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RESEARCH

Layout of Catenary Arches in the Spanish Enlightenment and Modernism

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Abstract Catenary arches and vaults were used in Spain during two historical periods. First, the theoretical concept was used in the eighteenth century by military engineers for the construction of gunpowder magazines. Subsequently, Catalan modernist architects, such as Antoni Gaudí i Cornet (1852–1926) and Cèsar Martinell i Brunet (1888–1973) used this shape throughout their buildings. The paper assesses the geometric approximations to the catenary made by eighteenth-century military engineers and twentieth-century architects in Spain. The investigation is based on two documentary sources: the designs for gunpowder magazines found in the Colección de Mapas, Planos y Dibujos del Archivo General de Simancas, and the design by Cèsar Martinell i Brunet for the Cooperative wine cellar in Pinell de Brai (1918), preserved in the Arxiu Històric del Col·legi d'Arquitectes de Catalunya. The assessment confirms the use of the concept of the chain during these two historical periods.

Keywords Catenary arches · Catenary vaults · Military architecture · Cèsar Martinell i Brunet · Spanish enlightenment · Spanish modernism

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Introduction

The first use of catenary arches and vaults in Spain appears in the eighteenth century by military engineers for the construction of gunpowder magazines. This figure was introduced by the influence of Bernard Forest of Bélidor (1698–1761), whose works can be found in the Real Academia de Matemáticas de Barcelona (1720) (Lluis i Ginovart 2015).

These forms were used subsequently by Antoni Gaudí i Cornet (1852–1926) to determine the geometric shapes of vaults and arches (Alsina and Gómez Serrano 2002). This point of view is supported by many architects; see, for instance, the lecture entitled 'La fábrica de ladrillo en la construcción catalana' by Josep Domènech i Estapà (1858–1917), in which it is claimed that the catenary shapes are the lines of equilibrium in a system of evenly distributed loads where the parabolic shape relates to the horizontal projection and the catenary shape relates to the arch length (Domènech i Estapà 1900). These principles were used by Catalan modernists, such as Cèsar Martinell i Brunet (1888–1973), who used catenary arches exhaustively in his cooperative wine cellars.

The aim of this present research is to assess the geometric approximations to the catenary shape made by eighteenth-century military engineers and twentieth-century architects in Spain. Thus, the layouts of the curves $f(x_i)$ drawn by engineers and architects that tend to catenary shapes $f(x_{c_i})$ are assessed.

The assessment of military architecture developed in Spain during the eighteenth century was initiated in a previous work (Lluis i Ginovart et al. 2014), in which the geometric construction of floor plans of defensive bastions was analysed. Later, interest in the investigation led to an exhaustive assessment of the morphology of gunpowder magazine designs. The analysis was focused on formal features of projects in Barcelona, Tortosa and A Coruña (Lluis i Ginovart 2015). Thus, the historical and theoretical framework of gunpowder magazines was defined. In addition, the use of the catenary concept was introduced. The present investigation furthers the geometric assessment of the designs related to the theory of the chain and incorporates an analysis of the architecture.

A Brief History of the Equilibrium Curve

The theory of the shape of a hanging chain was proposed by Robert Hooke (1635–1703) at the end of his treatise *A description of helioscopes, and some other instruments*; Hooke presented a solution that would be revealed as *Ut pendet continuun flexile, sic stabit contiguum rigidum inversum* (Hooke 1676: 31). The awareness of the shape of the catenary was applied by Christopher Wren (1632–1723) in the dome of St. Paul's Cathedral in London (1675), with the collaboration of Robert Hooke in the design (Heyman 2003). Simon Stevin (1548–1620), in *De Beghinselen der Weeghconst*, previously tested the law of the equilibrium of a body on an inclined plane with a hanging cable that had the shape of a catenary (Stevin 1586: Bk I, 41). Despite the evidence provided by the figure, there was no mathematical approach to catenaries. That is why Jakob Bernoulli (1654–1705), in *Actae Eruditorum*, issued a challenge to the mathematical community to solve this problem (Jakob Bernoulli 1690). The solution was published in *Actae Eruditorum* by Johann Bernoulli (1667–1748) with the title "Solutio problematis funicaularii" (Johann Bernoulli 1691) and also by Christiaan Huygens (1629–1695) with the title "Dynastae Zulichemii, solutio problematis funicularii" (Huygens 1691). The mathematical equation of the catenary would be formulated some years later by David Gregory (1659–1708) and published in the *Philosophical Transactions of the Royal Society*. Gregory affirms that the catenary form is the real shape of an arch, as it can sustain itself because a catenary can be drawn in its section (Gregory 1697). James Stirling (1692–1770), in the *Lineae Tertii Ordinis Neutonianae*, compiled the ideas of the English school building and a catenary with hanging spheres to simulate the behaviour of a constructive element (Stirling 1717:11–14, esp. 'Methodus disponendi quotcunque Sphaeras in Fornicem. Et inde Demostratur Propietas praecipua Curvae Catenariae', described at the end of the work with another page). This solution inspired an analysis in the *Memorie istoriche della Gran Cupola del Tempio Vaticano* by Giovanni Poleni (1683–1761), who developed a methodology similar to Stirling's to understand the cracking of the vault of St. Peter's Basilica in Rome (Poleni 1748: 30–50, esp. ch. VIII and IX, and plates D and E; Heyman 1988).

