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Research

Three Musical Interpretations of Le Corbusier's Modulor

Abstract. The frequent evocative comparisons between music and architecture in the text of the Modulor may explain why Le Corbusier, in this constant search for ordering principles, considered his proportional sequence to be equivalent to a musical scale. Yet, since the dimensional ratios of Le Corbusier's 'scale' are much larger than the pitch ratios of the latter, the Modulor comes much closer to another structural element of music, that of harmony. By adjusting the Modulor ratios to the corresponding ratios of musical chords, three variants of a "musical" Modulor can be generated – Chromatic, Major and Minor. If, in addition, the two sets of the male-based Modulor dimensions are replaced with a universal female-male set, the inherently complex Modulor numbers become simpler and clear. The Master's intention of producing a rational and practical system of proportions, which is related to both the scale of the human being, and to the order inherent in the physical nature of sound, can thus be realized.

Introduction

It is commonly known that in his mature works Le Corbusier applied the Modulor, a system of architectural proportions, which he had developed in the 1940s over a period of some seven years [Le Corbusier 1954 (2000): 83]. What is probably not as well known is the role that music may have played in this development.

The Modulor constitutes a mathematical structure of specific interrelated dimensional ratios. Two fundamental ratios, the Golden Section (1.618 : 1) and the double square (2 : 1) are combined to form the Modulor system of proportions. Two terms of the Golden Section ratio, 1.0 and 1.618, are inserted into a double square (the double of 1, i.e., the 2 : 1 ratio), and an additional term, the Golden Section ratio of 2 in reverse, 2 : 1.618, i.e., 1.236 [Le Corbusier 1954 (2000): 83]. From this pair of related Golden Section ratios two Golden Section series originate, each in two directions: …, 0.618, 1, 1.618, 2.618, 4.236, …, and …, 0.764, 1.236, 2, 3.236, …. As a result, in addition to the four ratios of the combined two Golden Section series (i.e., 1.0, 1.236, 1.618, 2.0), two intermediate ratios between the consecutive terms of the combined two series are obtained: new – 4.236 : 3.236 = 1.309, or 2.618 : 2 = 1.309, etc., and the already familiar – 3.236 : 2.618 = 1.236, or 2 : 1.618 = 1.236, etc.

In the elaborate text that elucidates this system of proportions, *The Modulor* [1954 (2000)], Le Corbusier makes frequent references to music. At the beginning of chapter 5 he establishes a direct link between music, mathematics and his architecture by stating that "…the sap of mathematics has flown through the veins of my work,…for music is always present within me" [1954 (2000): 129]. Elsewhere he relates his work directly to music. Musical terms, such as "…counterpoint … cadences … rhythms … intervals …" are used in the description of a factory project based on Modulor dimensions [1954 (2000): 160-163], while the terms "…prelude, chorale and fugue, melody…" attempt to evoke the intended nature of a town planning scheme [1954 (2000): 168]. The word

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harmony pervades the pages of *The Modulor*, especially as it pertains to music, e.g., "…music, one of the subtlest phenomena of all, bringer of joy (harmony)…" [1954 (2000): 31] and "…music rules all things … or, more precisely, harmony does that" [1954 (2000): 74]. The notion of measure and perfection inherent in music is also ever-present, as, for example, in the following statements: "Music and architecture alike are a matter of measure." [1954 (2000): 29] and "…the music of attainable perfections…" [1954 (2000): 80] or "…subtleties almost musical in nature" [1954 (2000): 162].

It becomes apparent that in his constant search for ordering architectural principles, e.g., his early "regulating lines" [1954 (2000): 27], this arguably most inventive and imaginative architect of the past hundred years looked to music for potential structural parallels to be applied in the design of the built environment. He especially focused his attention on the musical scale, which he seems to have considered as a prime example of a viable system of measure to be emulated: "…how many of us know that in the visual sphere – in the matter of *lengths* – our civilizations have not yet come to the stage they have reached in music?" [Le Corbusier 1954 (2000): 16]. Thus, in his work on the Modulor he attempted to develop an architectural equivalent to the musical scale, and in the introduction to the second edition of *The Modulor* declared that "The Modulor is a scale. Musicians have a scale; they make music, which may be trite or beautiful" [1954 (2000): 5]. This statement is significant. It contains a clear disclaimer: the Modulor is only a structuring guide and its application does not automatically guarantee quality.

The scale

But is the Modulor really a visual equivalent of a musical scale? In spite of a superficial similarity due to the fact that, like a musical scale, it consists of distinct consecutive segments (or steps), the Modulor does not represent a direct analogy to that basic constituent element of music. In music, the distances between any two pitches are ratios of the frequencies of sound of the respective notes and are known in musical terminology as "intervals." The intervals (or steps) between consecutive notes of a musical scale in the Tonal System (the system which evolved from ancient Greek "modes" and has been employed in Western music for approximately the past four hundred years) are very small. They follow distinct patterns, which determine the type of scale: Diatonic Major, Diatonic Minor (Harmonic, Natural, Melodic Ascending and Descending), and Chromatic.

