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The 'Regular Fortress' by Guarini and the Citadel of Turin

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Abstract In his *Trattato di Fortificatione,* Guarino Guarini described and drew a pentagonal bastioned scheme as an ideal 'regular fortress'. The closely connected geometry, architecture, and ballistics in the drawing form a true fortified system. Guarini draws his experience from knowledge of military architecture in Italy, Flanders, and France. He was in Turin when he wrote the treatise, so it could be hypothesized that the Citadel of Turin was one of his references. Using the tools of graphical analysis and history of representation, the aim of this paper is to identify Guarini's models for the regular fortress and the points of divergence between his theorization and the Citadel of Turin as they appear in several archival drawings.

Keywords Geometry · Military architecture · Ballistics · Fortification · *Trattato di Fortificatione* · Guarino Guarini · Citadel of Turin

Introduction

In Guarino Guarini's theoretical speculation and building practice, disciplines such as geometry, drawing, and design are closely interrelated, as mainly demonstrated in the treatise *Architettura Civile* (published posthumously in 1737) and in his civil and religious buildings. These concepts also seem central in his other writings about the art of building, including the *Trattato di Fortificatione* (1676) (Fig. 1). Guarini was a priest of Theatines who started dealing with the issue of fortifications during his stay in Turin from 1666 to 1683 as an architect, engineer, and mathematician in the court of Carlo Emanuele of Savoy. Although he was predominantly involved in civil and religious architecture, he wrote the *Trattato di Fortificatione* at that time,

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TRATTATO
DI
FORTIFICATIONE
Che hora si vsa in Fiandra, Francia, & Italia;
Composto in Ossequio
DEL SERENISS. PRENCIPE
LODOVICO GIVLIO
Cauagliere di Sauoia
DA
D. GVARINO GVARINI
CHIERICO REGOLARE.
IN TORINO, M.DC.LXXVI.
Appresso gl' Heredi di Carlo Gianelli.
Con Licenza de' Superiori.
STORIA 14

Fig. 1 Guarino Guarini (1676). *Trattato di Fortificatione*, title page

which was dedicated to his pupil, Ludovico Giulio, Prince of Carignano. He also designed the Porta di Po, which is functionally connected to the walls of the eastern enlargement of the city and was realized in 1674; it was later demolished by French troops under Napoleon.

The treatise on fortification is one of the least studied among Guarini's theoretical works.

An essay by Gianni Carlo Sciolla offers some graphical reconstructions of text and figures contained in the plates, and analyzes the content from an art-history point of view (Sciolla 1970). Amelio Fara makes several references to the treatise in his studies on the geometry of the fortifications and particularly highlights the debate on the importance of the second flank, of which Guarini was one of the supporters, in opposition to the French school (Fara 1993, 2001, 2014). The recent research by James McQuillan underlines Guarini and Vauban's contemporary presence in Paris and compares the contents of the treatise to Milliet Dechales's *Cursus seu Mundus Mathematicus* of 1674 and Blondel's *Resolution des quatres principaux problemes d'architecture* dated 1676 (McQuillan 2014).

Guarini's work began and developed during the seventeenth century, a time permeated by the *esprit de géométrie*, when the discoveries and the studies in the

field of mathematics and pure geometry led by Galileo, Descartes, Pascal, and Desargues influenced and became crucial within the philosophical method. Even in his less speculative writings, distinguished by the use of the vernacular instead of Latin, as in the treatise on fortification, the rigorous mathematical system was aimed at deriving techniques of military architecture from geometry and arithmetic, two liberal arts of the *quadrivium* treated in the Prelude I and Prelude II. In particular, he states that 'All mathematical sciences need Euclid's *Elements*… so anyone who wants to advance in the military art must believe that this is the basis, the beginning and the first element of which is made up, and on which every reasoning progresses and grows' (Guarini 1676: 5; my trans.). Only after these mathematical preludes does he begin to focus on the theme of Military Architecture.

Geometry, Drawing, and Design Between Theory and Art of Fortification in the *Trattato di Fortificatione*

The *Trattato di Fortificatione* is one of the numerous books about modern fortifications written in Europe since the fifteenth century in response to innovations in artillery. Guarini explained that 'against the new artillery machines invented… was necessary… the new art of fighting, pushing back through new findings to fortify, and this is the science that now I am going to expose' (Guarini 1676: 34; my trans.).

In this context, the shape of the fortress in relation to that of the bastions and shooting trajectories was the center of interest among military architects. At that time, the modern fortification arose as a geometric system in which the change of an element affected the whole (Fara 1989: 117). This concept was perfectly consistent with Guarini's design method based on geometric concatenations, which has already been noted by scholars in his most famous works (Portoghesi 1970; Millon 1970).

In Plate I of the treatise (Fig. 2), the scheme of the 'regular fortress' based on a regular pentagon is reminiscent of a series of fortifications with the same figure of reference built between the sixteenth and seventeenth centuries, such as Fortezza da Basso in Florence and the citadels in Turin, Mantova, Antwerp, Pamplona, Parma, Ferrara, Modena, and Messina.

In these cities with ancient foundations, the modern fortifications in the forms of citadels were built in strategic positions, usually at the corners of city walls, as a last protection against enemies both external and internal. In the second half of the sixteenth century, the pentagonal plan of the citadels (and less frequently quadrangular or hexagonal) was established and connected to the pentagonal bastions that replaced circular towers.

In the science of fortification, a need for pentagonal citadels was realized when it became clear that in quadrilateral citadels the salient angles of the bastions were too acute, resulting in limited space for operation of guns. Only rarely was a citadel drawn with geometric regularity (Fara 2001: 157). Fara notes the design of the Fortezza da Basso by Antonio da Sangallo the Younger, which was built in 1533 and is a fundamental example of the proliferation of pentagonal citadels in Europe (Fara 1989: 88–101). The Citadel of Turin, begun in 1564, was the first affirmation

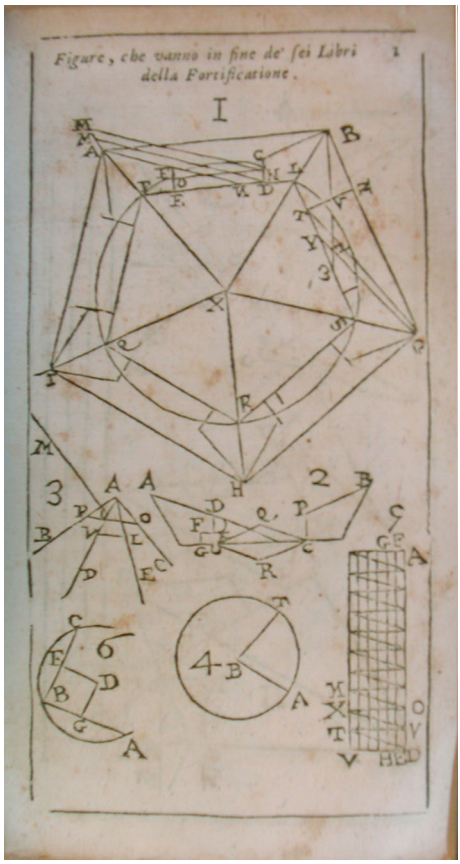


Fig. 2 Guarino Guarini (1676). *Trattato di Fortificatione*, Plate I. Images: Guarini (1676), courtesy of the Biblioteca di Storia ed Analisi dell'Architettura e degli Insediamenti, Politecnico di Torino

of a culture of pentagonal citadels and the linear protection of the flanks of the bastions, which would spread throughout Europe until the late seventeenth century (Fara 1993: 72).

