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Research

Graphical Tools for an Epistemological Shift. The Contribution of Protoaxonometrical Drawing to the Development of Stonecutting Treatises

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Abstract. In this paper we analyze the graphical methods employed in the stonecutting treatises from that of De l'Orme, published during the sixteenth century, to De La Rue's *Traité de la coupe des pierres*, published in 1728. We will deal with the graphical means that made possible an epistemological shift between treatises of the sixteenth century, written for practising stonecutting masons in a period where the subject was covered by a veil of secrecy, which in some specific cases included a very intuitive form of representation, "protoaxonometries", and eighteenth-century treatises, which show a clear scientific and pedagogical approach to the geometrical problems posed by stonecutting.

Introduction

In 1728 the French architect Jean Baptiste De La Rue published the first edition of his *Traité de la Coupe des Pierres*. Focused on the stonecutting technique or stereotomy, this treatise had a great influence on the subsequent development of this science and its didactic character, basically in France.

Although its author was not given much consideration by the French architectural historiography, his treatise constituted the basis for the pedagogical discourse of Gaspar Monge, as certain authors report, and it was reprinted twice during the eighteenth and nineteenth centuries (1764 and 1858).

What was the reason for the success of this treatise? Certainly, the care with which it was prepared and the high quality of its graphics, which found no parallel in previous treatises, helped its diffusion, but its intention to make this science more accessible to all professionals, and not only to the masons who were specialized in the stonecutting techniques, constituted the real value of De La Rue's treatise.

Among the graphical tools employed to achieve this object, we will point out the systematic use of axonometrical perspectives to depict the construction elements developed and their component parts. It is here that are found the significant contribution of De La Rue and the main reason for the success of his treatise.

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A code for experts

Le secret de l'architecture, as Jousse entitled his treatise [1642], or *el perpetuo silencio*, mentioned by Martínez de Aranda (c.1600), are expressions that show how close in spirit the first stonecutting treatises and manuscripts, such as those of De l'Orme, Vandelvira, Martínez de Aranda, Jousse, and others, were to the medieval tradition. As a consequence of this professional secrecy, mentioned by Philibert De l'Orme [1567: 50], who proclaimed the advantages of knowledge transmission through direct practice, the graphical methods used in this period are based on a very abstract mode of representation, difficult for anyone to understand, whose principal object is to obtain relevant geometrical information for construction.

These geometric constructions of the first stonecutting treatises are based on orthographic projections: plan and elevation. Auxiliary constructions, such as changes of projection plane and rotations, are other, complementary graphic devices. According to several authors [Sakarovitch 1998; Rabasa 1999; Calvo López 1999], these procedures can be considered as the germ of the science of descriptive geometry.

However, this method doesn't appear for the first time in the technical treatises of the sixteenth century in France and Spain. Rather, it derives from the Gothic method used for the construction of ribbed vaults, based on the tracing on the ground of the full-size figure of the plan of the vault and the pattern of its ribs, obtained by rabatting the elevation onto a horizontal plane, so that one could easily relate the levels of the different arches.

These drawings, as described by Rodrigo Gil (BNE Ms. 8884, c. 1540), were traced on working platforms of the scaffolding situated under the vault to be constructed, wooden works that were removed after the construction, so that very few examples of this kind of tracings have survived. Fortunately, we have two Spanish drawings that seem to be sketches similar to those used in the tracing of medieval cross-ribbed vaults: one of them in the manuscript of Hernán Ruiz, and the other one from the vault on the choir of Priego de Córdoba (although this was made for the measurement of the construction works) [Rabasa 1999b: 197; Rabasa 1999a: 130-131]. In any case, these full-size schemes, where the plan and the levels of the different arches can be related on the same drawing, were very abstract and simple.

While some authors consider this Gothic method to be the immediate predecessor of the graphical procedures employed during the Renaissance for the construction of stone vaults, they didn't still constitute a real language, since it was not possible to deduce from them a general and universal method to be understood by everyone [Sakarovitch 1998; Pérez Gómez 1983].

During the eighteenth century, the medieval "veil of secrecy" gave way to illustrated science, and the pedagogical interest of stereotomy became relevant. The treatises were not focused solely on practical stonemasonry, as in the previous period; a new social group, more refined and cultivated, such as mathematicians, engineers and architects, also showed concern for these geometrical problems.

In most cases, the general view of the architectural element was represented through double orthogonal projection, and the details, such as the shape of the voussoirs, was developed by means of some kind of axonometric view, since this provided a visual representation and not only facilitated understanding of the volume of the piece, but also the process of stone carving, especially in the case of the squaring method. These kinds of graphical resource will become fundamental tools for the conceptual changes of the new treatises on stonecutting.

De La Rue's *Traité de la coupe des pierres* [1728] is the first one in which the general use of very accurate axonometries is found. Each architectural element is graphically developed with orthogonal projections and a general view in perspective, a mode of representation that was already found in Desargues [1640]. To explain the process of stone carving De La Rue most often uses a military axonometry, a cavalier perspective or a pseudo-Egyptian perspective. But this resource had been already explored in the sixteenth-century treatises, which in some specific cases included a very intuitive form of this method of representation, which we will call protoaxonometries.

In this paper, we analyze the graphical methods employed in the stonecutting treatises from the sixteenth to the eighteenth centuries in order to obtain clues about the influence of the axonometry in the epistemological shift that took place in the science of stonecutting.

Axonometry and parallel projection

The term axonometry was used for the first time in 1852 in Meyer and Meyer's *Lehrbuch der axonometrischen Projectionslehre* [1852] to describe the drawing method based on parallel projection, where the projectors are perpendicular to the picture plane, to distinguish them from the oblique projection, where the projectors are oblique to the picture plane, which is the case of cavalier or military perspective, of common use in the French tradition [Loria 1921; Bryon 2008].

The drawings that we analyze here were made before the eighteenth century and are therefore previous to the scientific systematization of the method. They were made in a very intuitive way, and were probably not understood in their day as projections at all, but as situating the different points to represent in an axis system or simply by maintaining one plane undistorted (the elevation in the case of the cavalier perspective, or the plan in the military perspective) and practising a kind of extrusion of this face.

For the drawings of the first stonecutting treatises mentioned here, we consider the term protoaxonometries more appropriate, or, in some cases, parallel projections. The terms cavalier and military perspectives, as used traditionally in France, will also be employed to describe the parallel projections where the picture plane is parallel to one of the faces of the represented object.

In the case of the drawings of De La Rue's treatise, it is clear that, because of their graphical development and their quality, we can already speak of axonometries.

The use of parallel projection in the history of the drawing techniques

We can find multiple examples of drawings using oblique projection as early as Antiquity, where we find them in Greek vase paintings and in the Roman frescoes of Pompei.

It is, however, in China where this technique was introduced as the main method for depicting three-dimensional space and where the construction drawings developed its early use, since central perspective was not known in China until the seventeenth century, when it was introduced by the Jesuits.

Long before Leonardo introduced the aerial view or bird's-eye perspective in Western painting, the Chinese used it with a great ability. The painter and writer Huo His (tenth century) was the first one to systematize the three-dimensional representation in landscape views [Scolari 1984].