In Spain, the development and application of this theory took place in the context of the Real Academia Militar de Matemáticas de Barcelona (founded in 1720), which is a main point of reference for the work of Bernard Forest of Bélidor (1698–1761). In *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile*, Bélidor proposes the curve that must be given to a vault so that all its parts weigh the same and stand in equilibrium (Bélidor 1729: Bk. II, ch. iii, prop. 5); the resulting curve will have the shape of a catenary, and he specifies the method to lay out the true shape of the catenary vault (Bélidor 1729: II, 43–45). Thus, he determined up to five types of different vaults for military construction: round, pointed, elliptic drawn as a segmental arch, flat, and forms derived from the catenary (Bélidor 1729: II, 1–65). At the same time, in *De la poussée des voûtes* (Couplet 1731), Pierre Couplet (ca. 1640–1744) also refers to the *chaînette,* the hanging chain, as the best of all shapes for the construction of vaults. He also says that if building on any part of the vault was desired, a proportional weight of the construction would have to be added to the corresponding part of this hanging rope so that the resultant curve will be the ideal one. Amedée François Frézier defined the *Chaînesse* as a very favourable curve for the equilibrium of the voussoirs, despite the fact that it is not beautiful because it causes a break in the springing (Frézier 1738: vol. II, Sheet 33, fig. 50, 97–98).

These concepts were also introduced in the training of architects through the Escuela Especial de Arquitectura de Madrid. The work *Traité Théorique et Pratique de L'art de bâtir* (1802–1817) by Jean-Baptiste Rondelet (1742–1829) presented the methodology to lay out catenary arches by means of the theory of the chain and another complicated procedure (Rondelet 1804: 137–145). In addition, the textbook *Elements of Civil Engineering* by John Millington (1779–1868) (Millington 1839) was also used in architecture schools. It was translated into Spanish as *Elementos de arquitectura* and contained Hooke's theory and the layout of the catenary (Millington 1848: vol. II, fig. 153, 472–477). Juan Torras i Guardiola (1827–1910) developed the scientific basis for the calculation of these structures in the Escuela de Arquitectura de Barcelona (1875) (Graus and Martin-Nieva 2015).

The Analysis of Catenary Arches

The present investigation is based on two documentary sources. The first source is Colección de Mapas, Planos y Dibujos del Archivo General de Simancas (MPD), which is part of the Catálogo Colectivo de las Colecciones de Mapas, Planos y Dibujos de los Archivos Estatales.[1] Three gunpowder magazine projects were selected:

1. $f(x_1)$; $f(x_{c_1})$. Gunpowder magazine in Barcelona (1731) (MPD 07, 057) de Miguel Marín (Fig. 1-1).
2. $f(x_2)$; $f(x_{c_2})$. Gunpowder magazine in Tortosa (1733) (MPD 13, 035) de Miguel Marín (Fig. 1-2).
3. $f(x_3)$; $f(x_{c_3})$. Gunpowder magazine in A Coruña (1736) (MPD 17, 057) (1736) de Juan de la Feriére y Valentín (Fig. 1-3).

The second source is the design by Cèsar Martinell i Brunet (1888–1973) for the cooperative wine cellar of Pinell de Brai (1918), from the Arxiu Històric del Col·legi d'Arquitectes de Catalunya (Historical Archive of Catalonia's Professional Association of Architects, AHCOAC). The building is part of the so-called Wine Cathedrals (1918–1924) built in Catalunya during the times of the Mancomunidad Catalana and are considered to be the last built examples of Catalan masonry (Llorens Duran 2013).

The documentation of the project includes the specifications, budget figures, several sketches in the AHCOAC C222-170, and final drawings H108I/18/170. To determine the shape that best matches the geometric layout of the arches, we analysed the following documents: H101I/6/reg.2502 (Fig. 2); C222/170/1.1; C222/170/1.2; C222/170/2.4 and C222/170/2.6.

