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# Projecting an Architectural Perspective: Euclidean Propositions and Common Practices at Gonçalves Sena's Workshop

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**Abstract** The projection of architectural perspectives, from the drawing plane to the tectonic support, might be the central problem of *quadratura*. However, specialized treatises tend to treat it through ideal statements. Highlighting the materialization of abstract reasoning, those demonstrations serve the scientific legitimation of *quadratura* practice, possibly circumscribed to enlightened scientific and artistic circles, rather than the dissemination of procedures aiming to train people in the projection craftsmanship. This may even be the best-kept secret of the *quadratura* painters. In this context, this paper is focused on Sena's *quadratura* (1754), at the church of Santarém's Jesuit College, where under Vieira's statements exposed in his treatise on perspective (*Tractado de Prospectiva*, Codex 5170, Biblioteca Nacional de Lisboa, 1716), we will disclose procedures based upon common practices of the painter's workshop concerning the outline and projection of the image.

**Keywords** Perspective · Optics · *Quadratura* painting · Projective geometry · Virtual architecture

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## Introduction

Considering the projection of architectural perspectives as the central problem of *quadratura* practice, this article aims to unveil practical procedures hidden under theoretical statements widespread through specialized treatises. As such, the scope of *quadratura* conical projection is taken as practical procedures that allow for the outlining of linear structures onto curved surfaces. Although the theoretical framework is based upon ideal schemes and narratives of scientific demonstration, the common practice at the painter's workshop is assumed here, along with the material circumstances of the *quadratura* projection. The recent essays by Hoffmann (2009) and Camerota (2010) have looked at the problem without the constraints imposed by idealized procedures (such as Pozzo's *Modo di far la graticola nelle volte* or Danti's narrative of the *fili tesi* process), advancing into practical hypotheses. On the other hand, and concerning the primary sources reported here, studies on Inácio Vieira's treatise (1716) do not fully analyze the *quadratura* projection resolution, despite their value in rescuing the document, identifying its sources and organizing its main contents (Leitão and Mello 2005; Mello 2002; Raggi 2004; Trindade 2015). In the same order, Gonçalves Sena's *quadratura* (1754) was brought into daylight by recent art history studies on Portuguese *quadratura* (Mello 2001, 2002; Raggi 2004) although they call for further developments of its projective and architectural features.

In this sense, and under the recent considerations on both objects (Cabeleira 2015), a thorough review of Vieira's manuscript and Sena's *quadratura* identifies procedures for creating the image outline and its projection. Their cross-referencing leads to theoretical hypotheses tested upon graphical models that allow us to ascertain its validity as well as to communicate and visualize the aroused assumptions.

Based upon Euclidean propositions, the theoretical enunciation by Vieira coincides with the empirical reasoning undertaken at Sena's workshop, a consideration that allows us to detach the method from ideal schemes and advance towards a practical solution. Delimiting the exposed reasoning to this issue, unveiling the best-kept secret by *quadratura* painters, we aim to clarify the *quadratura* projection practice through a simplified process that sends us back to the very fundamentals of geometric knowledge, the Euclidean propositions.

## Approaching the Problem Through Sena's *Quadratura*

The study of Inácio Vieira's manuscript on perspective (1716) and the perspective restitution of Gonçalves Sena's painted architectural perspective (1754) at Santarém's Jesuit College (Fig. 1), demonstrates that alternative models of *quadratura* projection may be considered.[1] Although both authors assimilate

[1] Gonçalves Sena (1713/1790) was a self-taught spirit whose apprenticeship and artistic update arises from the study of coeval treatises (Benedicto 1791: 8), necessary to the scientific and iconographic codification of the pictorial practice. He started as an easel painter, developing, progressively, capacities as a mural painter and finally as a quadraturist.

**Fig. 1** *Ascension of the Virgin* (1754) Gonçalves Sena. Nossa Senhora da Conceição church, Santarém

perspective knowledge from contemporary treatises, the resolution of *quadratura* projection seems to move away from published models, incorporating Euclid's propositions and common practices at the painter's workshop.

Although the spatial purposes of Sena's *quadratura* are largely achieved, inconsistencies in the projective structure are revealed. Deflections from the fundamentals of perspective are consequent to the limited scientific speculation and artistic practice of perspective in the Portuguese context, as well as to Sena's knowledge and methodology.

Through the analysis of Sena's *quadratura* a coincidence is detected towards patterns taken from Pozzo's *Perspectivae pictorum et architectorum* (1693), a fact that may express the assimilation of Pozzo's aesthetic or, perhaps, the imposition of a model by the local Ignatian community aiming to achieve a coincident fictional program with the architectural perspectives of the Collegio Romano's church (Fig. 2). What is certain is that this incorporation refers to the image composition, assembling and rearranging Pozzo's engravings, instead of the absorption of Pozzo's methodology and technical model.

Through this a compositional matrix (Pozzo 1693: 89), is detected from which the architectural image is reset: the plan of the painted architectures was reorganized, the elevations were reconfigured, the employed constructive

**Fig. 2** Confrontation of *St. Ignatius entering paradise* (1691–94), and the *Ascension of the Virgin* (1754). The image of Pozzo's *quadratura* was compressed (1/3 of the columns height) in order to adjust the horizontal guidelines of Sena's architecture (gallery, balustrade, cornice and pediment). Overlay of engravings from *Perspectivæ pictorum et architectorum* (1693) evidencing Sena's assembly process

vocabulary was adjusted and the apparent depth was reduced. Architectural themes were rearranged in order to respond to the desired spatial image (consequent to the supporting built environment), reforming, in the same sequence, the color palette (coordinated with the polychrome of painted woods and inlaid marbles of the supporting space) and figuration (according to the commissioned iconographic program), renewing the whole image. Hence, the relevance of the work results from a *modus operandi* capable of shaping models according to the specific circumstances of the spatial support and the intention of the imagery. However, if compositional and perspective discrepancies are verifiable, such as deviations from the precepts of a *soda architettura* and from the management of depths, yet the image embodies its spatial intentions. Arrangements fall into a strictly imagery-based scope, pursuing the perceptual likelihood of the spatial effect rather than its metric truth and constructive logic constrained under perspective precepts. Even so, the painter is able to unify the representation through the engagement of the outlined structure to a single convergence point providing an apparent stability and an effective space induction. Nevertheless, despite his rudimentary mastery of perspective, how did Sena solve and execute the *quadratura*? How was he able to project the outlined image into the semi-cylindrical soffit of the vault? Without a clear definition of the viewpoint, regulating the outline of the depth and the image projection, how did Sena circumvent the problem of the *quadratura* projection?