Graphical Analysis of the 'Regular Fortress'

Guarini described the regular fortress as a geometric system in which all the elements are equivalent (Guarini 1676: 34–35). In the discourse, Guarini used the geometric construction of the regular fortress as a pretext to define the 'planimetric terms of Military Architecture' (Guarini 1676: 33) and, more precisely, the nomenclature of the elements that formed the fortress and the lines of its defense. The different parts of the fortress were treated as lines, angles, and plane figures. An earlier deconstruction of Guarini's regular fortress through a sequence of linked figures was the subject of previously research (Spallone 2015). Here, that analysis is refined as a result of further investigations that have allowed a closer relationship between geometry, architecture, and ballistics to be established and to more closely

approximate the dimensional values of the geometric construction recommended by Guarini. Moreover, the identified sequence of linked figures has enabled the nomenclature of the elements described in the text to be defined (Fig. 3; Table 1).

The deconstruction has generated twelve steps of linked figures, while the construction of the bastion shape (the focus of the interest of military treatises of the time) has been related in a more compelling way to the razant line of defense (step 6). This line starts from a third of the curtain and passes by the flank of the bastion. With the capital line, it determines the face and the vertex of the bastion. The vertex coincides with one of the vertices of the exterior pentagon (step 7). The razant line

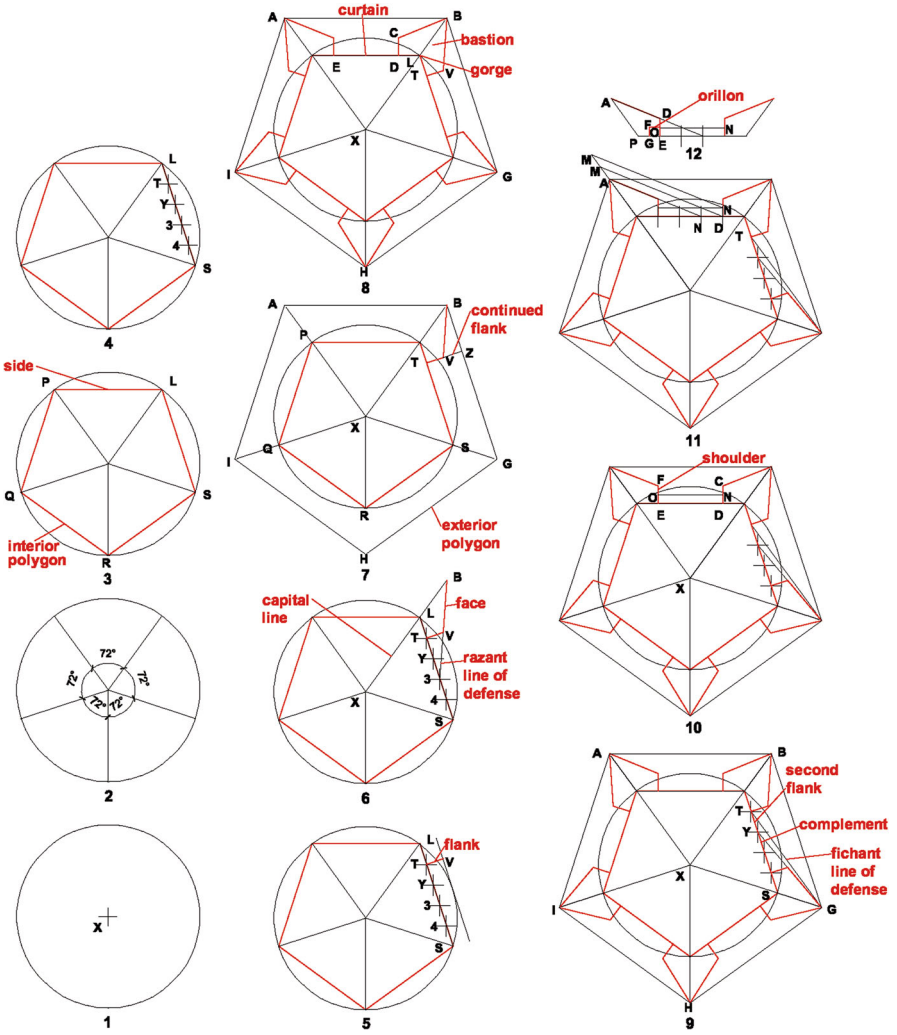


Fig. 3 Geometric deconstruction of Guarini's regular fortress. Drawing: Author

Table 1 Steps of the geometric deconstruction, identification of the fortress parts and definition of the nomenclature

N.	Geometric construction	Part of fortress/nomenclature
1	Draw a circle with center point X	
2	Divide the angle at the center of the circle into five equal parts and obtain angles of 72°	
3	Draw a regular pentagon LFQRS	*Regular fortress/interior polygon* (*Fortezza regolare/poligono interiore*) LPQRS
		Side of the fortress (*Lato della fortrzza*) LP
4	Divide into five equal parts the pentagon sides (see sides LS and points T, Y, 3, 4)	
5	Draw a line parallel to side LS and tangent to the circle	*Flank* (*Ala o fianco*) VT
	Draw a line segment from point T perpendicular to LS and obtain V	
6	Draw a line through 3 and V that meets in point B the extension of line segment XL	*Razant line of defense* (*linea di difessa radente*) 3B
		Capital-line (*Linea capitale*) XB *Face* (*Faccia o Fronte*) BV
7	Draw a regular pentagon GBA1H and distance TZ from LPQRS to GBAIH	*Exterior polygon* (*Potigono esteriore*) GBAIH
		Continued flank (*Ala continuata*) TZ
8	Construct the five bastions as in step 7	*Bastion* (*Bastione o balloardo*) BCDTV
		Capital-lines (*Linee capitali*) XA, XG, XH, XI
		Gorge (*Collo o gola*) TLD
		Curtain (*Cortina o corda*) DE
9	Draw line segments GT and GY	*Razant line of defense* (*Linea di difessa radente*) GY
		Fichant line of defense (*Linea fictante*) GT
		Second flank or curtain-flank (*Ala secondaria o finaco*) TY
		Complement (*Complemento*) YS
10	Draw midpoints of CD and EF, and connect them with line segment NO	*Shoulder of the bastion* (*Spalla del balloardo*) *FO*
11	Draw line segments MN, MD, AN	*Razant line of defense* (*Linea di difessa radente*) NA
12	Extend line segment NO, draw a perpendicular from midpoint G of line segment PE and identify F	*Orillon,* variant of the shoulder (*Orecchia,* variante della spalla) OFG

of defense is related to the fichant line of defense, which starts from the angle between the curtain and the flank, and it ends at the vertex of the bastion (step 9).

The portion of the curtain delimited by these two defensive lines represents the second flank which, as Guarini stated ‘should never be forgotten… because as the first wing is mostly occupied by the artillery; if there were no second flank, few musketeers would remain to defend the opposite side, with the consequence of a serious danger’ (Guarini 1676: 40; my trans.).

The second flank allows the fichant shot on the salient and became a central issue in the discussion of the art of fortification in the sixteenth and seventeenth centuries. Guarini's opinion was part of the debate 'which opposed two schools of thought: one that supported the validity of the layout with second flanks and the other that condemned it' (Fara 2014: 786).

In the treatise Guarini explains that the tracing of the regular pentagon results from the subdivision the angle at the center of the circle into five equal parts (steps 2-3). The defensive line had to consider the range of musket shots, which at that time did not exceed 750 to 850 feet (between 225 and 255 meters) (Guarini 1737: 43–44; Martini 1883: 783), thus its length was determined within this value (Guarini 1676: 37–38). In the present deconstruction, imposing the curtain on the minimum dimension suggested by Guarini in Chapter II (300 feet), the length of the face of the bastion (which he described as about 240 feet) turns out to be about 243 feet, while the maximum defensive line measures about 658 feet. This allows us to assess the consistency of the overall proportions of the reconstruction and to appreciate the precision of the original drawing by Guarini (Fig. 4).

