György Darvas

Perspective as a Symmetry Transformation

From the quattrocento to the end of the nineteenth century perspective has been the main tool of artists aiming to paint a naturalistic representation of our environment. In painters' perspective we find a combination of affine projection and similitude. We recognise the original object in the painting because perspective is a symmetry transformation preserving certain features. The subject of the transformation, in the case of perspectival representation, is visible reality, and the transformed object is the artwork. The application of symmetry transformations developed from the origin of perspective through the centuries to the present day. The single vanishing point could be moved (translated), and even doubled, developments that made it possible to represent an object from different points of view. In the twentieth century, the application of topological symmetry combined with similitude resulted in new ways of seeing, new tools for artists such as cubists and futurists.

A Brief History of the Concept of Symmetry

The meaning of the intriguing term 'symmetry' has undergone considerable transformation in its use throughout the centuries. The proper translation of the Greek term *symmetria* (*συμμετρια*) is 'common measure', from the prefix *sym* (*συμ*), common, and the noun *metros* (*μετροσ*), measure. The Greeks interpreted this word to mean the harmony between the different parts of an object, the good proportions between its constituent parts. Later this meaning was applied to such things as the rhythm of poems, of music, and even the cosmos ("well-ordered system of the universe, in contrast with chaos"). Up until the Renaissance, Latin and the emerging modern European languages translated "symmetry" as harmony or proportion. In a wider sense, the terms balance and equilibrium also belonged to this family of synonyms. It is not too difficult to deduce that in many ways symmetry was always related to beauty, truth and the good. These related meanings determined its application in the arts, the sciences, and ethics respectively. Symmetry was not only related to such positive values, it even became a symbol of the search for perfection, as in the writings of Plato.

The earliest surviving description of symmetry was given by the first century B.C.E. Roman architect Vitruvius, in his *Ten Books on Architecture*, which laid the foundations for the particular meaning of the notion. His usage suggests symmetry is a general term related to the meaning of words such as harmony, proportion, rhythm, etc. It was not by chance that the humanists of the quattrocento rediscovered Vitruvius's treatise for themselves. With the publication of several translations, the term "symmetry" replaced earlier translated versions of the word, and took its place within modern European languages, first in *volgare* [Italian: *simmetria*], later in German [*Symmetrie*]. It was also not by chance that this adoption of the Greek term took place in parallel with the growing self-consciousness of the arts, the development of art theory, the appearance and application of perspective, and geometry in the visual arts in general. A number of written works contributed to establishing the contemporary meaning of the term symmetry in the fifteenth century.[1]

Thus, after the Golden Age of Greece (fifth century B.C.E.), the paths of science and the arts crossed again in the Renaissance, only to divide once again—for centuries—and then to meet for a third time in the cultural melting pot of the twentieth century. This has provided fertile ground for the application of symmetry in both spheres, art and science, of human creativity, to products of two opposite cerebral hemispheres.

Manifestations of Symmetry

If one were to ask general readers what is meant by symmetry, most are likely to answer that it is something like ***reflection***. Indeed, reflection, also called bilateral symmetry, is a very frequent (although not the most frequent) manifestation of symmetry. Take a shape in the plane, and a straight line in the same plane. We can reflect all points of this shape through the straight line, and we will obtain the mirror-shape of the original. We call the two shapes symmetrical because the mirror-shape preserves many properties of the original. The length of the lines between their corresponding points, the distance of the individual points from the reflecting straight line, the angles of intersecting straight lines connecting the corresponding points, and the form of the shape do not change. There are other properties as well that do not change, such as colour, etc. At the same time, however, the direction moving around it, and the left and right sides change. We can describe what has happened thus: first, we have taken a geometric object; second, we have made a geometric transformation (reflection through the straight line), which changed some properties; third, we found that certain other geometric properties remain invariant under the given transformation. After two reflections in respect of the same straight line, we return to the original object. Similar observations can be made when reflecting three-dimensional objects (Fig. 1).