Considering that the outcome does not result from a happy coincidence, a possible resolution and materialization of the architectural perspective is pursued through simple assumptions, both theoretical and technical, rather than the commonly accepted and disseminated procedures.

## Euclides and the Hidden Key of *Quadratura* Projection

The influence of *quadratura*'s theoretical models can be seen in the Portuguese panorama via Inácio Vieira's lessons at the *Aula da Sphera* of the Jesuit College of Santo Antão in Lisbon (Leitão 2008). Compiling the theory on the sciences of vision, the *Tractado de Prospectiva* (Vieira 1716) confronts the practical statements of Andrea Pozzo's, *Perspectiva pictorum et architectorum* 1693–1700 (Pozzo 1693, 1700), with the theoretical ordination of Claude Dechales, *Cursus seu mundus mathematicus* 1674 (Leitão and Mello 2005; Dechales 1674), directing the discourse towards the artistic/scientific prodigy of visual deceit. It is precisely from this theoretical body that a resolution of the *quadratura* projection emerges, fleeing away from instituted technical models.

The *quadratura* procedures are unveiled with the determination of the convergence point for any given picture plane, *Como acharemos as apparencias q[uan]do a taboa está inclinada* (Vieira 1716: 248), ('how to find the perspective appearances in a sloping picture plane') (Fig. 3). A key statement of the reasoning is held in the chapter entitled *Dos tetos e abóbedas* (Vieira 1716: 270), ('ceilings and vaults'), where, following closely the *Liber Quintus* of Dechales's *Perspective,* Vieira explores the nature of the picture surface and warns about the preservation of projective rules. As such, by displacing the picture surface the relative positioning of the geometric data is reviewed, conditioning the perspective structure.

Based on these assumptions, Vieira describes the *quadratura* outline process, particularly the *sotto in sú perspective*, taking into account the fundamentals of perspective such as the viewpoint projection, horizontal line, base line and the distance point, "which should always be further than the outlined image" (Vieira 1716: 274). A final consideration that may be interpreted under two possibilities: if, on the one hand the projective rule is explained, relating the distance point with the distance from the viewpoint to the picture plane, on the other hand it seems to cogitate over the proportional adjustment of the visual cone, seeking the annulment of peripheral distortion. However, this elasticity in handling with the distance point, emphasizing perceptual conditions over projective rules, is never made explicit by Vieira, being even contrary to the *modus operandi* of his reference authors.

Illustrating the procedure with the 282nd and 283rd schemes (Fig. 4) Vieira draws the architectural composition plan coincident with the picture plane. Through delineation of the plan, vertical lines would be drawn converging into F (main convergence point), and the element's depth would be dictated by the foreshortened image of CE, EH, HI e IJ. So, bearing in mind that the represented space is made up by a succession of homothetic figures, managed by the main convergence point, it is possible to draw the different perimeters of the illusory architecture. However, the presented theory is still limited to the image outlined in the picture plane, remaining unclear about its projection onto curved, irregular or compound surfaces.

It is then, at *Das abobedas, e planos irregulares* (Vieira 1716: 283) ('from vaults and irregular planes'), that Vieira explains the image projection onto curved surfaces. Repeating almost entirely statements from Dechales (1674: 519–520), which in turn holds its origins in the practices of Dubreuil (1649: 48), Vieira

