Nexus Netw J (2015) 17:875–898
DOI 10.1007/s00004-015-0265-5

NEXUS NETWORK JOURNAL
Architecture and Mathematics

RESEARCH

A Grammar-Based Model for the Mass Customisation of Chairs: Modelling the Optimisation Part

Mário Barros[1] · **José Pinto Duarte**[1] · **B. M. Chaparro**[2]

Abstract This research presents a methodology to develop and implement a generative design system as the technological model for mass customisation in the furniture industry. The generative design system comprises two subsystems that permit the generation and the evaluation of customised designs within a predefined design language. The shape generation subsystem is defined by shape grammars and parametric design models. The shape evaluation subsystem encompasses simulation and optimisation to guarantee the structural feasibility of the customised designs under operating conditions. This paper focuses on the modelling activities regarding the constitution of the optimisation part of the shape evaluation subsystem. Structural optimisation using simulated annealing is applied to assist the designer in the automatic search for an optimal grammar-based detailed solution. The theoretical model is illustrated by its application to a symbolic mass production problem: Thonet bentwood chairs, which were changed to comply with the mass customisation paradigm.

Keywords Design analysis · Shape grammars · CAD · Structural engineering

---

✉ José Pinto Duarte
jduarte@fa.ulisboa.pt

Mário Barros
milbarros@fa.ulisboa.pt

B. M. Chaparro
bruno.chaparro@sapo.pt

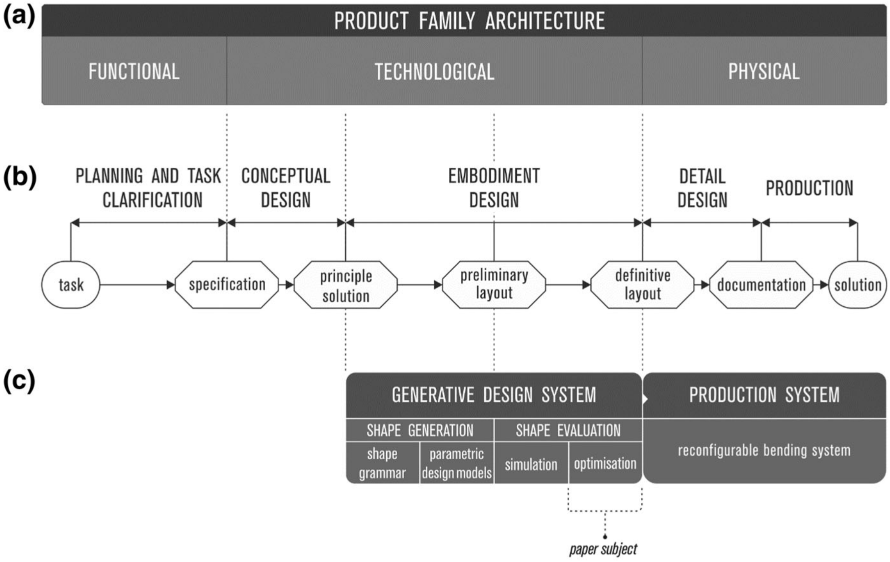
[1] CIAUD, Faculdade de Arquitetura, Universidade de Lisboa, Rua Sá Nogueira, Pólo Universitário, Alto da Ajuda, 1349-055 Lisbon, Portugal

[2] Escola Superior de Tecnologia de Abrantes, Instituto Politécnico de Tomar, Rua 17 de Agosto de 1808, 2200-370 Abrantes, Portugal

Introduction

Companies that mass customise their products must continually refine their organisational practices in order to achieve the desired levels of balance between the extent of customisation in products, the robustness of internal processes, and the fulfilment of consumers' expectations (Salvador et al. 2009).

The leverage of the aforementioned goals is achieved by product family architecture (Erens and Verhulst 1997). This methodology decomposes the complexity of the overall product development process and accommodates the different areas of knowledge involved. It comprises three complementary product models: functional, technological and physical. The functional model translates customers' needs into functional product requirements that serve as assessment criteria and will guide the development of the technological and the physical models. The technological model is defined through collaboration between the design and engineering disciplines. During the development of this particular model, product features must be defined and developed in accordance with the manufacturing capabilities of the company. Moreover, in light of mass customisation principles, the result must include a strategy for creating multiple outputs from the product family architecture. This condition can be characterised as the extent of customisation. Its definition and refinement are crucial issues of product family modelling. The physical model integrates the information generated in the



a) Product family architecture [Erens and Verhulst 1997]
b) Model of the design process, adapted from [Pahl et al. 2007: 130]
c) Model for mass customisation in the furniture design industry

Fig. 1 Positioning of the described research relative to: **a** the product family architecture; **b** the model of the design process; and **c** the proposed model for mass customisation in the furniture industry previous models, with the ones related with the manufacturing, assembly and distribution operations.

Product family architecture can be analysed by mapping it to a conventional model of the design process (Fig. 1). According to the model of Pahl et al. (2007: 130) the design process is divided into five main phases. The planning and task clarification phase constitutes the functional model. The conceptual design and embodiment design phases correspond to the technological model. The definitive layout achieved by the technological model constitutes the input information for the detail design and production phases, which correspond to the physical model.

Product family architecture organises the backend operations of a company. For consumers, the interaction with the company and the selection of a customised solution are made using configurators. These frontend applications are interconnected with the information created by the product family architecture (Forza and Salvador 2006: 74). However, frontend applications may comprise a different structure than the one used in backend operations, in order to improve usability for the consumer (Blecker et al. 2004). The goal is to enable consumers to change the fit, style, and functionality of the product before its acquisition (Piller 2004).

The ultimate goal of the present research is to enable the design of products that can be customised by an individual consumer according to his/her own idiosyncratic intentions. The current focus is on the development of the technological model of a product family architecture, particularly on the encoding of the activities and decisions involved in the embodiment design phase as a generative design system. The proposed generative design system for the furniture industry (Fig. 2) aims to

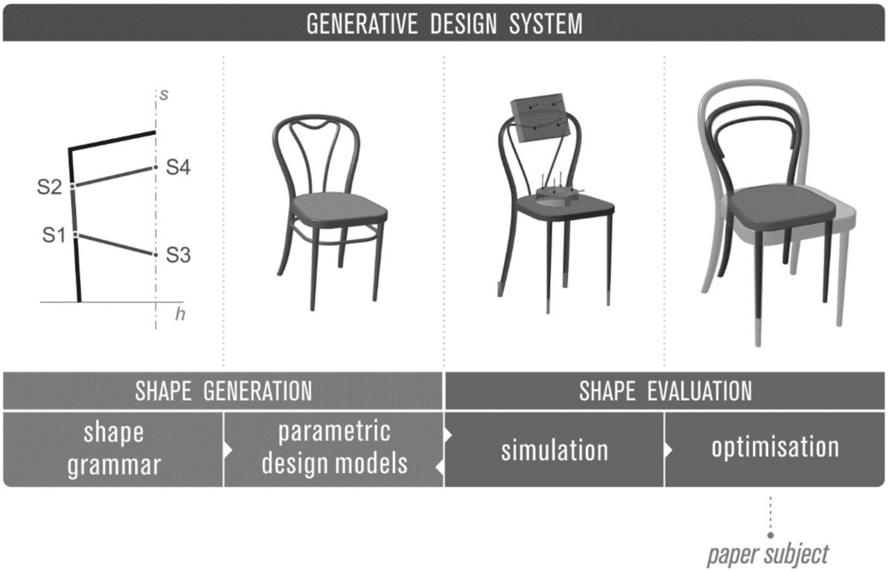


Fig. 2 Model of the generative design system proposed for mass customisation in the furniture design industry

improve the patterns of collaboration between the design and the engineering disciplines in order to permit the establishment of a customisable design language.