Fig. 1. Reflection. Drawing by the author

A considerable proportion of readers may mention ***rotation***, another frequent manifestation of symmetry. Take a shape in the plane and a straight line—this time perpendicular to the plane—as the axis of rotation. If we rotate all points of the shape by some (equal) angle of our choice around the axis, the rotated shape will preserve its properties; only its placement in the plane and orientation will change. Again, first, we took a geometric object; second, we made a geometric transformation (rotation around the straight line), which changed certain properties; third, we found that other properties did not change under the given transformation. If we choose an angle of rotation that is an integer (*n*) divisor of 360°, we return to the original object after n transformations (rotations). In these cases we speak of *n*-fold symmetry. Similar observations can be made when rotating three-dimensional objects (Fig. 2).

Fig. 2. Rotation. Drawing by the author

Fewer readers will mention the most frequent manifestation of symmetry: ***translation***. Take an object, a direction, and a distance. Move the object in the fixed direction by the given distance, repeating this transformation any number of times, in principle ad infinitum. We will never get back to the original, but we generate a series of copies of the original object. The object preserves all its properties, except for its location in the plane (in space). What happened? Again, first, we took a geometric object; second, we made a geometrical transformation (translation in a set direction by a given distance), which changed the object's location; third, we found, that its geometrical properties remain invariant under the given transformation. The same object can be translated in several directions. The decorative arts often make use of translation-symmetric motifs, as in frieze patterns, mosaics, window patterns on a building, or crystals in the sciences. Any repetition can represent a symmetry, such as rhythm in poetry and in music, or all periodic phenomena, like periodicity in the celestial mechanics or in the calendar (Fig. 3).

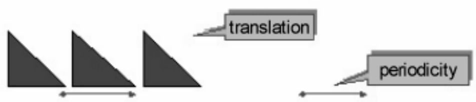

Fig. 3. Translation. Drawing by the author

The conservation of other geometric properties may serve as the basis for other types of symmetry, such as similitude, affine projection and topological symmetry. Similitude is a symmetry transformation whereby the distances between the corresponding points of two objects change, but the ratios between the lengths and the angles are preserved; thus the shape of the object remains similar to the original (Fig. 4).

Fig. 4. Similtude. Drawing by the author

Affine projection is a symmetry transformation in which straight lines are transformed into other straight lines but angles are not conserved in this transformation (Fig. 5).

Fig. 5. Affine projection. Drawing by the author

Topological symmetry is a symmetry transformation in which the neighborhood relations between the points of the object are left intact, the distances between them as well as the angles between the lines connecting them are altered. Straight lines do not necessarily remain straight. A good example of topological symmetry is the lattice of the points of a squashed sponge (Fig. 6).

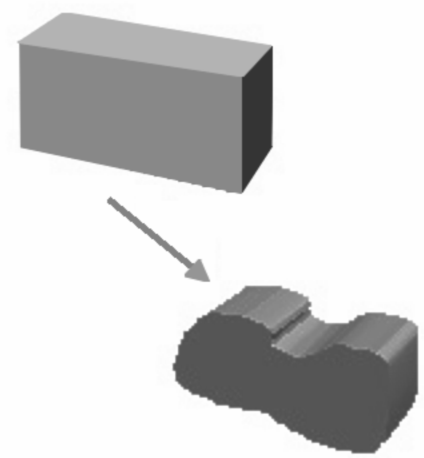

Fig. 6. Topological Symmetry. Drawing by the author

Generalisation of the Concept of Symmetry

All the symmetry types mentioned are geometrical symmetries. What they have in common is that in all cases we perform a certain geometric operation, a transformation. During this transformation one (or more) geometric properties of this geometric object remain unaltered. We say that this property is ***invariant under the given transformation***.

To generalise the concept of symmetry, first, replace the geometric transformation with any kind of transformation; second, apply such transformations not only to geometric objects, but to any kind of object; third, investigate not only their geometric properties, but consider any kind of property of the objects.

In other words, in a generalised interpretation of its meaning, we can speak of symmetry *if*

- *under any* (not necessarily geometric) *transformation* (operation)
- *at least one* (not necessarily geometric) *property*
- *of the* (not necessarily geometric) *object*

is invariant. Thus we make a generalization in respect of three things: any transformation, any object, and any of its properties.

This generalized conception of symmetry makes it possible for us to apply symmetry to the materialised objects of the physical and organic world, as well as to products of the mind. In addition to geometrical (morphological) symmetries, we can now discuss functional symmetries and asymmetries (such as may occur in the human brain, for example); gauge symmetries in physical phenomena; and properties like color, tone, light and shadow, weight, in objects of art.

