Nexus Netw J (2014) 16:523–541
DOI 10.1007/s00004-014-0213-9

NEXUS NETWORK JOURNAL
Architecture and Mathematics

RESEARCH

Alberti and Military Architecture in Transition

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Published online: 2 October 2014

Abstract Alberti's contribution to the nascent science of fortifications in the 1450s is often ignored, but a careful reading of his descriptions of fortifications show that he was the first to describe the elements of fortification formally in the precise mathematical terms of shape, measurements, relationships, proportions. In forming his ideas, Alberti embraced both the old and the new. The old he re-elaborated and set in modern terms, and although he cites numerous ancient authors as sources for his information about fortifications, the obvious historical source for Alberti's description of the elements of fortification is Vitruvius. On the other hand, Vitruvius alone cannot account for all of Alberti's knowledge of military architecture. Plans with well-define geometric shapes, solid walls, scarped bases, curtains, towers open to the interior appropriated located on the bases of flanks, loopholes for grazing fire conjoined to systems of defense for dropping stones and other missiles vertically, structures in earth: all of these show that Alberti was in possession of a thorough knowledge of the state of fortifications in his day. All of these features are present in fortresses that had been built some years earlier based on designs attributed to Filippo Brunelleschi. We examine the fortress at Vicopisano to show a built example that Alberti may have seen and drawn on.

Keywords Leon Battista Alberti · Filippo Brunelleschi · Military architecture · Fortifications · De re aedificatoria · Vicopisano

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Introduction: Alberti on the Cusp of a Revolution

Leon Battista Alberti (1404–1472) lived in a very exciting time, on the very cusp between a medieval world and a modern one. On the one hand, Renaissance humanists were bringing back to light the knowledge of antiquity, including of course Vitruvius's ten books on architecture. On the other hand, every day brought the invention of new knowledge, such as linear perspective, which revolutionized art and architecture.

Alberti embraced both the old and the new. The old he re-elaborated and set in modern terms (as shown by his re-fashioning of Vitruvius's treatise on architecture, where he was more inspired by the aim than by the results). The new he set forth in his own terms (as shown by his treatise *On Painting*, where he worked the new practices of perspective into a proper theory). In both cases, the 'terms' we are speaking of are mathematical. In his own words, 'Of the arts the ones that are useful, even vital, to the architect are painting and mathematics' (Alberti 1988, p. 371). Historian Paul Rose noted, 'Apart from Piero della Francesca, no artist viewed life more exhaustively in mathematical terms than Leon Battista Alberti' (Rose 1975, p. 6). Alberti sought to endow several of the arts with a solid mathematical foundation. For art, he composed the *Elementa Picturae* (*Elements of Painting*), a compendium of definitions of terms and lists of exercises of drawing skills important for the painter (Alberti 2010, pp. 141–152 and 153–168). Also for artists, his *De Pictura* (*On Painting*) (Alberti 2004) is a treatise on principles of distance, dimension and proportion. For sculptors, he wrote his *De statua*, in which he describes the use of an instrument that he calls a *diffinitore* for measuring the distances between two points on a model in order to transfer them via a change of scale (di Pasquale 2014, p. 21). For urban planners, his *Descriptio Urbis Romae* provides instructions for use of an astrolabe-like surveying disk for determining coordinates to fix positions of significant points in an urban landscape. His booklet *Ludi matematici* is a compendium of mathematical 'games' using readily available props (spears, for example) to measure distances, heights, weights and more. Finally, his masterpiece on architecture, *De re aedificatoria*, organises and codifies the various elements of building in order to place that profession on a scientific basis. In the words of Martin Kemp, 'His vision of mathematical order expressed through tangible activities found its perfect realisation in the work of the architect' (Kemp 2004, p. 6).

Here we will examine how Alberti applied his mathematical thinking to problems of a military nature. We use the broad general term 'fortifications' to refer to the military architecture discussed in the *De re aedificatoria.*[1] This includes fortified city walls, citadels and other small strongholds, military camps and watchtowers.

It might appear at first glance that Alberti did not create or invent anything new, but his descriptions of fortifications deserve a more careful reading, because he was the first to formally describe the elements of fortification in precise mathematical

[1] The primary source for the *De re aedificatoria* is the 1565 edition in vulgar by Cosimo Bartoli (Alberti 1565). English translations of the citations are taken from the edition by Joseph Rykwert, Neil Leach and Robert Tavernor (Alberti 1988). Some citations are personally translated by Kim Williams, as specified each time in the text.

terms: shape, measurements, relationships, proportions. Alberti was perfectly in tune with the progressive mathematising of all expressions of architecture—religious, civil, urban and military—that characterised his times. The application of geometry as a governing system to fortifications in particular, would continue to mature throughout the fifteenth and sixteenth centuries, ultimately becoming the foundation of the new science of fortifications. Further, as we have described elsewhere (Williams and Bevilacqua 2013), Alberti was also the first to extend the principles of geometry to another very practical problem involving ballistics: that of how to aim a bombard, discussed in his *Ludi matematici.*

Thus, we see Alberti as the first 'modern' thinker about both defensive and offensive tactics. His *De re aedificatoria* was a point of reference for the more specialised treatments that followed. In the final analysis, Alberti can be seen as a first significant figure in the creation of a new military science.

Alberti and the Role of the Architect in Military Matters

In tumultuous fifteenth-century Italy, where a mosaic of small *Signorie*[2] contended with each other for territory and dominion, military arts of attack and fortification were of prime importance. It is therefore not surprising that Alberti thought and wrote about them. It is here that we see Alberti precisely at the turning point between old and new. There is a formal Alberti, compiler of a set of rules to be presented to his patrons, and a modern Alberti, grappling with the 'dirty' problems of warfare—albeit at a safe distance.

The formal Alberti appears to us in his *De re aedificatoria* (Fig. 1). This treatise, in Latin, was written for an erudite, noble readership composed of those who were patrons rather than practitioners of architecture. As Joseph Rkywert has pointed out,

> In order for [Alberti's] lessons to have the proper authority … the tone and the audience to which he addresses himself must be established. … [W]hereas Vitruvius, for all his encyclopedic and philosophical pretension, writes to confirm his positions as the custodian of a tradition, and to claim imperial patronage in its name, Alberti writes to claim a high place in the social fabric for the re-formed discipline of the architect, which has to be established anew (Alberti 1988, pp. ix-x).

In the view of the 're-formed discipline' that Alberti presented to his noble readership, the architect's role in military matters is an extremely important one:

> Nor should you forget ballistic engines and machines of war, fortresses and whatever else may have served to protect and strengthen the liberty of our country, and the good and honor of the state, to extend and confirm its dominion. It is my view moreover that, should you question all the various cities which within human memory have fallen into enemy hands by siege, and inquire who defeated and conquered them, they would not deny that it was

[2] The *Signoria* (pl. *Signorie*), or lordship, was a form of rule that arose with the definitive decline of the medieval commune forms of governance and power-wielding.