The envisaged model is illustrated by its application to a symbolic mass production problem: Thonet bentwood chairs, which were changed to comply with the paradigm of mass customisation.

The novelty of the research is twofold. First, it proposes the use of shape grammars as the basis of a methodology to support the embodiment design phase of the furniture design process for mass customisation. Shape grammars are used to encode the rules of an existing design style, thus defining a design language. The term “design style” refers to the set of form and functional features shared by a set of designs, whereas “design language” defines the set of designs generated by the shape grammar. The designer may explore alternative preliminary layouts by applying the rules and by varying its parameters to generate custom solutions. Alternative preliminary layouts may correspond to different topologies within the predefined design language.

Second, it implements the Thonet shape grammar as a set of parametric design models, each permitting additional exploration within a predefined topology. The parametric design models are then linked to simulation and optimisation tools to guarantee that the custom solutions generated satisfy performance requirements under operating conditions. The generative design system is a device that augments the designer’s ability to define and explore the universe of preliminary layouts within a design language and supports the automatic search for definitive layouts.

Previous papers focused on the development of the shape grammar (Barros et al. 2011a), the implementation and refinement of the parametric design models (Barros et al. 2011b), the link to simulation (Barros et al. 2011c), and the preparatory formulation of the optimisation part (Barros et al. 2014).

This paper presents the modelling of the optimisation part of the generative design system, which serves the purpose of guaranteeing the structural feasibility of the generated designs. Regarding the conventional design process, the goal of the optimisation part is to automate the search for an optimal solution for an alternative preliminary layout, which then becomes a definitive layout. The achieved solution can then proceed to the detail design and production phases.

The paper is structured as follows. Section “Shape Grammars” presents shape grammars as a formalism suitable to encode the shape generation subsystem. Industrial design applications are reviewed, with a particular emphasis on the step of the design process where shape grammars are introduced. Moreover, the combination of shape grammars and automated search methods to assist the generation process are analysed. Section “The Thonet Generative Design System” presents an overview of the generative design system, detailing the specific goals of each of its constituent subsystems and parts. Section “Optimisation” describes the modelling of the optimisation part, presenting two alternative strategies for formulating the optimisation problem and describing experiments undertaken to assess their suitability. Next, in Sect. “Analysis of Results“, the results of the experiments are presented and in Sect. “Discussion” they are discussed to identify the more suitable strategy. Section “Discussion” also discusses how the current design language could be transformed and the role of optimisation in guaranteeing syntactic and semantic adequacy. The paper ends with Sect. "Conclusion", where the issues of combining shape grammars with optimisation techniques are clarified, and paths for future research are identified.

Shape Grammars

Shape grammars is a formalism defined by Stiny and Gips (1972) having as a reference Chomsky's generative grammar (Stiny 2006: 18). It permits the creation and description of design languages through four key concepts: a set of geometrical shapes; a set of spatial relations between them; a set of composition rules that recreates such spatial relations; and an initial shape to which these rules can be applied recursively to generate different designs (Stiny 1980). Shape grammars operate as spatial algorithms that explain the generative properties of shapes, their decomposition into sub-shapes, and Boolean operations on them (Stiny 1991).

The application of shape grammars comprises two major types (Knight 2000): (1) creation of new design languages and (2) analysis of existing design styles to encode them as design languages. Transformation of shape grammars is a theoretical premise suitable to both types (Knight 1981). It permits the iteration of the process of encoding a shape grammar to define new languages based on the previous encoded ones.

The analysis of existing design styles is the prevalent approach in shape grammar studies. Since results can be compared and validated against existing knowledge, it provides means to study other aspects of the design process. The goal is to develop methods that can be adapted to model design problems that share similar features.

Knight (1999) categorised the design strategies employed to develop the vocabulary of shapes and shape rules as abstract grid, subdivision, and additive. In the abstract grid strategy, the grammar first generates a grid that structures the space of the design, and then adds details by further rule application. The subdivision strategy is employed when the language of designs shares the same boundary. Rule application determines solutions by subdividing an initial shape into sub-shapes. The additive strategy is the one selected when the language of designs has irregular or different boundaries. The initial shape is the core of the structure and other components or elements are then added. This strategy has been prevalent in industrial design applications (Prats 2007: 68).

Industrial Design Applications

In his review of the application of artificial intelligence techniques to mass customisation, Simpson (2004) reported on the success of shape grammars in automating the generation process in the context of industrial design. Shape grammars can be used as the basis of the methodology to encode the technological model of a product family architecture. In accordance with the model of (Pahl et al. 2007), shape grammars have been developed as a method to formalise the activities performed in the conceptual design and the embodiment design phases.

The Hepplewhite-style chair back shape grammar developed by Terry Knight (1980) is the first example of the application of shape grammars to a furniture design problem. It addresses a common issue in classic chair design (Oates 1981), that is, the detailing of the shape of the backrest and its elements as the main constituents of the design language. The representation of the chair back was simplified, first by removing decorative motifs and then by representing it as a basic curvilinear shape. After this depiction, the curvilinear representation was simplified into an equivalent rectilinear representation and only half the structure was represented due to symmetry properties. Therefore, the shape grammar is devised based on a higher abstraction model. The parametric shape grammar employs the subdivision strategy to generate several solutions that belong to the defined design language.

The coffeemaker grammar (Agarwal and Cagan 1998) employs the additive strategy to develop the design language. The overall shape of the coffeemaker is divided into three units, and different sets of rules are applied separately to each unit. Labels control the functional requirements and precede the application of rules for shape generation. The initial grammar was extended to include manufacturing costs within its properties (Agarwal et al. 1999). The cost information serves as an evaluation criterion of the customised design. The evaluation criterion supports the final activities of the embodiment design phase, by improving a preliminary layout into a definitive layout.

The motorcycle grammar (Pugliese and Cagan 2002) is a more generic grammar comprising a two-dimensional side representation. Besides the generation of different motorcycles, it encompasses a set of rules that enables the generation of solutions in a specific style, namely a classic Harley-Davidson. The front view of Buick vehicles (McCormack et al. 2004) is a specific grammar developed to explain a particular design style. The cross-over vehicle grammar (Orsborn et al. 2006) also applies the shape grammar formalism to generate vehicles, but is built upon a different approach. It analyses a sample of automobiles, which are categorised into three classes. The grammar operates in front, side, and rear view representations, and it enables the generation of crossover vehicles by a novel combination of shapes. Considering the support for the design process, these grammars codify the "principle solution"—the first step of the embodiment design phase in the model of (Pahl et al. 2007)—into a design space by addressing functional requirements during shape generation. They enable a better support of the initial steps of the embodiment design phase permitting the generation of preliminary layouts. However, they exclude issues related to three-dimensional representation and material properties.

The application of shape grammars in other industrial design examples confirms its suitability to codify the characteristics of a style and assist designers in the process of form finding. The work of Chau et al. (2004) for the creation of Coca-Cola bottles and Head & Shoulders packaging; the kettle grammar developed by Prats et al. (2006); the ultrasound transducer handle grammar (Culbertson 2012); and the tableware grammar (Castro e Costa and Duarte 2013) fit this description.

Evaluation and Shape Grammars

Evaluation is an assessment mechanism that is developed to take given criteria into account in the process of generation. Its combined use with shape grammars can be summarised by two main categories: syntactic and semantic. In the syntactic category, evaluation is (mainly) based on the required performance, such as cost or structural issues. In the semantic category, evaluation is based on a set of characteristics defined to interpret the generated designs.

Cagan (2001) discusses the integration of evaluation with shape grammars, arguing that, depending on the goal of the shape grammar, evaluation can be incorporated in the shape grammars' properties or postponed to after exploration of the solution space.

