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Optimal block designs for diallel crosses

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Abstract Dey and Midha (Biometrika 83(2):484–489, 1996) constructed optimal block designs for complete diallel cross experiment using triangular partially balanced incomplete block designs with two associate classes. They listed optimal block designs for the lines in the range from $5 \leq v \leq 10$. In this paper, we are also proposing additional optimal block designs for complete diallel cross experiment using two associate class partially balanced block designs for some additional values of v. Our method yields designs for proper and non-proper settings for complete diallel cross experiments. The proper and non proper designs are optimal in the sense of Kempthorne (Genetics 41:451–459, 1956) and non-proper designs are universally optimal in the sense of Kiefer (A survey of statistical design and linear models, North Holland, Amsterdam, 1975). The list of practically important designs is also given.

Keywords Partially balanced incomplete block design · Complete diallel cross · General combining ability · Mating–environment design · Auxiliary design · Efficiency

1 Introduction

A diallel cross consists of all possible crosses between a numbers of varieties. Reciprocal crosses and the selfed parents may or may not be omitted. Diallel crosses as a mating design is used to study the genetic properties of inbred lines in plant breeding experiments.

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Among the four types of diallels discussed by Griffing (1956), system IV is the most commonly used in plant breeding. Suppose there are v lines and let us consider a cross of the form $i \times j$ with $i < j = 1, \ldots, v$. With all possible $n_c = v(v-1)/2$ crosses. This is sometimes referred to as the modified or half-diallel. We shall refer to it as a complete diallel cross (CDC).

The common practice with diallel cross experiment is to evaluate the crosses in completely randomized designs or randomized complete block designs as environment designs, e.g. Kempthorne and Curnow (1961). Due to limitation of homogeneous experimental units in a block to accommodate all the chosen crosses, the estimate of genetic parameters would not be precise enough if a complete block design was adopted for large number of crosses. To overcome this problem, many researchers used balanced incomplete block (BIB) designs, partially balanced incomplete block (PBIB) designs with two associate classes etc. by treating the crosses as treatments. These designs have interesting optimality properties when making inferences on a complete set of orthonormalised treatment contrasts. However, in diallel cross experiments the interest of the experimenter is in making comparisons among general combining ability (gca) effects of lines and not crosses and therefore, using these designs as mating designs may result into poor precision of the comparison among lines. Further, the analysis of a diallel cross experiment in incomplete block depends on the incidence of lines rather than the incidence of the crosses as treatments with in a block. It is therefore apparent that special techniques are required to obtain good designs for diallel crosses experiments.

Several authors such as Gupta and Kageyama (1994), Dey and Midha (1996), Mukerjee (1997), Das et al. (1998), Parsad et al. (1999) and Sharma (2004) addressed the problem of finding optimal designs by using nested incomplete block designs (NBIB), triangular PBIB designs, nested balanced block (NBB) designs, GD PBIB designs and circular designs, etc. Gupta and Kageyama (1994) and Sharma (2004) reported optimal designs in which every cross is replicated once but their designs differ in their parametric values with proposed designs. Das et al. (1998) and Parsad et al. (1999) reported optimal designs for single as well more replications. Dey and Midha (1996) reported optimal and efficient designs in which the crosses are replicated in the range $3 \leq r \leq 10$. These designs also differ in parametric values of our proposed designs.

We, in this paper, derive additional incomplete block designs for the same mating designs, using two associate class PBIB designs such that none of the λ's is zero. We have also listed these designs for reasonable practically usable values along with designs reported by these authors. The model considered involves only the gca effects .The specific combining ability (sca) effects being excluded from the model because the derived designs are not connected for cross effects. The paper is structured as: (1) the method of construction of designs is presented in Sect. 2, (2) in Sect. 3 analysis and optimality of the designs is considered and we show that the non-proper designs have strong optimality properties, (3) in Sect. 4, the efficiency factor of both designs (proper and non-proper) as compared to randomized block designs is considered. For definition and properties of PBIB design, see Dey (1986)

2 Method of construction

The method is simply stated as: from tables of Clatworthy (1973), take for v lines under evaluation and numbered randomly, a two associate PBIB design with parameters $v = b, r = k, \lambda_1, \lambda_2, n_1, n_2, p^i_{jk}$ $(i, j, k = 1, 2)$ having none of the λ's equal to zero and also with the property that any pair of treatments does not occur more than once in any column of the design when we consider block size k as a blocks. We call this design as an auxiliary design. This feature of the auxiliary design helps us to construct a block design for CDC.