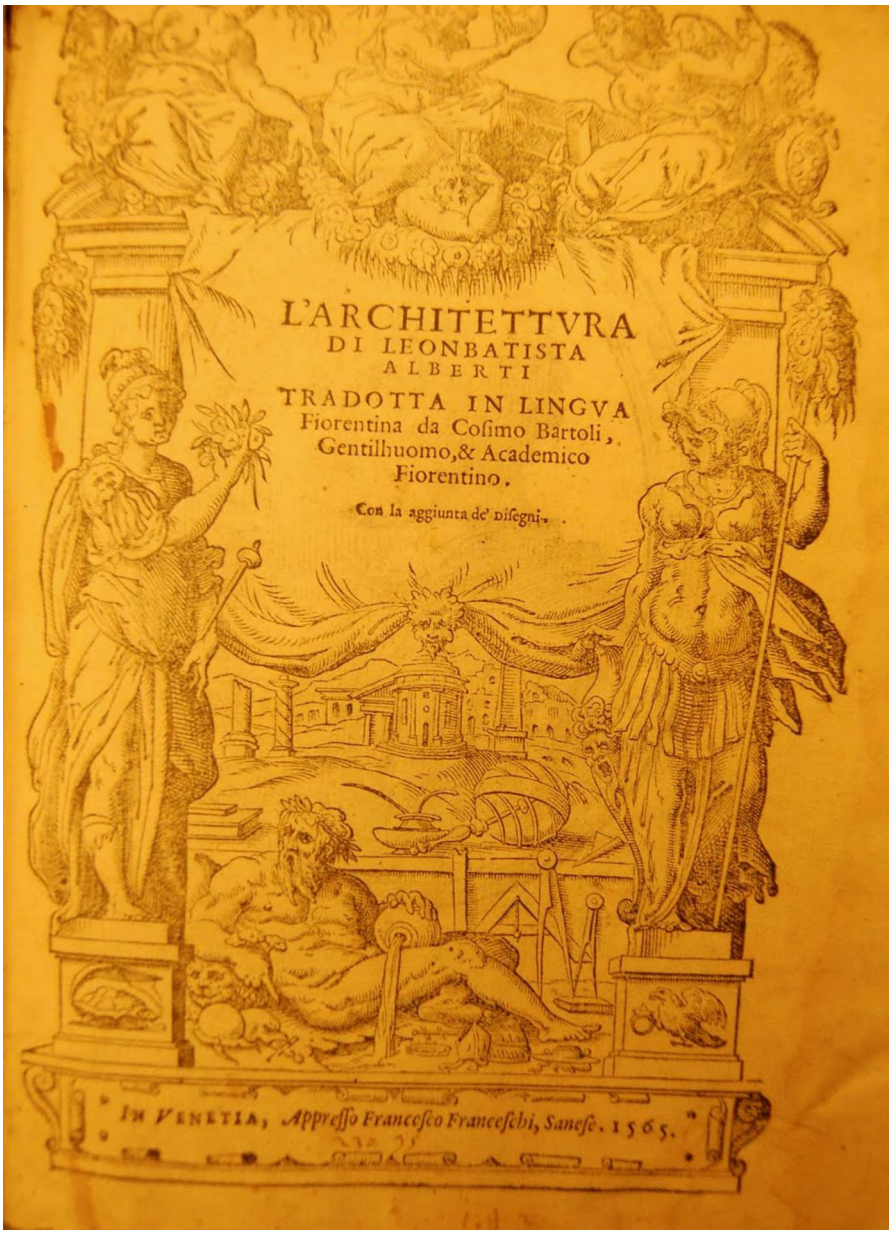

Fig. 1 Frontispiece of the 1565 edition in vulgar of *De re aedificatoria* by Cosimo Bartoli, from which the citations in this paper are drawn

the architect; and that they could easily have scorned an enemy armed with weapons alone but could no longer have resisted the power of invention, the bulk of war machines and the force of ballistic engines, with which the

> architect had harassed, oppressed, and overwhelmed them. On the other hand, those besieged would consider no protection better than the ingenuity and skill of the architect. Should you examine the various military campaigns undertaken, you would perhaps discover that the skill and ability of the architect have been responsible for more victories than have the command and foresight of any general; and that the enemy was more often overcome by the ingenuity of the first without the other's weapons, than by the latter's sword without the former's good counsel. And what is more important, the architect achieves his victory with but a handful of men and without loss of life (Alberti 1988, p. 4).[3]

Modern Requirements for Fortifications

Historical sources alone were not sufficient to allow Alberti to prescribe methods of fortification that were sufficient for his own day, because the introduction of more effective and powerful firearms was changing the nature of warfare.

The discovery of gunpowder, between the ninth and eleventh centuries, is attributed to the Chinese. In Europe, Roger Bacon (1214–1292) is credited with having encoded its formula in an anagram (Bevilacqua 2007, p. 249). The earliest forms of European firearms had begun to be used around the end of the thirteenth and beginning of the fourteenth centuries, most notably during the Hundred Years' War. By the mid-fifteenth century, when Alberti was composing his *De re aedificatoria*, the efficacy of adopting firearms as a wartime strategy—relatively undervalued before that time—was becoming increasingly obvious, and would eventually overturn the centuries-old tradition of building fortified cities and strongholds, leading finally to a radical new conception of all military techniques. The true tipping point in Italy came as a result of the invasion by the army of King

[3] *Aggiugni à questo gli Instrumenti & Machine da guerra; le Fortezze, & quelle cose che fanno di bisogno à difendere la libertà della Patria, & à mantenere l'honore, & ad accrescere la grandezza della Città: & ad acquistare & à stabilire uno Imperio. Io certo mi penso che se si dimandassero tutte quelle città, lequali dapoi inqua che è memoria de gli huomini son venute per assedio, sotto lo imperio d'altri; da chi esse fussino state soggiogate & vinte; elleno certo direbbono, dallo Architettore: Et di essere state sofficienti asprezare facilmente lo armato inimico; ma non già di essere state possenti di durare contro alla forza dello ingegno; & alla grandezza delle Machine, & allo impeto de gli instrumenti bellici; con le quali cose lo Architettore le strigneva, le infestava, & le rovinava. Et cosi per il contrario diranno quelle che sono state assediate, di non si essere difese con alcuna altra cosa, piu che con lo aiuto, & con le Arti dello Architettore.*

Et se tu andrai esaminando le fatte espeditioni; troverai forse che la maggior parte delle vittorie si sono acquistate piu tosto per le arti; & per le virtuti de gli Architettori: che per i governi, o per le fortune de Capitani; Et che lo inimico è stato piu volte superato, & vinto dallo ingegno degli Architettori, senza le armi de Capitani; che dalle armi de Capitani, senza l'ingegno de gli Architettori. Et quel che grandemente importa, è che lo Architettore con poca gente, & senza perdere i soldati, vince (Alberti 1565, *Proemio*, p. 6). Here Alberti agrees with Vitruvius's comment in Book X, when he begins by saying 'In matters of defence, one must equip oneself not just with machines but also, above all, with good tactics', then goes on to describe clever tactics devised by architects in Chios, Apollonia, and Marseilles, concluding,'… these victorious cites were liberated not with the help of machines but by the tactical skill of the architects deployed against mechanical devices' (Vitruvius 2009, pp. 318–319). See also (Alberti 1988, p. 367, note 8).