Cagan and Mitchell (1993) pioneered the automated search for designs using shape grammars, by creating shape annealing. This optimisation technique combines a simulated annealing algorithm (Kirkpatrick et al. 1983) with the shape grammar formalism to control rule application based on functional requirements. The concept was further developed by Shea and Cagan (1999) to generate and optimise the configuration of trusses, based on topological and structural optimisation. The algorithm searches for optimal designs based on structural conditions, by manipulating rules in a model with a high level of abstraction. In this approach, the grammar is responsible for the generation of the geometry, while the annealing algorithm optimises designs according to structural performance requirements, which comprise both syntactic and semantic goals. Shape annealing permits the codification of the conceptual design and embodiment design phases of the design process, thus permitting the generation of a definitive layout. The resulting design must be detailed in order to be produced.

Evaluation focusing only on semantic goals is identified in the work of Hsiao and Chen (1997). After morphological analysis, the authors construct a system to generate preliminary layouts of office chairs. Then, the shape grammar codifies semantic properties that can be applied to chair components after the preliminary layout is defined, in order to detail shape-related aspects. Lee and Tang (2004) applied the semantic approach in the development of a genetic algorithm that transforms the shapes of digital cameras based on an evaluation mechanism that included the user. The genetic algorithm supports design exploration after a preliminary layout is defined. Therefore, the generation of alternatives is based on a model with a lower level of abstraction that represents the shapes of the final design.

In order to model the shape grammar and its appropriate evaluation mechanism, there must be an assessment of several conditions. First, the features of the design language that will be subjected to evaluation must be selected. This condition influences the type of algorithm to be used. The encoding of the algorithm for the search process requires a decision about the type of representation of the designs, which, in turn, is related to other aspects of the design process, namely: (1) the computer implementation; (2) the step of the process where it is introduced; and (3) the integration with other tools used in subsequent steps or phases.

In the present research, the combined use of a shape grammar and evaluation follows a decomposition of the customisation problem into four sub-problems:

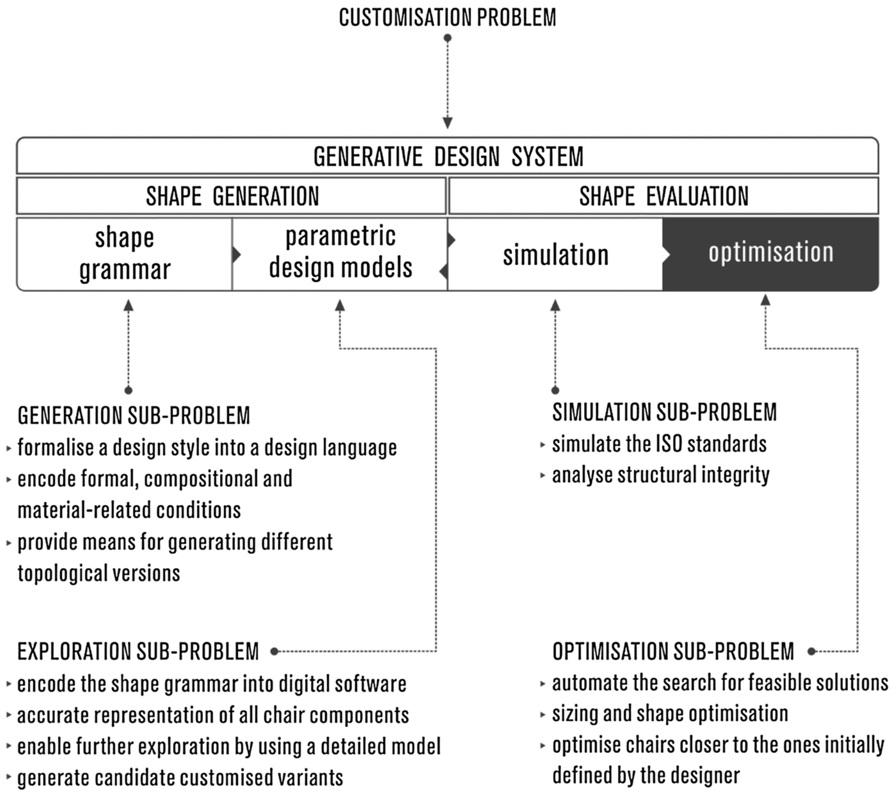


Fig. 3 Decomposition of the customisation problem into sub-problems that correspond to the constituent parts of the proposed generative design system

generation, exploration, simulation and optimisation. The decomposition into these four sub-problems permits the modelling of each sub-problem separately, facilitating its implementation (Fig. 3).

The generation sub-problem addresses the codification of the formal, compositional and material-related conditions verified in the analysis of the design style. These features are encoded into the shape grammar. The use of a higher abstraction model enables one to decompose the shape into its fundamental principles and simplifies the encoding of the functional requirements. Functional requirements are encoded as shape grammars' labels, which enables preliminary layouts to be generated within a constrained solution space, defined to guarantee a feasible arrangement of elements. Labels act as an implicit device to constrain the solution space, rather than an explicit evaluation mechanism.

The second sub-problem is the exploration of preliminary layouts to achieve the customised solution that might become the definitive layout. Exploration occurs in a complementary step through the use of a detailed model. The model represents the three-dimensional geometry, thereby enabling the selection and refinement of shape attributes based on dimensional parameters. This division is important to an industrial designer, because it permits the configuration of a functional preliminary layout through the use of a higher abstraction model followed by its refinement through the use of a detailed three-dimensional model.

Evaluation is postponed until after generation and exploration have taken place. In the present research, it is employed to evaluate the mechanical structure of the customised chairs. Therefore, evaluation comprises two sub-tasks: the first is simulation, used to calculate the stress in the structure under predetermined operating conditions; the second is optimisation, necessary to find the optimal solution of the structure under the specified conditions.

The Thonet Generative Design System

The potential application of the envisaged generative design system for mass customisation in the furniture industry is demonstrated by encoding an existing design style. The chair is selected as the experimentation typology for modelling a furniture design problem, as its design is directly related to the human body, and allows for investigation of aesthetic and structural requirements. Since Thonet chairs are considered to best represent the mass production paradigm (Wilhide 2010: 29), they were chosen to assess the degree of transformation that might occur when moving from the mass production to the mass customisation paradigm. Significant material constraints and non-standard production techniques constitute layers of complexity and establish firm boundaries for the research.

The generative design system aims to assist the designer in the embodiment design phase and create information suitable for the detail design and production phases of the design process. Generation and exploration issues are addressed in the shape generation subsystem. Simulation and optimisation constitute the shape evaluation subsystem. The purpose is to create structurally feasible designs defined by the designer according to a shape grammar-based methodology. In the current stage of the research, automatic integration between all parts of the system is not fully achieved, but it will be sought in future research, in order to enable its industrial application.

The following subsections detail the modelling of the generative design system, specifying its subsystems and its constituent parts.

Shape Generation Subsystem

As previously mentioned, the shape generation subsystem encompasses shape grammars and parametric design models. The Thonet chair grammar (Barros et al. 2011a) is a 2D parametric shape grammar with thirty-four rules, developed from a corpus of six chairs. The grammar comprises a simplified representation for the curves, similar to the one developed by Knight (1980) for the Hepplewhite chairs. The rules focus on the generation of the backrest structure or frame, which is the key feature of the design style.