Now in auxiliary design, take all possible distinct pair of combinations of treatments in each block, starting from the first treatment of the block. These give $k(k-1)/2$ pairs per block. Thus we get resulting design in which $bk(k-1)/2$ pairs arranged in b blocks, each containing $k(k-1)/2$ pairs. Now in resulting block design identify these pairs of treatments as crosses by treating treatments of the original design as lines. Now we consider in resulting design, the number of plots $(= k(k-1)/2)$ as blocks and number of blocks $(b = v)$ as the block size of each $k(k-1)/2$ blocks. We call this arrangement as mating design and denote it as d. Since λ's are unequal, it leads to unequal repetition of the crosses in design d. The total number of crosses in mating design d, is $bk(k-1)/2$ and out of these $vn_i/2$ crosses are appearing in $n_i\lambda_i/2$ blocks $(i = 1, 2)$. Since replications of crosses are unequal, it makes mating design unbalanced for CDC experiment. To make the design balanced for CDC experiment, we will have to make the replications of the crosses equal because in balanced designs each elementary contrast among gca effects is estimated with equal precision under the assumptions of homogeneous error variance across all blocks. To do this, if $\lambda_1 > \lambda_2$ we delete $\frac{1}{2}[n_1(\lambda_1 - \lambda_2)]$ blocks, (or, if $\lambda_2 > \lambda_1$ then $\frac{1}{2}[n_2(\lambda_2 - \lambda_1)]$) from $k(k-1)/2$ blocks in design d.

In some auxiliary designs, for v even lines, if $(\lambda_1 - \lambda_2)$ (or $(\lambda_2 - \lambda_1)$) is odd, then n_1 (or n_2) is also odd, then the expression of number of blocks to be deleted, will not be a positive integer but it will be equal to some positive integer ± 0.5. So in this case we will have to delete blocks equal to some value of positive integer and 0.5 fraction of one of the block which contains repeated crosses i.e. number of crosses to be deleted are equal to $v\times$ (value of the integer $\pm\ v/2$). The process of deletion of blocks will be done with the help of association schemes of auxiliary designs (i.e PBIB designs). Now we may classify our auxiliary designs into two classes (1) where $[n_1(\lambda_1 - \lambda_2)]$ and $[n_2(\lambda_2 - \lambda_1)]$ are both positive even integers and (2) where $[n_1(\lambda_1 - \lambda_2)]$ and $[n_2(\lambda_2 - \lambda_1)]$ are both odd positive integers. We denote both these auxiliary designs as $d_{(1)}$ and $d_{(2)}$.

Thus the process of deletion of blocks of repeated crosses will yield two types of mating–environment designs (proper and non-proper) for CDC experiments with parameters.

(i) $v_1 = v(v-1)/2, b_1 = \lambda_2(v-1)$ if $\lambda_1 > \lambda_2$ (or $\lambda_1(v-1)$ if $\lambda_2 > \lambda_1$), $r_1 = \lambda_2$ (or λ_1), $k_1 = v$

(ii) $v_2 = v(v-1)/2, b_2 = \lambda_2(v-1)$ if $\lambda_1 > \lambda_2$(or $\lambda_1(v-1)$ if $\lambda_2 > \lambda_1$), $r_2 = \lambda_2$ (or λ_1), $k_2 = (v, v/2)$

Plan (auxiliary design $d_{(1)}$)

B_1	1	2	4
B_2	2	3	5
B_3	3	4	1
B_4	4	5	2
B_5	5	1	3

Treatment	First associate	Second associate
1	2, 5	3, 4
2	1, 3	4, 5
3	2, 4	1, 5
4	3, 5	1, 2
5	1, 4	2, 3

