informed decisions

Zambrano (2019) studies a principal-agent framework where a risk-neutral principal delegates to a risk-neutral agent the decision of whether to pursue a risky project or a safe one. The return from the risky project is unknown, and the agent can acquire costly unobservable information about it before taking the decision. The problem has features of moral hazard and hidden information since the acquisition of information and its content is unobservable to the principal.

The principal-agent setup under consideration is quite general, the only restriction imposed being that signals are ordered in the likelihood ratio order. Under risk neutrality, limited liability, and payoff monotonicity for the principal, optimal wages are option-like with a strike price greater than the safe return. Moreover, it is optimal to distort the project choice in favor of the risky project as a strategy to reveal the uncertainty directly.

The optimal contract suggests that the principal should only reward the agent for outcomes that are significantly better than the safe return. It is also optimal to distort the project choice in favor of the risky one as a mechanism to induce the direct revelation of the uncertain state. In terms of implications for the theory of organizations, the findings explain why compensation options induce better decision-making from CEOs, as well as why excessive risk-taking might be optimal. These features are likely to be of broad relevance in various settings, in particular in managerial contexts.

The theory of incentives has incorporated notions of supermodularity from its very inception, e.g., in the definition of the single-crossing property. In this paper, the supermodularity notions emerge through the usual channels of partial orders on probability distributions commonly used in information economics, broadly defined. Specifically, the monotone likelihood ratio order amounts to the log supermodularity of the parameterized probability density function, induces first-order stochastic dominance, and leads to a suitably monotonic updating process (see, e.g., Athey 2002).

8 Comparative statics and heterogeneity

In nice environments, the equilibrium (solution) $x \in X \subset \mathbb{R}^n$ of an economic model (decision problem) is characterized implicitly by n smooth nonlinear equations $\frac{\partial f_i(x,t)}{\partial x_i} = 0$ for $i = 1, \ldots, n$. Denote this system $\nabla f(x, t)$. If the parameter t is complementary to the endogenous variables, Christensen (2019) shows that monotone comparative statics (MCS) are assured at equilibrium if the Jacobian $D_x \nabla f(x, t)$ satisfies

$$\frac{1}{n} \sum_{j=1}^{n} \frac{\partial^2 f_i(x,t)}{\partial x_i \partial x_j} \leq \min \left\{ 0, \frac{\partial^2 f_i(x,t)}{\partial x_i \partial x_j} | j \neq i \right\} \quad \text{for} \quad i = 1, \ldots, n. \tag{1}$$

Moreover, Christensen (2019) demonstrates in Theorems 2–3 how this condition can be used to prove stability, existence, and uniqueness. Using recent results in linear algebra that are new to the economics literature, this study develops novel insights and results for MCS that are rooted in the implicit function theorem and apply in models where the interaction effects are not uni-signed (also see Beggs 2018).

When $f(x, t)$ is supermodular in x, meaning $\frac{\partial^2 f_i}{\partial x_i \partial x_j} \geq 0$ $i \neq j$, $i = 1, \ldots, n$, Christensen and Cornwell (2018) have shown that, under some additional conditions, stability is equivalent to monotone comparative statics.[4] In this case condition (1) is too strong. In models where $f(x, t)$ is not supermodular in x, the usual strategy to obtain MCS is to impose diagonal dominance on the Jacobian in combination with other conditions which vary by application (Dixit 1986 or Vives 1999). Condition (1) is distinct from diagonal dominance and has the advantage of being sufficient for MCS by itself; no additional conditions are required. Section 3.1 in Christensen (2019) offers a comparison of condition (1) to diagonal dominance and illustrates how condition (1) captures a trade-off between the heterogeneity and magnitude of endogenous interactions in maintaining MCS.

The scope of application of this method to economic models is quite broad. For the consumer's problem, condition (1) yields a sufficient condition under which all goods are normal. In contrast to existing sufficient conditions (Chipman 1977; Quah 2007; Barthel and Sabarwal 2018), this one does not require complementarity. Condition (1) also improves upon some of the conditions for MCS in oligopoly models (Dixit 1986). Finally, condition (1) provides an alternative to gross substitutability in general equilibrium models and allows some or all goods to be complements.

9 On maxmin dynamic programming and the rate of discount

In this article, Drugeon et al. (2019) develop an extended dynamic programming argument for a maxmin optimization problem where, in addition to the usual maximization over the set of feasible policies, the agent/planner also undertakes a minimization operation of a normalized reward functional over an exogenously given set of discount factors. Even though the consideration of such a maxmin criterion results in a program that is in general neither convex nor stationary over time, the authors prove that a careful reference to extended dynamic programming principles and to a maxmin functional equation however allows for circumventing these difficulties and recovering an optimal sequence that is time-consistent.

In order to accomplish this, the structure needed on the primitives of the family of Ramsey-type dynamic programming problems includes the customary concavity and regularity conditions on the one-period utility function and the feasibility correspondence, as well as a supermodularity condition on the former and an ascending condition on the latter. The non-customary, lattice-theoretic part of this structure guarantees in particular the key property of monotone optimal paths (Amir 1996b).

The main result of the paper is thus the derivation for this problem of a Bellman-like functional equation where the usual maximum operation is coupled with a minimiza-

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[4] See Echenique (2002) for a lattice-theoretic version of this result.

tion over the set of discount factors, thereby reflecting the stationary nature of the optimal solution.

The class of problems at hand relates to the recent decision-theoretic literature that addresses in an axiomatic way the scope for multiple optimal discount rates in dynamic models, e.g., Chambers and Echenique (2018). Under some monotonicity, homogeneity, constant additivity and, most importantly, convexity and stability assumptions on a preference order, the evaluation of the inter-temporal utility assumes a representation that directly echoes the above considerations.

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