Perfection And Symmetry

With this expansion of the meaning of symmetry — which opens the door to the consideration of a wide range of different phenomena as symmetries — one might ask the question: can everything around us, therefore, be considered symmetric? Before answering this in the negative, it would be wise to remind ourselves that symmetry is never perfect. Objects can be invariant in respect of certain transformations, but never under all; certain properties of objects can be invariant under a particular transformation, but not all properties. The tendency towards symmetry does not mean it is achieved perfectly everywhere or in everything.

Fig. 7. Dissymmetry in an arabesque fragment from the Alhambra. Photograph by the author

Dissymmetry. Symmetry often manifests itself in combined and generalized forms, especially in the arts. Here we should emphasize the role of dissymmetry, an expression denoting a property of objects showing symmetry in their general features, albeit slightly distorted. For example, a door is generally a mirror-symmetric object, but the asymmetrical handle distorts its bilateral symmetry. The patterns of windows on the facade of a building may show symmetry, but the curtains in the windows may be pleated in different folds in each of the symmetrically-placed windows; the windows themselves may be open or closed; the window sills may be decorated with different potted plants. This dissymmetry lends a certain liveliness to an otherwise inanimate structure. Motifs on an arabesque follow symmetrical shapes, but individual leaves may differ from their symmetrically-placed pair, and tendrils spin around each other, distorting perfect mirror symmetry. Nevertheless, we find dissymmetry beautiful; usually more beautiful than absolutely perfect symmetry (Fig. 7).

Over a hundred years ago Pierre Curie, the great physicist and crystallographer, claimed, “Dissymmetry makes the phenomenon.” What could he have meant? For a scientist, the subject of study should be sought where symmetry is distorted; what is there to study in a “perfect” object? Indeed, the dislocations of matter provide the most interesting phenomena for crystallographers. They provide one of the most interesting examples of dissymmetry: modern-day semiconductors, without which most of our devices would not work. The epoch-making discovery that allowed their manufacture was that very, very small amounts of contamination (that is, very small distortions of a once-perfect crystal) can completely change the electrical conductivity. While jewellers look for perfect gem crystals and never find them, scientists produce (almost) perfect artificial crystals and “distort” them, because they have learned how to exploit the advantages of this kind of “dis-perfection.” The history of particle physics in the recent half-century can be considered as a discovery of symmetry-breaking.

Combined symmetries. Returning to the geometric manifestations of symmetry, we say that combined symmetries occur when we perform two or more symmetry transformations on the same object and one or more properties remain invariant. For example, take an object, translate (glide) it in a particular direction for a certain distance, and then reflect it through a straight line parallel with the direction of the glide, then repeat this couple of transformations any number of times. This operation is called ***glide reflection***, and is frequently applied in frieze patterns (Fig. 8).

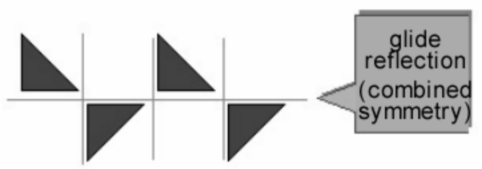

Fig. 8 (above) Glide Reflection. Drawing by the author. Fig. 9 (left) Double helix by Watson and Crick

Other geometric symmetry transformations can also be combined. To give but one example, a helix can be obtained by combining translation and rotation about the axis (Fig. 9).

Perspective And Symmetry

In the perspective of artists[2] we find a combination of the symmetry transformations affine projection and similitude. Were perspective not a symmetry transformation preserving certain features, we would not be able to recognise the original from the picture (Figs. 10 and 11).[3] An example from the Renaissance is Raphael's *School of Athens*.