Design d

B_1	B_2	B_3
1×2	1×4	2×4
2×3	2×5	3×5
3×4	3×1	4×1
4×5	4×2	5×2
5×1	5×3	1×3

Design d_1 (mating–environment design)

B_1	B_2
1×2	1×4
2×3	2×5
3×4	3×1
4×5	4×2
5×1	5×3

Plan (auxiliary design $d_{(2)}$)

B_1	1	2	4
B_2	2	3	5
B_3	3	4	6
B_4	4	5	1
B_5	5	6	2
B_6	6	1	3

Treatment	First associate	Second associate
1	4	2, 3, 5, 6
2	5	1, 3, 4, 6
3	6	1, 2, 4, 5
4	1	2, 3, 5, 6
5	2	1, 3, 4, 6
6	3	1, 2, 4, 6

Design d

B_1	B_2	B_3
1×2	1×4	2×4
2×3	2×5	3×5
3×4	3×6	4×6
4×5	4×1	5×1
5×6	5×2	6×2
6×1	6×3	1×3

Design d_2 (mating–environment design)

B_1	B_2	B_3
1×2	1×4	2×4
2×3	2×5	3×5
3×4	3×6	4×6
4×5		5×1
5×6		6×2
6×1		1×3

We denote both designs as d_1 and d_2. The method of construction is illustrated below by two examples.

Example 1 For illustration, we consider design C12 (Clatworthy 1973) with parameters $v = b = 5$, $r = k = 3$, $n_1 = n_2 = 2$, $\lambda_1 = 1$ and $\lambda_2 = 2$. The plan and association scheme of the design is given below:

Since $\lambda_2 = 2$, the crosses (1×3), (1×4), (2×4), (2×5) and (3×5) appeared in both blocks 2 and 3 of design d. So we will delete one of the blocks to obtain M–E design d_1.

Example 2 For second type of design, we consider design R 42 (Clatworthy 1973) with parameters $v = 6$, $r = 3$, $k = 3$, $b = 6$, $m = 3$, $n = 2$, $\lambda_1 = 2$ and $\lambda_2 = 1$.

Since $\lambda_1 = 2$, the crosses (1×4), (2×5), and (3×6) appeared repeatedly in block 2 of design d. So we will delete these crosses from block 2 to keep the replication same for all crosses. Thus we obtain d_2.

3 Analysis and optimality

For convenience in further discussion, we will denote both designs d_1 and d_2 as d^*.The data obtained from the design d^*, we take the following model:

$$\mathbf{Y} = \mu\mathbf{1_n} + \mathbf{\Delta}_1\mathbf{g} + \mathbf{\Delta}_2\boldsymbol{\beta} + \mathbf{e} \tag{3.1}$$

where $\mathbf{Y}$ is the $n \times 1$ observational vector, μ is the general mean, $\mathbf{1}_n$ denotes an n-component vector of 1's, $\mathbf{g} = (g_1, \ldots, g_v)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_b)$ are the vector of v gca effects and $b(= b_1 = b_2)$ block effects respectively. $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ are the corresponding design matrices of of $n \times v$ and $n \times b$, respectively; that is (s, i)th element of $\mathbf{\Delta}_1$ is 1 if the cross in the sth experimental unit has one parent i and is 0 otherwise. Similarly (s, u)th element of $\mathbf{\Delta}_2$ is 1 if the cross in the sth experimental unit comes from uth block and 0 otherwise. $\mathbf{e}$ is a random vector of error components and takes care of specific combining ability as well as unassignable variation and distributed with mean 0 and constant variance σ^2.

For $1 \leq i < j \leq v$, let g_{d^*ij} is the number of times cross $(i \times j)$ occurs in d^*. Let s_{d^*i} be the number of times the ith line occurs in design d^*.