The Thonet design grammar is the Cartesian product of different algebras. Labels describe the topological relationship between the inner and outer frames. The

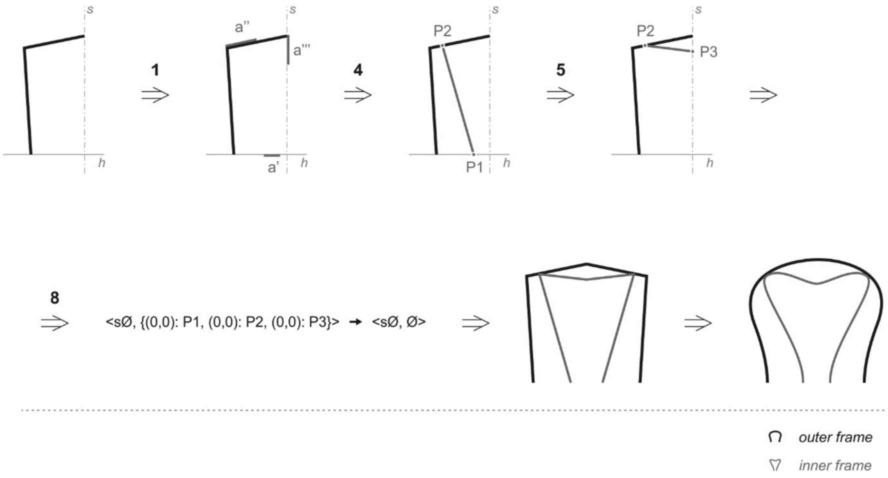


Fig. 4 Derivation of a topological version by shape grammar rule application

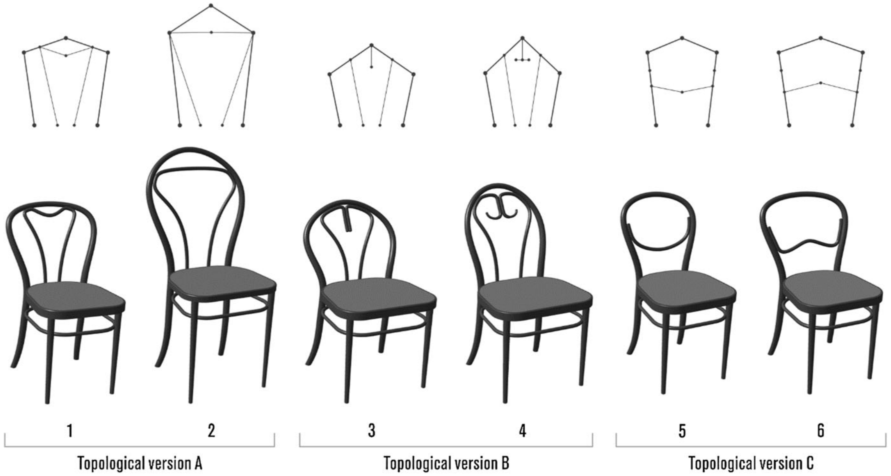


Fig. 5 Customised variants created by exploring parametric design models within three topological versions

specific range of solutions codified by the labels follows an analysis of the relations in the corpus and enables topological variation within a finite design space. This condition guarantees the generation of backrest designs with an adequate degree of rigidity. Weights indicate variations in material aspects of the designs.

Rule application determines the generation of different designs in the language (Fig. 4). Each resulting topological version constitutes a distinctive topological relationship between the inner and the outer backrest frames. On this first level of design generation, the representation of designs uses a higher level of abstraction.

The conversion of the shape grammar into a parametric design model to facilitate implementation requires the following procedures: (a) convert the topological variation space specified by the shape grammar labels into the dimensional search space encoded by the parametric design models; (b) translate rectilinear representation into curvilinear representation; (c) geometrically represent the remaining chair components; (d) establish relationships among the chair components in the reconfigurable model.

A set of parametric design models is implemented in CATIA. Each parametric design model corresponds to a distinct topology encoded by the shape grammar. The higher level representation used in the shape grammar is complemented by detailed geometry representing all chair components. Exploration is made by assigning different values to the parameters, thereby creating customised variants (Fig. 5).

To summarise, the shape generation subsystem permits the definition of a design language, the generation of multiple topological versions, and the exploration of detailed designs within the language by creating customised variants.

Shape Evaluation Subsystem

The shape evaluation subsystem complements the shape generation subsystem by introducing the performance requirements into the generative design system. Since the ultimate goal of the system is the generation of custom chairs that can be produced, computational techniques are employed to guarantee that each customised variant produced in the previous steps is feasible, so that it can proceed to the production phase. Shape evaluation comprises simulation and optimisation. The first part simulates the 1989 ISO tests for the structural performance of domestic chairs (ISO 7173, Ameublement—chaises et tabourets—détermination de la résistance et de la durabilité). Optimisation complements simulation by automating the search for an optimal solution.

The input information for simulation is retrieved from the parametric design models, which include additional features to represent the devices used in the ISO tests for domestic chairs. The beechwood chair structure is defined as having the following material properties: a Young's modulus of 15 GPa and a mass density of 737 kg/m^3. The simulation is performed in the Generative Structural Analysis (GSA) workbench of CATIA and the model is completed with information on finite element properties, translation constraints and applied loads as diagrammed in Fig. 6.

The tests are examined using the von Mises yield criterion. Wood is an anisotropic material, but it can be analysed as an orthotropic material in numerical simulation (Mackerle 2005: 580). Beechwood material resistance is calculated using the Eurocode 5 (Porteous and Kermani 2007: 70) standardised function, which takes into account the effect of load duration and a material property factor in the ultimate limit state conditions. Accordingly, the compressive yield strength parallel to grain is 21.2 MPa.

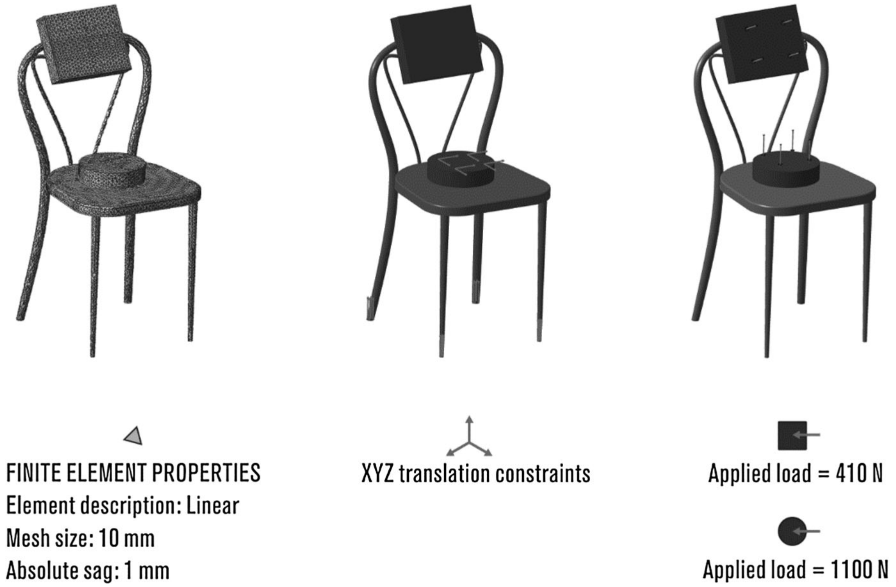


Fig. 6 Definition of the simulation model

Finite element analysis results show that a customised variant may or may not be structurally sound due to a small variation of certain parameters' values. Given the possible combinations of parameters, there is a large range of possible solutions, meaning that intuitive and manual testing of these possibilities to reach a better understanding of the solutions domain is a time-consuming activity. Furthermore, it does not fulfil the goal of full automation and integration of the system's parts.

The optimisation part is included to overcome these limitations by enabling automated search within a predefined set of conditions. Optimisation is included in the generative design system to guarantee structural performance and acts upon the information generated by the previous parts.