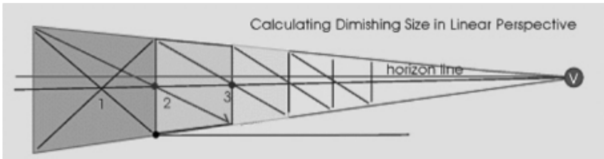

Fig. 10. Similitude and affine projection. From "Calculating Diminishing Size in Linear Perspective" © 1995, Ralph Murrell Larmann

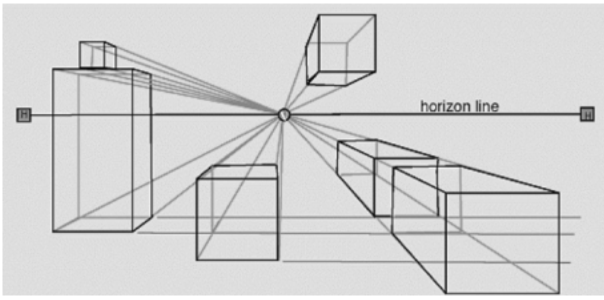

Fig. 11. One vanishing point. From "Exteriors in 1-Point Perspective" © 1995, Ralph Murrell Larmann

Similarly, we may think of aerial perspective as combining similitude and colour change (Fig. 12).

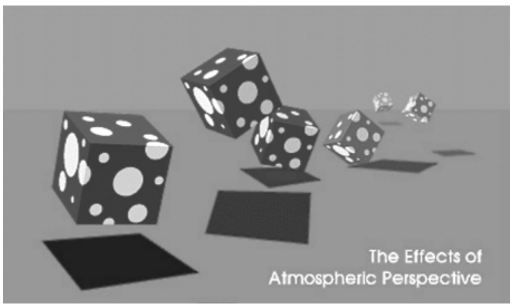

Fig. 12. Aerial perspective. From "Atmospheric or Aerial Perspective" © 1995, Ralph Murrell Larmann

Developing perspective in the arts. The application of symmetry transformations has undergone an evolution from the origin of perspective down through the centuries. For example, the single vanishing point could be doubled, and even moved (translated). Developments made it possible to represent an object from different points of view (Figs. 13, 14, and 15).

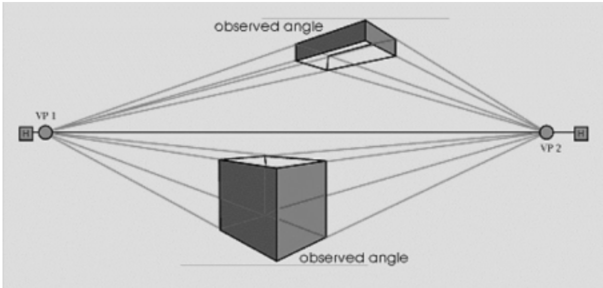

Fig. 13. Two vanishing points. From "Exteriors in 2-Point Perspective" © 1995, Ralph Murrell Larmann

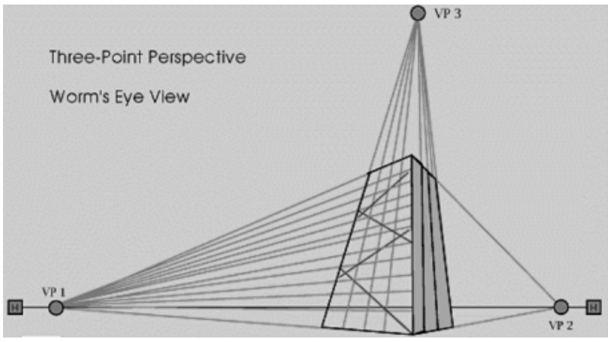

Fig. 14. Three vanishing points, worm's eye view. From "Exteriors in 3-Point Perspective" © 1995, Ralph Murrell Larmann

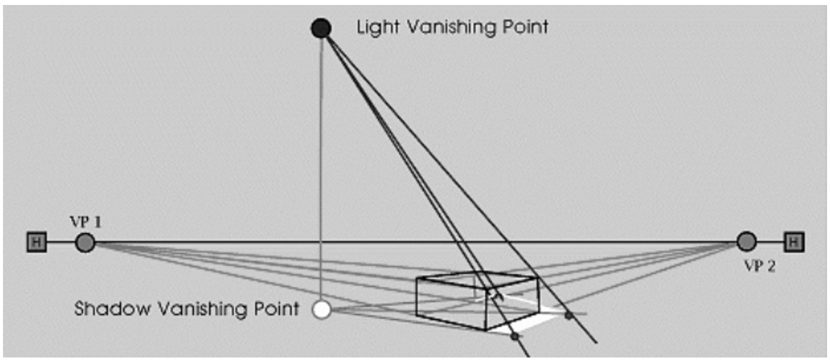

Fig. 15. More vanishing points; shadow and light vanishing points. From "The Perspective of Shadows" © 1995, Ralph Murrell Larmann

In the twentieth century, such developments were taken further, as can be seen in the work of cubists like Georges Braque and Pablo Picasso, and futurists such as Umberto Boccioni. The application of topological symmetry combined with similitude resulted in new ways of seeing, and new tools for artists. Thus they were able to give up the straight lines and fixed direction demanded by the affine projection, and replace this by a topology, making it possible for them to stress certain important features of the represented object. An example of this would be M.C. Escher's *Ascending and Descending*.