There are eight notes in a Diatonic scale. The pitch (frequency) of its Keynote (note of the lowest frequency in a scale, e.g., C, F, etc.), to which all other notes in that scale are related, determines the name of its Tonality, e.g., C, F, G, etc. – either Major or Minor. The seven intervals between this lowest note and other notes of the scale are designated in an ascending sequence as the Second, Third, Fourth, Fifth, Sixth, Seventh and Octave (double frequency of the lowest note), respectively. Each of these intervals has one of the following additional designations depending on the type of scale (Major or one of the three Minor scales): Perfect, Major, Minor, Augmented or Diminished. In addition to the seven intervals between the lowest note (Keynote) and the Octave of a Diatonic scale five additional intermediate intervals can be obtained between any two specific notes of the scale, bringing the total to twelve. The Chromatic scale has the same twelve intervals, but in a different sequence – all between the lowest note and its Octave.

The phenomenon of "pitch class" makes the notes sound similar at the double or quadruple, etc. (or half or quarter, etc.) of their original frequency, i.e., at a repeated ratio of 2 : 1 – the interval of an Octave. Therefore, the basic structure of a scale, which is always determined within that interval, is maintained. Its constituent notes may then be repeated at double, quadruple, half, etc., of their original frequencies. The letter designations of notes of particular frequencies, e.g., C, D, E, F, G, etc., within each octave range of the scale remain consequently the same.

The intervals most familiar to our ears are "tempered" intervals,[1] whose frequency ratios are produced mechanically, as for example by the exact tuning of a piano or an organ. In the tempered Chromatic scale each consecutive interval is the same – a tempered Minor Second, with a frequency ratio of 1.059.[2] In tempered Diatonic scales, whose intervals are based on multiples of the tempered Minor Second, the consecutive intervals are either a Minor Second (ratio of 1.059) or a Major Second (double of the ratio of 1.059, i.e., 1.121). The exact sequence varies, depending on the type of scale. The Augmented Second (triple of the ratio of 1.059, i.e., 1.188) occurs only once in the Harmonic Minor scale. In "just" Diatonic scales the "perfect" and "just" intervals are based on whole number ratios of the respective pitches, e.g., the ratio of a just Minor Second is 16 : 15 (1.067), and that of a just Major Second 9 : 8 (1.125). Thus, they deviate slightly (and unequally) from their respective tempered intervals.[3]

If ratios of architectural dimensions are compared to intervals between (i.e., ratios of) musical pitches, then the difference between Le Corbusier's "scale" and the musical scale becomes easily apparent. The Modulor has only four ascending ratios between 1 and its double (2 : 1 – Octave), including its own ratio (1:1): 1.0, 1.236, 1.618 and 2.0 [Le Corbusier 1954 (2000): 50], as compared to eight (including the ratio of 1 : 1 – the Unison) in a Diatonic musical scale, and thirteen (including the ratio of 1 : 1) in a Chromatic scale. Also – and this is significant – the smallest dimensional ratio in the Modulor is 1.236 – considerably larger than the typical 1.067 or 1.125 between adjacent notes in a just musical scale. Therefore, the Modulor cannot be considered as a direct equivalent of a musical scale. But its potential 'musical' aspect lies in a related, but much more important element of music, frequently invoked by Le Corbusier.

Harmony

Single notes sounding in sequence at various intervals, and over various lengths of time are referred to as "melody." The simultaneous sounding of two or more notes of different pitches is referred to as "harmony." Certain ratios between such simultaneously sounding notes constitute intervals which are perceived as pleasant or "consonant," as, for example, the interval of the Perfect Fifth (3 : 2 or 1.5) or of the Major Sixth (5 : 3 or 1.666). There are other intervals which are perceived as "dissonant," e.g., the interval of the Minor Seventh (9 : 5 or 1.8) or of the Major Second (9 : 8 or 1.125). Two or more intervals can be combined into "chords" – three or more notes sounding simultaneously. In the Tonal system, the many possible harmonic combinations constitute a highly complex structure, in which chords based on three notes (Triads) or more, with consonant intervals of Thirds, follow each other according to distinct rules, which have been evolving over centuries. A comprehensive technical grasp of this structure allows a composer to create highly imaginative and meaningful, yet fully disciplined, works of music. Compositions of Mozart, Chopin, Brahms, as also most of the music of other composers of the Baroque, Classical and Romantic periods, and the "popular" music of today, are based on this structure, while some twentieth-century systems of composition are based on structures with different rules for interval combinations, e.g., Arnold Schoenberg's Twelve Tone system.

While the relatively few and large ratios of the Modulor cannot be considered to be similar to those in the consecutive sequence of a musical scale, they are close to certain harmonic musical intervals inherent in that scale.[4] In devising the Modulor, Le

Corbusier, probably unwittingly, came closer to this much more important aspect of music than the intended equivalent of a scale, which, after all, only furnishes a certain number of the possible frequencies for acceptable harmonic ratios. A few of these ratios which, following the Pythagorean-Platonic tradition, were articulated and applied by the great Renaissance thinkers-practitioners, notably Alberti and Palladio, are of comparable size as those of the Modulor, but not precisely the same. The number of ratios in these systems was, however, larger than Le Corbusier's four.