Optimisation

Optimisation aims to guarantee that a customised variant that belongs to the formalised design language can perform under operating conditions. Therefore, a search is undertaken within a constrained domain of solutions, since the candidate customised variant has been already specified by the user in previous steps using other parts of the system. The automated search for an optimal solution serves to ensure the minimum thickness of beech profiles. This premise follows the explicit goal of guaranteeing compliance of each customised variant with the ISO standards, and the implicit goal of achieving the visual elegance present in the original design style.

The search for optimal designs proceeds by applying a simulated annealing algorithm to the sizing and shape optimisation problems. The algorithm must explore the design space defined by the range of values set for explicit parameters in order to optimise the size and shape of the customised variant, so as to meet predefined performance specifications. This is accomplished by using the available computational optimisation techniques embedded in CATIA. The existence of a graphic user interface offers a shorter learning curve for pre-processing and post-processing data, whilst accomplishing the goal of design automation to assist the designer in searching within a large combinatorial solutions space.

Problem Formulation

The goal of optimisation is to minimise equivalent stress to a target value of 18 MPa, which corresponds to a safety factor of 15 % below the resistance limit. The respective objective function of the problem formulation is expressed as:

$$\max\left(\sigma_{eq\,i}\right) \sim 18\ \text{MPa}$$

The range of values of the parameters manipulated by the simulated annealing algorithm in the search for an optimal solution are listed in Table 1. These parameters are used to design a customised variant, and can be divided in two groups (Fig. 7). The first group of parameters (p_1–p_3) comprises the parameters associated with the shape grammar labels used in the generation of the topological version, which can be explored in the parametric design models that encode the grammar to customise the chosen topological version. The grammar defines the variation space, which is explored by the simulated annealing algorithm in the search for the optimal topological configuration of the inner frame. The second group (p_4–p_{12}) is related to the configuration of the remaining generic frame,

Table 1 Parameters considered in the generation of customised variants, values of the initial variant, range of variation of each parameter, and increments of values used in the search

Parameter		Parameter values of the initial variant	Lower bound	Upper bound	Increment
p_1	Ratio of P1 (%)	50	0	100	1
p_2	Ratio of P2 (%)	20	0	100	1
p_3	Ratio of P3 (%)	60	0	100	1
p_4	Inner frame diameter (mm)	10	10	30	1
p_5	Seat frame height (mm)	450	450	480	1
p_6	Seat frame width (mm)	420	420	510	1
p_7	Seat frame depth (mm)	420	420	510	1
p_8	Seat frame thickness (mm)	30	30	60	1
p_9	Outer frame backrest height (mm)	420	420	530	1
p_{10}	Outer frame diameter (mm)	27	27	40	1
p_{11}	Outer frame backrest angle (°)	0	0	11	1
p_{12}	Outer frame rear leg angle (°)	4	4	10	1

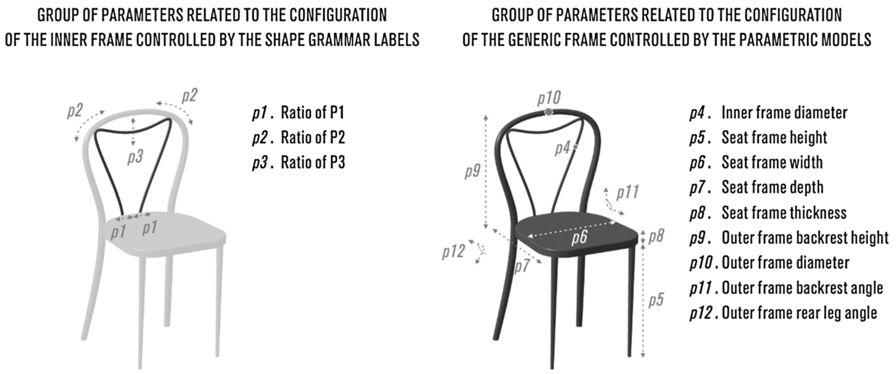


Fig. 7 Parameters considered in the generation of customised variants, divided into two groups: one controlled by the shape grammar labels and another only controlled by the parametric models

common to all Thonet chairs. These parameters permit the customisation of the aspects not described by the shape grammar, such as the height, width, and angle of the backrest, seat frame, rear legs and front legs.

The definition of the range for the optimisation complies with the design principles observed in the original Thonet design style. The lower bound of the design space was defined by setting the lowest values for the parameters associated with the shape grammar labels. The values for the seat frame height, width and depth, and for the outer frame backrest height, angle and rear leg angle are the lower values found in the analysis of Thonet chairs. The values for the outer frame diameter, the inner frame diameter and the seat frame thickness were equally extracted from the analysis of Thonet chairs, and lowered by 10, 15 and 50 %, respectively. A customised variant that uses the values corresponding to the lower bound does not meet the structural requirements. It has the proportions of a standard Thonet chair, but thinner beech profiles are employed in the outer frame, the inner frame and the seat frame. The upper bound of the design space is defined according to the dimensions observed in a larger Thonet armchair featured in the 1904 Catalogue (Thonet Company 1980). The outer frame has a thicker, solid, bendable beechwood profile and the variation of its backrest angle is set according to seating recommendations specified in anthropometric literature (Diffrient et al. 1985).

Two sets of tests were performed a priori to formulate in an adequate manner the problem of minimising the equivalent stress to a safety factor, the goal of the optimisation part. The purpose was to gain understanding regarding the most appropriate modelling strategy for optimising Thonet chairs, so that the optimised chair remained closer to the one initially defined by the designer. In the first set of tests all parameters were optimised. In the second set of tests, the parameters associated with the shape grammar labels were not optimised. This second approach was intended to respect the topology selected by the designer using the previous parts of the generative design system.

The customised variant using variables values corresponding to the lower bounds was submitted to the two sets of tests. The compressive yield strength parallel to grain for this initial variant was calculated as 36.7 MPa, 73 % above the yield limit. This approach of using the lower bound values, rather than using random or feasible values, was employed to test the algorithm's capacity to search the entire space of solutions.

A simulated annealing algorithm was selected to search for optimal designs. In the Product Engineering Optimizer workbench (Dassault Systèmes 2008), the simulated annealing optimisation setup provides the user with the ability to select the speed of convergence, ranging from slow to medium, to fast and to infinite. Convergence occurs when the algorithm reaches a solution. Termination criteria can be set as time, the maximum number of iterations, or the number of iterations without improvement.

The "lower bound chair" was submitted to optimisation experiments with different available convergence speeds, with time setups of 25, 100 and of 400 min established as termination criteria.

Analysis of Results

Optimisation of the Complete Chair

Across the tests configured with a slow convergence speed, the same optimised chair was achieved in the sixth iteration (Table 2; Fig. 8). When visually compared with the initial variant, the optimised one displays a greater backrest angle. The total of nine parameters, common to all Thonet chairs, was optimised. The backrest angle varied 10° and the backrest height 15 mm from the initial variant. The remaining parameters varied within a range of 5 mm. Out of the three parameters related with

Table 2 Results of the optimisation of the complete chair, sought in the first set of tests

A	B	C	D	Value of optimised parameters (mm)												E
				p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	
Slow	25	32	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
	100	428	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
	400	569	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
Medium	25	125	49	58	20	58	17	420	420	421	32	421	28	0	4	18.02
	100	139	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
	400	2138	49	58	20	58	17	420	420	421	32	421	28	0	4	18.02
Fast	25	125	49	58	20	58	17	420	420	421	32	421	28	0	4	18.02
	100	226	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
	400	569	6	50	19	70	15	452	423	424	33	435	30	10	7	18.02
Infinite	25	98	26	55	24	66	11	450	421	422	30	427	30	2	7	17.95
	100	459	397	53	24	63	12	451	520	422	30	426	30	2	4	17.99
	400	1927	397	53	24	63	12	451	520	422	30	426	30	2	4	17.99