Charles VIII of France in 1494, some half a century after Alberti's *De re aedificatoria*. The fact that Alberti was thinking about and responding to the new exegeses of artillery long before invasion made this imperative is a sign of his farsightedness.

The subject of firearms is one of the areas where we most clearly see Alberti's two faces, the traditionalist and the modernist. In Book IV of the *De re aedificatoria*, we find a key statement:'…there are two types of war machine: one demolishes the wall by ramming and striking it, the other undermines it and digs away its foundations' (Alberti 1988, p. 103).[4] Alberti was quite aware, however, of the extraordinarily powerful third strategy in addition to ramming and mining: bombarding. That he had given the problem some serious consideration is shown by one of the twenty problems in his less formal booklet, *Ludi matematici*, which describes a geometrically-based technique for aiming a bombard. Alberti's singular treatment of a problem involving how to aim a bombard, usually given scant importance and in any case incorrectly interpreted (Williams and Bevilacqua 2013), turns out to be a precursor of important developments in both military science and in military architecture. It shows him to be on the cutting edge of the developments in artillery then in progress, and also presages the increased use of geometry that the new form of warfare would entail. His use of geometry to generalize the problem of aiming a cannon, with the recovery of the ancient *archipendulum* and his method of using it in several of the 'games' that have military applications would reappear later in texts by Tartaglia and others, before eventually evolving into the military compass perfected by Galileo.

It is intriguing that Alberti doesn't include any of this knowledge in his *De re aedificatoria*. A key to explaining this might lie in the fact, mentioned earlier, that his formal treatise was intended for erudite readership. The *Ludi*, in contrast, was written as a mathematical 'pastime' for the amusement of Meliaduse d'Este, the older but less influential brother of Lionello d'Este, who was an important patron of Alberti's (Alberti 2010, p. 75, note 2). Then too, this omission in the formal treatise may echo the notion that there was something 'vile' or 'unchivalrous' about artillery, something in any case that made it unworthy of appearing in the formal treatise. Petrarch (1304–1374) expressed this idea in his *De remediis utriusque fortunae* (*Remedies for Fortune Fair and Foul*):

> Joy: I have war machines and *ballistae* beyond number.
> Reason: I would be surprised if you did not also have those balls of iron which are ejected by fire with terrifying thunder. For little men like you the wrath of immortal God thundering from the sky is not enough: O cruelty wedded to pride! The earth itself thunders now and the flash that cannot imitated, as Virgil said, is now being imitated by human frenzy. What used to be thrust forth by the clouds of heaven is now being thrust forth by a machine conceived in hell. … Until recently this fiendish tool was rare and viewed with great

[4] *Le macchine certamente sono di due sorti; una è quella, con la quale percotendo, & battendo si gettano a terra le muraglie; L'altra è quella, mediante la quale accostandosi a le mura, le si scalzano sotto, & si rovinano* (Alberti 1565, Bk. IV, ch. III, p. 108).

> amazement. Now artillery of all kinds is common as any other weapons because our minds are quick to learn the very worst (Petrarca 1991, p. 270).

A hint of this attitude is found in how he prefaced the bombard problem for Meliaduse in the *Ludi matematici*. A few pages earlier Alberti had described an instrument that he calls an *equilibra*, which could be used measure weights and angles, but now he shows that it can also be used to aim a bombard. He tells his patron that he will discuss the problem 'more to show you a use for your *equilibra* than to reason about things [alien] to your dignity and authority' (Alberti 2010, p. 49).

Alberti's Mathematising of Military Architecture in the *De re aedificatoria*

We have seen that Alberti believed that mathematics was capable of forming a solid foundation for all arts, and that he considered it vital for architects, and we have heard him express the importance of the architect in military success. Now let us look at how he combines these two concepts in the *De re aedificatoria*.

A first mathematical notion found in Alberti's treatment of fortifications is that of shape. While acknowledging that the choice of the site—whether in the mountains or on a flat plain, for example—is determinant for the outline and distribution of the parts of a town (Alberti 1565, Bk. IV, ch. III, p. 105), the shapes of cities are ideally regular and geometric: the circle provides the most capacious city; the best defended has walls that undulate, like those of Jerusalem. He recounts that the walls of Babylon were quadrangular, those of Memphis 'in the shape of a Δ', i.e., triangular (1565, Bk. IV, ch. III, p. 108). Also effective in protecting the wall from projectiles is a sawtooth outline ('s'assomiglino ai denti delle Seghe', 1565, Bk. IV, ch. IV, p. 110). Towers placed at intervals in town walls should be circular in plan (1565, Bk. IV, ch. IV, p. 110). The citadel, which is a stronghold within the city walls, should also be circular, 'in the shape of an O' (1565, Bk. V, ch. IV, p. 128). Military camps, more or less permanent but still based on the same principles as cities, should be circular or quadrangular (1565, Bk. V, ch. XI, p. 139), though again considerations of site are predominant. Finally, watchtowers, which are not subject to site conditions and can therefore be more exactly prescribed, are to be either circular or square in plan (1565, Bk. VIII, ch. V, p. 282).

A second mathematical notion in the treatment of fortification is that of precise measurement. Alberti is diligent in quoting dimensions of the historical examples he draws on. Here we find that he often cites historical authors precisely where he gives dimensional data. This indicates that he was purposely seeking such information in the works of the ancients that he studied. There are a great number of such citations. According to 'ancient sources', Thebes was 20 miles in perimeter, Memphis 18 and 6/8, Babylon 43 and 6/8, Nineveh 60 (1565, Bk. IV, ch. III, p. 106). According to Arrian, the walls of Tyre were 112 and a half ells high (1565, Bk. IV, ch. III, p. 107). According to Megasthenes, Polimbothra was 12 miles long and 3 miles wide (1565, Bk. IV, ch. III, p. 108). According the Law of the Twelve Tables (in the commentary by Gaius), main roads were to be six ells wide where straight and eight

ells wide where curved (1565, Bk. IV, ch. V, p. 111). According to Appian, the technique used by Octavius in besieging Lucius in Perugia involved building a ditch 7 miles long, 15 ells wide and 15 ells deep, along which ran a high wall topped with 1,050 towers (1565, Bk. V, ch. XI, p. 141–142).

For new constructions as well as old, the information he gives is precisely dimensioned. The city walls are to consist of two walls, 20 feet apart (according to the ancients), the space between them filled with compacted earth (1565, Bk. IV, ch. IV, p. 109). Round towers are to be placed every 37 and a half ells (1565, Bk. IV, ch. IV, p. 110). For added protection outworks with triangular bases (again shape!) are to be built along the wall at intervals of 7 and a half ells (1565, Bk. IV, ch. IV, pp. 109, 110). In military camps, walls are to be topped by turrets placed every 100 feet (1565, Bk. V, ch. XI, p. 140).