Following Gupta and Kageyama (1994), it can be shown that the information matrix for $\mathbf{g}$ under d^* is

$$\mathbf{C}_{d^*} = \mathbf{G}_{d^*} - v^{-1}\mathbf{N}_{d^*}\mathbf{N}_{d^*} \tag{3.2}$$

where $\mathbf{\Delta}_1'\mathbf{\Delta}_1 = \mathbf{G}_{d^*} = (\mathbf{g}_{d^*ij}), \mathbf{g}_{d^*ii} = s_{d^*i}$ and $\mathbf{\Delta}_1'\mathbf{\Delta}_2 = \mathbf{N}_{d^*} = (n_{d^*ij})$ is the $v \times b$ matrix of parental lines versus blocks, n_{d^*ij} is the number of times line i occurs in block u.

Under the model, the reduced normal equations for gca, using design d^*, are

$$\mathbf{C}_{d^*}\hat{\mathbf{g}} = \mathbf{Q} \tag{3.3}$$

where $\mathbf{Q} = \mathbf{T} - v^{-1}\mathbf{N}_{d^*}\mathbf{B}$. Here $\mathbf{T}$ is the vector of line total and $\mathbf{B}$ is the vector of block totals. Following Dey and Midha (1996) we now have the following results which will help in obtaining $\mathbf{C}_{d^*}$-matrix for both types designs i.e. d^*.

Lemma 3.1 *For the design d^*, the following are true.*

(i)
$$\sum_{u=1}^{b} n_{il} = \lambda_1(n_1 + n_2) = \lambda_1(v-1) \quad \textit{if}\ \lambda_2 > \lambda_1$$
$$= \lambda_2(n_1 + n_2) = \lambda_2(v-1) \quad \textit{if}\ \lambda_1 > \lambda_2$$
$$\sum_{i=1}^{v} n_{il} = 2v$$

(ii) $$\sum_{u=1}^{b} n_{il}^2 = 2\lambda_1(n_1+n_2) = 2\lambda_1(v-1) \quad \textit{if}\, \lambda_2 > \lambda_1$$

$$= 2\lambda_2(n_1+n_2) = 2\lambda_2(v-1) \quad \textit{if}\, \lambda_1 > \lambda_2$$

(iii) $$\sum_{u=1}^{b} n_{il}\, n_{i'l} = 2\lambda_1(n_1+n_2) = 2\lambda_1(v-1) \quad \textit{if}\, \lambda_2 > \lambda_1$$

$$= 2\lambda_2(n_1+n_2) = 2\lambda_2(v-1) \quad \textit{if}\, \lambda_1 > \lambda_2$$

The proofs of all identities are easy are omitted.

From Lemma (3.1), it follows that, for the design d^*, $\mathbf{C}_{d^*}$ is given by

$$\mathbf{C}_{d^*} = \theta\left(\mathbf{I}_v - v^{-1}\mathbf{1}_v\mathbf{1'}_v\right) \tag{3.4}$$

where $\theta = \lambda_1(v-2)$ or $\lambda_2(v-2)$, $\mathbf{I}_v$ is the identity matrix of order v and $\mathbf{1}_v$ is the v component vector of all 1's.

From (3.4) it is easy to see that generalized inverse of $\mathbf{C}_{d^*}$ is.

$$\mathbf{C}_{d^*}^{-} = \theta^{-1}\mathbf{I}_v \tag{3.5}$$

It is obvious from (3.5) that d^* is a variance balanced and therefore all elementary contrasts among gca effects are estimated under the assumption of homogeneous error variance across all blocks with a variance $2\sigma^2/\theta$. Hence design d^* is connected and has rank equal to $v-1$. Since design $d_{(2)}$ has blocks of unequal block sizes, therefore the error variance of $d_{(2)}$ will not be the same as of design $d_{(1)}$. Let σ_1^2 and σ_2^2 be the error variances of designs $d_{(1)}$ and $d_{(2)}$, respectively. Now we can interpret that all elementary contrasts among gca effects in designs $d_{(1)}$ and $d_{(2)}$ are estimated with a variances $\frac{2\sigma_1^2}{\theta}$ and $\frac{2\sigma_2^2}{\theta}$, respectively. Further the adjusted sum of squares due to gca effects is simply $\theta^{-1}\mathbf{Q'Q} = \theta^{-1}(Q_1^2 + \ldots + Q_v^2)$, where $i = 1, 2 \ldots v$, Q_i is the adjusted total of the ith line; that is Q_i is the ith component of the vector $\mathbf{Q}$, defined in (3.3). We thus have the following result.