The peculiarities verified in Guarini's scheme are the geometric concatenation, the regularity of the plan, and the proportion between the parts. The linking between geometry and ballistics highlighted in the regular fortress appears in several archival drawings concerning the Citadel of Turin, which will be analyzed and compared to Guarini's scheme below. Starting from the geometric characteristics discovered in the scheme, the main issues considered in the comparison by Guarini are:

- the construction of the pentagon through the division of the angle at the center into five equal parts;
- the division of the side of the interior pentagon into five equal parts, with the middle three delimiting the curtain and the lateral ones indicating the position of the bastion flank perpendicular to the curtain;
- the presence of the second flank between the razant and fichant lines of defense.

The Citadel of Turin as an *Exemplum* for Theorizing a Model

In the dedication to the reader, Guarini writes, 'I've not taken careless license to introduce new forms of fortification, although sometimes I designed them, because I am not able to offer them authenticated by experience… I therefore offer ancient examples of Italy, Holland, France, but selected and confirmed by experience, and with the applause of the World' (Guarini 1676: 3; my trans.).

Among the examples, there seems to be a clear reference to the Citadel of Turin, which was designed by Francesco Paciotto, built between 1564 and 1577, and gradually dismantled starting in the mid-nineteenth century. Later, there is a series of drawings pre-dating Guarini's treatise, which he could have consulted in the Savoy Court Military Archives. He also could have examined or even surveyed the Citadel directly, as suggested by a drawing attributed to him by Fara (2001: 173–177). He probably also knew the iconographies of the *Theatrum Sabaudiae*, since he had been called to write some of the captions, such as that of his design for

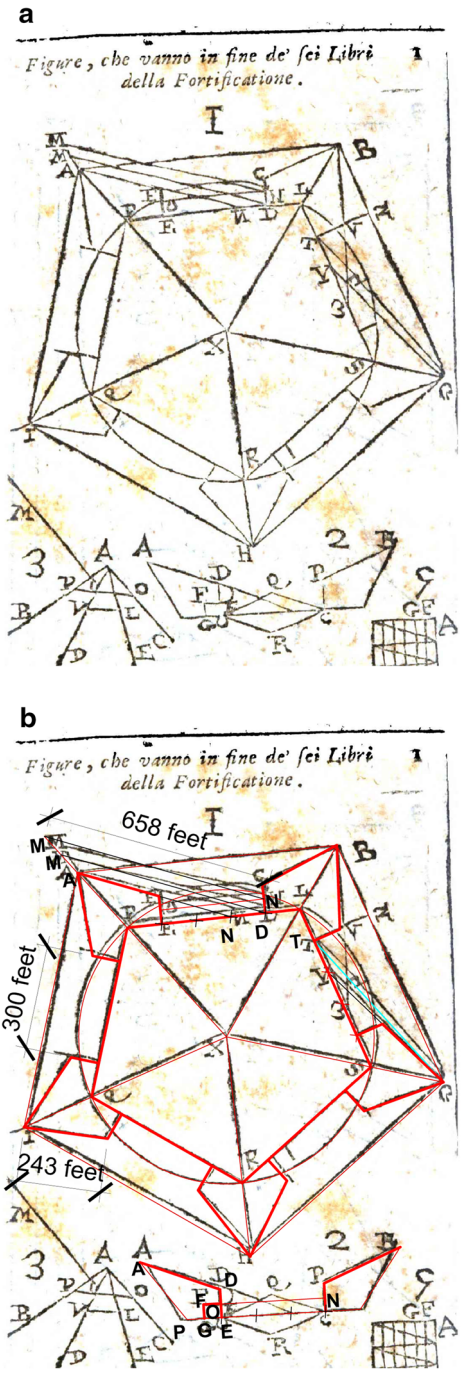


Fig. 4 Scheme of the regular fortress and variant of the shoulder. **a** Detail of plate I; **b** dimensional check of Guarini's scheme. Image: Guarini (1676). Graphic overlay: Author

the Porta di Po, and he could have seen an accurate plan of the Citadel, which was published in the page after the perspective view of the gate. For these reasons, it seems important to deepen the comparison between Guarini's fortress and the Citadel, which no longer exists but is preserved in many archival drawings, although these are not always fully consistent with each other due to the different aims for which they were drawn.

In effect, among the numerous archival drawings, the most interesting are those that allow us to compare different stages of the Citadel's life with Guarini's scheme, from its design to its execution and its modifications. The drawings represent only the Citadel or the whole system of fortified walls. They include concept drawings, survey drawings, proposals for transformation and improvement, and other iconographies aimed at celebratory representations. The geometric and metric reliability of such sources should therefore be critically evaluated, while taking into account the purpose of the drawing, its reduction scale, the measurement units (Bevilacqua 2015: 95–101), and the survey techniques of the time.

Design Drawings, Survey Drawings, and Views of the Citadel: Comparison with the Regular Fortress

Scholars attribute the design drawing of the Citadel to Francesco Paciotto or his entourage and mainly date it to about 1564 (Comoli Mandracci 1983: 11, Fara 2012: 198). However, Aurora Scotti Tosini dates it to 1565–66 by relating to a phase that coincides with the work in progress (Comoli Mandracci et al. 1998: 434). It presents quadrangular walls equipped with bastions of the ancient square-shaped city with the Citadel set on the diagonal. The sheet shows metalpoint lines and compass holes, which have been very useful for graphical analyses. The pentagonal fortification is connected with existing walls and originates from the side of the pentagon where the *mastio* (a fortified tower built within the fortification walls) is located.

To locate the real positions of points and segments, a survey of the original drawing of the Citadel was carried out through the trilateration method from the side of the exterior pentagon connected to the walls (which might even be a reasonable starting point for the designer). The application of this method has revealed an extraordinary fact: two of the three deepest metalpoint lines in the original drawing coincide with the sides of the triangles traced in the present survey. The third passes through the intersection of one of the lines with one of the capital lines, and it seems to determine the position of the flank of a bastion. The compass holes used to mark off equal measures identify the center and vertices of the interior pentagon and the endpoints of the face and flank of each bastion. Given these marks and the absence of any known pentagon constructions at that time (such as those of Ptolemy and Dürer), it might be cautiously supposed that the drawing is actually a transcript or a copy of another one. The superimposition of the survey on the archival drawing following a minimal compensation of measurements reveals a remarkable coincidence (Fig. 5).

The measurements of the reconstructive drawing highlight that the exterior side of the pentagon where the mastio was and the two sides facing the countryside have