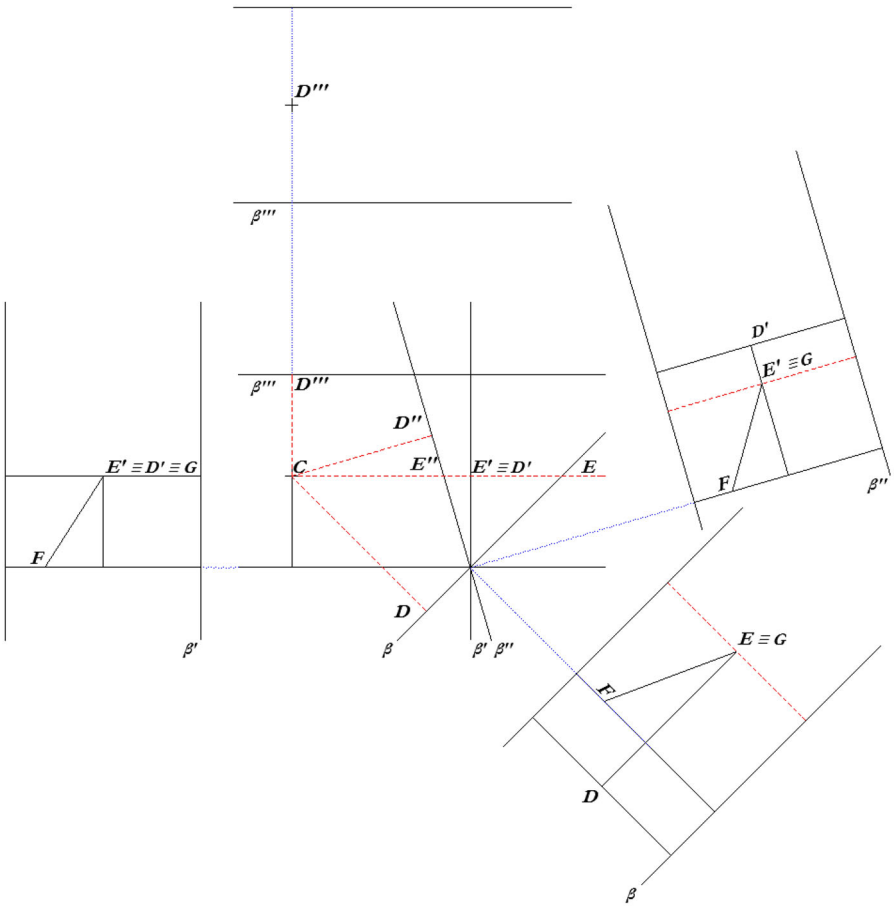


**Fig. 3** Graphic interpretation of Vieira's 261st illustration and statement concerning the displacement of the picture plane β, and consequent adjustment of the perspective structure (horizontal line, base line, convergence point E and projection of the viewpoint D) as well as the represented geometrical data (line FG and its parallels)

supports the procedure with an abstract grid. That is, a geometrical structure that, once drawn over the prototype, will determine the vault organization (Vieira 1716: 283) in the same number of squares although deformed by the conical projection.

Although a *modus operandi* concerning the quadrature projection is not immediately clarified, the debate touches upon three canonical possibilities: "To prescribe this in an easy way, strings may be used… or with view and nothing else, or at night by putting a light in the viewpoint…" (Vieira 1716: 283–284). These theoretical procedures all have evident drawbacks to an effective implementation: the scaffolding installation that, ultimately, should result from a set of beams permeable to the passage of ropes, light and sight; and the quality of the light source that should be strong and stable enough to project shadows with the necessary accuracy.

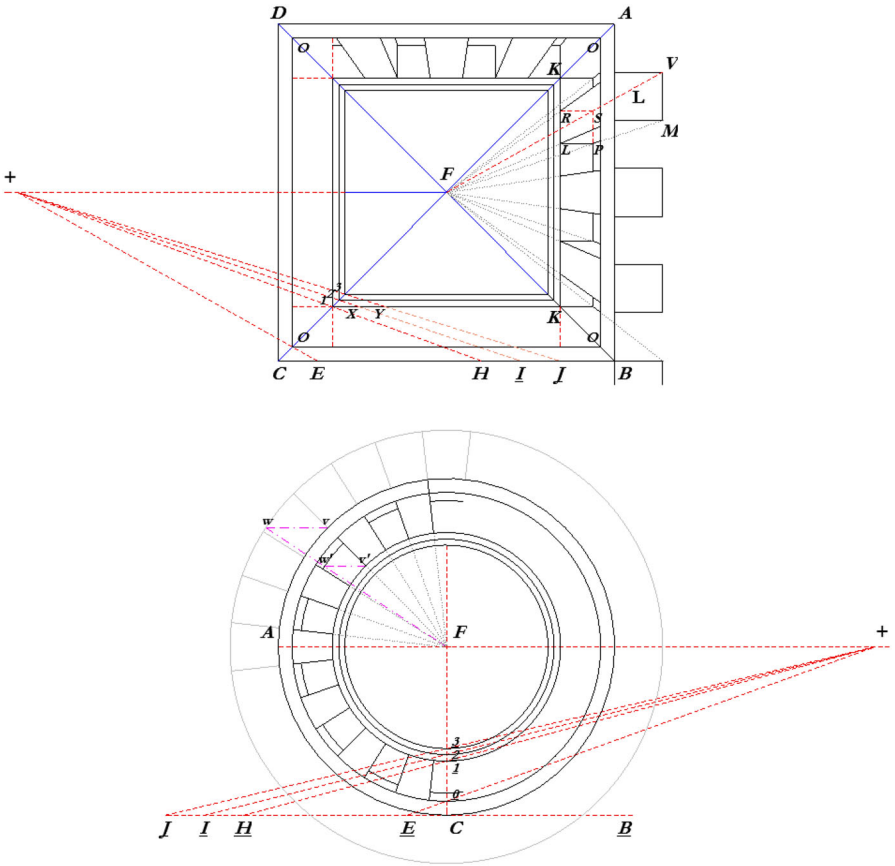


**Fig. 4** Interpretation of Vieira's 282nd and 283rd scheme

Just in the 492nd and 493rd paragraph (Vieira 1716, f. 284–285) attention is directed to a method that, although paraphrasing Dechales (1674: 520), regards the *quadratura* projection under tangible procedures, deviating from theoretical models. The explanation starts by reviewing the use of light, "to better understand what was said" (Vieira 1716: 284), evidencing the geometric transformation of the projected grid. Simultaneously Vieira mentions the use of ropes materializing the visual rays, to amplify the disclosure of the geometrical mechanism, and identifies the main geometric elements and their functions represented the 292nd scheme (Fig. 5): E, viewpoint; J, viewpoint projection onto the picture surface (placed upright on E). From these elements, an ABCD portion of the vault is divided into correspondent squares with the CDFG grid, forming into the eye a matching image (Vieira 1716: fol. 286v). From this data, the statement of Dechales/Vieira organizes the grid projection into separate steps for vertical and horizontal lines.