The identification of the catenary arches of the building itself was performed by a topographic survey, a terrestrial laser scanner done with Leica Scan Station P20. The catenary arches are the large arches T1 in the central nave and T2 in the lateral nave (Fig. 3). It revealed that these large arches of the central and lateral naves are catenaries, whereas the small ones are not, despite the fact that they have a similar shape. Thus, the main arches T1 in the central nave and T2 in the lateral nave were assessed while the small ones were not (Fig. 2). The nomenclature for each arch was defined by the name of the corresponding drawing:

4. $f(x_4)$; $f(x_{c_4})$. $A_{2502.d}$. Plano H101I/6/reg.2502, T2.
5. $f(x_5)$; $f(x_{c_5})$. $A_{170\text{-}2.4.b}$. Plano C222/170/2.4, T1.
6. $f(x_6)$; $f(x_{c_6})$. $A_{170\text{-}2.4.d}$. Plano C222/170/2.4, T2.
7. $f(x_7)$; $f(x_{c_7})$. $A_{170\text{-}1.1.a}$. Plano C222/170/1.1, T2.
8. $f(x_8)$; $f(x_{c_8})$. $A_{170\text{-}1.2.c}$. Plano C222/170/1.2, T2.
9. $f(x_9)$; $f(x_{c_9})$. $A_{170\text{-}2.6.a}$. Plano C222/170/2.6, T1.

[1] Established 14 April 2014, http://www.mcu.es/ccbae/gl/mapas/principal.cmd. Conserves the cartographic sources of the AGS, with 5281 files, and the graphical material AGS with 2042 references.

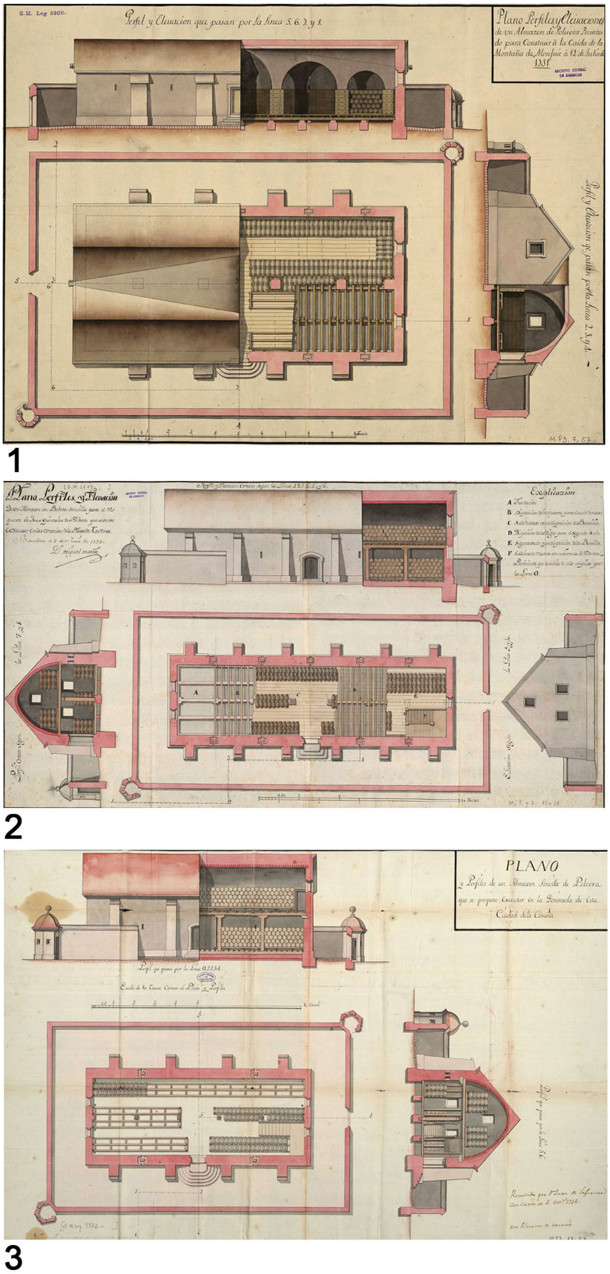

Fig. 1 Gunpowder magazines selected for the study: **1** Barcelona (1731), **2** Tortosa (1733), **3** A Coruña (1736). Source: Spain. Ministerio de Educación, Cultura y Deporte. Archivo General de Simancas. Colecciones de Mapas, Planos y Dibujos de los Archivos Estatales, de la Colección de Mapas, Planos y Dibujos del Archivo General de Simancas (MPD)