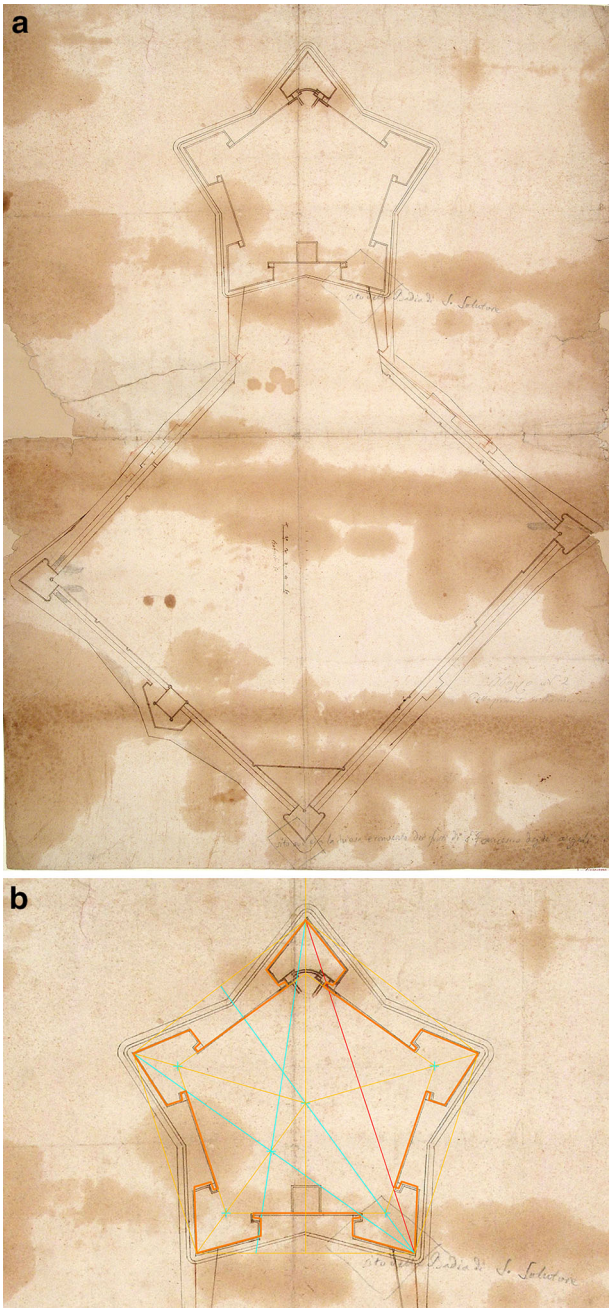


Fig. 5 Design drawing of the Citadel of Turin. **a** [Francesco Paciotto], [1564–66], *Abozzo n. 2 della pianta di Torino antica.* 665 × 465 mm. Graphic scale 50 trabucchi ≈51 mm; **b** superimposition of the graphical analysis and geometric construction (in *orange*). Traced over metalpoint lines, compass holes (in *cyan*) and construction lines (in *red*). Image: Biblioteca Reale di Torino, Disegni, II, 25, reproduced by permission; graphic overlay: Author (color figure online)

roughly the same measures, while the intermediate ones are smaller. The faces, the flanks perpendicular to the curtain, and the orillons of the five bastions have similar measures, whereas the vertex angles of the bastions vary between 77 and 80°. The length of the curtain is about 3/5 of the side of the interior pentagon, similar to the scheme by Guarini. The second flank is present on only the curtain of the mastio and was reduced to about 1/12 of the curtain, but there is no trace of the razant line of defense. The comparison with Guarini's scheme shows that the exterior and interior pentagons in the Citadel design drawing are closer together and lose their regularity, and the vertex angles of the bastions are larger (Fig. 6).

One of the most important survey drawings of the Citadel is a plate in *Avvertimenti sopra le fortezze di S.R.A.* by Carlo Morello (1656), a military engineer at the Savoy Court. Morello's description of the Citadel is particularly interesting for the definition of its proportions.

> The Citadel of Turin is pentagonal; but not equilateral. It has two curtains of a measure, two of another one, and that of the mastio of another one, which is somewhat longer for motive of defense. It is nevertheless true that the mastio located in the middle can defend more closely some parts; therefore the difference between these curtains cannot decrease the strength of this figure. This plan is the best that it could be found (Morello (1656): 18; my trans.).

The graphic reconstruction of Morello's drawing shows different dimensions of the curtains that are substantially comparable to those hypothesized by Fara, who attributes the pentagon irregularities to the existing foundations of Badia di San Solutore. Fara further hypothesizes the application of Dürer's method for constructing the pentagon (which is thought to be regular) on the plot of land, as demonstrated by the location of a well at the intersection between two circles with the endpoints of the side where the mastio was located as the center (Fara 2001: 157–160). The razant lines of defense that originate the second flank are traced in the original drawing and vary in length from about 1/5 of the curtain to almost zero along the shorter curtains. The length of the curtain is about 3/5 of the side of the interior pentagon (Fig. 7).

The graphical reconstruction of Guarini's scheme superimposed on that of Morello's survey highlights a kind of deformation of the regular pentagon by 'squeezing', which mostly affected the two curtains (each about 46 *trabucchi*; 1 *trabucco* is about 3.0825 meters) (Martini 1883: 783) adjacent to the longest curtain (about 52 *trabucchi*). The two curtains facing the countryside have intermediate lengths (about 49 *trabucchi*). Therefore the main elements show some dimensional changes, except for the flanks, all of which measure about 12 *trabucchi*. The most significant difference between Guarini's scheme and Carlo Morello's drawing is the measure of the vertex angle of the bastions, which consequently leads to a different distance between the interior and exterior pentagons. As mentioned, in Guarini's construction, this angle originated from the tripartite division of the curtain and measured about 64°, in contrast to the average of 76° in Morello's drawing (Fig. 8).

Some years later, Michel Angelo Morello drew the Citadel as a part of an incomplete collection of 'Military Drawings,' which were identified by Micaela Viglino Davico and Claudia Bonardi Tomesani (2001). Morello was the son of

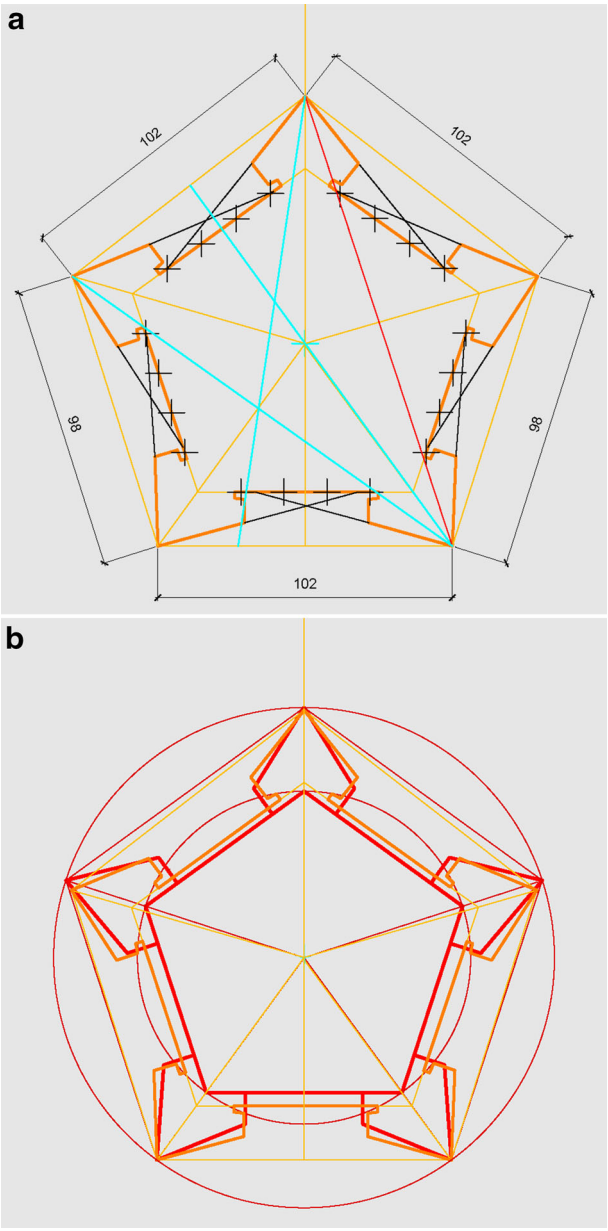


Fig. 6 Graphical analyses of the design drawing by Paciotto. **a** geometric construction with the measurements of the exterior pentagon sides. Units: trabucchi. 1 trabucco ≈3.0825 m; **b** comparison between the geometric constructions of Paciotto's drawing (in *orange*) and Guarini's scheme (in *red*). Drawings: Author (color figure online)