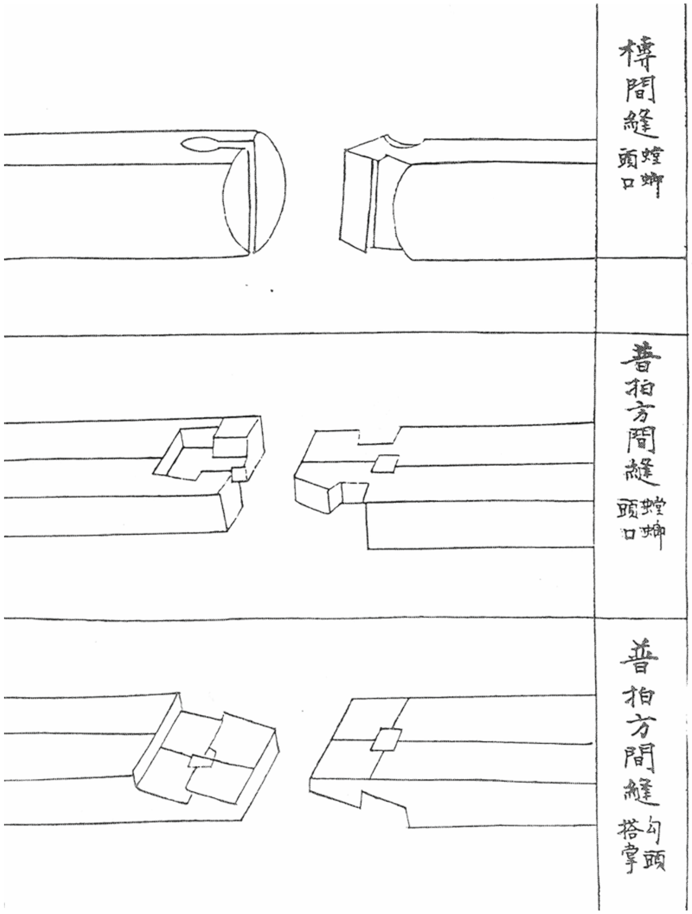


Fig. 1. Joining in tie-beams, *Ying Tsao Fa Shih* (ca. 1103), ch. 30, p. 17b

The Chinese *Ying Tsao Fa Shih* construction treatise of 1103 already contained excellent construction drawings. We can almost speak of working drawings in a modern sense (fig. 1), perhaps for the first time in any civilization [Needham 1971], unparalleled in the West, developed by means of the parallel projection technique, a system in which lines that were parallel in fact remained so in the drawing, in a way that we can say is very similar to that used today by architects and engineers for mechanical or structural "working drawings." This way of representation is indicative of an attitude towards Nature that is at once humbler and more social than that of Western man [Needham 1971].

European culture had to wait until Leonardo's exploded views (fig. 2) to find technical drawings using some kind of parallel view, which is a type of drawing very common later on.

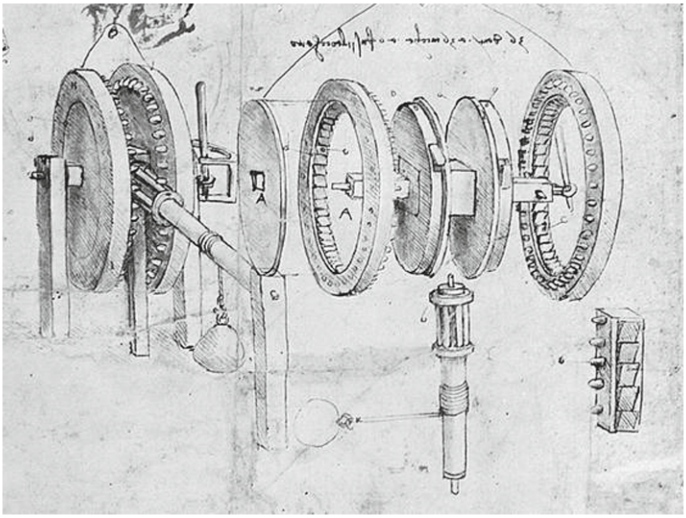
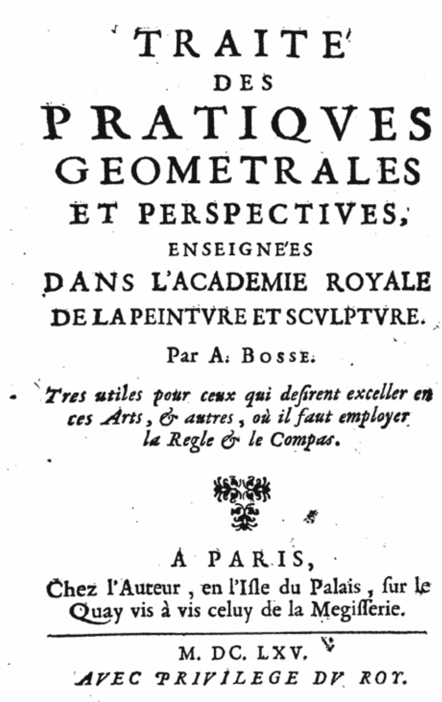


Fig. 2. Leonardo da Vinci, *Codex Atlanticus*, detail, fol. 30v/8v-b



TRAITE'
DES
PRATIQVES
GEOMETRALES
ET PERSPECTIVES,
ENSEIGNE'ES
DANS L'ACADEMIE ROYALE
DE LA PEINTVRE ET SCVLPTVRE.

Par A. BOSSE.

Tres utiles pour ceux qui desirent exceller en ces Arts, & autres, où il faut employer la Regle & le Compas.

A PARIS,
Chez l'Auteur, en l'Isle du Palais, sur le Quay vis à vis celuy de la Megisserie.

M. DC. LXV.
AVEC PRIVILEGE DV ROY.

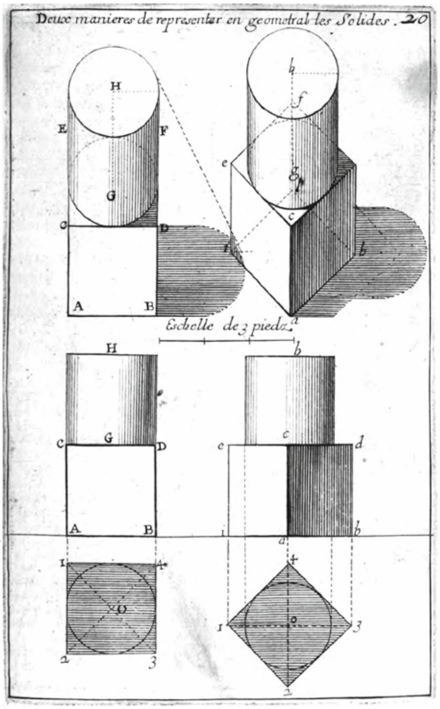


Fig. 3. Abraham Bosse, *Traité des pratiques geometrales et perspectives* [1665],
Title page and pl. 20

Abraham Bosse's *Traite des pratiques geometrales et perspectives*, published in Paris in 1665 and organised in two parts, can be understood as a textbook or a notebook of his lessons in the Academie Royale de la Peinture et de la Sculpture. The first part, focused on theory and with few graphical references, is addressed to painters and designers, and contains some notes about the military perspective. Bosse mentions two forms of representation: *geometral* and *perspectif*. The term *geometral*, which is used nowadays in France to designate the orthographic projections, can be understood as a synonym of *petit pied* or scale, since he uses it to refer to the possibility of measuring and, therefore, to a type of drawing in which in some way certain geometrical attributes of the object remain undistorted, as opposed to the term *perspectif*, in which they appear modified.

The so-called geometrical method refers to the representation by means of both orthographic projections (the plan and the elevation), and by means of the development or unfolding of the faces on a horizontal plane, as we can see in the military perspective of a cube (fig. 3). Bosse insists that both forms of representation are *deux especes d'un mesme genre, et non pas deux genres divers* (two spaces of the same kind, not two different kinds) [1665: 3]. He also explains the method to obtain the shadows to allow an increasingly solid appearance.