The use of two or more centres of projection, or even the dissolution altogether of fixed centers, liberated the painter from central perspective and opened up new vistas for artists to provide the spectator with multiple-sided representations of the delineated object.[4]

We ought to note that pictures liberated from central perspective and other constraints, appear at first not to be symmetrical at all. In the old-fashioned, everyday, meaning of the term which restricts symmetry operations to reflection, rotation, translation, and perhaps similitude and affine projection, the naive spectator would be right in assuming there is no symmetry. Having admitted combined symmetries and topological symmetry, however, our concept of symmetry has been expanded, making it possible for us to understand the view of artists who broke with conservative traditions and allowing us to find the beauty (via symmetry, that is, the harmony of details and balance between the parts) in artworks that dare to go beyond traditional composition. To validate this approach, and go even further, we should adopt the generalisation of the concept of symmetry described above.

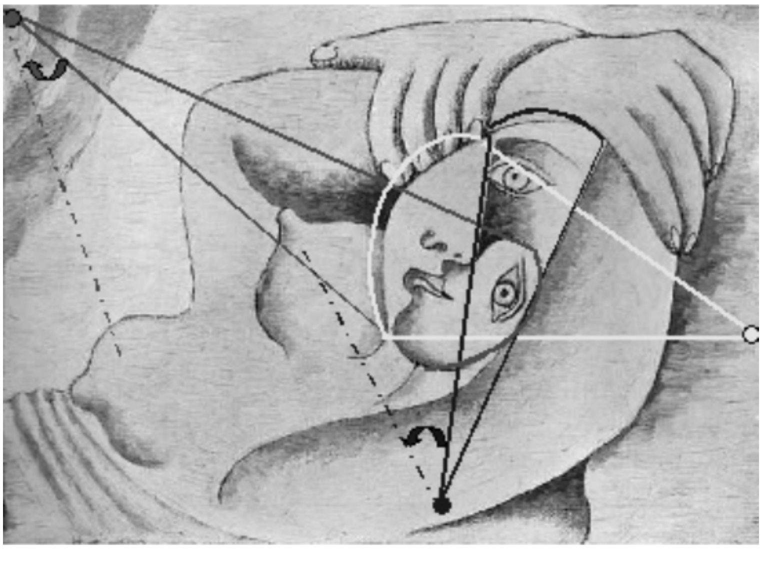

Fig. 16. Pablo Picasso, *Nude on a Beach*, 1929

There were at least two transitional phases in which the change of the traditional perspective concept appeared in early twentieth-century art. The first step was the multiplication of the viewpoints of the artist, while the vanishing point(s) were fixed. That is, the artist would see the individual details of his or her object from different directions. An example of this is Picasso's *Nude on a Beach* (1929). In Fig. 16 I have added colored straight lines to help recognize three different directions of view for the nude's face, and dotted lines to mark how one sees her breasts under equal angles from each of the three directions.

The next phase was when both the vanishing points and the artist's viewpoints moved. For example, in Picasso's *Portrait of Marie-Thérèse* (1937) (Fig. 17), one can easily recognize the different viewpoints of the artist on the face of the woman; in addition, I have added dotted lines along the edges to help recognize the moved vanishing points in the "distorted" (!) perspective of the room. The symmetry in these kinds of perspective is one where the represented object does not change, although its appearance is not that to which we are accustomed in traditional perspective artistic representation.

Fig. 17. Pablo Picasso, *Portrait of Marie-Thérèse*, 1937

These transformations appear multiplied in several cases. Multiple perspective is used by cubists in horizontal perspective, as in Braque's *Girl with mandolin* (1910) and Picasso's *Man with clarinet* (1911-12); futurists used multiple perspective in vertical perspective, as in Boccioni's *Simultaneous visions* (1911).