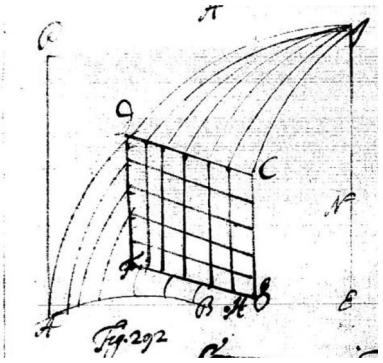

**Fig. 5** Vieira's 292nd scheme

To project the vertical lines into the curved surface, the statement is grounded upon the 18th proposition from Euclid's eleventh Book (Vieira 1716: 286)[2] concerning conditions of perpendicularity among planes. As such, and according to Vieira's 292nd scheme, once the grid vertical lines CG and DF are parallel to EJ (defined by the viewpoint and its projection into the vault) their projective planes could be defined. Freeing the painter from ideal models, the statement allows a practicable delineation of linear entities in the curved surface. The only requirement is the spatial materialization of the projective plane, based upon rope triangulation, achieving the desired segment (Fig. 6) through the intersection of the projective plane (EDJ) and the projection surface (ABCD). As advised by Vieira, the intersection of both surfaces results from this method: "extend the first string ED and, from the point J, extend another wire that touches the first string at any given point so it will produce in the vault a point belonging to such plane—with this method we'll have the grid vertical lines" (Vieira 1716: 285).

Following this reasoning the conical projection onto curved surfaces is solved through recognition and materialization of a Euclidean proposition. Hence, the possibility of outlining drawn structures onto the curved surface emerges using up the free space between the scaffold and the vault.

In its sequence, and regarding projection of the grid horizontal lines, Vieira reuses a vertical segment CG (Fig. 7), now divided into regular parts (Vieira 1716: 285). Although its implementation is not completely clarified, it is implied in the use of an auxiliary sketch where the foreshortening among projected horizontal lines into the vault's curvature could be calculated.

By this simple and demystified process, the *quadratura* projection is outlined as approachable, feasible and accurate given its operational substance. First, the grid projection onto the curvature is performed entirely above the scaffolding. Second, curvilinear projections, consequent to the conic transformation of linear segments, are operated by the intersection of a projective plane and the curved picture surface. Third, linear projections, whose transformation is strictly relative to its metric nature, are achieved by employment of an auxiliary drawing. Through this

[2] If a straight line be at right angles to any plane, all the planes through it will also be at right angles to the same plane (Euclid 1956, vol. III: 302).

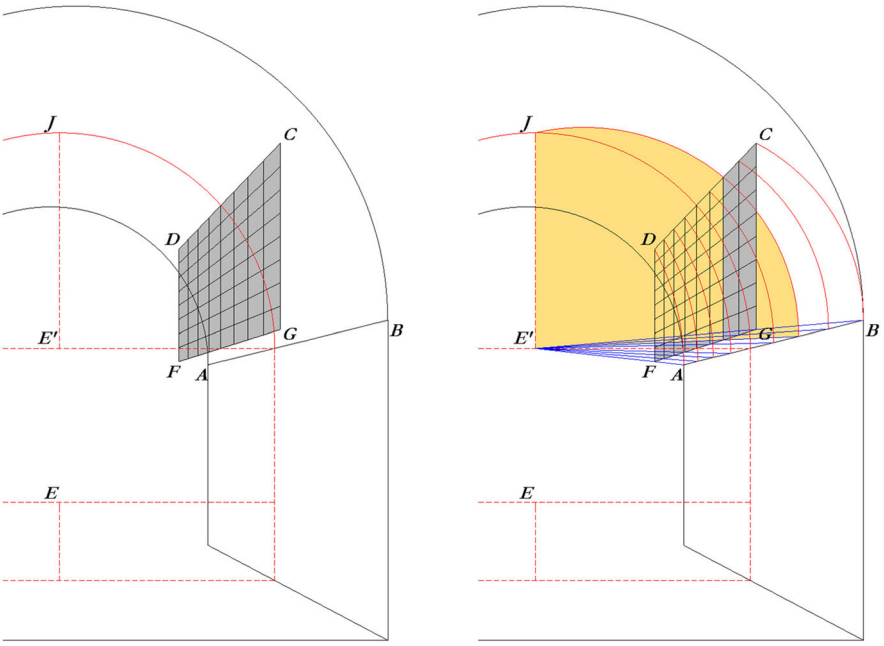


**Fig. 6** Graphic interpretation of Vieira's 292nd scheme: displacement of the CDFG grid; projection of a vertical segment through definition of its projective plane applying the 18th proposition from Euclid's eleventh Book

procedure the operation renounces the spatial materialization of the eye, although obedience to a viewpoint is conceptually required.

According to this sequence, and although Vieira's statements result from a path trodden by Dechales (1674: 520), the emancipation from the widespread models based upon employment of extended ropes or light sources from the viewpoint should be valued. An attainment of the *quadratura* projection rooted in Euclidean propositions beside the common resources of the painter's workshop.

## The Painter's Workshop Practice

Returning to Sena's *quadratura*, it is accepted that the painted image was previously solved in the plane, where the sources of the imagery were combined and regulated under a single point of convergence. At the same time, abstract alignments would be established in order to support the image projection onto the tectonic picture surface. But how would the painter actually transfer the draft onto the vault soffit?

Due to its compositional symmetry (in four quadrants) it is possible to solve the projection out of ¼ of the image and, after that, obtain its totality by means of card and *spolvero* techniques. Simultaneously, assuming the hypothesis of projection already considered, it seems tangible that the operation is freed from the constraints of a projective center where a light source was placed or ropes would be attached.

