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Erratum

Erratum to: Excitation Spectrum of Interacting Bosons in the Mean-Field Infinite-Volume Limit

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We correct the assumptions on the interaction potential v in the original article. We also present a careful discussion of the periodization of a potential—the discussion contained in the original article was not quite precise.

1. Conditions on Potentials

In the first section of the original article we stated that the *interaction potential* v is an even real function on $\mathbb{R}^d$ satisfying the following assumptions:

1. $v \geq 0$,
2. $v \in L^1(\mathbb{R}^d)$,
3. $\hat{v} \geq 0$,
4. $\hat{v} \in L^1(\mathbb{R}^d)$,

where $\hat{v}$ denotes the Fourier transform given by $\hat{v}(\mathbf{p}) = \int_{\mathbb{R}^d} v(\mathbf{x})\, \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{x}}\, \mathrm{d}\mathbf{x}$. Unfortunately, these assumptions seem not sufficient for the proof of the main result of the original article. However, all the arguments of that paper are correct if we replace the condition (4) by the condition

$$(4') \qquad \text{there exists}\, C \text{ such that, for } L \geq 1, \quad \frac{1}{L^d} \sum_{\mathbf{p} \in \frac{2\pi}{L}\mathbb{Z}^d} \hat{v}(\mathbf{p}) \leq C.$$

The online version of the original article can be found under doi:10.1007/s00023-013-0302-4.

Note that, even though (4) and (4′) are closely related, neither of these conditions implies the other one.

Some readers may complain that (4′) looks somewhat complicated. Therefore, we give yet another condition, which looks easier and which implies (4′):

(4″) there exists C and $\mu > d$ such that $|\hat{v}(\mathbf{p})| \leq C(1+|\mathbf{p}|)^{-\mu}$.

In fact, (2) implies that $\hat{v}$ is continuous and (4″) implies that $\hat{v} \in L^1$. Then an easy argument involving Riemann sums and the Lebesgue Dominated Convergence Theorem yields

$$\lim_{L\to\infty} \frac{1}{L^d} \sum_{\mathbf{p}\in \frac{2\pi}{L}\mathbb{Z}^d} \hat{v}(\mathbf{p}) = (2\pi)^{-d} \int \hat{v}(\mathbf{p})\mathrm{d}\mathbf{p}. \tag{1.1}$$

Clearly, (4′) is an immediate consequence of (1.1).

2. Periodization of Potentials

One of the concepts used in the original article is the *periodization* of a potential. Below we would like to give a discussion of this concept which is somewhat more careful from the one contained in the original article.

Suppose that $v \in L^1(\mathbb{R}^d)$ and $L > 0$. Then the following formula

$$v^L(\mathbf{x}) = \sum_{\mathbf{n}\in\mathbb{Z}^d} v(\mathbf{x} + \mathbf{n}L) \tag{2.1}$$

defines a function on $\mathbb{R}^d$ periodic with respect to the lattice $L\mathbb{Z}^d$ and satisfying $v^L \in L^1([-L/2, L/2[^d)$, which will be called the *periodization of* v *with period* L (see [1, sect. 4.2.1]).

Suppose now in addition that

$$\sum_{\mathbf{p}\in \frac{2\pi}{L}\mathbb{Z}^d} |\hat{v}(\mathbf{p})| < \infty. \tag{2.2}$$

Then the *Poisson summation formula* shows that

$$v^L(\mathbf{x}) = \frac{1}{L^d} \sum_{\mathbf{p}\in \frac{2\pi}{L}\mathbb{Z}^d} \mathrm{e}^{\mathrm{i}\mathbf{p}\mathbf{x}}\, \hat{v}(\mathbf{p}). \tag{2.3}$$

(See, e.g. [2, Thm. 2.4] or [1, Sect. 4.2.2].)

In the original article we used (2.3) to define v^L. Strictly speaking, this was not a mistake, at least under the conditions (1), (4′) (which clearly imply (2.2) for all L), since then the definitions (2.3) and (2.1) are equivalent. However, one can argue that the definition (2.1) is more natural and slightly more general and thus we should have used it in the original article.

In the original article we wrote that $v^L(\mathbf{x}) \to v(\mathbf{x})$ as $L \to \infty$. The meaning of that statement can be the following: if $v \in L^1(\mathbb{R}^d)$ and I is a compact subset of $\mathbb{R}^d$, then $v^L \to v$ in $L^1(I)$. In fact, let $I \subset [-L_0/2, L_0/2[^d$. Then for $L > L_0$ we have

$$\int_I |v(\mathbf{x}) - v^L(\mathbf{x})|\mathrm{d}\mathbf{x} \leq \sum_{n\in\mathbb{Z}^d\setminus\{\mathbf{0}\}} \int_I |v(\mathbf{x}+nL)|\mathrm{d}\mathbf{x} \xrightarrow[L\to\infty]{} 0.$$

Note also that $v \in L^1(\mathbb{R}^d), v \geq 0$ and (2.1) imply immediately that

$$v(\mathbf{x}) \geq 0 \Rightarrow v^L(\mathbf{x}) \geq 0.$$

We use this fact in the Proof of Lemma 4.1 of the original article.

References

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