A type of convergence speed, *B* time (min), *C* number of iterations, *D* iteration of convergence, *E* stress value (MPa)

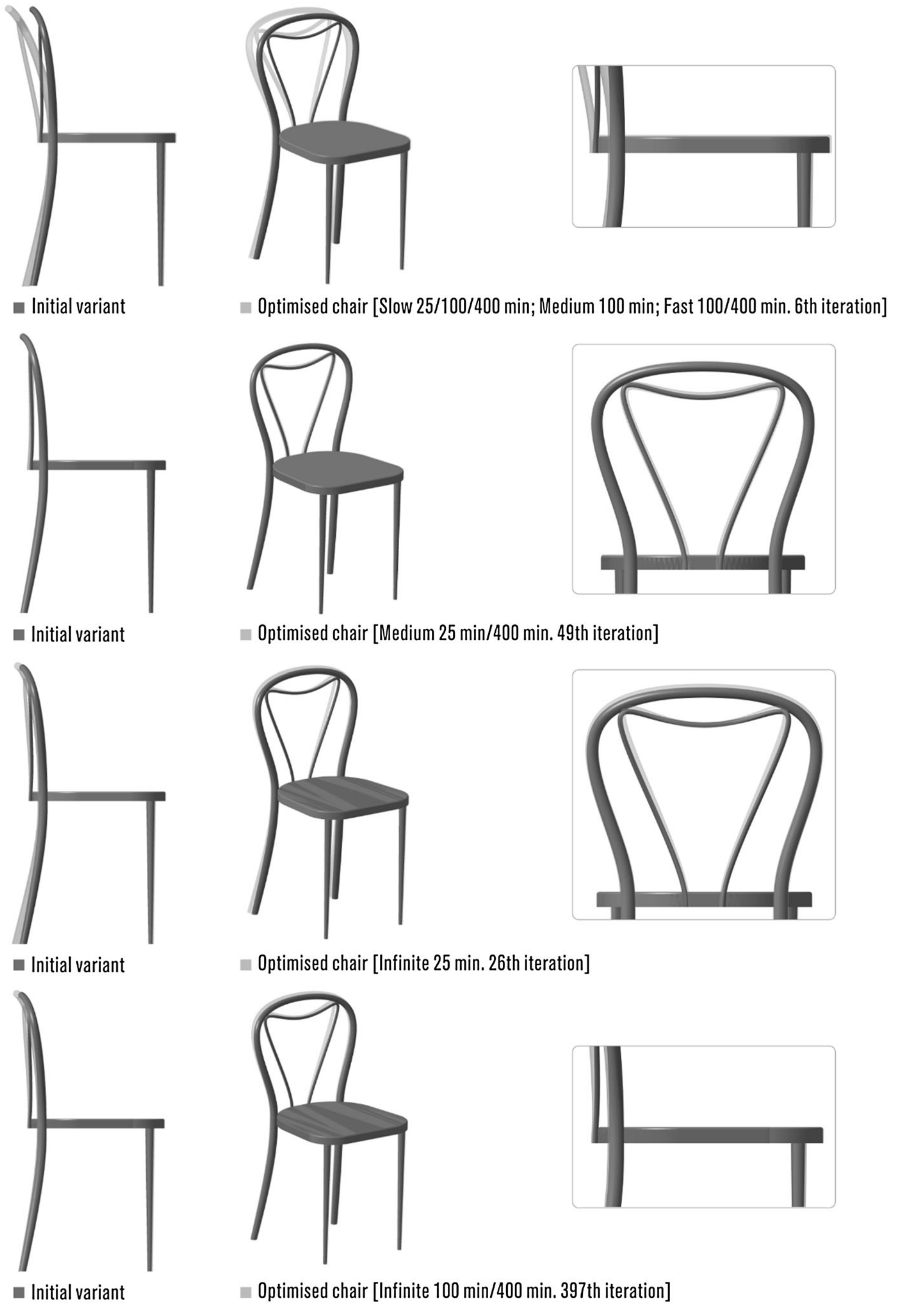


Fig. 8 Optimised chair that resulted from the first set of tests, which sought the optimisation of the complete chair, overlaid on the initial variant

the shape grammar's labels, two were optimised: the ratio of P2 decreased 1 %; and the ratio of P3 increased 10 %.

Tests configured with medium convergence speed yielded two configurations of optimised chairs. In the 25-min and 400-min tests, the convergence was achieved in the 49th iteration. The 100-min test generated the same chair created in the set configured with slow convergence speed.

The visual comparison between the two chairs reveals that the optimised chair from the 49th iteration is closer to the initial variant. The analysis of the numerical results confirm the visual correlation: of the nine parameters common to all Thonet chairs, only five were optimised. Apart from the inner frame diameter, which increased 7 mm, the remaining parameters varied between 1 and 2 mm. Furthermore, the proportion between the inner frame diameter and the outer frame diameter was 0.61 (against 0.5 achieved in the sixth-iteration chair), which is closer to the 0.65 proportion verified in the original Thonet design style. Out of the set of grammar-related parameters, two were optimised. Nevertheless, they are very similar to the initial configuration.

The set of tests with a faster convergence speed produced two optimised chairs equal to the ones created in the medium convergence speed tests. In the 25-min test, the optimal chair was achieved in the 49th iteration, matching the one achieved in the 25-min and 400-min tests configured with medium convergence speed. In the other time configurations, the chair was optimised in the sixth iteration, and it is equivalent to the one created in the 100-min test configured with medium convergence speed.

The set of tests configured with infinite convergence speed produced two novel optimised chairs: one in the 25-min test and another in both the 100-min and the 400-min tests. Visually, the optimised chairs are very similar. The distinct feature is the rear leg angle, which is greater in the optimised chair in the 25-min test. The analysis of the numerical values for the parameters confirm the similarity between the nine parameters related with the generic frame of the chairs, which vary between 1 and 2 mm from the ones used in the initial variant.

In a visual comparison between the four optimised chairs in this set of tests, it is observable that the chair generated in the 49th iteration of the 25-min test configured with medium convergence speed is closer to the initial variant. The variation of the inner frame diameter and of the parameters that control the shape grammar's labels is key to the generation of an optimal chair that is more similar to the initial variant.

Optimisation of the Generic Frame of Thonet Chairs

The optimisation of the generic frame common to all Thonet chairs also yielded four different chairs across the twelve tests (Table 3; Fig. 9).

In the tests configured with slow and medium convergence speed, the same optimised chair was generated in the thirtieth iteration. From the nine available parameters, seven were optimised, and they differ between 1 mm and 8 mm from the ones used in the initial variant.

In the tests configured with fast convergence speed there were two results: in the 25- and 400-min tests, the optimised chair was the same as the one described in the

Table 3 Results of the optimisation of the generic frame of Thonet chairs only, sought in the second set of tests

A	B	C	D	Value of optimised parameters (mm)									E
				p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	
Slow	25	115	30	11	451	420	421	30	428	32	5	9	18.03
	100	142	30	11	451	420	421	30	428	32	5	9	18.03
	400	684	30	11	451	420	421	30	428	32	5	9	18.03
Medium	25	113	30	11	451	420	421	30	428	32	5	9	18.03
	100	484	30	11	451	420	421	30	428	32	5	9	18.03
	400	687	30	11	451	420	421	30	428	32	5	9	18.03
Fast	25	116	30	11	451	420	421	30	428	32	5	9	18.03
	100	496	249	10	450	420	420	30	420	33	5	6	18.02
	400	1984	30	11	451	420	421	30	428	32	5	9	18.03
Infinite	25	107	53	11	456	420	421	32	421	33	4	9	17.99
	100	148	53	11	456	420	421	32	421	33	4	9	17.99
	400	641	98	10	452	421	420	31	425	32	3	4	17.99

A type of convergence speed, *B* time (min), *C* number of iterations, *D* iteration of convergence, *E* stress value (MPa)

previous tests. In the chair generated in the 100-min test only three parameters were optimised. This optimal chair is closer to the initial variant, particularly due to the smaller variation of the rear leg angle and to the absence of change in the backrest height.