In the practical part of the treatise, profusely illustrated with graphical details, he uses military perspective to explain the development of a solid by rabbating its faces to a horizontal plane, a method that, as he indicates, was used by some professionals, such as the stonecutting masons, to obtain the templates that they needed to cut the stone blocks. He also maintains that military perspective is the usual method employed by the engineers. He illustrates his explanation of this drawing method with figures of prisms or groups of vertical prisms and some pyramids, consisting in tracing verticals from the plan, in a very elementary way, to go on with more complicated figures such as sloped prisms or complex solids, to finally obtain the shadows.

One documented predecessor of this procedure is Buonaiuto Lorini's *Delle fortificationi* [1596]. It is a book destined for military engineers which contains a brief exposition of the method of military perspective. It appears again in Hendrik Hondius's *Instruction en la science de perspective* [1625] with the name of "geometric depth".

Jean Du Breuil, in his treatise *La perspective pratique* [1651] provides for the first time a broad description of a normalized method to draw military perspectives, but he does not mention the shadows.

More than a quarter of Bosse's text is dedicated to military perspective, showing the significant development achieved by this mode of representation, which had an empirical character and was taught in the *Academie Royale de la Peinture et de la Sculpture*, but, as we know, had certain difficulties in being adopted by painters or even in the field of architecture.

Despite this, the architect Jacques Androuet du Cerceau had made already use of the cavalier and military perspective about a century before. In *Les plus excellents bastiments de France* [1576-1579] and in the third volume of his *Livre d'architecture* [1559-1582] we find cavalier, military and Egyptian perspectives (fig. 4). In any case, his use of this kind of parallel projection drawing is very intuitive, and in many cases he makes some errors or uses both parallel and central perspective on the same drawing, which shows that he had not entirely mastered this method of representation.

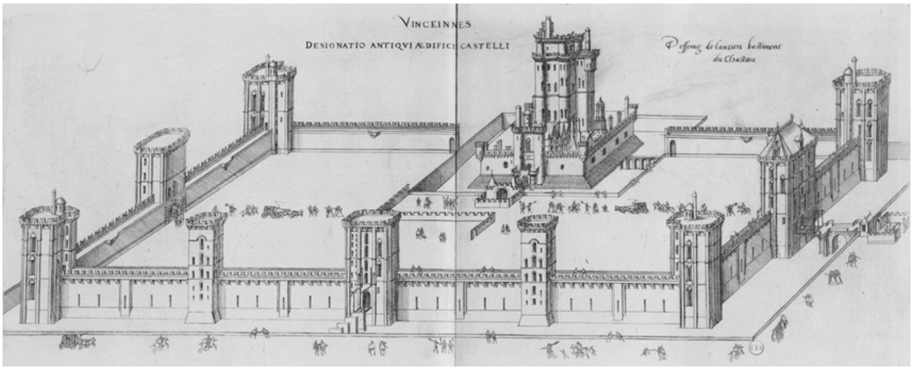


Fig. 4. Jacques Androuet Du Cerceau, Vincennes, from *Les plus excellents bastiments de France* [1576-1579], Bk. I, pl. XXVI

In *Des fortifications et artifices architecture et perspective* from Jacques Perret [1601] we find again the use of military perspective but in this case applied to the field of military architecture, since it deals with polygonal bulwarks that are represented in the plates by tracing verticals from the plan, a procedure which can be related to the method described later by Bosse [Alonso 1991].

First predecessors in the use of protoaxonometrical drawings in stonecutting treatises

In the sixteenth-century treatises appear some drawings, which we will call protoaxonometries, made by the extrusion of the two-dimensional image of the elevation or the plan, resulting in an oblique parallel projection. In these representations, the plane of the object that is parallel to the picture plane remains undistorted, making it a form of representation particularly suitable for illustrating stonecutting techniques. In these first examples that we will analyze, the method employed to trace them is very intuitive and practical, rather than geometric or mathematic, since we will have to wait until the following century [Bosse 1665] to find the first attempts to systematize the technique of drawing parallel projections.

Among the first construction drawings made as parallel projections, or protoaxonometries, are those found in Vandelvira's manuscript of 1575 ca. [Ms. R31] to explain the cutting of the steps in spiral staircases. Fig. 5 (left), which corresponds to *declaración del caracol exento*, shows two steps seen from below, but rotated to show the intrados surface, in a protoaxonometrical drawing of the horizontal picture plane. The two steps, while not connected with vertical lines, are situated so as to show how they join together; this could thus be considered an "assembly drawing" in a modern sense.

This graphical resource is frequently found in construction treatises, especially in wood joint drawings, such as those of the Chinese treatise *Ying Tsao Fa Shih* from the twelfth century. At the beginning of the nineteenth century, Rondelet universalized it in his *Traité théorique et pratique de l'art de bâtir* [1802], one of the first books on general construction. In the stonecutting treatises it will be frequently used beginning with Frezier [1737-1739], but the first to make use of it was De La Rue [1728].

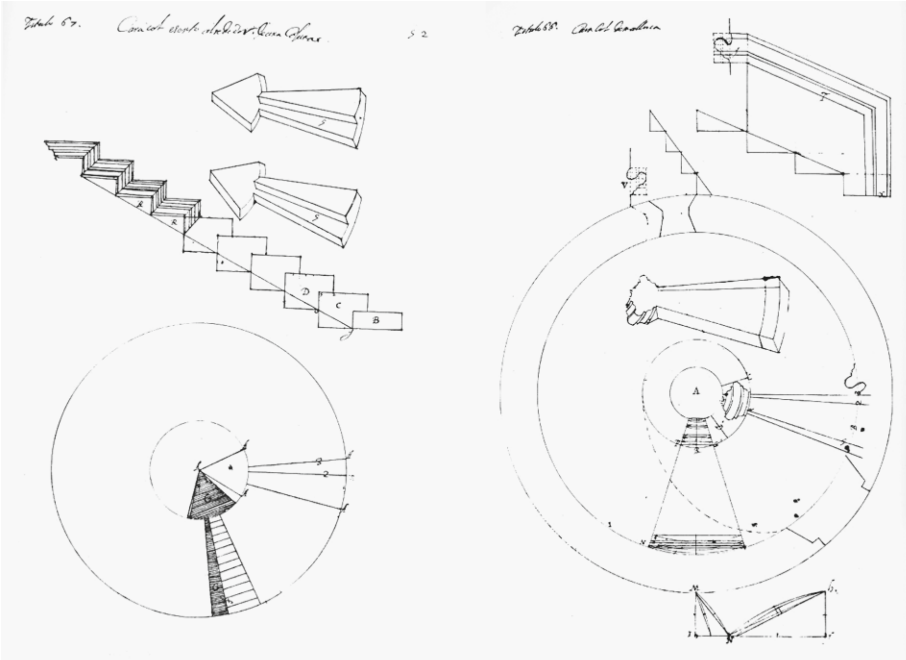


Fig. 5. Ms. R31, known as *Libro de trazas de cortes de Piedras*, attributed to Alonso de Vandelvira. Left, *Declaración del caracol exento*, fol. 51v; right, *Declaración del caracol de Mallorca*, fol. 49v

It is significant that, to represent the intrados surface of the steps, instead of constructing a worm's eye perspective, a technique that he probably did not know, Vandelvira draws a bird's-eye perspective, which forces him to turn the piece. This recourse will appear again in other authors, such as De La Rue, as we will see later.

In Vandelvira's drawing dedicated to the *caracol de Mallorca* (fig. 5, right) we find another step represented in a protoaxonometrical view of the horizontal picture plane, where the dimensions of the piece were taken directly from the plan. In this case, Vandelvira makes a drawing that we would now call *wireframe*, and indicates the part of the step that fits into the wall. It is the initial volume previous to the carving of the helicoid of the soffit surface.