Theorem 3.1 *The design d^* (i.e. d_1 and d_2) is variance—balanced for general combining ability effects.*

Now we take the optimality aspects. The optimality criterion is the minimization of average variance of the best linear unbiased estimators of all elementary comparisons between gca effects.

Let $\mathbf{D}$ $(v, b, k_1, \ldots, k_b)$ denote the class of all connected block designs d^* with v lines, b blocks such that jth block is of size k_j. Similarly $\mathbf{D}_0$ (v, b, n) denote the class of all connected block designs d^* with v lines, b blocks and n experimental units. Here the block sizes are arbitrary but for a given design $d^*\varepsilon\mathbf{D}_0(v, b, n)$, the block sizes are equal.

Now using the following theorem (Parsad et al. 1999, page 41).

Theorem 3.2 *Let* $d^* \in \mathbf{D}(v, b, k_1, \ldots, k_b)[\mathbf{D}_0(v, b, n)]$ *be a block design for diallel crosses and suppose that* d^* *satisfies*

(i) Trace $(C_{d^*}) = 2\sum_{j=1}^{b} k_j - \sum_{j=1}^{b} \frac{1}{k_j}[2k_j(2x_j+1) - vx_j(x_j+1)]$ if and only if $n_{d^*ij} = x_j$ or $x_j + 1$ for all $i = 1, \ldots, v$; $j = 1, \ldots, b$, where x_j is a positive integer. And for $n_{d^*ij} = 0$ or 1, this reduces to [Trace $(C_{d^*}) = 2(n-b)$], where n and b are total number of experimental units and number of blocks in diallel cross design $[\mathbf{D}_0(v, b, n)]$, respectively.
(ii) C_{d^*} is completely symmetric.

Then d^* is universally optimal over $\mathbf{D}(v, b, k_1, \ldots, k_b)[\mathbf{D}_0(v, b, n)]$

For any member of $\mathbf{D}(v, b, k_1, \ldots, k_b)$ $[\mathbf{D}_0(v, b, n)]$, we have Trace $(C_{d^*}) = \lambda_2(v-1)(v-2)$ if $\lambda_1 > \lambda_2$ or $\lambda_1(v-1)(v-2)$ if $\lambda_2 > \lambda_1$ and for any member $\mathbf{D}_0(v, b, n)$ to be universally optimal, the Trace (C_{d^*}) must be equal to $2(n-b)$, where $2(n-b) = \lambda_2(v-1)^2$ if $\lambda_1 > \lambda_2$ (or $\lambda_1(v-1)^2$ if $\lambda_2 > \lambda_1$). The equality of Trace (C_{d^*}) follows for any member of $\mathbf{D}$ $(v, b, k_1, \ldots, k_b)$ but for any member of $[\mathbf{D}_0(v, b, n)]$, the Trace (C_{d^*}) is less than $2(n-b)$.

Hence we have the following theorem.

Theorem 3.3 *The designs* d_2 *obtained from* $d_{(2)}$, *a two associate PBIB designs, where none of the* λ*'s is zero, are universally optimal over* $\mathbf{D}(v, b, k_1, \ldots, k_b)$.

4 Efficiency factor

Now we will show that the designs $d_{(1)}$ and $d_{(2)}$ are optimal in the sense of Kempthorne (1956). If instead of the designs $d_{(1)}$ and $d_{(2)}$, one adopts a randomized complete block design with $r_1(r_2)$ blocks, each block having all the $v(v-1)/2$ crosses , the $\mathbf{C}$ matrix of the randomized block design i.e. $\mathbf{C_R}$-matrix can easily shown to be, see Dey and Midha (1996).

$$\mathbf{C_R} = r_i(v-2)(\mathbf{I}_v - v^{-1}\mathbf{1}_v\mathbf{1}'_v) \quad \text{where } i = 1 \text{ or } 2 \tag{4.1}$$

Hence the variance of the best linear unbiased estimator of any elementary contrast among gca effects in the case of randomized block experiment is $2\sigma^2/r_i(v-2)$, where σ^2 is the per observation variance. Thus the efficiency factors e_1 and e_2, respectively of the design $d_{(1)}$ and $d_{(2)}$, relative to randomized complete block designs under the assumption of equal intrablock variances is given by

$$e_1 = \theta/r_1(v-2) = r_1(v-2)/r_1(v-2) = 1 \tag{4.2}$$

and

$$e_2 = r_2(v-2)/r_2(v-2) = 1 \tag{4.3}$$

where $\theta = r_1(v-2)$ (or $r_2(v-2)$) for design $d_{(1)}$ (or $d_{(2)}$).