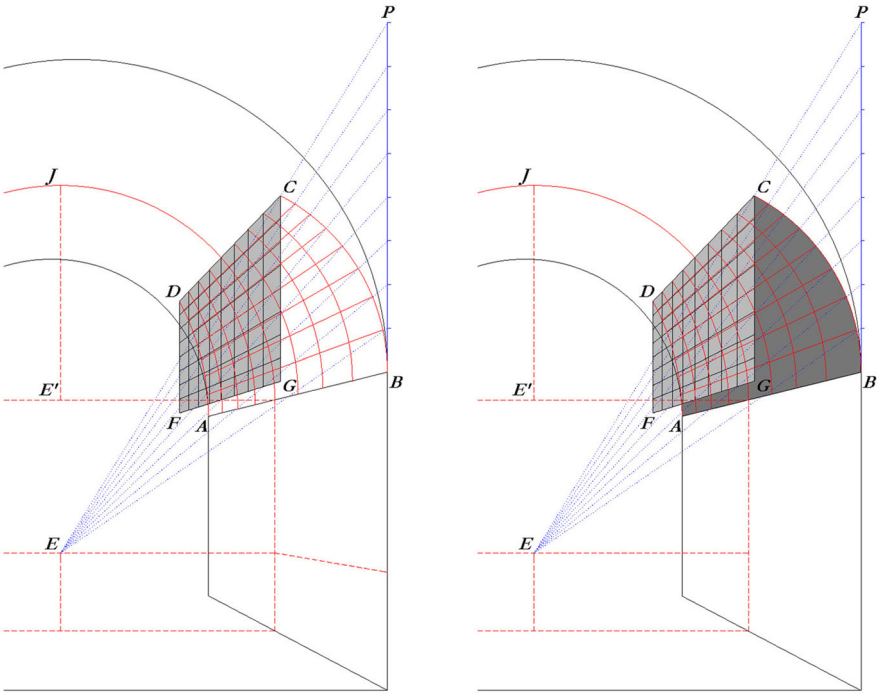


**Fig. 7** Graphic interpretation of Vieira's 292nd scheme: definition of the CG rope knots; projection through calculation of CB division based upon preliminary drawing

This disaffiliation of the projection process from the projective center is consistent with Sena's drafting process (whose assembly of images seems to suppress fundamental conditions of a projective space) tending towards the implementation of procedures equivalent to the Dechales/Vieira statements.

If Sena's expertise on perspective seems rudimentary it suggests the need to verify the possibility of performing the *quadratura* image projection by application of pictorially common procedures. From the resources of the painter's workshop it is possible to highlight the application of reference grids (used in the calculation and manipulation of scale besides the combination and transfer of imagery models), the employment of card and *spolvero* techniques (to transfer and replicate defined configurations) as well as the use of accepted drafting tools (such as lines or ropes in the guidance and control of the image layout).

The use of geometrical grids serves the delineation of architectural perspectives, regulating the preliminary draft (enlarging and reducing Pozzo's images, composing, combining and manipulating the scale of represented architectural features according to an apparent depth) and simultaneously supporting the projection onto the vault surface (being a common procedure applied at the image transference among preparatory drawings and the picture plane, in the case of easel painting).

The employment of a *quadrettatura* expedient (at the root of the *quadratura* term) is thus a technical condition that, beyond the scope of easel painting,

integrates the *quadratura* purposes and practice. It regulates the designed composition and its projection onto the tectonic surface, as enunciated by the specialized treatises that bail out the statement in order to communicate the geometric transformations consequent to a conic projection.

On the other hand, the application of card and *spolvero* techniques, intrinsic to the practice of mural painting (at which Sena had a large experience), would allow the artist to transfer the outlined data from ¼ of the vault onto the remaining surface, shortening the whole process.

As for the applied instruments (such as ropes, plumb line, ruler and compass), these allow the *quadratura* painter to respond to the essential problems caused by the projection of linear entities, coordinated under practical geometrical procedures, onto curved surfaces.

Having in mind that the definition of the perspective draft in the plane is a *sine qua non* condition, alignments and axes of the represented structures (setting a compositional grid) or a juxtaposed regular grid (providing an abstract reference grid) can be individualized (Fig. 8). Dismantling Sena's *quadratura* reveals a possible compositional grid regulating the image through vertical lines (with origin at A, B and C) and horizontal alignments (with origins at points 1 till 10).

Starting from this inscribed grid over the perspective draft, Sena would project it onto the vault through the definition of projective planes without the necessary materialization of a projective center. Operating exclusively above the scaffolding,

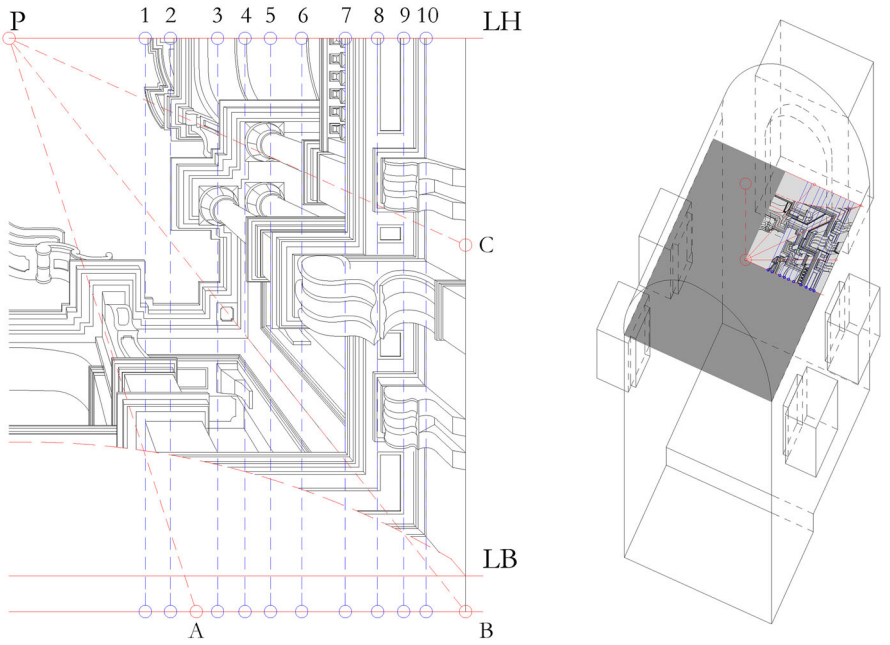


**Fig. 8** Proposed compositional grid over ¼ of Sena's *quadratura*. Overlapping of the grid with the perspective draft and its installation on the scaffolding

the painter would be assisted by a plumb line (suspended from the center of the vault) and a layout of the compositional grid (inscribed over the scaffold through extended ropes or by direct sketch). Applying the Euclidean proposition, underlined by the procedure (Vieira 1716: 286), the plumb line and a segment of the materialized grid, at the scaffold, would define the required projective plane for projection of the geometric entity onto the vault.