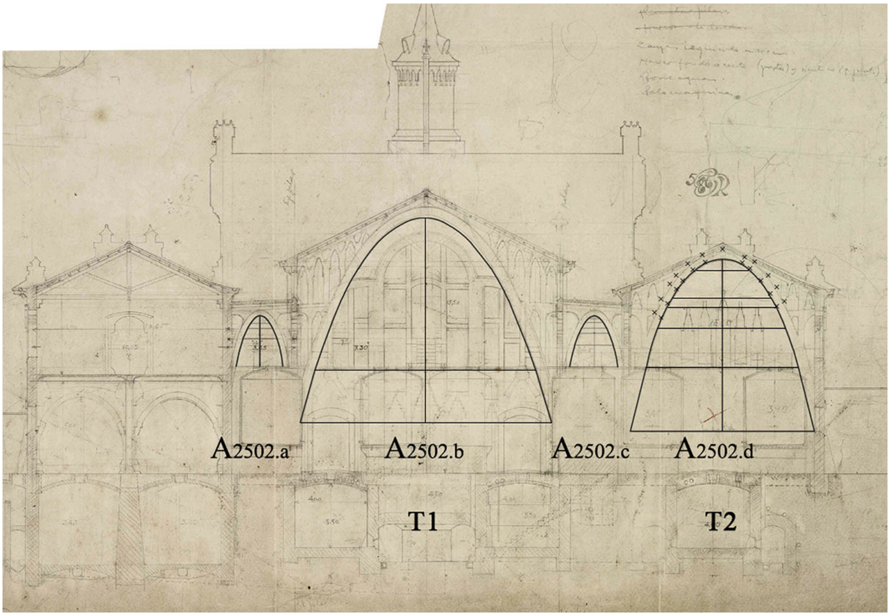

Fig. 2 Drawing. Transversal section of H101I/6/Reg 2502. Source: Spain. Historical Archive of Catalonia's Professional Association of Architects (AHCOAC)

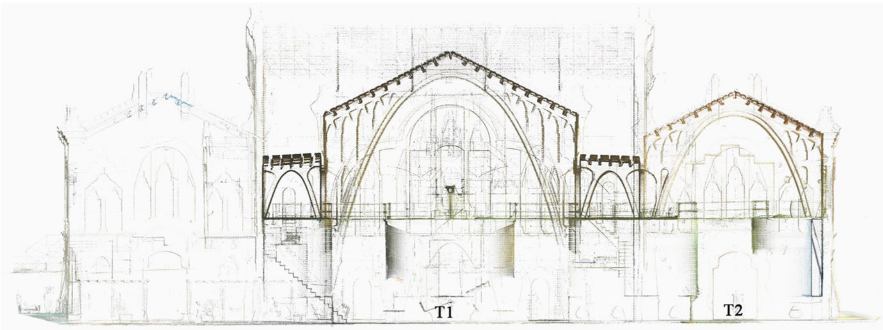

Fig. 3 Terrestrial laser scanner of Pinell de Bray, done with Leica Scan Station P20 (2015–2016)

Methodology

The sketches and drawings provide insight into the aids used for the layout of the arches (auxiliary lines, compass centre points, cuts on the paper and graphite pencil markings) that were used. They were reproduced with AutoCAD and then re-scaled to deduce the drawing process of the arches in both cases.

The shape layouts by engineers and architects are determined by an equation $f(x_i)$, which is compared to the catenary function $f(x_{c_1})$:

$$f(x_c) = m \cdot Cosh\left(\frac{x}{m}\right), \tag{1}$$

where $x = ½$ span, $m = T_o/p$, and T_o is the horizontal component of the tension, which is constant, and p is the weight per length unit of a hanging element.

The formal differences between the curves of the projects and the corresponding catenary curve were assessed graphically using a plugin for AutoCAD called Innersoft, which allows vector curves to be drawn with polylines.[2] The error was defined as (1×10^{-11}), and the number of divisions of the polyline depended on the curve's length.

The curves were laid out with the same rise and span. Thus, three common points were defined: the vertex (x_1, y_1), which is the rise of the arch, and the two springing points $(x_2, 0)$ and $(x_3, 0)$, which correspond to the span of the arch. The parameters used were length L and area S.

The difference in length between $Lf(x_a)$ and $Lf(x_c)$ are:

$$Lf(x_c) = \int_{x_2}^{x_3} Cosh\left(\frac{x}{m}\right) \cdot dx. \tag{2}$$

In addition, the areas of the arches of the design $Sf(x_a)$ were compared with the areas of the corresponding catenary $Sf(x_c)$:

$$\begin{aligned} Lf(x_c) &= \int_{x_2}^{x_3} Cosh\left(\frac{x}{m}\right) \cdot dx \\ Sf(x_c) &= \int_{x_2}^{x_3} \int_{0}^{y_1} Cosh\left(\frac{x}{m}\right) \cdot dxdy. \end{aligned} \tag{3}$$

Results and Discussion

The Catenary in Gunpowder Magazines

The metrology identified in the drawings is a *toesa* of 1949.0 mm (divisible into 6 feet of 324.8 mm) and Castilian *vara* of 935.9 mm (divisible into 4 palms of 20.90 mm). The vault geometries of the gunpowder magazines in Barcelona (1731), Tortosa (1733) and A Coruña (1736) were assessed by means of an inverted hanging chain that passed through the imposts and the keystone of the vault. The study revealed that the curves obtained—C_1 Tortosa, C_2 Barcelona, C_3 A Coruña—were very similar to the designs (Fig. 4), but not perfectly coincident. There are small deviations near the impost line, indicating that the designs were not drawn using the catenary shape.