We are also giving the list of practicable useful designs for $5 \le v \le 30$ in Table 1.

Table 1

Sl. no.	Ref. no. of the design	No. of lines	No. of blocks to be deleted	No. of crosses to be deleted	No of replication of the crosses (r)	Total no. of experimental units	Design names reported by different authors
1	S2	6	1	3	2	30	F2, Parsad et al. (1999) and Das et al. (1998, Th. 4.1)
2	S19	8	1	4	4	112	N
3	S23	9	3	9	3	108	F2, Parsad et al. (1999)
4	S29	12	4	12	2	132	N
5	S33	14	2	7	2	182	N
6	S42	21	5	21	1	210	Ser. 1, Gupta and Kageyama (1994) and Sharma (2004)
7[a]	S44	26	2.5	13	1	325	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
8	S52	10	1	5	6	270	N
9	S60	14	2	7	4	364	N
10	S68	20	9	30	2	380	N
11	S72	26	3	13	2	650	N
12	S90	21	6	21	3	630	F2, Parsad et al. (1999)
13	S93	30	7	30	2	870	N
14	S99	12	1	6	8	528	N
15	S104	15	10	30	5	525	F2, Parsad et al. (1999)
16[a]	S105	18	2.5	9	5	765	F2, Parsad et al. (1999)
17	S111	22	3	11	4	924	F2, Parsad et al. (1999)
18	S115	30	3	60	2	870	N
19	SR9	9	3	27	3	108	F2, Parsad et al. (1999)
20	SR68	12	4	48	2	132	N
21[a]	R42	6	0.5	3	1	15	Ser.2, Gupta and Kageyama (1994) and Sharma (2004)
22	R94	6	1	6	2	30	F2, Parsad et al. (1999)
23	R104	9	2	9	1	36	F1, Parsad et al. (1999) and F5, Das et al. (1998)

Table 1 continued

Sl. no.	Ref. no. of the design	No. of lines	No. of blocks to be deleted	No. of crosses to be deleted	No of replication of the crosses (r)	Total no. of experimental units	Design names reported by different authors
24[a]	R109	12	0.5	6	1	66	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
25	R133	8	3	12	2	56	N
26	R134	8	3	24	2	56	N
27	R137	9	2	9	2	72	F4, Das et al. (1998)
28	R139	10	1	5	2	90	F2, Parsad et al. (1999)
29[a]	R145	12	4.5	54	1	66	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
30	R166	10	6	20	2	90	F2, Parsad et al. (1999)
31	R168	15	8	30	1	105	F1, Gupta and Kageyama (1994) and Sharma (2004)
32	R170	27	2	27	1	351	F1, Parsad et al. (1999) and F5, Das et al. (1998)
33[a]	R171	28	1.5	42	1	378	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
34	R172	9	1	9	5	180	F2, Parsad et al. (1999)
35	R173	12	10	30	5	330	F2, Parsad et al. (1999)
36[a]	R174	12	4.5	54	3	198	F2, Parsad et al. (1999)
37	R175	12	10	30	2	132	F2, Parsad et al. (1999)
38[a]	R176	12	4.5	54	3	198	N
39[a]	R177	14	1.5	7	3	273	N
40[a]	R178	18	12.5	45	1	153	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
41	R179	20	2	40	2	380	N
42	R180	20	2	10	2	380	N
43[a]	R186	12	0.5	6	5	330	F2, Parsad et al. (1999)