Of Alberti's nine recommended "musical" ratios for "areas" (floor plans) of rooms (Alberti, *De re aedificatoria,* Bk. IX, ch. iii [1988: 305-306]), five are within the range of an Octave (the ratio of 2 : 1), including the 1 : 1 ratio: 1.0 (1 : 1), 1.333 (4 : 3), 1.5 (3 : 2), 1.777 (16 : 9) and 2.0 (2 : 1). The other four are within the ratio of 4 : 1, the range of the second of two Octaves: 2.25 (9 : 4), 2.666 (8 : 3), 3.0 (3 : 1) and 4.0 (4 : 1). All of Palladio's seven recommended ratios are within the range of an Octave, including its own ratio (2 : 1), as well as the ratio of 1:1 and the circle (which can also be considered as a unity): the circle, 1.0, 1.333, 1.414, 1.5, 1.666, 2.0 (Palladio, *I Quattro libri dell'architettura*, Bk. I, ch. xxii [1997: 57]).

This music-architecture relationship has been succinctly explored by Rudolf Wittkower in his *Architectural Principles in the Age of Humanism* [1971: 101-154], and more recently by Deborah Howard and Malcolm Longair in their examination of all plan drawings in Book II of Palladio's *Quattro Libri* [1982]. Since Wittkower's ground-breaking study was first published in 1948, the same year that the manuscript of *The Modulor* was completed, it is not likely that Le Corbusier was aware of it. Otherwise he might have modified his system, in order to bring it much closer to his "musical" ideal.

At about the same time, following Wittkower's findings, his student, Colin Rowe, made an attempt to link Le Corbusier's work to that of Palladio. In his much quoted essay, *The Mathematics of the Ideal Villa* [1976: 1-27], he compares Le Corbusier's early iconic house, the Villa Stein at Garches, with Palladio's Villa Foscari at Malcontenta. The latter is one of the five Palladian villas, whose plan dimensions in completed buildings Howard and Longair found to be based to a higher degree than others on musical ratios, and concluded that "It can hardly be fortuitous that these are probably the most famous and best loved of all Palladio's villas and palaces" [Howard and Longair 1982: 126-127]. Among the aspects that Rowe considered are the key dimensions between the main structural elements in the floor plans of the Corbusier villa, which show that some of the resulting ratios (e.g., 2 : 1, 3 : 2, 4 : 3) correspond to "perfect" consonant musical intervals, which were also used by Palladio as dimensional ratios in Villa Foscari [Rowe 1976: 5]. However, in spite of starting his article with a general reference to the Renaissance emphasis on the music-architecture relationships, it appears that in his comparison Rowe was more interested in general patterns of floor plans and elevations, and other non-musical and non-mathematical aspects, rather than in the musical significance of dimensional ratios. Rowe also ignores the importance of vertical dimensions in reference to the proportions of the Palladian villa. Of course, Wittkower only glosses over this aspect [1971: 109-110], in spite of the fact that it was explained at length and clearly by both Alberti and Palladio [Alberti 1988: IX, iii, p. 306-309; Palladio 1997: I, xxii, p. 58-59], and misses perhaps the most important of its musical implications, that of harmonic chord formation.[5]

It is, therefore, not surprising that Rowe does not speculate as to whether or not Le Corbusier used the plan ratios deliberately in reference to music. It is quite likely that Le Corbusier did not, since he would not have made a blatantly non-musical (and non-mathematical) assertion with reference to an important dimensional ratio in the elevations of Villa Stein. Rowe draws our attention to this assertion, but points to the mathematical inconsistency only. Le Corbusier equates the ratios of 5 : 3 (1.666) and of 8 : 5 (1.6) with each other and with the Golden Section ratio (1.618) by writing A : B = B : (A+B), the mathematical formula for the latter, on the elevation drawing, where A = 3, and B = 5 [Rowe 1976: 9]. Yet, 1.666 does not equal 1.6, and neither the one nor the other equals 1.618. Also, the ratios 1.666 and 1.6 are two distinct musical intervals: 1.666 is the ratio of 5 : 3, the just consonant musical interval of the Major Sixth (and one of the six dimensional ratios recommended by Palladio), and 1.6 is the ratio of 8 : 5, the just consonant interval of the Minor Sixth. This error may be attributed to Le Corbusier's unfamiliarity with the historical theoretical significance of the 5 : 3 ratio, or it may have been due to his limited knowledge of musical basics, which he readily admitted [Le Corbusier 1954 (2000): 29]. The Golden Section ratio is not a recognized musical interval, either consonant or dissonant, as it lies unevenly between the intervals of the Major Sixth (1.666 in the just, and 1.682 in the tempered tuning) and of the Minor Sixth (1.6 in the just, and 1.587 in the tempered tuning). The maximum deviation is close to 4% and even the minimum deviation is higher than 1%, which in musical terms is significant.