The assessment of the auxiliary marks on the designs revealed that the layouts of these arches were made using the shape of an oval, but in these three cases, the horizontal axis is located below the springline. Thus, these three vaults do not have a vertical tangent on the impost line (Fig. 5).

[2] Innersoft is a plugin for Auto CAD that installs a suite of productivity tools. The catenary can be drawn using a spline, a 3D-polyline, or a LW-polyline with arcs.

1
2
3

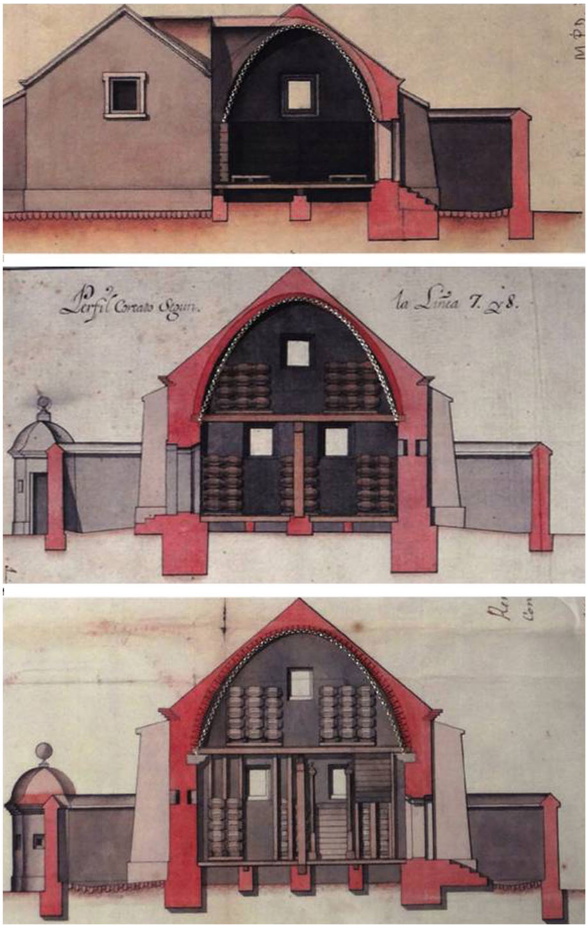

Fig. 4 Assessment of the geometry by means of an inverted hanging chain: **1** Barcelona (1731), **2** Tortosa (1733), **3** A Coruña (1736). Source: Spain. Ministerio de Educación, Cultura y Deporte. Archivo General de Simancas. Colecciones de Mapas, Planos y Dibujos de los Archivos Estatales, de la Colección de Mapas, Planos y Dibujos del Archivo General de Simancas (MPD)

By knowing the rise and span of the vault, the architectonic shape is determined with a hanging chain. Thus, a scale model can be built that can easily be taken to the construction site. In contrast, the layout of the catenary in the projects of military engineers is more complex, which necessitates the use of an approximation of the catenary through the geometric shape of a surbased oval.

The geometric layout of these arches and vaults that are based on ovals were well known by eighteenth-century military engineers (Lluis i Ginovart et al. 2014). These can be defined through the main axes, according to Vol. I of the *Compendio mathematico* (1707) by Fray Vicente Tosca (1651–1723) (Tosca 1707: I, 292–295).

The study of the auxiliary marks on the designs allowed us to conclude that the geometric figures are not rounded arches, curves drawn through pointed arches, or