However, Vandelvira, who wrote his treatise between 1575 and 1580, was not the first to employ the protoaxonometrical drawing for stonecutting tracings. These types of protoaxonometries are also found in three plates in the first stonecutting treatise published in 1567 by Philibert De l'Orme: the *porte biaise*, *the porte sur l'angle*, and *the porte dans une paroi de plan courbe* (fig. 6). In all three cases, he develops the detail of one or two voussoirs in parallel projection to show its volume, in order to make the object understandable as a three-dimensional entity in space. These are cavalier projections that exhibit an orthographic view or elevation of the front size, and therefore this face is represented in true shape.

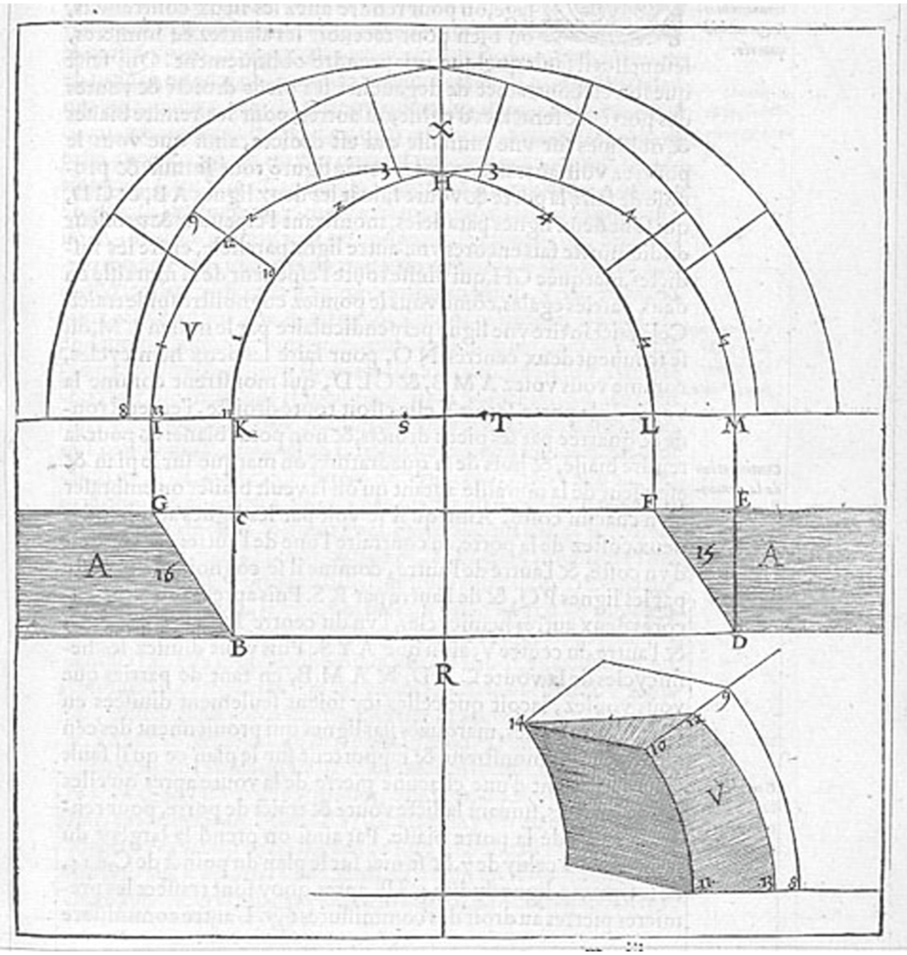


Fig. 6. Philibert De l'Orme, *Premier tome de l'Architecture* [1567], Bk. III, fol. 69r, *porte biaise*

Something similar is found in Martínez de Aranda's manuscript [1600 ca.]. At the beginning of the second part, focused on arches and splays, he uses parallel projection to explain the squaring method in four cavalier perspectives (fig. 8, left), but in this case seen from below. Two of them show the initial prismatic volume on the front face of which is drawn the face template of a voussoir as a sector of circular crown which, after being squared straight, results as a voussoir of an arch, and after being worked curved results as a dome. In the first case, the edges are straight lines; in the second one they are arches. Except for their inclusion at the beginning of the second part, this kind of drawing does not appear anywhere else in the treatise. These drawings are interesting since they constitute the earliest examples of worm's eye views found in stonecutting treatises.

The didactical approach: Desargues – Bosse – De La Rue

In 1640 Girard Desargues published a short work entitled *Brouillon Project d'exemple d'une manière universelle touchant la pratique du trait à preuves pour la coupe* *des pierres…* in which he tries to solve the particular problems on stereotomy through a unique rule: the *manière universelle*.

In four pages and five figures he analyses a sloped vault that is at the same time skewed and opened to a sloped wall. This is the only model he uses to explain his method.

Stonecutting treatises, up to then, had been organized as a series of models which were more or less hierarchical, but did not establish a general method. In this sense, Desargues's approach was in some way didactical and epistemological, focused on describing a methodology [Bessot 1994: 297], clearly in contrast to the previous treatises, which described stonecutting work from a geometrical point of view.

This method contains an important graphical development. Desargues starts by presenting a global image through a central perspective of the architectonical model that he wants to study. Up to then, stonecutting treatises had not been devoted to these matters [Sakarovitch 1994: 351; Sakarovitch 1998: 174]. In 1643 Abraham Bosse, a disciple of Desargues who dedicated himself to problems of axonometrical representation, published *La pratique du trait a preuves de M. Desargues pour la coupe des pierres en l'architecture*, where he tries to apply the method of his teacher to a great number of particular cases: sloped vaults, skewed arches, corner arches, squinches, splays, etc. Bosse carefully follows the indications of Desargues, not only applying his method but also in his presentation of the different models to study.

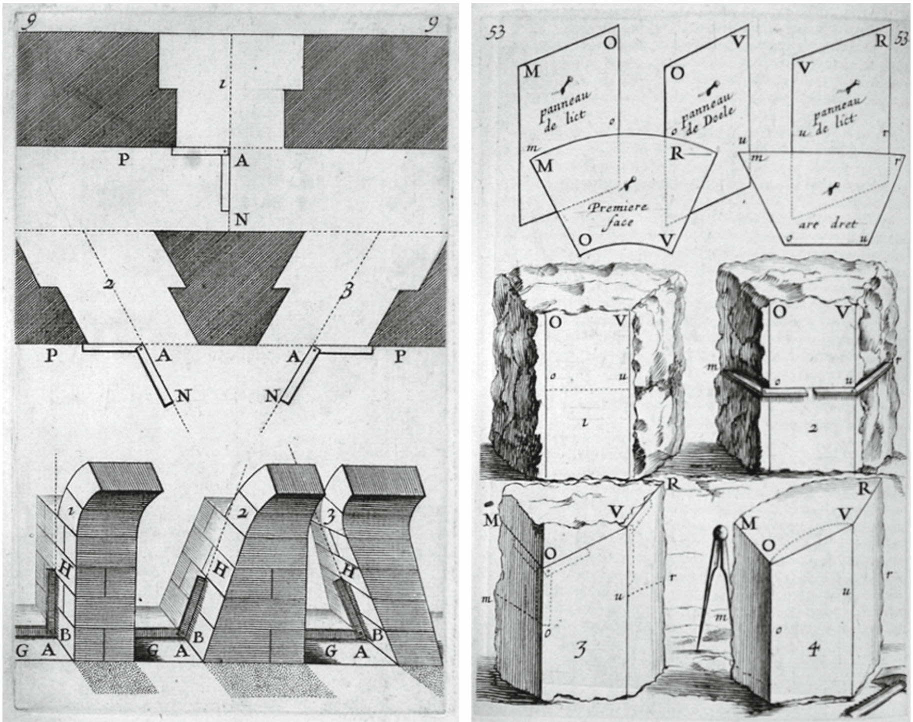


Fig. 7. Abraham Bosse, *La pratique du trait à preuves de M. Desargues,…* [1643], Pl. 53

He makes many global perspectives of the construction elements that he analyses, some of them in cavalier perspective (fig. 7 left), but he also shows a new mode of representation that will be used by Derand [1643] and further developed by De La Rue. We are referring to the parallel projection views that show the stonecarving method showing the tools used on the process, as we can see in plate 53 (fig. 7 right). While De La Rue will not accept Desargues's thesis, he will adopt and develop this pedagogical mode of representation in his treatise.