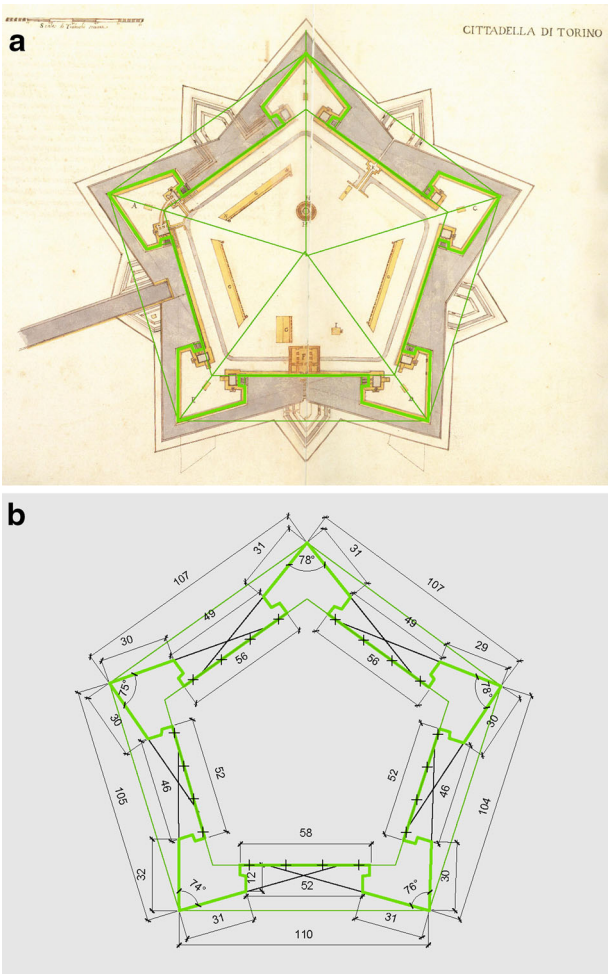


Fig. 7 Survey drawing of the Citadel of Turin. **a** Carlo Morello, 'Cittadella di Torino' from (Morello 1656) [400 × 460 mm]. Graphic scale 60 trabucchi ≈96.5 mm. Superimposition of the graphical analysis and geometric construction (in *green*). Image: Biblioteca Reale di Torino, Ms. Militari 178 c. 15v-16r, reproduced by permission; Graphic overlay: Author; **b** geometric construction with the measurements of the main parts of the fortress. Units: trabucchi. Drawing: Author (color figure online)

Carlo and an architect, military engineer, and artillery captain in service of the Duke of Savoy from 1640 to 1676. The identification was also due to the comparison with the plates by his father Carlo and the four known views realized by Michel Angelo in 1664 for the *Theatrum Sabaudiae*, a collection of celebratory iconographies of Savoy Duchy printed in 1682 (Peyrot 2000: 31–56). The collection of drawings about the Citadel is presumed to be either a copy of or preparatory drawings for the *Theatrum* (Viglino Davico and Bonardi Tomesani 2001: 16) and is dated to between 1663 and 1664 by Ada Peyrot (2000: 47). The graphical reconstruction of Michel Angelo Morello's drawing leads to the hypothesis that it was born from a regular

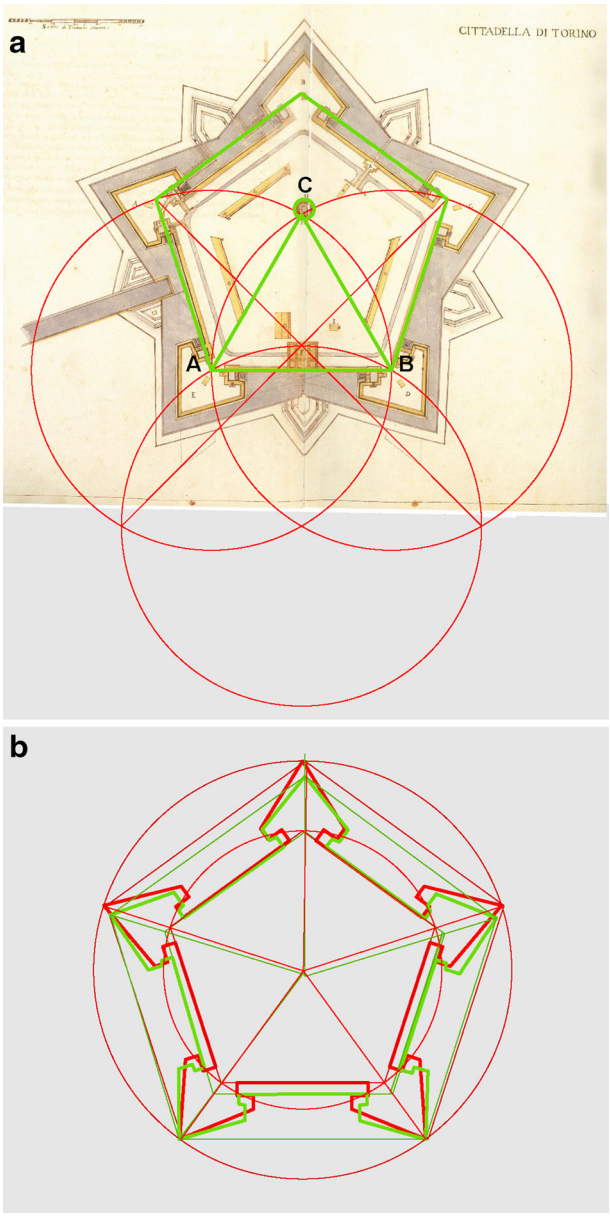


Fig. 8 Graphical analyses of the survey drawing by Carlo Morello. **a** Durer's construction of the regular pentagon and location of the well. Image: Biblioteca Reale di Torino, Manoscritto Militari 178 c. 15v-16r. Reproduced with permission. Graphic overlay: Author; **b** comparison between the geometric constructions of Carlo Morello's drawing (in *green*) and Guarini's scheme (in *red*). Drawing: Author (color figure online)

pentagon positioned on the side of the mastio. The adjacent sides coincide with the sides of this regular pentagon, although they are shorter, and the sides towards the countryside are parallel with those of the regular pentagon. Each side of the interior

polygon is divided into five parts, as Guarini proposed a few years later, with 1/5 delimiting the position of the bastions and leaving 3/5 for the development of the curtain. Even the razant lines of defense all converge in the angles between the flanks of the bastions and the curtains, eliminating the second flanks (Fig. 9).