A third mathematical notion made evident in the treatment of fortifications is that of the relationships between parts. Geometric relationships are evident between parts, especially circular parts. One such relationship is concentricity. A first instance of this is found in the discussion of multiple city walls: Carthage is said to have had three sets of walls; Cebetana seven sets (1565, Bk. IV, ch. III, p. 107); the citadel of the Temple of Hamon was surrounded by three walls (1565, Bk. V, ch. III, p. 128). A second instance of concentricity is found in Alberti's description of the division of a city as a means of either separating classes or preventing a popular uprising. In this case, 'The best means of dividing a city is to build a wall through it. This wall, I believe, should not run diametrically across the city but should form a kind of circle within a circle'[5] (Alberti 1988, p. 118). Concentricity is also inherent in the alignment of the seven stories of the watchtower prescribed in Book VIII, where storeys first square and then circular are placed one on top of the next (1565, Bk. VIII, ch. V, p. 282). A second relationship is that of nested tangent circles. This is found in the location of the citadel within the city. The citadel is a round shape located within 'a huge C with bent horns', in other words, a series of nested tangent circles. The shift in the relationship of the circles from concentric to tangent allows Alberti to characterise the relationship as 'neither inside nor outside' (1565, Bk, V, ch. IV, p. 128). Another relationship that is 'neither inside nor outside' is the house of a tyrant, that is, one who rules by having conquered a city (1565, Bk. V, ch. III, p. 127).

Finally, a fourth mathematical notion used in discussing fortifications is that of proportion. The precise dimensions just outlined naturally allows proportional relationships to be established. For instance, along the city walls, the relationship between the triangular outworks and the round towers is 7.5 to 37.5 ells, or 1 to 5.

Where proportions are most exhaustively described is the section on watchtowers in Book VIII. Here, unconstrained by site restrictions, Alberti is free to dictate a series of proportions to control the plan, height and thickness of the multi-storied tower. In the tower with a square plan, the width to height ratio is 1:6; in the round tower, the ratio of diameter to height is 1:4. If the towers are to be 'stout', those

[5] *[le Città] fussino divise da alcuna muraglia commodissimamente; & tal divisione non penso io che si habbia a tirare come un Diametro a traverso d'una pianta; ma come rinchiudere un circuito in un altro* (Alberti 1565, Bk. V, ch. I, p. 123).

ratios will increase respectively to 1:4 and 1:3. The proportions of the successive stories are described so minutely that it is worthwhile to reproduce the passage at length:

> First of all from the square plan is raised above the ground a basement, the height of which shall be the tenth part of the entire work from the top to the foot; the width shall be a fourth of this same height, in the middle of each facade above this basement shall be put two columns, and a column for each separate corner with their adornments. … And above that basement shall be placed then the quadrangle constructed like a *tempietto* [small temple]; the width of which shall be equal to two heights of the basement, and the height of which equal to the width. And there shall be put, on the exterior side, three, four and five steps of columns like those that we said in temples, above this square shall be placed the round *tempietti*. There will thus be of these round *tempietti* up to three in number, which we, taking their similarity to canes, will call knots. The length of any of these knots will be equal to their own width plus a twelfth of it, which we want to serve for a basement. But the width shall be derived from that square *tempietto* that we placed on the first basement in that way. That is, divide the façade of this square *tempietto* into twelve parts, eleven of which will be assigned to the first knot. Then divide the diameter of the first knot into twelve parts, eleven of which will assigned to the second knot, and the third knot similarly will be thinner by a twelfth part of the second, and with this order we will have carried out what some good ancient masters praised immensely in the columns, that the part of the shaft of this work at the bottom will be larger by a fourth than the upper part.[6]

In short, nothing about the proportional relationships of the watchtower is left to chance, and indeed they are considered every bit as carefully as those of the orders in Books III and IV.

These four mathematical notions—shape, measure, relationship and proportion—allow Alberti to define, clarify and quantify the indications he is giving for

[6] *Inanzi tratto dalla pianta quadrata si rilievi da terra uno imbasamento, l'altezza della qual' sia per la decima parte del tutto dell'opera dal capo al piede la larghezza sia per il quarto di questa stessa altezza, nel mezo di ciascuna facciata sopra questo imbasamento, si mettino due colonne, & una colonna per ciascuna cantonata distinte con i loro addornamenti…… Et in sul medesimo imbasamento si ponga di poi il quadrangolo fatto, come un Tempietto; la larghezza del quale sia per due altezze dello imbasamento, & la altezza sia quanto la larghezza, & ci si metteranno, dallo lato di fuori tre, quattro, & cinque gradi di colonne come quelle che noi dicemmo ne Tempii, sopra questo quadrato si porranno i Tempietti tondi. Seranno adunque questi tempietti tondi fino a tre di numero, i quali noi presa la similitudine dalle canne chiameremo nodi. La lunghezza di qual si voglia di questi nodi sarà quanto è la lor' propria larghezza aggiuntovi uno duodecimo di essa, ilche vogliamo serva per imbasamento. Ma la larghezza si caverà da quel tempietto quadrato che noi ponemmo sul primo imbasamento in questo modo, cio è. Dividasi le faccia di questo Tempietto quadrato in dodici parti, undici delle quali assegneremo al primo nodo; Dividasi di poi il Diametro di detto primo nodo in dodici parti, le undici delle quali si assegnino al secondo nodo, & il terzo nodo similmente sarai più sottile la duodecima parte che il secondo, & con questo ordine ci verrà fatto che conseguiremo quelche i buoni maestri antichi lodarono nelle colonne grandissimamente; che la parte del fuso di sì fatto lavoro da basso, sarà più grossa il quarto che la parte di sopra* (Alberti 1565, Bk. VIII, ch. V, pp. 282–283; trans. by Kim Williams); cfr. (Alberti 1988 pp. 258–259).

fortifications. In essence, this represents the beginning of the scientific treatment of military architecture.

Alberti's Descriptions of the Elements of Fortification

While the reference to Vitruvius in the definitions of the principles of fortification is undeniable, a more careful reading makes it possible to discern Alberti's awareness of the science of fortification as it stood in the first half of the 1400s.[7]

For example, the in-depth description that Alberti gives of fortresses in Book V, in particular of how these were located with respect to urban fabric and walls, is a reflection of the requirements of a political context that was undergoing a profound change in those years in Italy:

> In every case the citadel must have an unobstructed outlet, by road, river, lake, or sea, through which unimpeded it may seek or admit reinforcements or help from outside, against the enemy or, in the case of treachery or mutiny, against its own citizens and soldiers. The most suitable layout for the citadel is for all sections of town wall to be linked in the form of O, which is either in turn grasped, but not enclosed, by a huge C with bent horns … or from which several radial walls emanate to the circumference. Thus the citadel, as we have just recommended, is neither inside nor outside the town (Alberti 1988, p. 123).[8]

At the time, the young Italian *Signorie* were engaged in solidifying their regional powers by means of conquering and occupying territory and negotiating political agreements throughout Europe.