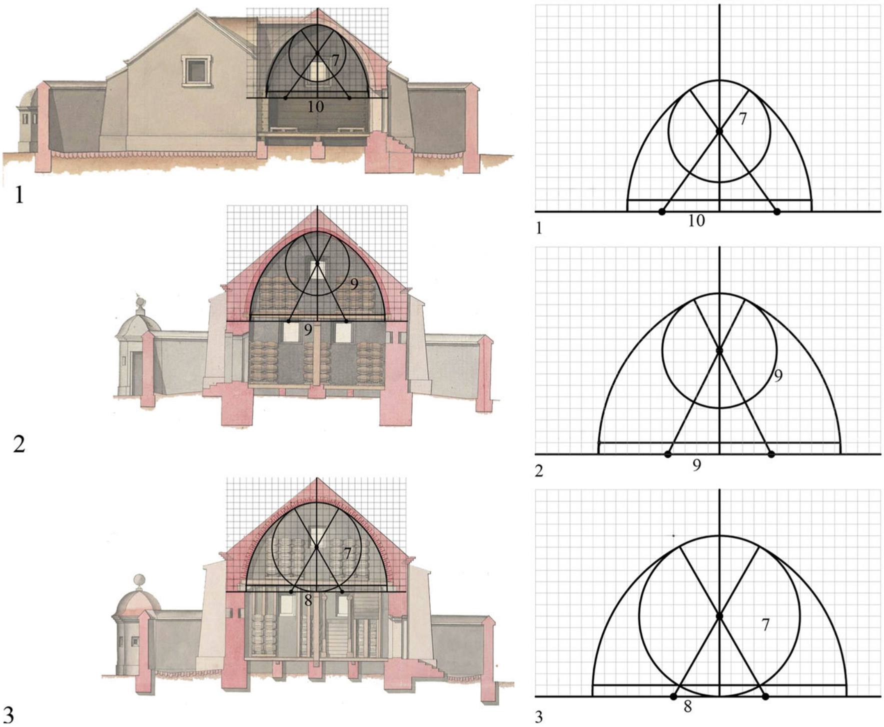

Fig. 5 The layout of the ovals of the projects: **1** Barcelona, **2** Tortosa, **3** A Coruña. Source: Spain. Ministerio de Educación, Cultura y Deporte. Archivo General de Simancas. Colecciones de Mapas, Planos y Dibujos de los Archivos Estatales, de la Colección de Mapas, Planos y Dibujos del Archivo General de Simancas (MPD)

ellipses or ovals tangent to the springline of the vault. The assessment of the original drawing revealed three marks of a compass. One point is over the vertical axis of symmetry of the figure and the other two are over the perpendicular axis, slightly below the spring-line of the vault. Thus, the layout of the vaults was made by means of ovals that are not tangent to the springline.

The formal discrepancies between the ovals of the designs and the corresponding catenary curve were assessed. Two parameters were analysed: the length of the curves and the area defined by them. The study is complemented with the assessment of the figure of the ellipse. Both ellipses and catenaries have a unique solution for a given rise and span, the values of which were deduced from the projects.

Table 1 shows the results of the comparison between the assessed figures for each gunpowder magazine. It reveals that the elliptical curve is more similar to the oval, but the horizontal axis has to be located below the impost line. Thus, a non-tangent curve to the spring line is obtained.

Table 1 Comparison of the assessed figures for each gunpowder magazine

	Barcelona	Tortosa	A Coruña
Area (cm^2)			
Oval	1464.708	2338.402	2510.289
Ellipse	1517.087	2436.081	2554.150
Catenary	1365.526	2190.158	2290.489
Dif. ellipse	−52.379	−97.678	−43.861
%	−3.58	−4.18	−1.75
Dif. catenary	99.182	14.245	219.800
%	6.77	6.34	8.76
Length (cm)			
Oval	98.3942	123.2283	126.9311
Elipse	99.9639	125.4628	127.7957
Catenary	96.4542	121.0489	123.0926
Dif. ellipse	−1.570	−2.235	−0.865
%	−1.60	−1.81	−0.68
Dif. catenary	1.940	2.179	3.839
%	1.97	1.77	3.02

Design for a Wine Cellar and Oil Press, Commissioned by the Agricultural Union at Pinell de Brai

The folder that contains the design for a wine cellar and oil press commissioned by the Agricultural Union at Pinell de Brai C222-170 includes a cross-sectional blueprint on a 1:100 scale in which three naves of the cellar are covered by wood trusses C222/170/1.5, but this blueprint was corrected with a graphite pencil to substitute the trusses with arches. This correction resulted in a new 1:100 scale sketch H101I/6/reg.2502 (Fig. 3), and the corresponding blueprint H103A/14/reg.2290 featured four brick arches and a truss. The drawing of the intrados of arch $A_{2502.d}$ shows 13 perforations on the paper, and this sequence is repeated on a parallel curve. There is evidence that this arch was drawn based on a chain (Fig. 6a). In addition, several horizontal lines cut the curve and define 13 points.

The cross-sectional sketch C222/170/2.4 at a 1:50 scale shows all four arches of the structural line ($A_{170/2.4.a}$ … $A_{170/2.4.d}$) (Fig. 6b). The auxiliary lines of the sketch reveal that the arches' points were translated using a system of coordinate points (x,y), 17 points for arch $A_{170/2.4.b}$ and 19 points for arch $A_{170/2.4.d}$. Sketch C222/170/1.1 shows the main arch of the central nave $A_{170/1.1.a}$ and the adjoining arch $A_{170/1.1.b}$, which was not analysed. The scale of this sketch is not indicated, but we determined that it is 1:50. The arch was drawn by translating 17 coordinate points (Fig. 6c). The sketch C222/170/1.2 shows the building's cross section on a 1:50 scale, including the $A_{170/1.2.c}$ arch on one side of the nave. This half-arch was drawn by translating and scaling 7 coordinate points (Fig. 6d). Finally, sketch C222/170/2.6 (Fig. 6e) shows only one arch, $A_{170/2.6}$, in a 1:50 scale that was drawn using a translation and scaling system consisting of 17 coordinate points.