The Golden Section

The reason for this error probably lies elsewhere. Like countless others, Le Corbusier could not resist the mathematical wonder of the Golden Section. Indeed, it is impossible for any rational thinker not to be seduced by this unique progression, which yields highly coherent numerical and geometrical relationships. Thus, perversely, in his quest for the ultimate rational ordering system, Le Corbusier makes such cavalier non-rational approximation. He simply may have thought that that the 'magic' of the Golden Section had greater validity than the order inherent in music. In spite of his claim that "music is always present within me," the Golden Section persisted in his thinking. Thus it eventually became a basic structural component of the Modulor.

But even if this ratio yields in progression the extraordinary relationships of rational growth, its potential for combination with other ratios of similar magnitude is extremely limited. 1.0 and 1.618 are the only ratios in this progression which lie within the range of an Octave (i.e., between the ratios of 1:1 and 2:1), the next ratio in the series being 2.618. Compared to the thirteen ratios of a musical scale within the same range, the five ratios recommended by Alberti, or the six (excluding the circle) recommended by Palladio, these two ratios do not offer much variety. Constantly repeated, only one or two ratios, no matter how exquisite, cannot help but become tedious, even if they are repeated at various scales (the persistent squares in the work of some prominent twentieth-century architects come to mind).

Le Corbusier must have become aware of this limitation. And so, as we have observed above, in a brilliant move he inserted into a double square (the double of 1.0, i.e., 2.0 – Octave – which he introduced early in this work) [1954 (2000): 37], the two terms of the Golden Section ratio, 1.0 and 1.618, and the additional term of the Golden Section ratio of 2 in reverse, 2 : 1.618, i.e., 1.236. The combination of this new term with 1.618 yields an additional ratio, 1.618 : 1.236, i.e., 1.309. It should be noted that except for the two "perfect" consonant musical ratios, 1.0 and 2.0, the other Modulor ratios above are all discordant non-musical ratios.

The mathematical deviations of the two intermediate ratios from the neighbouring musical ratios, the Major Third and Perfect Fourth, are as follows: 1.25 (just Major

Third) – 1.236 = 0.014 (1.4%), and 1.333 (just Perfect Fourth) – 1.309 = 0.024 (2.4%). Both are above the assumed 1% tolerance. Deviations from tempered intervals are even higher: 1.26 – 1.236 = 0.024 (2.4%) and 1.335 – 1.309 = 0.026 (2.6%), respectively.

The human figure

The actual numbers which constitute the two overlapping series of the Modulor are derived from what Le Corbusier considered to be the typical dimensions of a standing male figure. For the first series, which he called "red," they are: 108 cm. (1 x 108 = 108) – the position of the solar plexus above the ground, and 175 cm. (1.618 x 108 = 174.744) – the top of the head. For the second series, which he called "blue," they are: 216 cm. (2 x 108 = 216) – arm raised above the head, and 82.5 cm. (216 – 216 : 1.618 [133.498] = 82.5) – the palm of the lowered arm [1954 (2000): 50-52].

The exact size of these dimensions appears to have been less of a concern to the master than the mathematical schema of the Modulor series. And so, with an eye to reconciling the metric and the imperial systems of measurement, and the concomitant opportunities offered by the emerging international trends of prefabrication in the construction industry, and especially mass housing in the United States [1954 (2000): 52-54], he altered the fundamental human dimensions from which the initial two Modulor series originate. Using as an excuse the argument that the original ones which coincided with the height of the average Frenchman were too specifically Continental, he chose the height of a six-foot-tall British policeman [sic] as being typical of the presumably more universal Anglo-Saxon male [1954 (2000): 56]. The critical height of the solar plexus was consequently changed from 108 cm. to 113 cm., and other dimensions followed according to the ratios inherent in the two Modulor series. Ultimately the primary reason for choosing these new dimensions was the fact that they can be relatively easily converted to the imperial system of measurement [1954 (2000): 57].

Neither the earlier nor the later set of measurements considered the dimensions of the female figure. Both sets also remain complicated in terms of their constituent numbers due to the complex decimal positions. This, of course, makes practical dimensioning cumbersome and led Le Corbusier to make numerous adjustments in the first version, which were inconsistent with the mathematical rigour of the Golden Section [Le Corbusier 1954 (2000): 51] (Table 1). This had been corrected in the second version, with only minimal adjustments, but the numbers remain complex and difficult to read [Le Corbusier 1954 (2000): 8] (Table 2), even if less so in the imperial version. This may be the main reason for the apparent reluctance on the part of others to employ the Modulor in practice and why it failed to be accepted as an international standard of measurement. As has been seen above, it also fails as the intended equivalent of a musical scale, due to the fact that its 'intervals' (i.e., ratios between its consecutive numbers) are significantly larger than those of the former.

However, these failures should not automatically lead to the dismissal of its other initially intended role of a proportioning grid. The idea of such a grid goes back to the very beginnings of Le Corbusier's work on the Modulor [Le Corbusier 1954 (2000): 37], and evolved into the system of proportions inherent in it. The acknowledged formal sophistication of Le Corbusier's late work, which was proportioned according to Modulor numbers [Gast 2000: 96], stands as a lasting testimony to the Modulor's validity and warrants its serious consideration, further study, and renewed application.