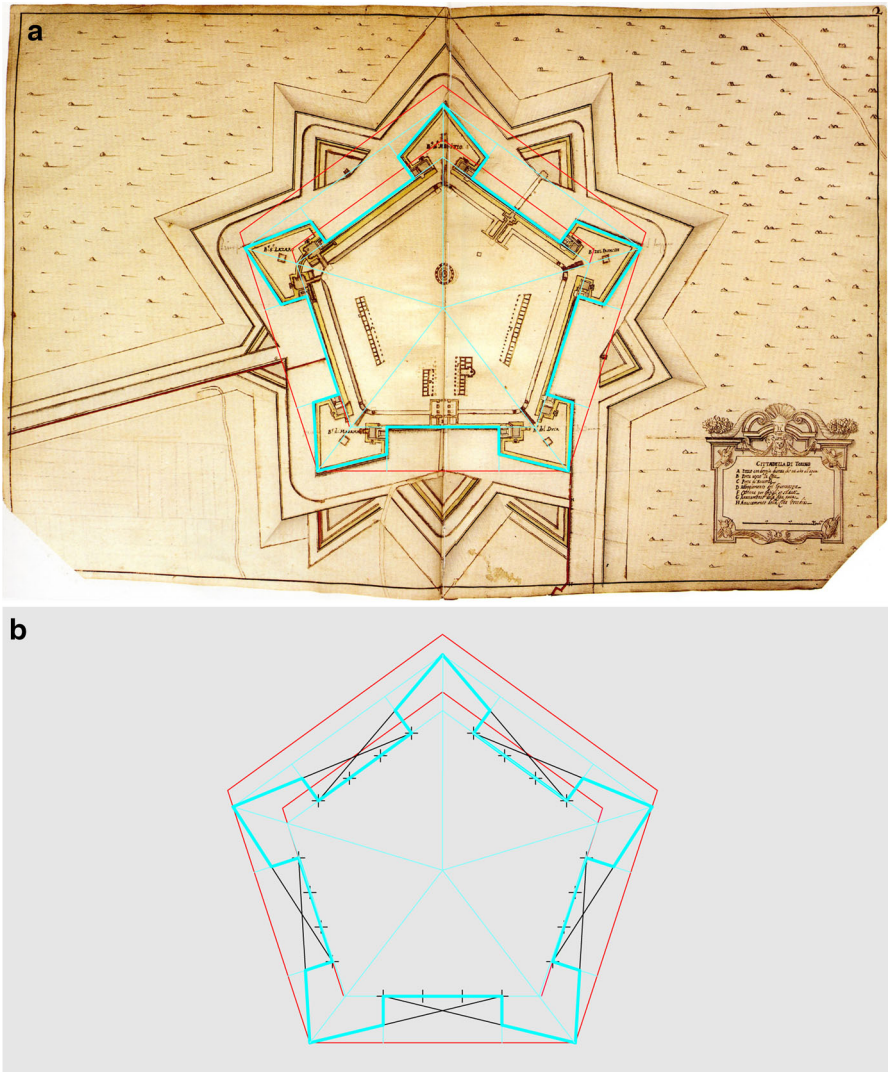


Fig. 9 Preparatory drawing for the *Theatrum Sabaudie*. **a** Michel Angelo Morello, [1663–1664], *Cittadella di Torino*. [370 × 470 mm]. Superimposition of the graphical analysis and geometric construction (in *cyan*). Image: Istituto di Storia e Cultura dell'Arma del Genio di Roma, Biblioteca 40/b-n. 18798. Reproduced with permission. Graphic overlay: Author; **b** comparison between graphical construction (in *cyan*) and regular interior and exterior pentagons (in *red*). Drawing: Author (color figure online)

Michel Angelo's drawing reveals similarities with the survey by Carlo, especially in regard to the ratios between the side lengths of the exterior and interior pentagons and that of the curtains, showing a similar 'squeezing effect,' but also a search for regularity, which would be even more pronounced in the engraving of the *Theatrum*. The superimposition of the graphical analysis of Carlo Morello's drawing on that by Michel Angelo is not perfect, especially in regard to the tracing of the bastions, which lacks the second flank in Michel Angelo's drawing (Fig. 10).

The view of the Citadel in the *Theatrum Sabaudiae* shows an attempt to regularize the shape into a regular pentagon, resulting in a geometric scheme similar to that by Guarini, except for the angle at the vertex of the bastions. In the caption of the plate, the Citadel is carefully described as having a pentagonal shape, with five forward bastions expertly realized. Together with the earthworks, they cover the entire space inside the fortress, which is intentionally lower so that not only the soldiers but also the military buildings are completely protected from external fire. In the description, there is no reference to the irregularity of the pentagon. The fortress is represented in an axonometric projection called *prospettiva soldatesca* (military perspective) (Alonso-Rodríguez and Calvo-López 2014), in which the plan is not deformed.

The representation method is similar to that described by Guarini in the last pages of his treatise where he explains the 'way of representing the plan and elevation of the fortresses by the drawing, because some people, not practiced in drawing, cannot understand how the plan drawn corresponds with the elevation' (Guarini 1676: 124; my trans.). After excluding the perspective projection for its difficulty, he illustrates an expeditious method for the construction of the elevation from the plan, which is simply an empirical application of oblique plan axonometry, which has been widely used since the early sixteenth century in military treatises (Scolari 1988). Viglino Davico compared the preparatory drawing by Michel Angelo Morello and the engraving of the *Theatrum* and observed that the representation is formally more exquisite in the engraved version, but it loses adherence to the true form and contains several errors in representation. This leads to the idea that it faces incongruous interventions of the engraver, who neglects the precision that is essential for an engineer in favor of a 'beautiful drawing' (Viglino Davico and Bonardi Tomesani 2001: 17).

In effect, the graphical analysis of the preparatory drawing is not perfectly superimposable, and the measurements realized on a *Theatrum* reproduction in its original size reveal numerous differences and a slight loss of symmetry. Starting from the exterior pentagon, the deconstruction reveals a search for regularity, which would suggest the drawing of a regular pentagon. The sides adjacent to that of the mastio were slightly opened, perhaps to demonstrate its greater length, but above all to create a perspective effect in the view. This effect is also evident in the representation of the trees, which decrease in height along the inner boulevard. The effect also appears to connect the plate to other bird's-eye views present in the volume. Regarding the axonometric graphic construction, it seems that starting from the plan, the engraver had traced down the lines necessary to generate the ramparts and had raised the lines that represent internal buildings and artifacts, making a