Territorial militarisation was thus of prime importance, both for affirming roles within this broad new balance of power at an international level, and for consolidating control of territory at home against any possible hint of rebellion. It is on account of this that the mark of occupation lay in citadels located in a position that was eccentric and tangent to city walls. These were conceived to guard against the greater probability of attacks from within than from without, and were

[7] It is important to note that Alberti's descriptions of fortifications are strictly verbal, and that the *De re aedificatoria* did not contain illustrations, a reflection of Alberti's inherent mistrust of copyists. His informal booklet, *Ludi matematici*, did contain illustrations, but it was intended for leisure, not scholarly study; see (Alberti 2010, pp. 3–4).

[8] *La fortezza bisogna che abbia sopra ogni altra cosa strade diritte, & espedite, donde si possa soccorrere addosso a i nimici, a Cittadini, & a suoi Terrazzani, se per alcuna seditione, or perfidia bisognasse. Et che è si possa metter dentro aiuti, & dei suoi, e dei Forestieri liberamente, & per terra, & per Fiume, Lago, o Mare. Sarà commodissimo quel disegno de la Fortezza, che come uno O, tondo si congiugnerà a tutte le mura de la Città, & le mura grandi si congiunghino con essa, come un C con corna piegate non la accerchiando intorno; o veramente quello del quale si partino più raggi, come per andare alla circunferentia, & in quella maniera quel che poco fa dicemmo che bisognava non saria la fortezza nè dentro nè fuori dela Città* (Alberti 1565, Bk. V, ch. IV, p. 128).

strategically placed to provide ease of access from the outside to reinforcements in case of siege.[9]

Another significant element, one whose roots this time are found in Vitruvius, is the reference in Book IV to flanking as the principal element defining systems of defence:

> The wall should be flanked by towers acting as buttresses every fifty cubits. These should be round, standing out from the wall, and somewhat taller, so that anyone venturing too close would expose his flank to missiles and be hit; thus the wall is protected by the towers and the towers by each other. Leave the side of the tower facing the town stripped of wall and open, to deny the enemy any protection should he happen to gain entry (Alberti 1988, p. 104).[10]

Towers open towards the inside of the citadel projected out from the exterior profile of the walls in order to more efficiently attack the enemy's flank. The distance between towers along the walls—based on the distance an arrow could fly—is defined so as to guarantee that from each tower it was possible to defend the next one as well as the intervening wall. The principle of flanking, already brought back to current use a century earlier and used in Alberti's day without reservation, was the hinge on which all the successive studies of fortifications in transition were based. This would lead to the definition of the modern bastioned profile in the first half of the 1500s.

The description of defensive works is further concentrated in the definition of formal and constructive elements of individual parts. In fact, the base of the walls of the citadel is described and explained in a particularly knowledgeable fashion in chapter IV of Book V:

> The base of the citadel must be solidly constructed of huge stones, and must have an inclined surface, so that any ladder propped against it will bend and be weakened, and any enemy who tackles it will be unable to escape the stones hurled down by clinging to the wall, and the missiles of the war machine will not ram it so hard, but will bounce off at an angle (Alberti 1988, p. 123).[11]

[9] In this key, for example, can be read a great number of the fortifications that the Florentine *Signoria* created to control their territories from the fifteenth through the mid-sixteenth century, from those of Pisa, Volterra and Sarzana to the well-known Fortezza da Basso and Fortezza del Belvedere in Florence.

[10] *Et quelle Torri ancora, che ad ogni trentasette braccia, & mezzo si congiungono alla muraglia, quasi come barbacani, risaltando tonde alo in fuora, & avanzando con l'altezza loro; l'altra muraglia, accioche chi fra loro ci volesse accostare alla muraglia, esponga alle Saette il fianco disarmato; & vi rimanga morto. Percio che in questa guisa le mura da il fiancheggiare dele Torri, & una torre dall'altra saranno difese. Da quella banda che le Torri, sguardano verso la Terra, fa che le siano senza mura,& aperte, acciocche per aventura, i nimici v'entrassero dentro, non vi stieno sicuri* (Alberti 1565, Bk. IV, ch. IV, p. 110).

[11] *Il Ricinto de la fortezza si debbe porre saldo, di Pietre grandi, con linea dal lato di fuori a scarpa, per la quale le scale, che vi fussino, poste, diventino deboli per l'havere a star troppo a pendio: Et accioche quello inimico, che accostatovisi si attacchasse a le mura non possa schifare i sassi, che di sopra gli fussero aventati. Et accioche le cose, che da le Macchine de nimici vi fussero gittate non colpischino in piena, ma smuccino per il traverso* (Alberti 1565, Bk. V, ch. IV, p. 129).

The walls, still very high, were to be solid with a scarped base to limit the possibility of their being scaled by assailants. The scarped base, a feature widely used in Alberti's day, served both to increase the resistance of the walls and to deflect projectiles.

Also described is the projecting apparatus supported on corbels and the catwalk at the top protected by crenellation, elements with a medieval root for defensive methods dropped from above. Alberti describes them in chapter IV of Book IV:

> The cornices of the towers and walls act both as ornaments and as a bond to strengthen them, while they prevent the use of scaling ladders. … There should be holes beneath the battlements as they stand out from the wall, through which stones and firebrands may be cast down at the enemy, or water, if the gate is on fire (Alberti 1988, p. 105).[12]

In addition to the apparatus for dropped defensive methods, the structure is to be provided with arrow slits or loopholes in the base of the walls to permit grazing fire, placed horizontally at regular intervals, close to the ground:

> It is essential with defensive walls to a town or citadel that every care be taken to prevent the enemy from approaching with impunity. This may be achieved with a deep, wide moat, as described above, together with loopholes positioned at the very base of the podium, where the enemy, although protected from above by his shield, will be struck where he is not covered. Indeed, this method of defense is superior to all others (Alberti 1988, p. 123).[13]

The effectiveness of such loopholes had been discovered during second half of the fourteenth century.[14] Grazing fire permitted a more precise aim at the target from an angle that was better than that from the traditional position at the top of the wall, and to hit groups of attackers more effectively. The loopholes opened to splayed embrasures cut into the thickness of the wall and close to the exterior ground level.

The profile of the wall section is always conceived in relation to the ditch, wet or dry, which Alberti mentions more than once. In Book IV he provides accurate

[12] *Le cornici a le Torri, & a le mura oltre a che le arrecano ornamento, & stabilità con la loro legatura, proibiscono ancora il salire, da le postevi scale…. Ne le merlature, che sportano in fuora, sianovi piombatoi, da quali si possino aventare a nimici, & pietre, & fuochi, & acqua ancora …* (Alberti 1565, Bk. IV, ch. IV, p. 110).