RED series				BLUE series
Ratio	Dimension In centimeters	Ratio between series	Dimension in centimeters	Ratio
			216	
		1.234		
	175			1.624
		1.316		
1.62			**133**	
		1.231		
	108			1.622
		1.317		
1.636			**82**	
		1.242		
	66			1.608
		1.294		
1.61			**51**	
		1.244		
	41			1.645
		1.322		
1.64			**31**	
		1.24		
	25			1.55
		1.25		
1.56			**20**	
		1.25		
	16			1.818
		1.455		
1.78			**11**	
		1.222		
	9			

Table 1. Original Modulor (108 – 216 cm.)

RED series				**BLUE series**
Ratio	Dimension in centimeters	Ratio between series	Dimension in centimeters	Ratio
			10619.6	
		1.236		
	8591.4			1.618
		1.309		
1.618			**6563.3**	
		1.236		
	5309.8			1.618
		1.309		
1.618			**4056.3**	
		1.236		
	3281.6			1.618
		1.309		
1.618			**2506.9**	
		1.236		
	2028.2			1.618
		1.309		
1.618			**1549.4**	
		1.236		
	1253.5			1.618
		1.309		
1.618			**957.6**	
		1.236		
	774.7			1.618
		1.309		
1.618			**591.8**	
		1.236		
	478.8			1.618
		1.309		
1.618			**365.8**	
		1.236		
	295.9			1.619
		1.309		
1.618			**226**	
		1.236		
	182.9			1.618
		1.309		
1.619			**139.7**	
		1.236		
	113			1.619
		1.309		
1.619			**86.3**	
		1.236		
	69.8			1.616
		1.307		
1.616			**53.4**	
		1.236		
	43.2			1.618
		1.309		
1.618			**33**	
		1.236		
	26.7			1.618
		1.309		
1.618			**20.4**	
		1.236		
	16.5			1.619
		1.310		
1.618			**12.6**	
		1.235		
	10.2			1.615
		1.308		
1.619			**7.8**	
		1.238		
	6.3			1.625
		1.313		
			4.8	

Table 2. Second Modulor (113 – 226 cm.)

Music and the Modulor

The complexity of the Modulor numbers remains the most obvious obstacle to the acceptance of this proportional system in practice. On the ideological level, their proximity to, yet critical deviations from, the whole number ratios of musical intervals, detract from the Modulor's potential 'musical' integrity and thus Le Corbusier's musical intentions. To bring the Modulor closer to these intentions, and to make its practical application simpler, three versions of a revised, 'musical' Modulor are here proposed.

In these revisions two essential premises will be observed: 1) The human figure will remain as a determining dimensional reference, and 2) The general structural principle of the dimensional relationships will be maintained. Since the numbers derived from tempered musical ratios are for the most part as complex in their many decimal positions as the Modulor numbers, whole number (i.e., just) ratios of consonant musical intervals will be used to obtain simpler dimensions. This would provide for a more easily adaptable architectural dimensioning tool and proportioning guide in practice, and would also establish a link to the classic Renaissance tradition.

Regarding the relation of the revised Modulor to the size of the human body, the fact that Le Corbusier used two sets of rather disparate dimensions suggests that neither one nor the other is binding, and would justify another alternative, especially if it were more inclusive. Consequently, the crucial controlling position of the solar plexus above the ground should be set at a dimension which would be close to a representative average. This dimension should also yield simple, workable numbers for easy application in practice.

In a small informal survey of young people of diverse ethnic backgrounds (10 men and 14 women) the dimensions for the position of the solar plexus were found to range from 76 cm. to 115 cm. and for the top of the head from 150 cm. to 185 cm. above the ground.[6] These results show a variation of 41% and of 21%, respectively, and thus allow for a high degree of leeway. They also suggest that culture- and region-specific Modulor dimensions may not be out of place, and prove that Le Corbusier was not that arbitrary in his choice of two different dimensional standards.

If a common reference standard is to be selected, however, then it must be based on reasonable averages. In this survey the averages, adjusted with respect to five tallest men and five tallest women, were as follows: the height of the solar plexus – 108.9 cm., and the top of the head – 177.6 cm.; thus the ratio of the top of the head to the solar plexus – 177.6 : 108.9, i.e., 1.631. Based on this small but representative sample, and taking the smaller dimensions of the remaining members of the group into account, the height of the solar plexus above the ground will be set at 108 cm. in the following revised versions of the Modulor, and will thus coincide with Le Corbusier's original dimension.

Musical Chromatic Modulor

In reconstructing the Modulor in musical terms, in the first instance, Le Corbusier's approximation: 5 : 3 = 8 : 5 = 1.618, may be viewed as a potential reverse cummulative approximation: 1.618 is close to 1.666 (5 : 3) and to 1.6 (8 : 5). It is also possible to obtain these two musical ratios from the Fibonacci series, which Le Corbusier often invokes and uses for some dimensional adjustments in the first version of the Modulor. The ratios between the lower terms of this series are all consonant (perfect and just) musical intervals, i.e., 1 : 1, 2 : 1, 3 : 2, 5 : 3, and 8 : 5. They are also among the ratios recommended by Palladio, except for 8 : 5, which he nevertheless applied in practice [Mitrović 1990: 283].