It is obvious that De La Rue knew Bosse's work regarding the stonecarving process. He mentions it in the preface of his treatise and we can also presume that he knew Bosse's treatise about geometrical representation [1665], since we have found important common points, such as the employment of the developed faces rabatted to a horizontal plane of a prism to show the application of the face or the bed templates of a voussoir.

De La Rue will follow the scheme of presenting first a general perspective or axonometry of the model and the plan or the elevation and, to facilitate comprehension of the stonecarving process, some views with the detail of the sometimes complex process. This scheme, as we can see, is based on the method proposed by Desargues and further developed by Bosse.

However, based on a new didactical spirit, De La Rue will develop these ideas much further, and therefore he contributed with them to the important epistemological shift that took place in stereotomy.

Graphical connections between Derand and De La Rue

In 1643 François Derand published a treatise devoted to stereotomy, *L'architecture des voutes ou l'art des traits et coupe des voutes*, probably the consequence of his teaching activity at the Collège in La Flèche; Jousse probably borrowed Derand's ideas to a great extent in *Le secret d'architecture*, although this was published one year before [Pérouse de Montclos 1982]. Both treatises deal with the art of stonecutting in relation with a collection of examples, but with a practical approach, very different from the scientific approach adopted by Desargues in his *Brouillon projet d'exemples d'une manière universelle*.

In Derand's treatise the use of parallel projection is not very common, but it does contain some drawings that can be considered precursors of the work developed later by De La Rue, such as, for example, those used to explain the squaring method in the voussoirs of cavalier and military perspectives, where depth is indicated by an oblique extrusion, from the elevation or from the plan. We also find in these drawings new and very expressive graphical resources: Derand shows the initial volume of the stone piece to cut the voussoir and indicates the parts to eliminate to obtain the final shape. We will find this new code, much more elaborated, also in De La Rue's treatise [1728].

In the first drawing where we find the use of the cavalier perspective, Derand makes use of it to explain in a general way the squaring method in the cutting of the voussoirs (fig. 8 right). In this drawing he traces a prism, in a way that we would call now wireframe, on the front face of which he draws the template, tracing parallel lines from its vertex to the edges of the prism, to obtain an image where the parts to eliminate from the solid are shown, in a very similar way to that developed previously by Martínez de Aranda (fig. 8 left).

Again, in plate IX dedicated to the *biaise par teste par équarissement*, he will resort to parallel projection to show the volume of the three voussoirs that are part of the arch. In this case he uses Egyptian perspective, and we can recognize the elevation, and therefore the face template and the plan both in true shape.

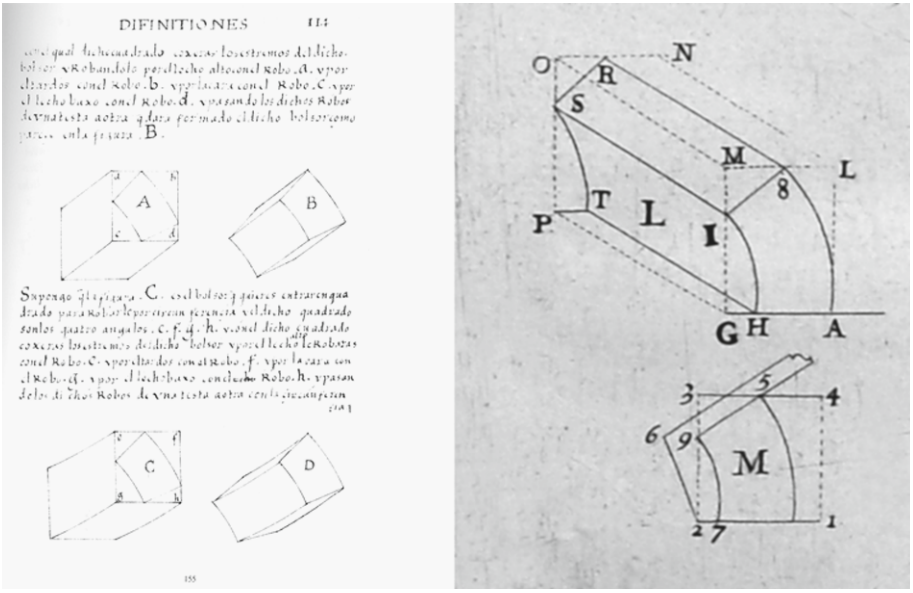


Fig. 8. Left, Martínez de Aranda, *Cerramientos y trazas de montea* [1600 ca.]; right, Derand, *L´Architecture des voûtes* [1643]

Although in the first voussoir he shows the whole volume of the initial solid as a wireframe prism, in the second one he shows only a little part of it, and in the last one he shows only the end shape of the voussoir. Each of the three voussoirs appears in its real orientation. We can affirm that the use of this kind of Egyptian perspective is not at all usual in these treatises, although we will find occasionally some of them in De La Rue.

We find the use of parallel perspectives again in the plate dedicated to the *biaise passé*. The drawing corresponds to the *Corne de boeuf* and the method he uses to arrive at the parallel perspective is obvious: from the elevation he takes the projection of the first voussoir, the front but also the back face, and traces from its vertex lines parallel to the depth, in a kind of extrusion as in a cavalier perspective, to define the vertex of the first voussoir of the arch, which leads to an image that shows at the same time the voussoir and of the solid volume from which it comes. He uses the same code again to indicate the parts to eliminate from this initial solid by a dashed line, as we saw in the previous plate and as we will see, more elaborated, in De La Rue.

In a subsequent plate, the one dedicated to the *Voute d'arestes barlongue par equarrisement*, Derand uses a military perspective from below for the two first voussoirs on the groin (fig. 9). In fact, what he shows is the whole initial solid in order to trace on its faces the silhouette that will allow it to perform as the groin of the vault. It is significant that in one of these perspectives are shown the *x*-, *y*- and *z*-axes, with the *x*-axis being the vertical one.

This treatise, in which drawings in parallel projection appear only very occasionally, will have a decisive influence, partly because of the use that Derand makes of it, but also because of the graphical resources that we can see here, still in a very primitive way.