[13] *Potentissima ragione sarà veramente, nel difender le mura, o d'una Città, o d'una fortezza se tu harai cura, che il nimico sopra ogni altra cosa non ti si possa accostare senza suo pericolo. Et questo si farà sì con fare i fossi larghi, & profondi, come ti dicemmo; sì ancora con lasciare nascose sotto le balestriere (per dir così) ne la grossezza d'esso ricinto, stabilite fessure, da le quali mentre che il nimico si cuopre con lo scudo delle offensioni che gli vengono di sopra, possa esser ferito per fianco da quella parte che li resta scoperta. Questo modo di difesa, è sopra tutto il principalissimo* (Alberti 1565, Bk. V, ch. IV, p. 129).

[14] In the area around Pisa, for example, are found systems for grazing fire in the fortifications of the *terre nove*, such as those of the walls of Cascina, and in outposts such as the Castello di Ripoli, of the second half of the 1300s. Here loopholes are found at several levels, once accessible by means of multiple balconies or wooden scaffolding long since destroyed (see Caciagli 1997, pp. 108–125. Leverotti 1989, p. 243–262).

descriptions of its function, the correct distance from the base of the all, and the details of the cross section:

> … what is required is a ditch, both wide and quite deep. This prevents a 'tortoise', mobile tower, or any such machine from being moved in close; also, if water or solid rock is met, it will thwart any attempt at mining. (Alberti 1988, p. 103).[15]
>
> But we truly, besides these, praise those walls situated in a manner that, even if in the end, by force of ramming, they are thrown to the ground, they have at their feet a flat place, where it lies in relation to them [the walls] almost like an embankment, and that with their ruin they do not fill the ditches.[16]
>
> If the outer bank of the ditch is raised above the level of the surrounding land, and sloped, that will be very useful. This will cause all missiles aimed at city walls to miss them by overflying (Alberti 1988, p. 104).[17]

The outworks described by Alberti fulfil the growing need to move the battlefront away from fortified perimeter, especially in relation to the strengthening of firearms. Made of earth, these are able to resist more efficiently the impact of projectiles. Alberti—albeit in reference to traditional war machines—introduces the topic by describing the system of buttresses in Book IV:

> Buttresses with triangular bases should be built at ten-cubit intervals along the line of the wall, with one angle pointing toward the enemy. Arches should be constructed to spring from one buttress to another, and then be vaulted over. The niches in between should be filled with a mixture of clay and straw, and packed down. Thus the softness of the clay will deaden the force and impact of the engines. In addition, despite the continual onslaught of the engines, the wall can only be weakened here and there, where it can be repaired quickly (Alberti 1988, p. 104).[18]

Books III and IV contain still other significant passages that describe both fortified structures as a whole, and a most detailed description of their parts. Taken

[15] *… vogliano che essa fossa sia oltra modo larga, & oltra modo profonda: Percioche essendo cosi, impedirà a la Testuggine andante, & a la Torre, o a simili altre Macchine, il potersi accostare a la muraglia. Et ritrovata l'acqua, o il sasso, sarà certo fatica indarno, il volervi far sotto Mine* (Alberti 1565, Bk. IV, ch. III, p. 108).

[16] *Ma noi veramente oltre a queste, lodiamo quelle mura collocate di maniera che, seppur a la fine, p*[*er*] *forza di batterie fussino gittate a terra, habbino a piedi loro un piano, dove le stiano quasi come un'argine, & che con la loro rovina non riempino i fossi* (Alberti 1565, Bk. IV, ch. IV, p. 109; trans. by Kim Williams); cfr. (Alberti 1988, p. 103).

[17] *Et inanzi ad ogni altra cosa, ti gioverà grandemente, se tu farai che la ripa de fossi di fuori, stia a pendio; & che l'argine del fosso sia alquanto piu alto che il resto del Terreno; Percioche i colpi de li Inimici non toccheranno le Mura; ma passeranno di sopra* (Alberti 1565, Bk. IV, ch. IV, p. 110).

[18] *Faccinsi Barbacani fuori del diritto de le mura, a guisa di Triangolo, con uno angolo volto a nimici, discosto l'uno dall'altro sette braccia, & mezzo, & poi da l'uno a l'altro tirinvisi archi in volta; Et i Vani, che quivi come zane rimangono, si riempino di Strame, & di Terra, pillata con stagioni. Et di quì ti avverrà, che la forza de le Macchine, & gli impetuosi colpi, saranno da la tenerezza del Terreno ingannati, & le Mura da la continouazione de la batteria, non si debiliteranno, se non quà, & là sparsamente, & quelle buche, che vi si faranno, si potranno riserrare in un subito* (Alberti 1565, Bk. IV, ch. IV, pp. 109–110).

as a whole, the intention appears to be to formulate rules, based on an analysis and comparison of what was done in ancient times, for a tradition of fortifications whose roots are medieval, but which has begun to take account of the use of firearms and of the impulse towards new and original technical and formal experimentation in the field of military architecture that was taking place in these same years.

Historical Sources for Alberti

Although Alberti cites numerous ancient authors as sources for his information about fortifications, the obvious historical source for Alberti's description of the elements of fortification is Vitruvius (ca. 80—15 B.C.), whose *De architectura* was written in the late first century B.C.[19] By the time Vitruvius's treatise arrived to the fifteenth century, readers, interpreters and translators of his text were plagued by its many difficulties: corruption of the text, loss of the figures cited, difficulty of understanding the terminology used, Vitruvius's obscure style of writing, and knowledge that was either outdated or simply lost. Alberti's choice in the face of these difficulties was, rather than struggle to render Vitruvius understandable, to write the entirely new treatise on architecture that he felt his times required. He did this by studying how things had been done in antiquity, and how they were being done in his own times, and then codifying the rules that he drew from his observations into a new set of norms.

There are two important differences between the approaches to military architecture in the treatises of Vitruvius and Alberti. First, while the only form of fortification described by Vitruvius are city walls, Alberti considers a range of types of military architecture. Second, for all of his praise of the architect's skill with military machines, Alberti includes no discussion of machines in his treatise.[20] However, in spite of its defects, Vitruvius was still an important point of reference, so it is worthwhile to see what Alberti drew on, in order to see where he went forward.

Vitruvius devotes Chapter 5 of his Book I (2009, pp. 24–26) to the fortified walls of cities, giving descriptions of and instructions for their construction, beginning with how to select a site and make the foundations. Regarding shape, without being as specifically geometric as Alberti, Vitruvius says the perimeter should be curved, without 'salient angles' to provide unimpeded views of the enemy. Alberti refers to this, without naming Vitruvius, when he says,

[19] Although it was known, consulted and cited through the middle ages, credit for its resuscitation in the Renaissance is given to the Florentine Poggio Bracciolini (1380–1459). However, scholars are not unanimous about Bracciolini's 'rediscovery' of Vitruvius: some say the manuscript was found in the Abbey of St. Gall in Switzerland, others say it was found in Montecassino in Italy. Sometimes the date is given as 1414, sometimes as 1416.