Using the last two of the above Fibonacci ratios, a progression consisting of intervals of one Major Sixth (1.666 or 5 : 3) and two Minor Sixths (1.6 or 8 : 5) can be produced. The resulting overall ratio of the three combined ratios is very close to the overall ratio of the progression of three consecutive Golden Section ratios – 4.265 (1.666 x 1.6 x 1.6) and 4.236 (1.618 x 1.618 x 1.618), respectively. Thus, these two large ratios, each comprising two Octaves and one Minor Second, are practically the same, with a deviation of 0.0068 between the original Modulor and the 'musical' Modulor, i.e., substantially less than the assumed tolerance of 1%. If equivalent tempered intervals were to be used, the two Modulors would coincide exactly at 4.236. The resulting additional 'dissonant' interval of a Minor Second at every second Octave would permit the generation of notes equivalent to a complete Chromatic scale if the series were to be extended and the pitch class phenomenon were to be taken into account. Consequently, this 'musical' version of the system will be referred to here as the Chromatic Modulor.[7] The internal ratios between the terms of the new chromatic Red and Blue series are also equivalent to consonant just intervals, although not part of the Fibonacci series, i.e., a Minor Third (1.2 or 6 : 5), a Major Third (1.25 or 5 : 4), and a Perfect Fourth – (1.333 or 4 : 3).

Following the above ratios and using the dimension of 108 cm. for the height of the solar plexus of a 'universal' male-female standing figure, the top of the head will consequently be at 180 cm. (108 x 1.666) and the raised hand at 216 cm. (108 x 2) above the ground. These dimensions are rather close, although not equal, to the averages resulting from the above survey, including the top of head to solar plexus ratio of 1.653.

When these numbers are applied to the Red and Blue series of the new Chromatic Modulor, several decimal-free dimensions are obtained in centimeters in the critical range pertaining to habitable space: from 81 cm. to 288 cm. Other dimensions are still complex. However, with respect to their 'music' the resulting interrelationships provide seven distinct potential consonant ratios in every second Octave: 1:1 (1.0), 4 : 3 (1.333), 5 : 4 (1.25), 6 : 5 (1.2), 5 : 3 (1.666), 8 : 5 (1.6) and 2 : 1 (2.0),[8] and four of these ratios in the alternating extended interval of an Octave + a Minor Second: 1 : 1 (1.0), 4 : 3 (1.333), 5 : 4 (1.25), 6 : 5 (1.2) and 8 : 5 (1.6) (Table 3). Although not all of the above consonant ratios are the same as those recommended by Alberti and Palladio, their number in the first Octave surpasses by two the number of ratios recommended by Alberti (five within the overall ratio of 2 : 1) and by one those recommended by Palladio (six, not counting the circle). This number also surpasses by two the number of ratios in LeCorbusier's Modulor (five within the ratio of 2 : 1), but fall substantially short of the thirteen ratios inherent in a musical scale.[9]

Musical Major and Minor Modulors

Two other versions of a musical Modulor can be obtained by means of a sequence of the already familiar consonant whole number ratios of a Major Sixth (1.666), a Minor Sixth (1.6), and an additional ratio obtained from the Fibonacci series – (3 : 2), a Perfect Fifth (1.5). Depending on the ascending order of the ratios in the two series (Red and Blue) either a Major or a Minor Modulor can be obtained: 1.666 – 1.6 – 1.5, and 1.666 – 1.5 – 1.6, respectively. The sequence of these three intervals produces an overall interval of two Octaves (ratio of 4 : 1). The deviation from Le Corbusier's Modulor at this point is that of a Minor Second (1.067), which, however, allows a continuos progression of Octaves alternating between the two series. The order of internal intervals between the terms of the two sequences (Major or Minor) follows respectively.

RED series						**BLUE series**
Assumed relative pitch	Ratio	Dimension in centimeters	Ratio between series	Dimension in centimeters	Ratio	Assumed relative pitch
				10485.8		**E**
			1.25			
C		**8388.6**			1.666	
			1.333			
	1.6			**6291.5**		**G**
			1.2			
E		**5242.9**			1.6	
			1.333			
	1.6			**3932.2**		**B**
			1.2			
A b		**3276.8**			1.6	
			1.333			
	1.666			**2457.6**		**E** b
			1.25			
B		**1966.1**			1.666	
			1.333			
	1.6			**1474.6**		**G** b
			1.2			
E b		**1228.8**			1.6	
			1.333			
	1.6			**921.6**		**B** b
			1.2			
G		**768**			1.6	
			1.333			
	1.666			**576**		**D**
			1.25			
B b		**460.8**			1.666	
			1.333			
	1.6			**345.6**		**F**
			1.2			
D		**288**			1.6	
			1.333			
	1.6			**216**		**A**
			1.2			
F #		**180**			1.6	
			1.333			
	1.666			**135**		**C** #
			1.25			
A		**108**			1.666	
			1.333			
	1.6			**81**		**E**
			1.2			
C #		**67.5**			1.6	
			1.333			
	1.6			**50.6**		**G** #
			1. 2			
F		**42.2**			1.6	
			1.333			
	1.666			**31.6**		**C**
			1.25			
G #		**25.3**			1.666	
			1.333			
	1.6			**19**		**D** #
			1.2			
C		**15.8**			1.6	
			1.333			
	1.6			**11.9**		**G**
			1.2			
E		**9.9**			1.6	
			1.333			
				7.4		**B**