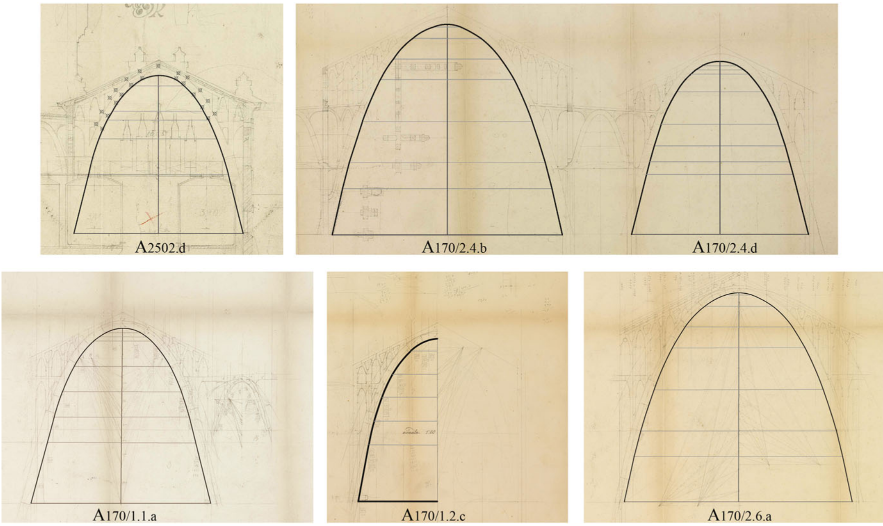

Fig. 6 Survey of the geometry and auxiliary lines of arches selected for the study: **a** $A_{2502.d}$, **b** $A_{170\text{-}2.4.b}$ and $A_{170\text{-}2.4.d}$, **c** $A_{170\text{-}1.1.a}$, **d** $A_{170\text{-}1.2.c}$, **e** $A_{170\text{-}2.6.a}$. Source: Spain. Historical Archive of Catalonia's Professional Association of Architects (AHCOAC)

The x and y coordinates of the identified points p_i are summarized in Table 2, where origin 0,0 is located in the middle of the span of the arch. The numbering of the points is defined consecutively. Point p_1 is located in the lower left coordinate, so x is the middle of the span $l/2$ and $y = 0$. The coordinates are only registered until the centre of the arch, where $x = 0$ and $y =$ rise.

By means of these points. we can determine which approximate shape was drawn. Martinell used straight sections at the arch's springing and curved sections at the top.

The layout methodology of catenary curves was assessed through comparison of the coordinates defined by the auxiliary lines. We identified a common pattern in the y-axis for the arches $A_{170/1.1a}$ and $A_{170/1.2c}$. There are four equidistant segments ($\overline{BC}\ldots\overline{EF}$). measuring a distance a another segment in the base that measures $2a + 1/3a$ and an upper segment that measures $0.75a$ or $0.25a \times 3$ (Fig. 7).

It is not possible to find another pattern in the other arches in the x or y coordinates, or a common proportion between them, so each arch is defined with different control points.

The results of the geometrical analysis of the projected arches are shown in (Table 3). where the curves $f(x_i)$ of each document are compared with the curves obtained from the catenary equations $f(x_c)$.

The results reveal that the arches of the projects $f(x_a)$ are generally very similar to the catenary equation $f(x_c)$. both for the area and the length. In addition, the values tend to be smaller in $f(x_c)$ than in $f(x_a)$. with the exception of arch $A_{170/1.2.c}$, where they were slightly larger.

The range of differences in area between the arches $f(x_a)$ and catenary shape $f(x_c)$ is -2.19 to $+0.73$ m^2 for arches $A_{170/1.1.a}$ and $A_{170/1.2.c}$ respectively. In relative terms, the range in percentages is -2.79 to $+0.98$ %.

Table 2 Coordinates identified through auxiliary lines

p_i	$A_{2502.d}$ (T2)		$A_{170/2.4.b}$ (T1)		$A_{170/2.4.d}$ (T2)		$A_{170/1.1.a}$ (T2)		$A_{170/1.2.c}$ (T2)		$A_{170/2.6.a}$ (T1)	
	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)
1	507.00	0.00	692.50	0.00	530.90	0.00	528.80	0.00	500.10	0.00	687.10	0.00
2	418.60	353.50	629.90	278.40	440.70	360.50	435.30	356.30	433.90	357.30	628.20	271.90
3	357.00	567.51	589.50	436.60	421.40	436.50	396.70	507.40	400.20	508.10	588.30	427.40
4	307.30	679.88	510.20	682.70	393.80	532.10	349.40	656.20	350.80	660.00	507.60	674.70
5	275.70	732.48	402.10	935.20	352.90	662.20	281.70	805.00	284.40	803.70	398.30	926.10
6	152.10	884.93	322.00	1058.30	259.10	858.10	167.70	954.80	166.80	952.80	324.00	1051.70
7	0.00	944.01	203.50	1181.40	174.50	963.90	135.70	978.60	0.00	1030.00	203.50	1176.30
8	–	–	79.10	1250.95	145.10	989.60	96.40	1003.30	–	–	77.80	1244.40
9	–	–	0.00	1266.20	105.50	1013.90	0.00	1029.20	–	–	0.00	1256.80
10	–	–	–	–	0.00	1040.10	–	–	–	–	–	–