The infinite speed tests generated two novel chairs when compared with the remaining tests in this set. The 25- and 100-min tests produced the same chair as in the fifty-third iteration. The chair optimised in the 400-min test was closer to the initial variant, since the rear leg angle was not optimised.

Figure 8 shows the initial variant superimposed on the optimised chairs obtained after the tests. The ones considered closest in terms of similarity are the optimised chair created in the 100-min test configured at fast convergence speed (second row) and the optimised one achieved in the 400-min test configured at infinite convergence speed (fourth row). The 400-min test generated a chair closer to the initial variant. This is observable in the sideways view by the minor deflection in the backrest-rear leg component, which is the direct combination of the backrest angle and rear leg angle parameters.

Discussion

Two problem formulations were submitted to stochastic optimisation of a detailed model of a custom Thonet chair, which is the result of the customisation process enabled by the manipulation of the shape generation subsystem. In the set of tests performed in the first problem formulation, all the parameters were considered in

■ Initial variant ■ Optimised chair [Slow/Medium 25/100/400 min; Fast 25/400 min. 30th iteration]

■ Initial variant ■ Optimised chair [Fast 100 min. 249th iteration]

■ Initial variant ■ Optimised chair [Infinite 25/100 min. 53th iteration]

■ Initial variant ■ Optimised chair [Infinite 400 min. 98th iteration]

Fig. 9 Optimised chairs that resulted from the second set of tests, which sought the optimisation of the generic frame of Thonet chairs only, overlaid on the initial variant the optimisation; whereas in the second one, only parameters associated with the generic frame were taken into account. Eight optimal chairs were produced in the two sets of tests. In each set, a chair was selected as the best response, considering the dual goal of minimising stress and remaining closer to the designer's original intent. The comparison between these two chairs offers insights about the adequacy of the two strategies used to model the optimisation problem. In the first set of tests, the 49th-iteration chair (Fig. 7, second row), which was generated in three tests, was closer to the initial variant. In the second set of tests, this premise was satisfied by the chair generated in the 400-min test configured with infinite convergence speed (Fig. 8, fourth row). A comparison between these two chairs reveals that the complete optimisation of the chair performed in the first set of tests was closer to the initial variant. The ratio between the inner and outer frame diameters is closer to the one in the original design style and the deflection created by the backrest angle and the rear leg angle is less pronounced. This suggests that the first modelling strategy, in which all the parameters are optimised, is more appropriate given the original goal of remaining faithful to the style, despite permitting a change of the original selected topology.

The optimisation presented in this paper suits the goals behind the development of the generative design system. It permits the optimisation of a customised chair according to a safety factor of 15 % below the resistance limit, which guarantees that it can perform under operating conditions. Therefore, the system is able to generate structurally sound customised designs.

The use of a stochastic optimisation technique determines that there is a range of optimal designs for both problem formulations instead of only one single solution. The analysis of the results offer valuable insights about the impact of each parameter on the overall design of the chair as a structure, and the particular parameters codified by the shape grammar's labels. Results from this analysis can be used to refine the generative design system before full automation and integration between generation and evaluation is sought. Results can also be used to guide the transformation of the language as explained below.

Regarding the refinement of the generative design system, it may be sought to broaden or restrict the space of possible solutions, thereby increasing or decreasing optimisation time. Such a control of the solution space can be done in the parametric design models by editing the relationships between parameters; in the optimisation setup by changing the ranges of values of the parameters, or even by freezing some parameters to narrow down search; or by editing the simulated annealing algorithm.

Transformation of the generative system can be pursued to create novel design languages based on the existing one, following the theoretical premises defined by (Knight 1981). As mentioned in Sect. "Evaluation and Shape Grammars", the combined use of evaluation and shape grammars falls into two categories, syntactic and semantic. Accordingly, shape grammar transformations relying on evaluation and shape grammars to generate appropriate or optimal designs can fall into the same categories.

Syntactic transformation could include topological transformations. In this scenario, Thonet's shape grammar would have to foresee additional topological relations, in which case labels should be unrestricted to amplify the domain of possible solutions. Since labels in the current version of the grammar guarantee functional rigidity, unrestricting them could lead to backrest designs without a proper degree of rigidity. Shape annealing would then verify syntactic correctness while searching for structurally optimal designs.

Semantic transformation could follow the methodology of coupling shape grammars with genetic algorithms. In such a case, the Thonet shape grammar would be changed to enable a greater number of design solutions. The solutions output by the optimisation approach presented in this article could be used as individuals in the development of the genetic algorithm to guarantee structural feasibility. The user would then be part of the search process by ranking the population of individuals output by the genetic algorithm according to his or her preferences. The search for the fittest solutions would then be undertaken taking into account user preferences in the selection of individuals for the next generation, and then the genetic algorithm would be run again. The user's subjective preferences constitute the semantic property of the genetic algorithm. If such subjective preferences can be made explicit and codified into the fitness function, the process can also be fully automated.

Conclusion

The relation between shape grammars and evaluation depends on the amount of knowledge that one is willing or able to put in each of them, the particular step of the design process where they are introduced, and the tools involved.

In the generative design system presented here, the shape grammar comprises an intrinsic property—the labels—that restricts the domain of solutions. This strategy accounts for a greater fidelity towards the original Thonet design style at the expense of suppressing more novel solutions. It serves the purpose of showing the process of encoding an existing design style into a generative design system for mass customisation, and permits verification against existing knowledge. Evaluation is applied on a different level of abstraction than the one on which the shape grammar operates, and in a subsequent step of the design process. It optimises the preliminary layout into a definitive layout and generates the information for the detail design and production phases. The optimisation addresses structural performance issues in a detailed model, searching for an optimal solution closer to the customised design initially configured by the designer.

The proposed methodology enables the modelling of a generative design system with different complementary parts for generation, exploration, simulation, and evaluation to explore a considerably wide range of feasible designs. These parts can be refined locally or the system can be transformed globally, in order to achieve the desired level of balance between the number of novel designs, the amount of control, and the fidelity towards the original design style.

Future research will address the complete automation of all the parts of the generative design system in order to enhance the usability of the technological model of the product family architecture. The connection to a production system and the frontend application will be developed to enable the use of the envisaged model for mass customisation in the furniture design industry.

References

Agarwal, M. and J. Cagan, J. 1998. A blend of different tastes: The language of coffeemakers. *Environment and Planning B: Planning and Design* **25**(2): 205–226.

Agarwal, M., J. Cagan and K. G. Constantine. 1999. Influencing generative design through continuous evaluation: associating cost with the coffeemaker shape grammar. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **13**(4): 253–275.

Barros, M., J.P. Duarte, and B.M. Chaparro. 2014. Integrated generative design tools for the mass customisation of furniture. In: *Design Computing and Cognition'12*, eds. John Gero, 285–300. Dordrecht: Springer.

Barros, M., J.P. Duarte, and B.M. Chaparro. 2011a. Thonet chair design grammar: A step towards the mass customization of furniture. In: *Designing Together: Proceedings of the 14th International Conference on Computer Aided Architectural Design Futures*, eds. Pierre Leclercq, A. Heylighen and G. Martin, 181–200. Liége: Les Éditions de l'Université de Liège.

Barros, M., J.P. Duarte, and B.M. Chaparro. 2011b. A design system for the mass customization of Thonet chairs. In: *SIM 2011: International Conference on Sustainable Intelligent Manufacturing*, eds. Helena Bártolo, N. Alves and J.P. Bártolo, 737–745. Lisboa: IST Press.