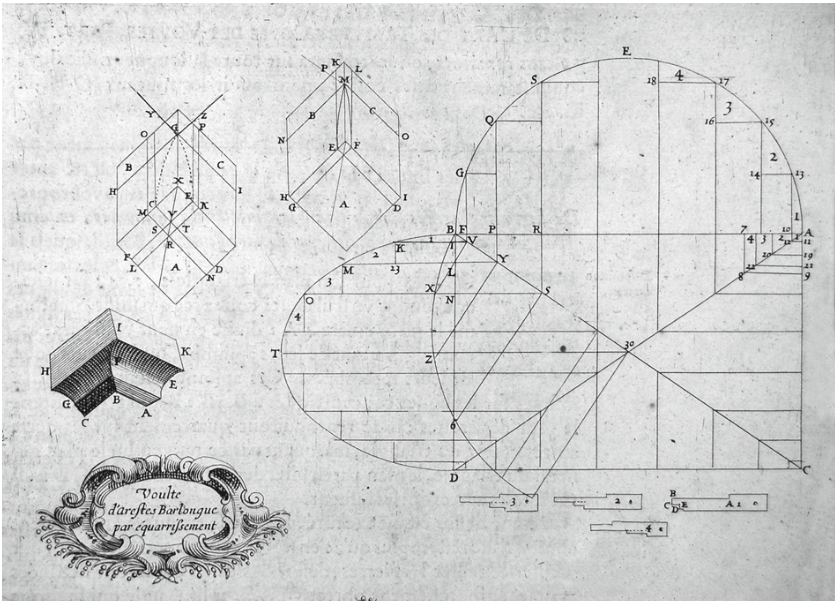


Fig. 9. Derand, *L´Architecture des voûtes* [1643], *Voute d'arestes barlongue par equarrisement*

Axonometrical drawing in De La Rue's Traité de la coupe des pierres

De La Rue divided his treatise into five chapters that were accompanied by a preface and a *petit traité de stereotomie*. In the preface, De La Rue states his intentions. After briefly discussing the state of the art, surveying the works that had been published up to that point, he arrives at the main objective of his publication: to make the treatise, and therefore, the science it intends to communicate, instructive and comprehensible. In order to obtain this objective, the treatise was carefully laid out, and each model studied is accompanied by an elevation, the design of the centering and an axonometric projection of the voussoirs in the different steps of construction.

> *Comme les Livres qui traitent de la Coupe des pierres sont très rares aujourd'huy; & que d'ailleurs, entre ceux qui ont paru jusqu'à present, il n'y en a pas un qui soit assez a`la portée, soit des Commençans ou des Ouvriers, j'ay crû qu'il ne seroit point inutile de presenter celuy-cy au Public. Je n'ay rien negligé pour le rendre aussi instructif qu'intelligible : & afin d'y parvenir plus sûrement, j'ay accompagné la plus grande partie des Epures, de leur élevation & de leur cintre ; j'y ay joint la représentation de plusieurs de pierres tracées dans des degrés differens, pour suppléer à la foible idée qu'en donne une Epure embarrassée des lignes qui la composent ; ce qu'on n'avoit pas encore fait jusqu'icy* [De La Rue 1728: Preface].

The *petit traité de stéréotomie* included in the book analyses several theoretical aspects that are related to spatial geometry and, in particular, to the plane sections of cones, cylinders and spheres and the development of cones and cylinders. It is in fact the first treatise to include the term "stereotomy" in its title. This term appeared for the first time in the booklet written by Jacques Curabelle against Desargues in 1644 [Curabelle 1644; Fallacara 2007: 36; Calvo 1999].

The main part of the treatise is arranged in five chapters devoted to the study of arches and splayed elements, vaults, squinches, sloped vaults and stairs. In total, sixty-four models are featured in the treatise, whereas Derand's treatise, his predecessor, featured 124. Of the sixty-four models, thirty-six include some kind of axonometric drawing to aid in the explanation. The fourth chapter does not include any axonometric views, whereas in the chapters about vaults, squinches and stairs, axonometric views appear in almost all the designs.

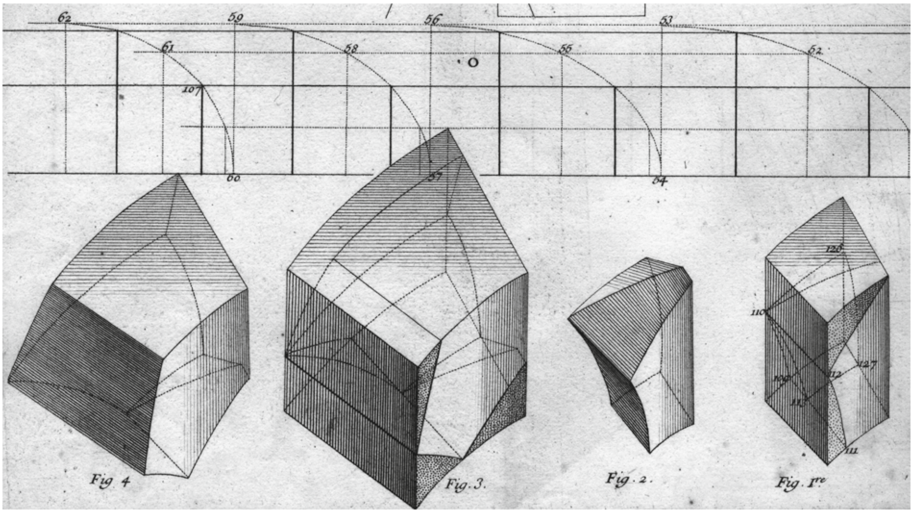


Fig. 10. De La Rue, *Traitè de la Coupe des Pierres* [1728], pl. XXX (detail)

The layout of each chapter also responds to a didactic criterion. De La Rue first explains those construction elements that are easier to carve. When the reader then comes to more complex problems, he already understands the basic techniques necessary for the comprehension of these harder problems. However, this is not an innovation, for this arrangement was already present in Spanish stonework manuscripts of the sixteenth and seventeenth centuries.

De La Rue frequently uses axonometric views to explain the carving process for the different types of voussoir and this becomes a key tool to comprehend the sometimes complex stone cuts (fig. 10).

The technique used De La Rue for drawing parallel projections of the pieces is rather simple. The different vertices of the piece are transferred from the top view or elevation onto the projected view using their space coordinates. This technique can be used to obtain any view, no matter how complex. Coefficients of reduction are not used, nor are projections of coordinate planes. It is a mere translation of points from one drawing to another. Therefore, if we were to apply modern definitions of descriptive geometry, we would not consider De La Rue's representations axonometric views in the strict sense of the term.

According to the type of piece and the face of the piece to be represented in true shape, De La Rue uses various types of axonometric views, all of which are oblique views, in keeping with the French tradition. It is worthwhile recalling that the orthogonal projections described by William Farish in 1822 would be mainly developed in Great Britain and Germany.

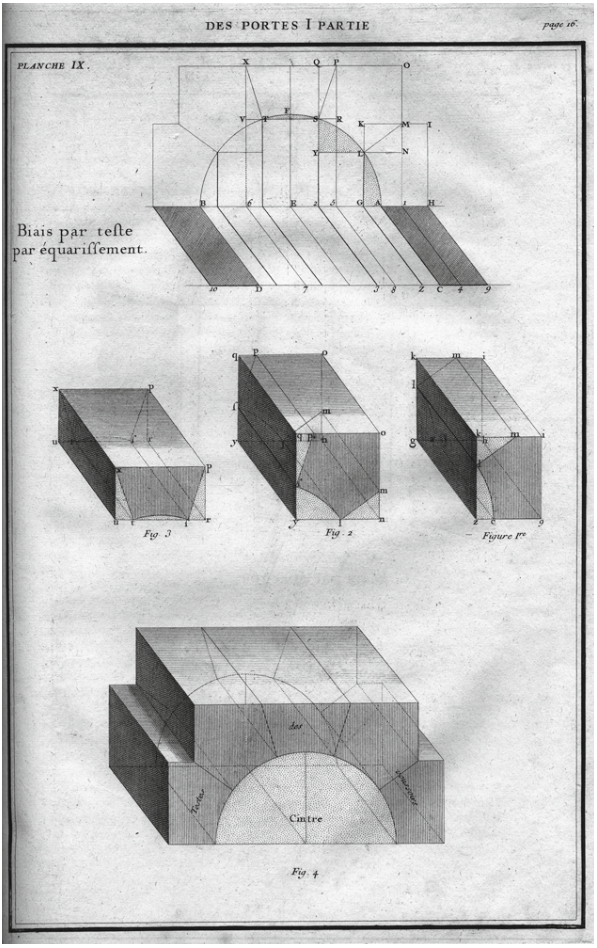


Fig. 11. De La Rue, *Traitè de la Coupe des Pierres* [1728], pl. IX, *biais par teste par équarissement*