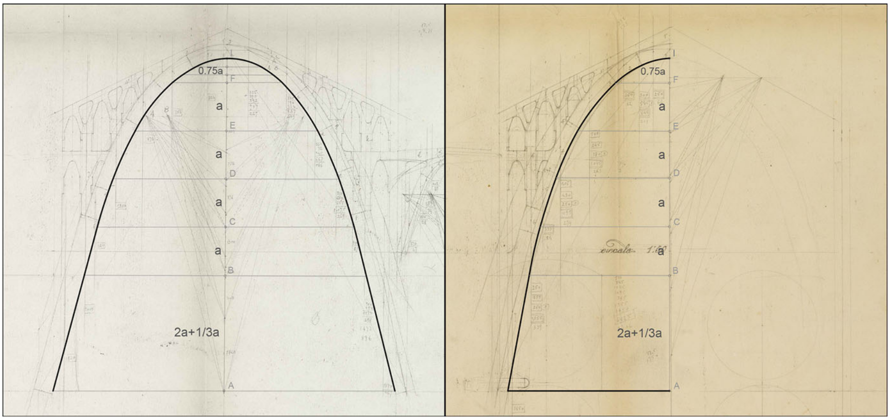

Fig. 7 Pattern in *y* for arches $A_{170/1.1a}$ and $A_{170/1.2c}$. Source: Spain. Historical Archive of Catalonia's Professional Association of Architects (AHCOAC)

Table 3 Comparison between curves for each arch

	A2502d	170/2.4b	170/2.4d	A170/1.1.a	A170/1.2.c	A170/2.6.a
Area (m^2)						
Arch	66.72	124.29	77.79	76.26	75.27	123.11
Catenary	67.96	125.98	79.78	78.45	74.54	123.95
Comparison	−1.24	−1.69	−1.99	−2.19	0.73	−0.84
%	−1.83	−1.34	−2.49	−2.79	0.98	−0.68
Length (m)						
Arch	22.55	30.51	24.71	24.44	24.30	30.36
Catenary	22.63	30.67	24.78	24.55	24.27	30.45
Comparison	−0.09	−0.16	−0.08	−0.11	0.03	−0.08
%	−0.38	−0.52	−0.30	−0.45	0.12	−0.27

Regarding the comparison of length, the range for the catenary $f(x_c)$ is -0.16 to $+0.03$ m for arches $A_{170/2.4.b}$ and $A_{170/1.2.c}$ respectively, and the relative range is -0.52 to $+0.12$ %.

The results allow for the conclusion that arch $A_{170/1.2.c}$ has the closest shape to the catenary, whereas arch $A_{170/1.1.ac}$ is the least similar. Notably, arch $A_{2502.c}$ is not the closest to a catenary. as the assessment of the design revealed several marks describing what appeared to be the coordinates of a catenary. The style of the drawing allows us to deduce that it is a working sketch, so the origin of the geometrical deviation could be an error in the location of the pins in the paper or the use of a non-uniformly loaded chain.

Martinell likely used a coordinate system for the layout of the catenary curves that was translated to the drawing of the project. The lower sections are straight and the others are curved, probably drawn freehand, to complete the approximation to the catenary. This could explain why none of the drawings are perfectly symmetrical. Thus, the hypothesis that Martinell used the homothetic property of the catenary for tracing the arches is discarded.

Conclusion

It is not possible to lay out Gregory's catenary (1697) with traditional drawing tools, such as a ruler and compass. Eighteenth-century military engineers used a method to approximate the curve through the construction of an oval that is not tangent to the impost, proving the intention to draw the curve of the vault using the *apaynelado* or *carpanel* arch of Tosca (1712) or the *anse de panier* of Bélidor (1729). Thus the x-axis is located below the spring line and therefore the tangent of the curve on the springing does not form a right angle with the horizontal. This is a feature of the definition of the catenary.

It was not until Antoni Gaudí and the Catalan modernist architects of the twentieth-century that the catenary shape was used in Spain for the geometric definition of masonry arches. Thus, the assessed projects have revealed that César Martinell used catenary arches in the cooperative wine cellar of Pinell de Brai, despite the fact that its layout was made using a method of approximation by the translation of coordinates. This method causes small formal deviations from the equation of the catenary $f(x_c)$ but the results are very close.

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