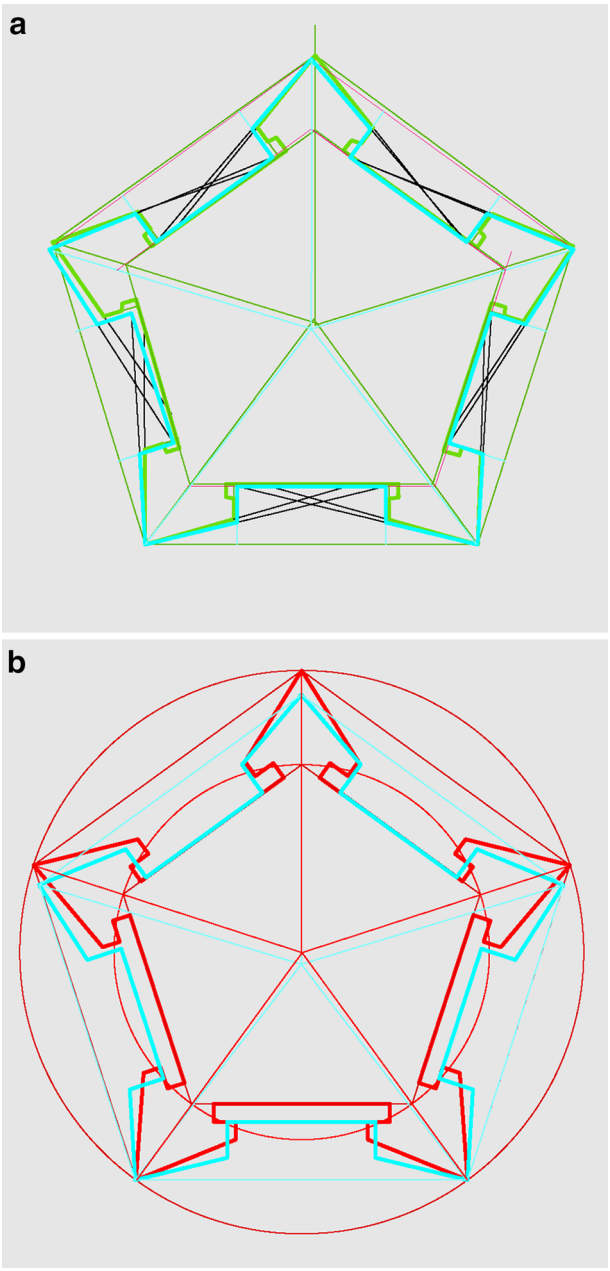
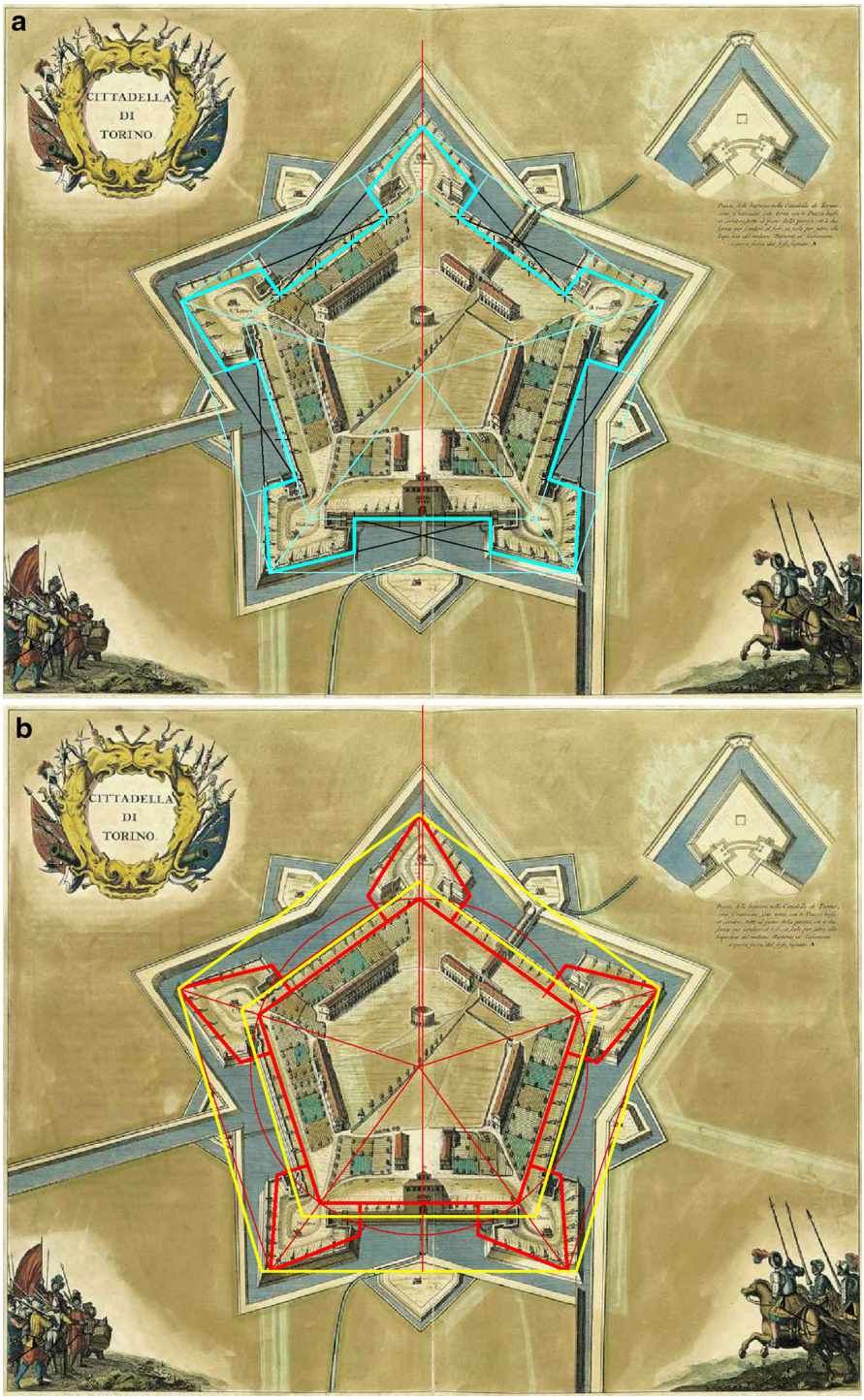


Fig. 10 Graphical analyses of the drawing by Michel Angelo Morello. **a** Comparison between the geometric constructions of the drawings by Michel Angelo Morello (in *cyan*) and Carlo Morello (in *green*); **b** comparison between the geometric constructions of Michel Angelo Morello’s drawing (in *cyan*) and Guarini’s scheme (in red). Drawings: Author (color figure online)


a
CITTADELLA
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◀**Fig. 11** View of the Citadel of Turin. **a** Cittadella di Torino, from *Theatrum Sabaudiae,* printed in 1682. [439 × 548 mm]. Superimposition of the preparatory drawing by Michel Angelo Morello (in *cyan*); **b** superimposition of the geometric construction of Guarini's scheme (in *red*) and the pentagons that circumscribe the Citadel (in *yellow*). Image: Archivio Storico del Comune di Torino, Fondo *Theatrum Sabaudiae*, reproduced by permission; graphic overlay: Author (color figure online)

projective error in the representation of the well, distorting circularity of the base (Fig. 11).

In about 1670, the Citadel was drawn as a detail of the design of the city wall system signed by Marquis Vauban. Sébastien Le Prestre de Vauban was probably the most famous military architect of the day and was superintendent of the fortifications in France. He had been sent by Marquis Louvois to the Duke of Savoy for strategic advice. A drawing signed by him seems to give definitive evidence about the curtains and bastions on a line that was actually realized on the eastern front, while the planned closure of the fortification on the west front was later changed (Comoli 1983: 43). The detail of the Citadel by Vauban is undeniably approximate because the fortress was not the focus of the design drawing. The interior pentagon is deformed in a manner similar to that by Michel Angelo Morello, but it loses the parallelism of one side to the regular pentagon of reference, and the bastions are visibly different in size and shape. One element is interesting, however: the faces of every bastion except one appear aligned with the razant line of defense originating from the angle between the curtain and the bastion flank, exactly as prescribed by Vauban, one of the main detractors of the second flank. Thus, he seems to transpose his ideological position into the drawing. Guarini might have also derived a stimulus from this rough drawing to affirm the importance of such a device in the military machine in the treatise (Fig. 12).

Realized in the same years, the unsigned design drawing for the transformation of the Citadel preserved in the *Bibliothèque Nationale de Paris* offers fascinating fields of investigation, not least because of the different hypotheses of attribution by scholars. In 1983, Vera Comoli Mandracci published it as an anonymous drawing from the seventeenth century (1983: 12). In 1989 she attributed it to Vauban, dating it to about 1670 (Comoli Mandracci 1989: 307). In 2000, she did not mention the author but dated the drawing to the beginning of the eighteenth century (Comoli Mandracci and Fasoli 2000: 17).