Barros, M., J.P. Duarte, and B.M. Chaparro. 2011c. Digital Thonet: An automated system for the generation and analysis of custom-made chairs. In: *Respecting Fragile Places: Proceedings of the 29th eCAADe Conference. Education in Computer Aided Architectural Design in Europe*, eds. Tadeja Zupancic, M. Juvanic, S. Verosek and A. Jutraz, 521–529. Ljubljana: University of Ljubljana.

Blecker, T., N. Abdelkafi, G. Kreutler and G. Friedrich. 2004. Product Configuration Systems: State of the Art, Conceptualization and Extensions. In: *Proceedings of the Eighth Maghrebian Conference on Software Engineering and Artificial Intelligence*, eds. A.B. Hamadou, F. Gargouri, M. JMaiel, 25–36, Tunis: Centre de Publication Universitaire, Tunis.

Cagan, J. 2001. Engineering shape grammars: where we have been and where we are going. In: *Formal Engineering Design Synthesis*, eds. Jonathan Cagan and E. K. Antonsson, 65–92. New York: Cambridge University Press.

Cagan, J. and W. J. Mitchell. 1993. Optimally directed shape generation by shape annealing. *Environment and Planning B: Planning and Design* **20**(1): 5–12.

Castro e Costa, E. and J. P. Duarte. 2013. Mass customization of ceramic tableware through digital technology. In: *Green Design, Materials and Manufacturing Processes*, eds. Helena Bartolo et al., 467–471. London: CRC Press.

Chau, H. H., X. Chen, A. McKay and A. Pennington. 2004. Evaluation of a 3D Shape Grammar Implementation. In: *Design Computing and Cognition'04*, ed. John Gero, 357–376. Dordrecht: Kluwer Academic Publishers.

Culbertson, T. 2012. Shape Grammar Composition and Salient Feature Identification for a Product Family. Master thesis, Pennsylvania State University.

Dassault Systèmes. 2008. *Product Engineering Optimizer: Version 5 Release 19*.

Diffrient, N., A. R. Tilley, J. C. Bardagjy and Henry Dreyfuss Associates. 1985. *Humanscale 1-2-3*. Cambridge: MIT Press.

Erens, F. and K. Verhulst. 1997. Architectures for product families. *Computers in Industry*, **33**: 165–178.

Forza, C. and F. Salvador. 2006. *Product Information Management for Mass Customization: Connecting Customer, Front-Office, and Back-Office for Fast and Efficient Customization*. Houndmills: Palgrave Macmillan.

Hsiao, S.-W. and C.-H. Chen. 1997. A semantic and shape grammar based approach for product design. *Design Studies* **18**(3): 275–296.

Kirkpatrick, S., Jr., C. D. Gelatt and M. P. Vecchi. 1983. Optimization by Simulated Annealing. *Science* 220: 671–679.

Knight, T. W. 1980. The generation of Hepplewhite-style chair-back designs. *Environment and Planning B: Planning and Design* **7**(2): 227–238.

Knight, T. W. 1981. Languages of designs: from known to new. *Environment and Planning B: Planning and Design* **8**(2): 213–238.

Knight, T. W. 1999. Applications in Architectural Design, and Education and Practice. *NSF/MIT Workshop on Shape Computation*. http://www.shapegrammar.org/education.pdf. Accessed 19 May 2015.

Knight, T. W. 2000. Shape Grammars in education and practice: history and prospects. *Shape grammars*. http://web.mit.edu/tknight/www/IJDC. Accessed 19 May 2015.

Lee, H. C. and M. X. Tang. 2004. Evolutionary Shape Grammars for Product Design. In: *Proceedings of the 7th International Conference on Generative Art*, 309–322.

Mackerle, J. 2005. Finite element analyses in wood research: a bibliography. *Wood Science and Technology* **39**(7); 579–600.

McCormack, J. P., J. Cagan and C. M. Vogel. 2004. Speaking the Buick language: capturing, understanding, and exploring brand identity with shape grammars. *Design Studies* **25**(1): 1–29.

Oates, P. B. 1981. *The Story of Western Furniture*. London: Herbert Press.

Orsborn, S., J. Cagan, R. Pawlicki and R. C. Smith. 2006. Creating cross-over vehicles: Defining and combining vehicle classes using shape grammars. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **20**(3): 217–246.

Pahl, G., W. Beitz, J. Feldhusen and K. H. Grote. 2007. *Engineering Design: A Systematic Approach*, 3rd ed. London: Springer-Verlag.

Piller, F. T. 2004. Mass Customization: Reflections on the State of the Concept. *International Journal of Flexible Manufacturing Systems* **16**(4): 313–334.

Porteous, J. and A. Kermani. 2007. *Structural Timber Design to Eurocode 5*. Oxford: Blackwell.

Prats, M. 2007. Shape Exploration in Product Design: Assisting Transformation in Pictorial Representations. Ph.D. thesis, The Open University.

Prats, M., C. Earl, S. Garner and I. Jowers. 2006. Shape exploration of designs in a style: Toward generation of product designs. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **20**(3): 201–215.

Pugliese, M. J. and J. Cagan. 2002. Capturing a rebel: modeling the Harley-Davidson brand through a motorcycle shape grammar. *Research in Engineering Design* **13**(3): 139–156.

Salvador, F., P. M. de Holan and F. T. Piller. 2009. Cracking the Code of Mass Customization. *MIT Sloan Management Review* **50**(3): 70–79.

Shea, K. and J. Cagan. 1999. Languages and semantics of grammatical discrete structures. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **13**(4): 241–251.

Simpson, T. W. 2004. Product platform design and customization: Status and promise. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **18**(1): 3–20.

Stiny, G. 1980. Introduction to shape and shape grammars. *Environment and Planning B: Planning and Design* **7**(3): 343–351.

Stiny, G. 1991. The algebras of design. *Research in Engineering Design* **2**(3): 171–181.

Stiny, G. 2006. *Shape: Talking About Seeing and Doing*. Cambridge: MIT Press.

Stiny, G. and J. Gips. 1972. Shape grammars and the generative specification of painting and sculpture. In: *Information Processing 71 Proceedings,* ed. C.V. Freiman, 1460–1465. Amsterdam: North-Holland.

Thonet Company. 1980. *Thonet Bentwood & Other Furniture: The 1904 Illustrated Catalogue*. New York: Dover.

Wilhide, E. 2010. *How to Design a Chair*. London: Octopus Books.

Mário Barros is a Ph.D. candidate at the University of Lisbon, Faculty of Architecture. His research interests include shape grammars, the digital design process and mass customisation in product design applications. His work aims to combine academic research with industrial applications. He was awarded a B.Sc. in Industrial Design by the Faculty of Fine Arts, University of Lisbon in 2004.

José Pinto Duarte is a Full Professor at the University of Lisbon, Faculty of Architecture and a Visiting Scientist at MIT. He holds a B.Arch. (1987) in architecture from the Technical University of Lisbon and a S.M.Arch.S. (1993) and Ph.D. (2001) in Design and Computation from MIT. He was awarded the Santander/TU Lisbon Prize for Outstanding Research in Architecture by the Technical University of Lisbon in 2008. His main research interests are mass customization with a special focus on housing, and the application of new technologies to architecture and urban design in general.

B.M. Chaparro is a Professor of Mechanical Engineering at the Polytechnic Institute of Tomar. He was awarded a B.Sc. (1997), M.Sc. (1999), and Ph.D. (2007), all in Mechanical Engineering, from the Faculty of Sciences and Technology, University of Coimbra. In 2010 he founded Critério Radical, an engineering service provider company working in the fields of industrial engineering and automation. His research interests include design optimisation and numerical simulation.