Table 3. Musical Modulor – Chromatic

RED series						BLUE series
Assumed relative pitch	Ratio	Dimension in centimeters	Ratio between series	Dimension in centimeters	Ratio	Assumed relative pitch
				9216		D
			1.333			
A		6912			1.6	
			1.2			
	1.5			5760		F#
			1.25			
D		4608			1.666	
			1.333			
	1.6			3456		A
			1.2			
F #		2880			1.5	
			1.25			
	1.666			2304		D
			1.333			
A		1728			1.6	
			1.2			
	1.5			1440		F #
			1.25			
D		1152			1.666	
			1.333			
	1.6			864		A
			1.2			
F #		720			1.5	
			1.25			
	1.666			576		D
			1.333			
A		432			1.6	
			1.2			
	1.5			360		F #
			1.25			
D		288			1.666	
			1.333			
	1.6			216		A
			1.2			
F #		180			1.5	
			1.25			
	1.666			144		D
			1.333			
A		108			1.6	
			1.2			
	1.5			90		F #
			1.25			
D		72			1.666	
			1.333			
	1.6			54		A
			1. 2			
F #		45			1.5	
			1.25			
	1.666			36		D
			1.333			
A		27			1.6	
			1.2			
	1.5			22.5		F #
			1.25			
D		18			1.666	
			1.333			
	1.6			13.5		A
			1.2			
F #		11.25			1.5	
			1.25			
	1.666			9		D
			1.333			
A		6.75			1.6	
			1.2			
				5.625		F #

Table 4. Musical Modulor – Major

RED series						**BLUE series**
Assumed relative pitch	Ratio	Dimension in centimeters	Ratio between series	Dimension in centimeters	Ratio	Assumed relative pitch
				8640		**C #**
			1.25			
A		**6912**			1.5	
			1.2			
	1.6			**5760**		**F #**
			1.333			
C #		**4320**			1.666	
			1.25			
	1.5			**3456**		**A**
			1.2			
F #		**2880**			1.6	
			1.333			
	1.666			**2160**		**C #**
			1.25			
A		**1728**			1.5	
			1.2			
	1.6			**1440**		**F #**
			1.333			
C #		**1080**			1.666	
			1.25			
	1.5			**864**		**A**
			1.2			
F #		**720**			1.6	
			1.333			
	1.666			**540**		**C #**
			1.25			
A		**432**			1.5	
			1.2			
	1.6			**360**		**F #**
			1.333			
C #		**270**			1.666	
			1.25			
	1.5			**216**		**A**
			1.2			
F #		**180**			1.6	
			1.333			
	1.666			**135**		**C #**
			1.25			
A		**108**			1.5	
			1.2			
	1.6			**90**		**F #**
			1.333			
C #		**67.5**			1.666	
			1.25			
	1.5			**54**		**A**
			1. 2			
F #		**45**			1.6	
			1.333			
	1.666			**33.75**		**C #**
			1.25			
A		**27**			1.5	
			1.2			
	1.6			**22.5**		**F #**
			1.333			
C #		**16.875**			1.666	
			1.25			
	1.5			**13.5**		**A**
			1.2			
F #		**11.25**			1.6	
			1.333			
	1.666			**8.4375**		**C #**
			1.25			
A		**6.75**			1.5	
			1.2			
				5.625		**F #**

Table. 5. Musical Modulor – Minor

The ratios of these internal intervals are also consonant, i.e., a Minor Third – 6:5 (1.2), a Major Third – 5:4 (1.25), and a Perfect Fourth – 4:3 (1.333). The number of ratios in each of the two Octaves is the same as in the first Octave of the Chromatic Modulor, i.e., seven.

When the 'originating' human module of 108 cm. is applied to the Red and Blue series of these two Modulors, almost exclusively decimal-free dimensions are obtained (in centimeters) in the Major Modulor: from 27 cm. to 6912 cm. and higher, i.e., in musical terms, within the range of at least eight Octaves (Table 4). A comparable range of decimal-free dimensions in the Minor Modulor starts at 90 cm. at the lower end, but if one number with a 0.5 decimal position (67.5 cm.) were to be accepted it could begin at 45 cm. (Table 5). The interrelationships of these dimensions provide eight potential consonant ratios (1:1, 6:5, 5:4, 4:3, 3:2, 5:3, 8:5 and 2:1) spread over two adjoining alternating Octaves, with seven such ratios in each Octave, in both the Major and Minor versions of the Modulor.