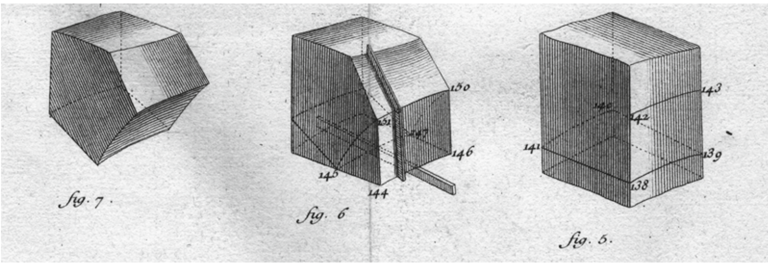


Fig. 12. De La Rue, *Traitè de la Coupe des Pierres* [1728], suite de la planche 65, *escalier suspendu et a repos* (detail)

There are examples in cavalier perspective, military perspective and pseudo-Egyptian perspective. When using cavalier perspective, the front vertical planes are in true shape and the top view is deformed. It is interesting the fact that the projection of the *y*-axis is drawn to the left in almost all cases, the opposite of what is common in this kind of representation. Examples of cavalier perspective can be found, for example, in *Porte droite en tour ronde par panneaux et par equarissement* (pl. XII), *Porte en tour biaise en talut etrachetant une voute sur le noyau par equarissement* (pl. XVI), *La corne de Boeuf* and *Biais Passé* (pl. XVII).

Military perspectives are also broadly used. In these cases, the top view and the planes parallel to it are in true shape. De La Rue uses two types of military perspective. In the first case, the top view is rotated by a certain angle and the vertical lines are drawn vertical; this is the most common technique used in this kind of axonometric views. De La Rue also employs this type of representation without rotating the top view, thus drawing vertical lines with a constant inclination with respect to the horizontal. Examples of military perspectives are found, for example, in *Voute d'Areste barlongue* (pl. XXIV), *Voute spherique ou Cu-deFour en plein Cintre* (pl. XXVII), *Voute sur le noyau* (pl. XXIX), and *Voute d'Areste en tour ronde* (pl. XXX).

Furthermore, De La Rue uses another type of view in which the projections of the *y*-axis and of the *z*-axis are superposed. Thus, the top view and top front view appear on top of each other. These representations could be referred to as pseudo-Egyptian (fig. 11).

They had been used before, as discussed above, by Du Cerceau, Perret, Dubrueil and Bosse. It is worth noting that De La Rue adopts the technique of using these pseudo-Egyptian projections for representing skewed objects. In this case, the inclination of the horizontal projections is the same as that of the plan. This is a special type of military perspective. The top view is not rotated, and the vertical lines are aligned with one of the directions of the top view.

The most outstanding axonometric projection found in the treatise is that of pl. LXV, concerned with the study of *l'escalier suspendu et a repos* (fig. 12). Here De La Rue tries to explain the carving process of one of the voussoirs by drawing what seems to be a dimetric orthographic axonometry: he uses the same angle for two of the axes and a different one for the third. In this case, our opinion is that De La Rue is drawing an oblique axonometry with coordinate planes that are not parallel to the picture plane, that is, a generic oblique axonometry. It seems that, once he had chosen the projection of the axes, he applied the same scale to all of them. This procedure will be broadly used, a century and a half later, by the French engineer Auguste Choisy. It is an application of Pohlke's theorem, formulated in 1858, which says: "three concurrent segments in a plane can always be considered as the cylindrical projection of the three edges of a cube" [Rabasa 1999]. It is obvious that De La Rue did not know this theorem, formulated almost two centuries later, but his procedure for constructing parallel projections is closer to these oblique representations than to the procedure for obtaining dimetric axonometries, such as the English or German axonometries of the nineteenth century. These kinds of oblique representations frequently appear in Frezier's treatise [1738].

A type of drawing mainly used by De La Rue when representing vaults is what we earlier called "assembly drawing" (fig. 13).

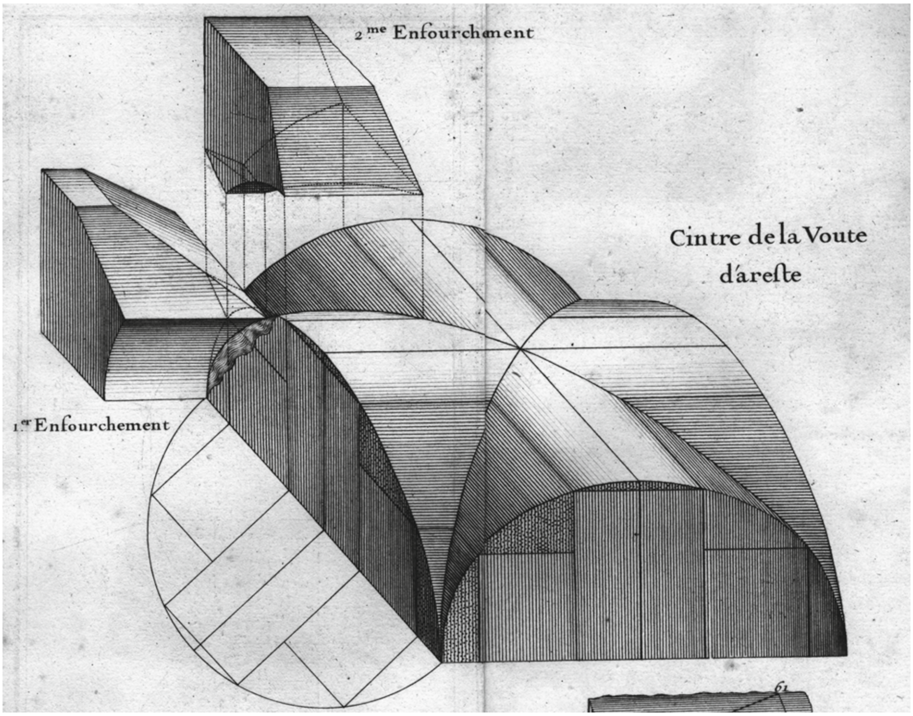


Fig. 13. De La Rue, *Traitè de la Coupe des Pierres* [1728], detail, pl. XXIV, *cintre de la voute d'areste barlongue*

As mentioned, this type of drawing has been widely used in engineering and construction. De La Rue usually makes use of this representation together with an axonometric projection of the vault centering. This way he achieves two objectives: on the one hand, the intrados surface of the vault is represented, as it is the same as that of the centering, and on the other hand, the centering is used as a support element, helping to organize the assembly of the different pieces that form the vault.