[20] To be precise, where Vitruvius declares:'Architecture has three divisions: the construction of buildings, of sundials and of machines' (Vitruvius 2009, p. 19), Alberti instead treats only buildings, and says nothing of sundials and machines. However, Alberti does say that he intends to treat war machines in a separate location (Alberti 1565, Bk. V, ch. XI, p. 141), but this was either never done, or has been lost.

> The architects of antiquity were against having angles in their town walls, because they felt that they would be of more benefit to an enemy on the attack than on inhabitants in defense, and that they had little resistance to war machines (Alberti 1988, p. 100).[21]

Vitruvius next describes the construction of curtains. This is addressed and endorsed by Alberti in Book IV, chap. IV (1565, pp. 109, 110). Vitruvius continues with a discussion of the towers that project from the curtains. These towers should be 'no further than a bowshot apart' and have removable floors to prevent the enemy from reaching all sectors of the wall in case of a breach in a single part (Vitruvius 2009, p. 25). They should be round or polygonal to prevent siege engines from breaking the corners and because they are more resistant to ramming (Vitruvius 2009, pp. 25–26).

These projecting towers are recuperated by Alberti in the round towers placed every 37 and a half ells along the wall (Alberti 1565, Bk. IV, ch. IV, p. 110). This principle of projecting towers may be the single most important element recovered by Alberti.

Alberti's Modern Sources

In light of what we have just seen of Vitruvius's rather limited description of fortifications, it appears clear that reference to Vitruvius alone cannot account for all of Alberti's knowledge of military architecture. Plans with well-defined geometric shapes, solid walls, scarped bases, curtains, towers open to the interior appropriated located on the bases of flanks, loopholes for grazing fire conjoined to systems of defense for dropping stones and other missiles vertically, structures in earth: all of these show that Alberti was in possession of a thorough knowledge of the state of fortifications in the mid-1400s, in addition to what he knew of Vitruvius.[22]

At the time in which Alberti was writing, military architecture was just at the beginning of a long evolutionary process in light of the increasingly consolidated adoption of firearms. Significant experiments had been carried out in the regulation of technical and formal aspects of medieval systems of fortification, and in the integration of new systems of defense.

Of particular interest to us here are some of the fortresses that had been built some years earlier based on designs attributed to Filippo Brunelleschi.[23] We cannot

[21] *Gli architettori antichi, nel cerchiare le Terre di muraglia, biasimarono le cantonate che escono fuori de diritti delle Mura, credendo che elle giovassero piu a gli nimici nel dare lo assalto, che a Terrazzani nel difenderli, & che le fussino debolissime a reggere contro a le percosse de le Macchine da guerra* (Alberti 1565, Bk. IV, ch. III, pp. 105, 106).

[22] It has been speculated, although we think not very convincingly, that Alberti was in some way engaged in the project for the Torre del Marzocco located in the port area of Livorno. References to this are found in Trotta (2005: 43–65) and Morolli (2006: 254, note 44).

[23] Works of military architecture attributed to Brunelleschi—probably in some cases only as a consultat - include works at the walls of Lastra a Signa (1424-26), Signa (1424) and Malmantile (1424), works at the fortress and the walls of Staggia Senese (1431), works at the walls of Rencine and Castellina (1430-31), the fortress of Vicopisano (1435-40), the fortified tower at the Porta del Parlascio in Pisa (1435-37), the new fortress in Pisa (1440-46).

exclude the possibility that these very fortresses were important references for Alberti as he compiled his *De re aedificatoria*.

For one thing, the high esteem in which Alberti held Brunelleschi is attested to by the dedication to Brunelleschi in the prologue of his treatise *De Pictura (On Painting)*:

> But after I came back here to this most beautiful of cities from the long exile in which we Albertis have grown old, I recognized in many, but above all in you, Filippo… a genius for every laudable enterprise in no way inferior to any of the ancients who gained fame in these arts. … I admit that for the ancients, who had many precedents to learn from and to imitate, it was less difficult to master those noble arts which for us today prove arduous; bit it follows that our fame should be all the greater if without preceptors and without any model to imitate we discover arts and sciences hitherto unheard of and unseen. … (Alberti 2004, pp. 34–35).

We see that Alberti's recognition of Brunelleschi's genius is total, and his admiration sincere.

Alberti might very possibly have had the opportunity to visit and analyse some of the fortifications designed by Brunelleschi, such as the well-known Rocca di Vicopisano, whose construction began in 1435. This fortress includes many of the elements described by Alberti in a project that is remarkable not least of all for the perfect regularity of its plan, based on a square module.[24]

The fortress consists in a large compound with a square perimeter, some 15 m to a side. The southwest corner is occupied by a very high (some 31 m) keep, whose area in plan is equal to a quarter of that of the entire the compound (Fig. 2). Access to the fortress is defended by a gate which is also based on a square module. A raised walkway, built on a wall with corbels and crenellations on both sides, connects the fortress to the tower "del Soccorso" on the city walls, originally near the Arno River.

Of particular interest, and quite authentic, are the connections between the various structures, which constitute—via drawbridges and easily demolished connecting structures—an effective compartmentalisation of internal spaces where defenders could withdraw for successive and extreme means of defense: access to the fortress was protected by the compound of the gatehouse; the walkway along the fortress walls could be isolated by demolishing the stair that gave access, built with vaults on slender walls of extremely thin masonry; access to the keep, the final stronghold of defense, equipped with water tank and storage of supplies, was by means of a walkway protected by a drawbridge.

The suspended walkway towards the tower "del Soccorso" along the city-wall, close to the Arno, permitted delivery of supplies and garrisons in almost total security. Particular innovative in the fortress is the system of openings for artillery for grazing fire, in the form of single round holes splayed to the inside, located at regular intervals along a line at the base of the wall.

The governing square module defines the proportions of the entire complex and allowed Brunelleschi to control military efficiency. It is difficult to consider the

[24] As is well known, Brunelleschi also used a square module for the churches of Santo Spirito and San Lorenzo in Florence; see (Cohen 2013).

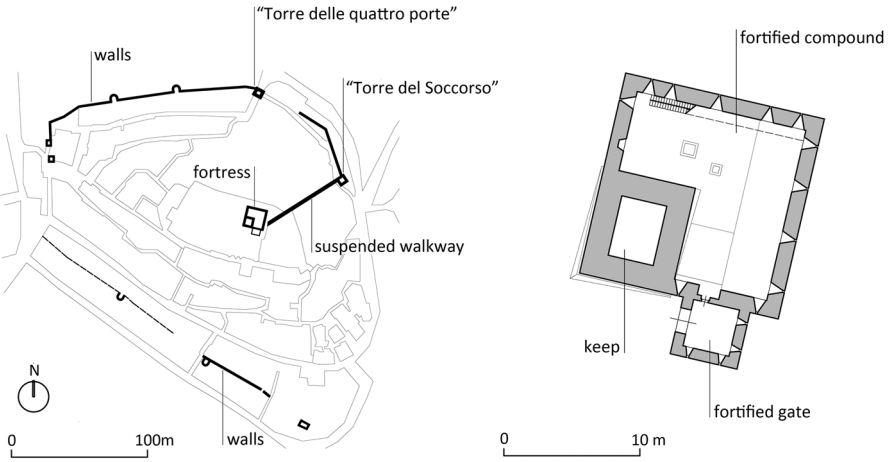

Fig. 2 On the left, plan of the system of fortification of Vicopisano. On the *right*, plan of the fortress by Filippo Brunelleschi. *Drawing* Marco Giorgio Bevilacqua

similarities between these formal expedients and Alberti's descriptions as coincidental.