In 2001, Fara refuted Comoli's 1989 opinion and attributed the drawing to Guarini, presuming it to be dated to 1674–1675. His hypothesis was motivated by the similarities between the interventions proposed in the drawing and the fortress elements described in some passages of Guarini's *Trattato di Fortificatione*. He also noted the relationship between the horn-shaped earthwork of the drawing and the wall of Porta di Po. In addition, he highlighted the correspondence between the coloring of the drawing and the 'way to color the drawing' (Guarini 1676: 128; my trans.) described in the treatise (Fara 2001: 173–177). The superimposition of the survey by Carlo Morello is almost perfect except for the curtain of the mastio, which appears shorter, and for the angles at the vertex of the bastions, which are slightly broader. The drawing by Morello may perhaps have served as the basis for this design drawing, or the Citadel could have been surveyed *ex novo* for this improvement. In any case, if the drawing is by Guarini, it would demonstrate his greater involvement in the military field in the Savoy court, and above all, it would

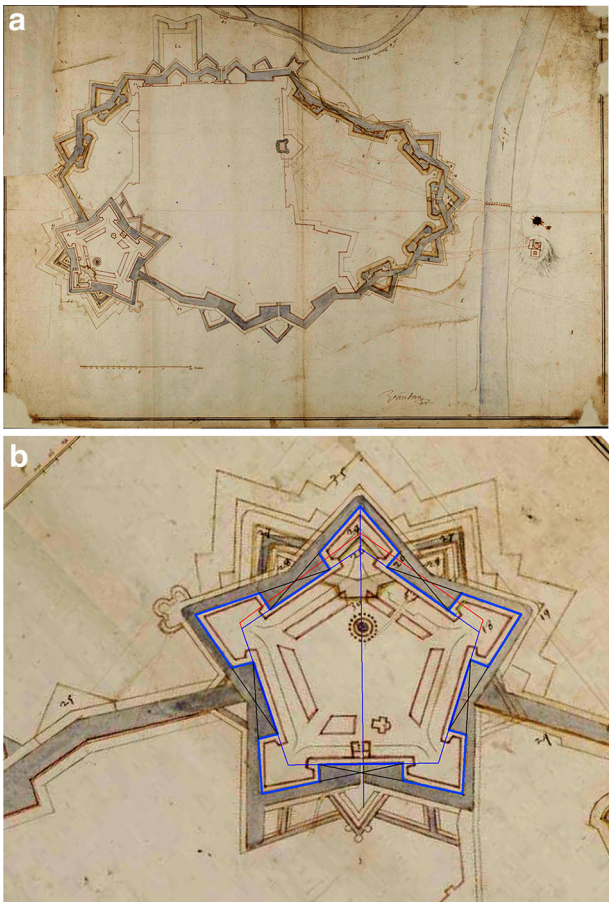

Fig. 12 Design drawing for the improvement of city walls. **a** Sébastien Le Prestre de Vauban, about 1670. Turin fortifications toward the river Po [565 × 798 mm]. Graphic scale 200 trabucchi ≈145 mm; **b** superimposition of graphical analysis and geometric construction (in *blue*) and comparison with a regular pentagon (in *red*). Image: Archivio di Stato di Torino, Corte, *Carte Topografiche* per A e B, cartella Torino 1, XII, reproduced by permission; graphic overlay: Author (color figure online)

be a further element of the Citadel study for him, which would strengthen its value as an *exemplum* for the formulation of the regular fortress scheme (Fig. 13).

Conclusion

Guarini claimed to refer to fortifications built in Italy, Flanders, and France. Therefore, research on such references can offer many interesting points for explicating his design method, which combines geometry, architecture, and ballistics. The pentagonal scheme of the regular fortress inevitably leads to comparisons with the pre-existing Citadel of Turin, which Guarini probably knew in

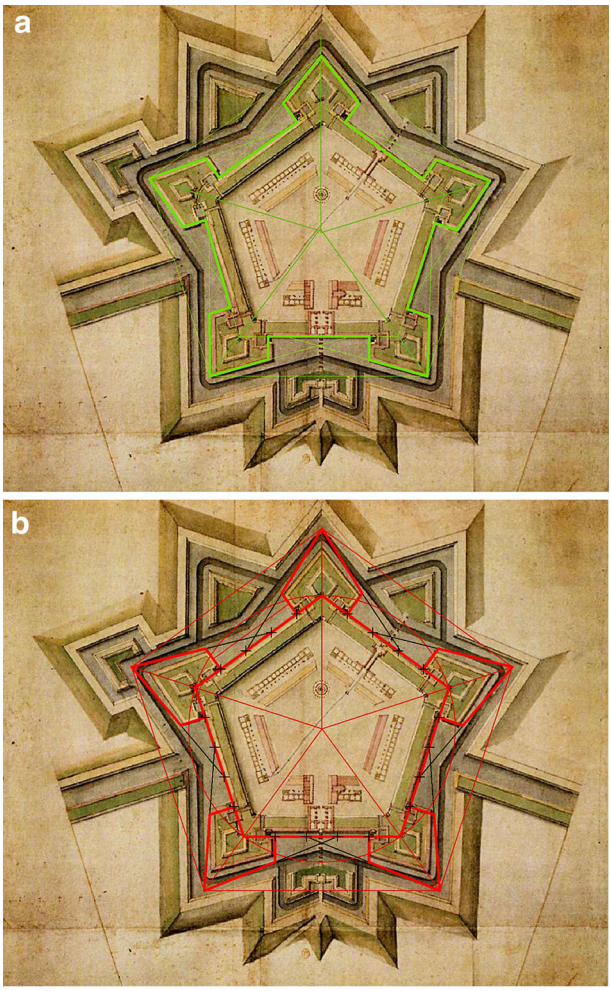

Fig. 13 Design drawing for the transformation of the Citadel. **a** 'Cittadella di Torino con il parere novo', about 1670 [555 × 685 mm]. Graphic scale 50 trabucchi ≈11.15 mm. Superimposition of the geometric construction of Carlo Morello's drawing (in *green*); **b** superimposition of the geometric construction of Guarini's scheme (in *red*). Image: Bibliothèque Nationale de France, Cabinet des Estampes, Topographie de l'Italie, Vb 7, 49; graphic overlay: Author (color figure online)

its actual shape and through its drawings. The present research highlights several open issues, and it also offers the possibility of proposing some consolidated results by applying the tools of the graphical analysis and history of representation.

Guarini's scheme for the regular fortress has been verified from the point of view of the proportional relationships between elements through the identified and traced geometric sequences. It also demonstrates Guarini's specific position in the debate between the supporters and detractors of the second flank as a useful tool to increase the defensive potential of the fortress during the sixteenth and seventeenth centuries. The comparison between Guarini's scheme and the Citadel is necessarily

speculative due to the fact that the building no longer exists except in its representations, but also opens many opportunities for philological research.

The numerous historical drawings found in the archives show buildings and designs growing out of the military culture of their time. Therefore, the broad range of geometric graphical reconstructions of archival drawings beyond the cases presented in this paper should make it possible to choose more reliable representations to practice the comparison. In summary of the main issues considered in the comparison, some conclusions could cautiously be advanced with assumptions about the geometric rules discovered in Guarini's scheme. There the construction of the regular pentagon is performed through the division into five equal parts of the angle at the center. In the drawings made after the erection of the Citadel the regular pentagon could result from the construction by Dürer, as shown by the location of the well. In the design drawing attributed to Paciotto, there are no traces of such construction, or of the well.

The irregularity of the fortress due to the presence of the foundations of Badia di San Solutore and the need to include the mastio along the side attached to the walls do not preclude the reference to the regular pentagon that is subject to a 'squeezing effect' in several drawings. The division of the sides of the interior pentagon into five equal parts is a proportional criterion present in the drawings by Paciotto, Carlo Morello and Michel Angelo Morello, which Guarini might have used as a reference. This proportional criterion would adapt well to the formulation of a theoretical model. The presence of the second flank between the razant and fichant lines of defense is ascertained in only some of the Citadel drawings. In the design drawing, it is difficult to say that it is present: only the longest curtain of the mastio could include it, but it does not appear in the original drawing. In the survey by Carlo Morello, which is believed to be among the most reliable, the razant line of defense that falls within the curtain and generates the second flank is drawn on each side. In later drawings, the razant line of defense is no longer traced, and the second flank can be identified in only the anonymous drawing attributed to Guarini by Fara, which recalls many of the geometries by Carlo Morello.

More ideas for research will come from the analysis of other pentagonal fortresses built in areas to which Guarini refers, starting from those he had known directly. Among these, the Citadel of Modena realized in 1635 would have been known by Guarini between 1671 and 1676, as he was engaged by the Duke Francesco II d'Este for the enlargement of the city. This Citadel and its relationship to the regular fortress is the subject of current research.

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