In this case, and in others that simply describe the cutting process for a voussoir, De La Rue uses bird's-eye views. We cannot say that he did not know how to make a worm's-eye view, since we can find at least three examples of this kind of perspective in the treatise – *l'Arriere Voussure de St. Antoine, en plein cintre* (pl. XX), *Voute spherique ou Cu-de-Four en plein Cintre* (pl. XXVII) and *Trompe en niche rampante* (pl. XLVII) – but there is no doubt that he was more comfortable drawing bird's-eye views. This is the case of the description of the carving process of certain voussoirs, such as those of the plate related to the groin vault (pl. XXIV, fig. 13), where he prefers to rotate the piece while maintaining the same type of projection than to change to a worm's-eye view to see the piece from below. This is very common in the few protoaxonometrical drawings in the stonecutting treatises before De La Rue's. With the exception of the drawing by Martínez de Aranda mentioned before, all previous drawings on stonecutting treatises are bird's-eye views. This fact makes the drawing by Martínez all the more interesting. Not until Frezier and the experts on descriptive geometry in the nineteenth century will worm's-eye view axonometries be used widely in stereotomy treatises.

Another remarkable characteristic of De La Rue's special way of representation is found in military perspectives with the face or bed templates rabatted to a horizontal plane to show them in true shape (fig. 14, right). This kind of drawing responds to a clear pedagogical intention and has its antecedents in the drawings that we have already mentioned from Bosse's *Traite des pratiques geometrales et perspectives* [1665], where he represents a series of prismatic figures with its faces rabatted to a horizontal plane (fig. 14, left).

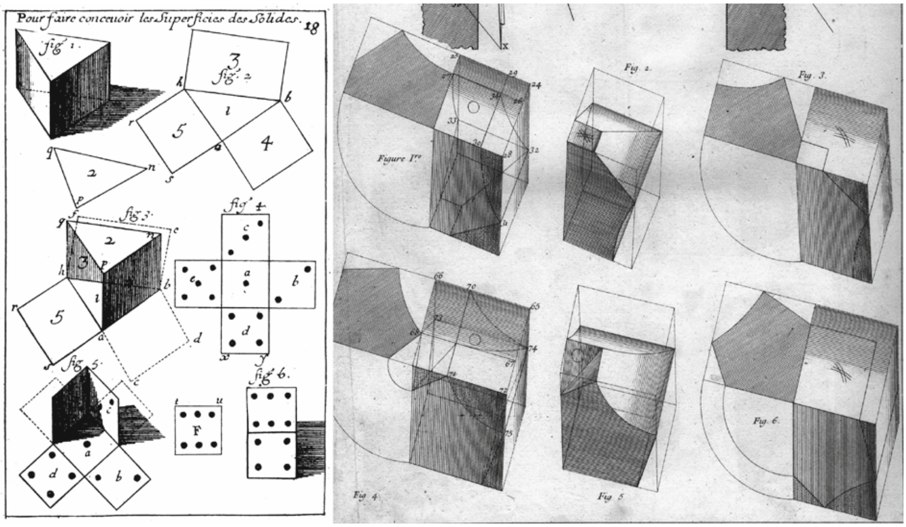


Fig. 14. Left, Bosse, *Traite des pratiques geometrales et perspectives* [1665]; right, La Rue, *Traitè de la Coupe des Pierres* [1728], pl. XXIV, *voute d'areste barlongue*

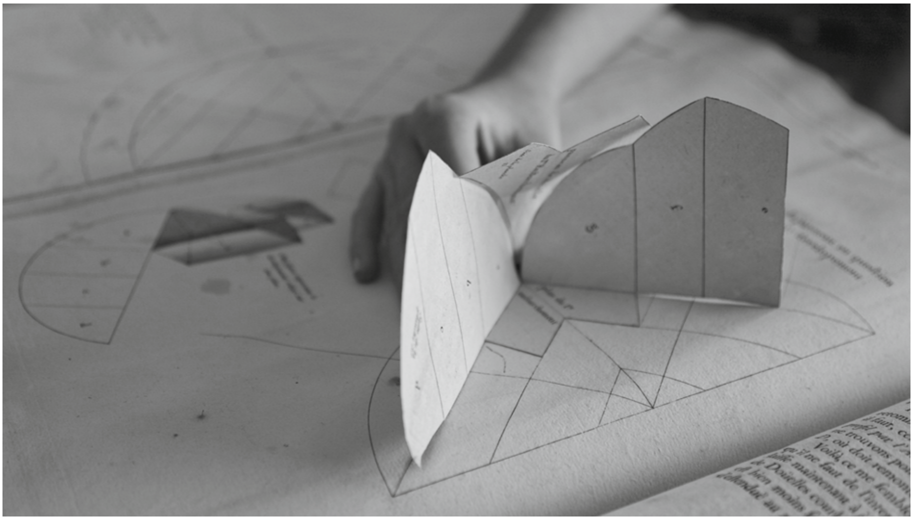


Fig. 15. De La Rue, *Traitè de la Coupe des Pierres* [1728], fold-outs, *Preuve de l'erreur du premier panneau d'enfourchement de la voute spherique fermée en quarré, dévelopé suivant Philibert Delorme, Mathurin Jousse a & le Pere Derand*

Certain graphical devices used by De La Rue are indicative of the same didactical interest, and constitute a genuine innovation in the history of stonecutting treatises. We are referring to the representation of the tools and instruments used in the stone carving process. This device makes it possible for the reader who is not familiar with these techniques to understand the process and to check the application of templets, bevels, straight edges, and so forth.

Another novelty, also indicative of the extraordinary quality of the edition of the treatise, are the fold-outs glued to the plates, used to demonstrate certain theories such as the case of the *Preuve de l'erreur du premier panneau d'enfourchement de la voute spherique fermée en quarré, dévelopé suivant Philibert Delorme, Mathurin Jousse a & le Pere Derand* (fig. 15), where he demonstrates the error commited by the authors named, proven by a development that can be visualized in space.

Apart from its scientific value, without a doubt the most important characteristic of the treatise, that which provides its graphical quality and caused it to be admired, studied and even mutilated so its plates could be sold individually all over Europe, is comprised in the variety of line types (continuous, dashed, dotted), hatching, shading, shadows, etc., which give the drawings a quality and hierarchy of elements never seen before.

Conclusions

De La Rue, as we have already mentioned, made use of a great number of graphical and spatial devices in his *Traité de la coupe des pierres* to achieve his objective of transmitting the "stonecutting science, one of the most important and useful sciences included in Architecture," and to make it more intelligible not only to the experts but also to those who wish to learn [De La Rue 1728: Preface].

To this end, he makes extensive recourse to the use of parallel projections or axonometries. By then, the old tradition of stonecutting treatises was giving way to the science of stereotomy, and the architects who were devoted to this field gave up their places to engineers. In this sense, Francois Derand's *L'Architecture des Voutes* can be considered the last great stonecutting treatise by an architect.

Traditionally, architect's drawings had been developed through orthographic projections: plan, elevation and section, according to Alberti and the letter to Leo X [Bois 1981]. Architects used to complete their projects with perspectives and also with scale models. Authors of Renaissance stonecutting treatises, also influenced by their medieval heritage, maintained these rules. In contrast, we know about the preference of the engineers for the axonometrical drawing, as Bosse and Dubreuil, among others, maintain.

At this moment of transition in which knowledge about the science of stonecutting was universalized and refined, axonometry plays a fundamental role at the hands of military engineers such as Frezier and, later, by the engineers of the École Polytechnique with Jules de La Gournerie.

Jean Baptiste De La Rue, architect and member of the Académie Royal d'Architecture was also dedicated professionally to the construction of bridges and the invention of machines. So we can affirm that he was an architect-engineer who was key for the epistemological shift which took place during the eighteenth century in France. It is in this sense that his *Traité de la Coupe des Pierres* can be considered the first great treatise on stereotomy by an engineer.

His treatise explored, with great success, the new graphical methods developed by mathematicians and geometers for the study of stereotomy, then further developed by engineers, before its demise as a practical science.

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