However, while in Vieira's treatise the application of the 18th proposition from Euclid's eleventh Book is based upon the definition of the projective plane through two parallel lines (taking into account the 35th proposition of Euclid's first Book), the methodological proposal in Sena's procedure is based upon the plane definition through two intersecting lines (the 2nd proposition from Euclid's eleventh Book). Thus, the parallel lines of Vieira's statement are the distance line and a vertical line of the vault suspended grid, while Sena's intersecting lines are the distance line and a vertical line of the perspective image outlined at the scaffold plane.

To depict the intersection between the projective plane and the picture surface an oblique rope can be used. This rope has its origin at the edge of the outlined vertical line (A, B and C) and is convergent with the viewpoint projection at the vault surface (Fig. 9). So, with the aid of a plumb line (ascertaining the verticality among the oblique rope and the straight line drawn in the scaffolding plane), or assisted by a ruler (achieving the necessary coincidence with the projective plane), the painter

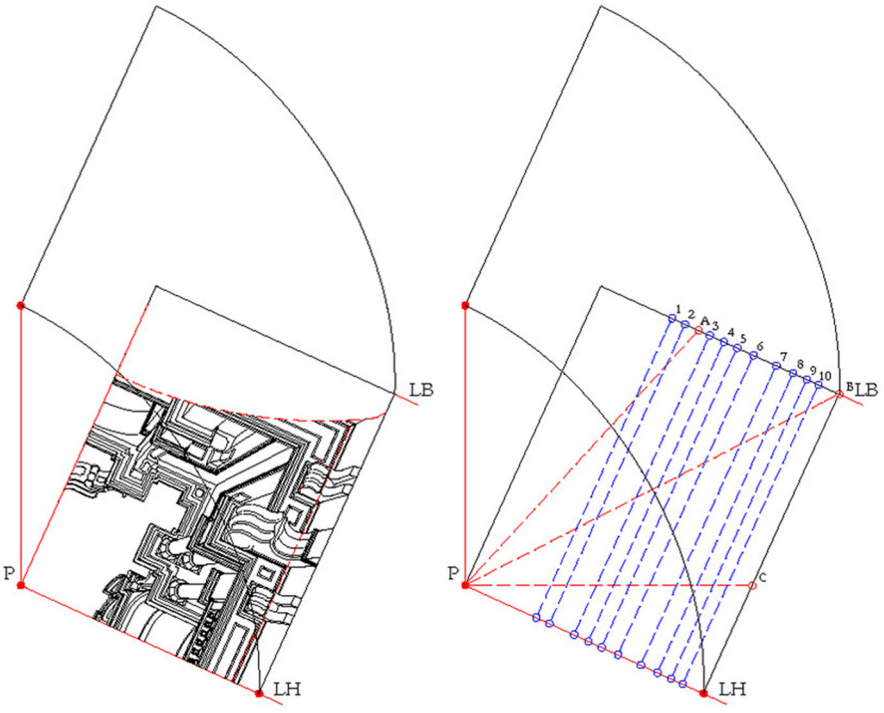


**Fig. 9** Image resolution at full scale in ¼ of the scaffolding plane

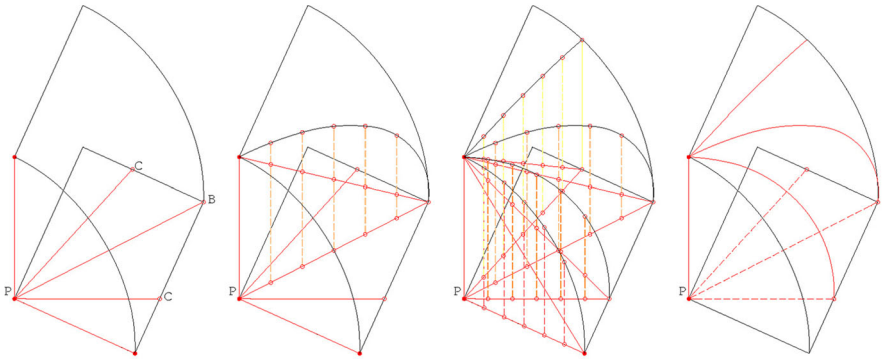


**Fig. 10** Graphical verification of the operational sequence in the projection of the vertical lines

can record the curvilinear intersection of both surfaces (Fig. 10). This is a procedure repeated in all vertical lines necessary to obtain the projection of a compositional grid, as dense as the amount of information to be projected.

Concerning the projection of horizontal alignments (Fig. 11), the procedure would require an auxiliary drawing in which the vault profile and the picture plane are inscribed, as well as an hypothetical viewpoint from which projective lines, visual rays, are drawn. The height of these alignments onto the built surface is provided by the graphical intersection of the projective lines with the vault's profile. Once these lines are parallel to the vault curvature (the picture surface) their projection does not transform its linear conformation, being its registration onto the vault achieved through leveled ropes. As such, following these steps, the control of the projection procedure obliterates the spatial materialization of a projective center far below the working plane.