Table 1 continued

Sl. no.	Ref. no. of the design	No. of lines	No. of blocks to be deleted	No. of crosses to be deleted	No of replication of the crosses (r)	Total no. of experimental units	Design names reported by different authors
44	R187	14	15	42	2	210	N
45	R188	21	18	63	1	210	Ser. 1, Gupta and Kageyama (1994) and Sharma (2004)
46	R189	24	5	60	2	552	N
47	R193	12	3	18	6	396	N
48	R194	15	8	30	4	420	F2, Parsad et al. (1999)
49	R195	16	21	56	2	240	F2, Parsad et al. (1999)
50	R196	18	2	9	4	612	N
51	R196	18	2	9	4	612	N
52[a]	R198	24	24.5	84	1	276	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
53	R200	28	9	84	2	756	F2, Parsad et al. (1999)
54	R204	14	6	42	6	546	F2, Parsad et al. (1999)
55	R205	14	6	84	6	546	F2, Parsad et al. (1999)
56	R206	18	28	81	2	756	F2, Parsad et al. (1999)
57	R207	27	32	108	1	351	F1, Parsad et al. (1999) and F5, Das et al. (1998)
58[a]	T33	10	1.5	15	1	45	Ser. 2, Gupta and Kageyama (1994) and Sharma (2004)
59	T58	10	3	45	2	90	F2, Parsad et al. (1999)
60[a]	T60	10	1.5	15	3	135	N
61	T61	15	8	60	1	105	Ser. 1, Gupta and Kageyama (1994) and Sharma (2004)
62	T71	10	3	15	4	180	N
63	T84	15	8	60	4	420	F2, Parsad et al. (1999)
64	T94	21	15	105	3	630	F2, Parsad et al. (1999)

Table 1 continued

Sl. no.	Ref. no. of the design	No. of lines	No. of blocks to be deleted	No. of crosses to be deleted	No of replication of the crosses (r)	Total no. of experimental units	Design names reported by different authors
65	T95	21	5	105	4	820	N
66	LS26	9	2	18	1	36	F1, Parsad et al. (1999) and F5, Das et al. (1998)
67	LS49	9	2	18	2	72	F4, Das et al. (1998)
68	LS72	9	2	18	2	72	F4, Das et al. (1998)
69	LS83	16	6	48	2	240	F2, Parsad et al. (1999)
70	LS99	16	9	72	3	360	F2, Parsad et al. (1999)
71	LS100	16	6	48	3	360	F2, Parsad et al. (1999)
72	LS101	25	4	100	2	600	N
73	LS116	16	6	48	4	480	N
74	LS117	25	12	100	2	600	N
75	C12	5	1	5	1	10	F1, Parsad et al. (1999) and F5, Das et al. (1998)
76	C23	13	3	39	2	156	N
77	C24	13	3	39	3	234	N
78	C25	29	7	203	1	406	F1, Parsad et al. (1999) and F4, Das et al. (1998)
79	C26	17	4	68	3	408	F2, Parsad et al. (1999)
80	C27	29	7	203	1	406	F1, Parsad et al. (1999) and F5, Das et al. (1998)
81	C28	17	4	68	4	544	N
82	C29	13	3	39	7	468	F2, Parsad et al. (1999)
83	M34	16	6	48	4	480	N
84[a]	M38	26	7.5	195	3	975	F2, Parsad et al. (1999)
85	M39	27	32	216	1	351	F1, Parsad et al. (1999) and F5, Das et al. (1998)

N new, *Ser.* series, *F* family
[a]Universally optimal designs

5 Conclusion

Optimal block designs with proper and non-proper settings have been proposed for CDC system IV using some PBIB designs. The non-proper setting designs found to be universally optimal in the sense of Kiefer (1975) and proper and non-proper setting designs are optimal in the sense of Kempthorne (1956). These designs retain full efficiency for the estimation of the contrast of interest. We investigated 85 PBIB designs (Clatworthy 1973). Out of which 20 and 29 PBIB designs gave block designs for CDC experiment in which each cross is replicated once and twice, respectively and in rest of the designs each cross is replicated more than twice. These designs are in the range of $5 \leq v \leq 30$, except for $v = 11, 17, 19, 22$, and 23.

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