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ERRATUM

Erratum to: A characterization of elementary abelian 2-groups

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**Erratum to: Arch. Math. 102 (2014), 11–14
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In this short note a correction to our previous paper (Tărnăuceanu in Arch Math 102:11–14, 2014) is made.

1. Introduction. The main result of [2] is Theorem 1.1 which gives a characterization of elementary abelian 2-groups in terms of their maximal (by inclusion) sum-free subsets. Unfortunately, as pointed out by Anabanti [1], it is not true without an additional condition. A correct version is the following.

Theorem 1.1. *Let G be a finite group. Then the set of maximal sum-free subsets coincides with the set of complements of maximal subgroups if and only if $G \cong \mathbb{Z}_2^n$ for some $n \leq 3$.*

According to this correction, Corollary 1.2 of [2] will be rewritten in the following manner.

Corollary 1.2. *The elementary abelian 2-group $\mathbb{Z}_2^n$ has $2^n - 1$ maximal sum-free subsets for $n \leq 3$, and at least 2^n maximal sum-free subsets for $n \geq 4$.*

2. Proof of Theorem 1.1. We can easily check that for $G \cong \mathbb{Z}_2^n, n \leq 3$, the maximal sum-free subsets coincide with the complements of maximal subgroups.

Conversely, let $M_1, M_2, \ldots, M_k$ be the maximal subgroups of G and assume that $G \backslash M_i, i = 1, 2, \ldots, k$, are the maximal sum-free subsets of G. Then $[G : M_i] = 2$, for any $i = 1, 2, \ldots, k$, by Lemma 2.1 of [2]. We infer that G is a

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Birkhäuser

nilpotent group, more precisely a 2-group. Since every non-trivial element of G is contained in a maximal sum-free subset of G, we have

$$G\backslash 1 = \bigcup_{i=1}^{k} G\backslash M_i = G \setminus \bigcap_{i=1}^{k} M_i = G\backslash \Phi(G),$$

that is, $\Phi(G) = 1$. Consequently, G is an elementary abelian 2-group, say $G \cong \mathbb{Z}_2^n$.

Next we will prove that $n \leq 3$. Assume that $n \geq 4$ and denote by $e_1, e_2, \ldots, e_n$ the canonical basis of G over $\mathbb{Z}_2$. It is clear that $A = \{e_1, e_2, e_3, e_4, e_1 + e_2 + e_3 + e_4\}$ is a sum-free subset of G. If there is a maximal subgroup M such that $A \subseteq G\backslash M$, then M must be of index 2, which implies that the sum of any two elements of A belongs to M. Therefore $e_1 + e_2, e_3 + e_4 \in M$ and so $e_1 + e_2 + e_3 + e_4 \in M$, a contradiction. This completes the proof. □

References

[1] C. Anabanti, A characterization of elementary abelian 3-groups, arXiv:1611.06546.

[2] M. Tărnăuceanu, A characterization of elementary abelian 2-groups, Arch. Math. **102** (2014), 11–14.

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