After the projection of the compositional grid, the transference of the outlined image is operated under evaluation of the proportional relations between the perspective image and its referential grid, either in the supporting draft or its homologous projection onto the vault. This is a procedure that does not require a complex scaffold (according to logics of flexibility and visual permeability) once the whole operation takes place exclusively above it. On the other hand, the transference of the outlined image onto the rest of the vault intrados can be operated through card and *spolvero* techniques, whose procedures are certainly controlled by the painter.

The projection of the compositional grid results from the conjunction of the horizontal and vertical alignments. However, it can be pointed out that the features considered are only related to the *quadratura* perspective structure and its projection, leaving aside pictorial aspects highly responsible for the image's effectiveness. Thus, besides the delineation of linear entities, the *quadratura* spatial effect arises from the pictorial potentialities, like the skill in material imitation, the use of color in the emphasis of depth and segregation of illusory spatial layers, the modelling of the *chiaroscuro* and the conformity between represented and physical light.

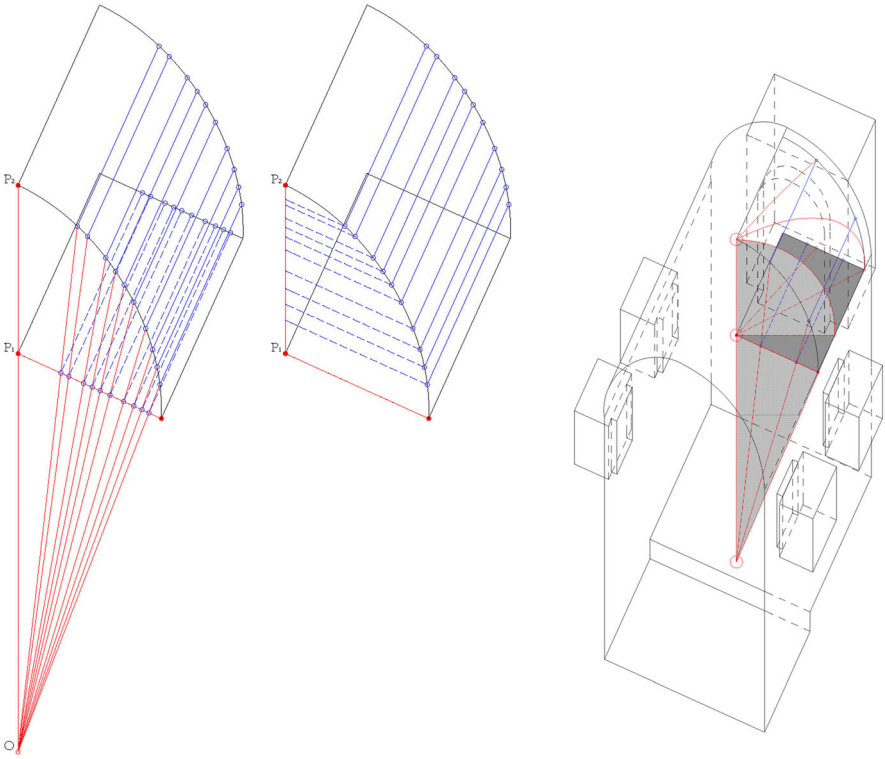

**Fig. 11** Graphical verification of the projection of horizontal alignments. Graphical simulation of the complete procedure identifying the scaffold (working plane), the projective plane and the viewpoint

## *Quadratura* Projection Between the Ideal and Real Constraints

As the problem of projection, between the plane draft and the curved picture surface, might be the key issue of *quadratura*, it is important to review the *modus operandi* followed by some authorities whose procedures may be at the origin of Vieira's statements.

In *Le due regole* (Vignola and Danti 1583) the *Fili tesi* method solves the conical projection into the tectonic surface through a matrix of horizontal and vertical alignments of the architectural image.

According to this, the horizontal alignments are calculated through an auxiliary drawing (determining the intersections of the visual rays with the vault profile), and delineated onto the vault by beating strings regarding the non-transformation of its linear configuration. On the other hand, the projection of vertical alignments will result into a curvilinear transformation. As such, their projection is supported by extended ropes, materializing visual rays, and confirmed through the eye. For each vertical segment, both extremes are signed into the vault: one coincides with the horizontal picture plane from which the image is generated, and the other

correspondent to the farthest point of the segment. Joining a rope from the first point until the viewpoint and suspending a plumb line from the second point, the method applies, implicitly, the same Euclidean proposition identified by Dechales and Vieira. Thus, the plumb line should intersect the oblique rope, evidencing the segment's projective plane. Once the accurate projection of the second point is verified, it is possible to outline the entire vertical entity with a flexible catenary adaptable to the surface curvature.

Anyhow, given the articulation between the projective plane and the picture surface, it is possible to avoid the viewpoint as the control center of the projection. According to the essay by Di Marzio (1999), the rope connecting the lower extreme of the vertical segment with the projected viewpoint, and the vertical plumb line suspended from the projected viewpoint, are enough to define the projective plane. Hence the method chases the scientific legitimacy of the *quadratura* practice rather than the disclosure of a practical procedure. Looking at Danti's statement, and taking into account Di Marzio's reasoning, the proposed variation is not far from the method presented by Vieira.

However, if the method exposed in *Le due regole* (Vignola and Danti 1583) is rooted in the specificities of a compositional matrix, *Le Moyen universel* (Bosse 1653) sets an universal model, either theoretical or practical, regulating the *quadratura* projection under constant projective rules.

In Bosse's method the image is subordinated to an abstract grid transformed by the conic projection. The same conceptual basis taken by Pozzo (1693), through the definition of three square grids, or adopted by Dechales (1674) and Vieira (1716), defining a square mesh suspended from the curved surface.