Consonance and Modularity

Had Le Corbusier been aware of the musical validity of the above proportional structures when designing Villa Stein, he would have readily acknowleged the presence of both the 5:3 and the 8:5 ratio in its façade, instead of erroneously trying to interpret them as a single Golden Section ratio. Since other ratios also found in the villa form part of both of the above musical Modulors, the "music … always present within" him must have subconsciously guided him in the creation of this early masterpiece.

In contrast to Le Corbusier's Modulor, where only two ratios within the octave range are consonant (1:1 and 2:1), while others are non-musical and therefore discordant, all ratios in the above three versions of the Musical Modulor are consonant and offer the opportunity for a wide variety of inventive 'harmonic' and harmonious proportional combinations.

The cumulative dimensions of Le Corbusier's Modulor, as well as of the Chromatic Modulor, preclude a progressive repetition (doubling) of their respective constituent dimensions. The Musical Major and Minor Modulors, however, allow such repetition at each Octave. Thus, modular coherence can be achieved in even very large projects, e.g., with respect to structural grids, spatial units, enclosure elements, etc. By using only dimensions -- small as well as large – which belong exclusively to one of the above musical Modulors, the integrity of the respective governing proportioning system can be maintained.

The above Musical Modulors may be seen, therefore, as an extension of Le Corbusier's early work, as well as a fulfillment of his intention of devising a universal, rational and practical system of architectural proportions, related to both the scale of the human being and to the mathematical order inherent in the physical nature of sound. In these comprehensive mathematical structures architecture and music merge.

Notes

1. Tempered intervals are obtained by dividing the interval of an Octave into twelve equal intervals, i.e., the intervals of a tempered Minor Second. Other tempered intervals are exact multiples of this interval.
2. The ratio is actually 1.059 : 1, but for the sake of simplicity, in ratios where the denominator is 1, this denominator will not be shown in this text.

3. The whole number dimensional ratios which form part of the Renaissance system of proportions, derived from the Pythagorean-Platonic tradition, have their equivalents in the "perfect" and "just" intervals – frequency ratios – in music. These intervals are produced naturally by singers or string instrumentalists, as opposed to tempered intervals produced on precisely pre-tuned instruments. While mathematically fully consistent, the tempered intervals deviate minimally from the perfect and just intervals within a range between 0.001335 (for the Perfect Fifth) and 0.010101 (for the Minor Seventh), or 0.009604 (for the Major Sixth). Since a singer or a violinist can adjust his or her natural inclination to produce sounds according to just intonation and follow the tempered system when accompanied on the piano, the upper range of this difference (1%) will be considered here as an acceptable deviation in an architectural system of proportions as well. As the perfect and just ratios not only relate to the Renaissance tradition of musical proportions, but also allow for simply expressed dimensions, they will be used in this text, unless otherwise noted. For detailed explanation of scales and other musical elements, see [Backus 1969].
4. Klaus-Peter Gast, in his recent extensive study on Le Corbusier, appears not to be aware of this fact when cursorily commenting on Le Corbusier's "scale"; see [Gast 2000: 94].
5. This aspect has been dealt with in some detail in [Zuk 2004: 184-185].
6. This voluntary self-administered survey was carried out by approximately 60% of students enrolled in the course "Geometry and Architecture" in the School of Architecture at McGill University, in the winter semester 2009.
7. While musically this extension of the series would produce frequencies which are beyond the audible range, the resulting large dimensions might be applicable in very large architectural projects. As the great distances and the related visual distortion make precise perception of dimensions difficult, the deviation between the "tempered" and the "just" series, which increases as the series expands, might become acceptable, even if it would exceed the acceptable limit of 1% established here.
8. The $5:4$ (1.25) and $6:5$ (1.2) ratios were also used by Palladio; see [Mitrović 1990: 282-284].
9. The above musical interpretation of the Modulor has been previously explored in [Zuk 2010]. The two interpretations that follow here are new.

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Radoslav Zuk attended high school and studied music in Graz, and earned his Bachelor of Architecture degree with honors from McGill University in Montreal. He won several prizes, including the Pilkington Traveling Scholarship, the highest award for a graduation design project in Canada. Later he earned a Master's in Architecture from MIT in Boston. More recently he was awarded an honorary doctorate degree by the Ukrainian Academy of Art in Kyiv. He has taught architecture at the University of Manitoba, the University of Toronto, and at McGill University, where he is an Emeritus Professor and a recipient of the Ida and Samuel Fromson Award for Outstanding Teaching in the Faculty of Engineering. A professor and an honorary professor, respectively, at two universities in Europe, he has also appeared as a guest lecturer and guest review critic at various universities in Canada, the United States, and a number of European countries. Winner and co-winner of several competition prizes, Radoslav Zuk has designed, among other projects, nine Ukrainian churches in North America and one in Ukraine, in association with or as consultant to a number of architectural firms. Most of these buildings have been recognized in the international architectural press and exhibited widely in North America and Europe. He has published on design theory, cultural aspects of architecture, and the relationships between architecture and music. A Fellow of the Royal Architectural Institute of Canada and of several scientific societies, he also has been honored with the Royal Architectural Institute of Canada Governor General's Medal for Architecture and Ukraine's State Prize for Architecture.