In addition to the aspects we have already considered, there are other passages of the *De re aedificatoria* that appear to refer directly to the model proposed by Brunelleschi:

> One main tower should be built within the citadel; it should be for the most part solid, of robust construction throughout, fortified on all sides, of greater height than anything else, difficult to reach and entered only by a drawbridge (Alberti 1988, p. 124).[25]

The outfitting of the fortress and the connection with outside towers, elements that are extremely original in the Vicopisano complex, appear to have been acknowledged by Alberti in Book V:

> … within the citadel itself the main tower … should be treated like a small-scale citadel and should be short of absolutely nothing expected of a citadel. It should have its own cistern and storeroom, well stocked with provisions and weapons for self-defence. It should have an outlet through which to launch an attack, even on one's own men, despite their opposition, and through which to receive any help requested (Alberti 1988, p. 125).[26]

[25] *Ne la Fortezza si debbe alzare un Turrione principale, saldissimo per tutto, & gagliardissimo, quanto ad ogni forte di muraglia, & fortissimo per tutto, piu alto che il resto de l'altra muraglia, difficile a lo andarvi, & che non habbia alcuna entrata, salvo che da uno Ponte levatoio* (Alberti 1565, Bk. V, ch. IV, p. 129).

[26] *Et in detta Fortezza quello Torrione che noi chiamiamo principale, sarà quasi come una fortezza minore, nel quale non deve mancar cosa alcuna di quelle che si desiderano in una fortezza. Debbe havere la Citerna, & i ripostigli di tutte le cose, mediante le quali egli si possa abbondantemente nutrire, & difendere. Debbe ancora havere uscite onde è si possa assaltare ancora i suoi medesimi a lor mal grado, & d'onde si possa metter dentro soccorsi* (Alberti 1565, Bk. V, ch. V, p. 130).

Conclusion

Before the development and increasing use of firepower, tactics of fortifications basically centered on considerations of shape, height, thickness and construction of fortified walls in order to make them resistant to ramming and mining. Alberti added to his description a consideration of earthworks such buttresses and ditches that contributed to defense against artillery, as well as innovations such as loopholes for grazing fire. The need to respond to the new threat posed by firearms would ultimately lead, in the centuries that followed, to the creation of an independent science of military architecture.

The force and effectiveness of firepower in warfare was made all too clear with the fall of Constantinople in 1453, just an instant after Alberti had composed his *De re aedificatoria*. It was brought home to Italy in 1494 as a result of the invasion by the army of King Charles VIII of France. As ballistics became increasingly more powerful and efficient, and ever more frequently used, much more technical prowess was required, especially knowledge of geometry and geometrical instruments. Such knowledge was used to aim firearms with as great an accuracy as possible, as in Alberti's bombard problem, but was also involved in solving the problem of defining the shape of a projectile's trajectory, compiling firing tables, in laying out the increasingly complicated star shapes of fortifications, and in developing precision instruments such as Galileo's military compass, of which, as noted, Alberti's *equilibra* was a forerunner. The fundamental importance of geometry as a compositional and governing matrix is often underlined in later specialised treatises, in which it is not unusual to find the first pages devoted to the description of elementary notions of geometry, aimed at facilitating the design of fortifications.

With his *De re aedificatoria*, Alberti was the first to impose a methodological and mathematical approach to the theory of fortification. The significance of his role in the evolution of systems of defense merits a new evaluation.

References

Alberti, Leon Battista. 1565. *L'architettura di Leonbatista Alberti; tradotta in lingua Fiorentina da Cosimo Bartoli, Gentilhuomo, & Academico Fiorentino. Con la aggiunta de disegni*. Venezia: Francesco Franceschi Sanese.

Alberti, Leon Battista. 1988. *On the Art of Building in Ten Books*, Joseph Rykwert, Neil Leach, Robert Tavernor, eds. Cambridge: MIT Press.

Alberti, Leon Battista. 2004. *On painting*. Cecil Grayson, trans. London: Penguin.

Alberti, Leon Battista. 2010. *The Mathematical Works of Leon Battista Alberti*. Kim Williams, Lionel March and Stephen R. Wassell, eds. Basel: Birkhäuser.

Bevilacqua, Marco Giorgio. 2007. The conception of ramparts in the sixteenth century: architecture, "Mathematics" and urban design. *Nexus Network Journal* 9(2): 249–261.

Caciagli, Costantino. 1997. Il castello di Ripoli. Rilevamento, storia e analisi del castello. In *La frontiera, la campagna, il mare*. ed. Massimo Dringoli, 108–125 Pisa: Pacini Editore.

Cohen, Matthew A. 2013. *Beyond beauty: reexamining architectural proportion through the Basilicas of San Lorenzo and Santo Spirito in Florence*. Venezia: Marsilio.

di Pasquale, Salvatore. 2014. Leon Battista Alberti and the art of building. In *Architecture and Mathematics from Antiquity to the Future*, vol. II, ed. Kim Williams, and Michael J. Ostwald. Birkhäuser: Basel.

Kemp Martin. 2004. Introduction. In *On Painting*. Cecil Grayson, trans., Leon Battista Alberti, eds. 1–29 London: Penguin.

Leverotti, F. 1989. Trasformazioni insediative nel Pisano alla fine del Trecento. In: *Archeologia Medievale* XVI: 243–62.

Morolli, Gabriele. 2006. Il ritorno della magnificenza antica. In *L'uomo del Rinascimento. Leon Battista Alberti e le arti a Firenze tra ragione e bellezza*, C. Acidini, G. Morolli, eds. 245–254 Firenze: Mandragora.

Petrarca, Francesco (Petrarch). 1991. *Petrarch's remedy for Fortune Fair and Foul: Book 1*. Conrad H. Rawsky, trans. Bloomington: Indiana University Press.

Rose, Paul Lawrence. 1975. *The Italian Renaissance of Mathematics: Studies of Humanists and Mathematicians from Petrarch to Galileo*. Geneva: Libraire Droz.

Trotta, Giampaolo. 2005. *L'antico Porto Pisano e la Torre del Marzocco a Livorno*. Livorno: Autorità Portuale.

Vitruvius. 2009. *On Architecture*, Richard Schofield, trans. London: Penguin.

Williams, Kim, and Marco Giorgio Bevilacqua. 2013. Leon Battista Alberti's bombard problem in *Ludi matematici*: geometry and Warfare. *The Mathematical Intelligencer* 35(4): 27–38.

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