This method advocates the delineation of the grid horizontal lines via auxiliary drawing, and the projection of vertical lines through light source. By launching the grid shadow into the projective surface, Bosse's method gives rise to a visual demonstration of the projective transformation. However, a major breakthrough is given by Bosse when the light source is positioned at any point of the vertical line containing the viewpoint. Again, we found similarities with the method of Dechales and Vieira (Fig. 12). By implied application of the Euclidian proposition, the 18th of the eleventh Book, Bosse decouples the projection control from the viewpoint being able to circumscribe the operation above the scaffolding.

From the ascertained methods, the *quadratura* projection may be based upon delineation of a compositional matrix, attached to the represented bodies (Vignola/Danti), or an abstract grid, juxtaposed to the draft image (Bosse/Pozzo and Dechales/Vieira). Focusing on the last case, it should be mentioned that the positions of the grids considered have differences although they may generate matching results.

As such, while in Bosse's procedure the baseline of the interposed grid matches with the vault width, in Vieira a correspondence of measure arises from a higher alignment of the projection surface, demanding the calculation of the grid lower limit constriction A requirement that, being forgotten by Vieira, reveals the metric awareness of Bosse's perspectival construction, while Vieira seems to depreciate the value of measure due to a strict management of an outlined image. However, Vieira's enunciation is reportedly based upon application of Euclid's Proposition,

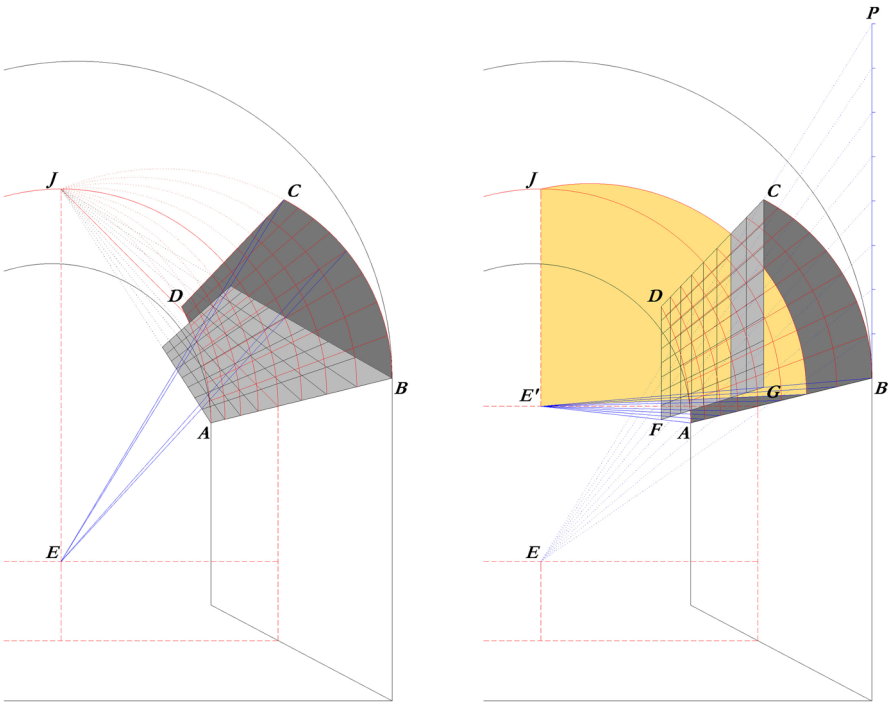


**Fig. 12** Graphical comparison of the exposed methods by Bosse (1653) and Vieira (1716)

the 18th of the eleventh Book, in a clear methodological coincidence with the distinct proposals disseminated throughout the specialized treatises. An enunciation that departs from dependence on the viewpoint, decreasing requirements for its execution (scaffolding installation, ropes extension and control from the viewpoint) at the same time that it responds to the required accuracy in outlining the referential drawing structures.

## Conclusion

The model taken from Vieira's treatise seems to converge with the recent speculations of Hoffmann (2009) and Camerota (2010). In the attempt to unveil Pozzo's practical procedures concerning the *quadratura* projection, especially the control of the delineated lines onto the vault curvature, both authors refer to the use of two ropes attached to the projection of the viewpoint at the center of the vault: one to guide "uprightly the rule" in the delineation of lines, and the other, as a suspended pendulum, to prevent any variation of the established direction. This is a model based upon statements and illustrations by Emmanuel Maignan (1648: 372), given for the need to control deformations of meridian lines, but neglected by the *quadratura* specialized treatises, other than Dechales (1674) and Vieira (1716).

Through the consideration of the theoretical and technical components examined here, the applicability of the method is surprising, giving to the *quadratura* projection a resolution distant from the elaborate theoretical assumptions expressed in specialized treatises. Those assumptions arise as speculative exercises in the research and demonstration of projective rules, mastered by high artistic and scientific erudition centers. As such, the wider practice of the *quadratura* projection would be solved through common instruments, procedures and knowledges acquired at the painter's workshop. An example of this parallelism between theory and practice is the case of the projection of straight lines into curved surfaces stated by Dechales and Vieira. A procedure solved through the spatial conformation of the 18th proposition of Euclid's eleventh Book and the method that the painter would have applied to connect pairs of points—the projection of the viewpoint in the vault, and points of the linear entities coincident with the impost line—with rope and ruler in order to guide the delineation.

Sena's *quadratura* at Santarém's Jesuit College openly asserts the imagery and resources of the Jesuit mission: "The painting is a brilliant mirror of virtues, a rhetorical scourge of this world of addictions, a spirituous incentive to the moral perfection and orientation of the wise, and catholic men." (Benedicto 1791: 14). But if the work reflects the desired virtues of its commissioners, Sena is not necessarily a virtuoso on perspective. Even so, this humble painter is able to solve the *quadratura* projection through the application of Euclidean propositions and pictorially common procedures.

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