The roles of the horizontal and the vertical directions in our view, and therefore in perspective representation, are not identical. We perceive the length of straight lines of equal length as different depending on whether they are horizontal or vertical. Imagine a tree 10 meters high at a distance of about 300 meters in front of you; you perceive (see) the tree as being much longer (higher), than a 10 meter long (horizontal) fence at the same distance. Since their roles in the view cannot be interchanged, we say that their roles are not symmetrical; that is, not invariant under this change. Thus the perspective will be different if we look at our object horizontally or in a vertical direction from above. The latter, vertical, view is characteristic of futurists works such as Boccioni's painting,*Simultaneous visions*. Disregarding this difference in the horizontal and vertical views, however, the multiple-vanishing-points perspective appears in a similar manner in both cubist and futurist works. This is apparent in the similarities between the portrait by the futurist Boccioni, *Materia* (1911-12) and that by the cubist Picasso, *Portrait of D-H. Kahnweiler* (1910).

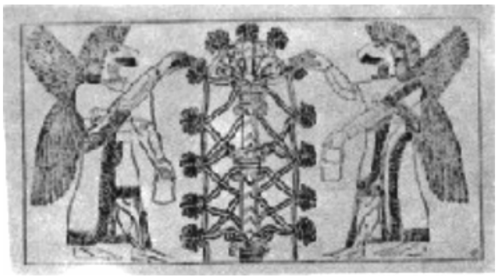

Fig. 18. Dissymmetry on a Sumerian picture (H. Weyl, *Symmetry*, fig. 4)

Reflecting Reality

How far are the images of art produced using symmetry transformations true and proper mirrors of reality (at least in generalized terms)? Any expectations the reader may entertain about a perfect and symmetrical world must now be challenged. The perfection of even the simplest mirror-images can be cast into doubt. For example, if you were to place a life-sized photograph of yourself next to a looking glass, and then were to stand first before the one and then before the other, you would find that both images would look like your perfect double. But the mirror reflection is different from the photograph. If you tried to shake hands with the photograph, your hand would cross the vertical axis between the two "bodies"; you and your photograph would form a rotational symmetric pair around this axis, your right hand would be across on the left on your photograph; the mole on the right side of your face would appear on the left side in the photograph. And what about the looking glass? Offering your right hand to the looking glass, the reflection's left hand, but to your right, would approach the mirror plane; that mole would be on the right on your image in the looking glass: you would find that your image in the looking glass (in contrast with your photograph) would be your mirror symmetric pair. If you now compared the two images, you would realize that the mirror image and the photograph are "mirror symmetric" of each other. Which is your real image? Which of the two represents your appearance more perfectly?

Does a perfect symmetrical image exist at all? In an ancient Sumerian picture previously analyzed by H. Weyl in 1952 and reproduced here in Fig. 18, the two figures stand almost symmetrically face to face. This symmetry is distorted, however, for both lift up their right hands, and the left figure's right hand is closer to us, while the right figure's right hand is further from us. Would the image be more symmetrical if one drew the picture again, changing the right figure only so that it lifts the hand closest to us? Graphically yes, by all means. However, we could say that the two figures are asymmetrical again, because one of them lifts its right hand, and the other its left. This scene, therefore, cannot be drawn in a perfectly symmetrical manner.

What are we to believe about the perfection of our world's symmetry? The facts do not demonstrate that our world lacks symmetry, but only that its symmetry is never perfect. If you look around you, you cannot fail to find many symmetrical objects, both natural and man-made. As the discussion above shows, there is symmetry, harmony and beauty in our environment. The world is almost perfect; there exist some distortions, admittedly, but it retains its dissymetry, which is still, above all, symmetry.

Symmetry is quite natural. Human beings are almsot symmetrical (who sees that our heart beats on one side, or that our motions are co-ordinated from the left hemisphere of our brain?). Flowers are symmetrical, butterflies even more so. Not perfectly, but practically symmetrical.

It seems quite natural to us that a tennis court is symmetrical. It must be symmetrical on two accounts: in geometrical terms, and in moral terms, since both the physical conditions and the rules of the game for the two players should be identical, that is, symmetrical. Nevertheless, these conditions and rules were not always so self-evident. In the sixteenth century, when this game originated, Henry VIII played it in his Hampton Court palace in an asymmetrical room, which together with the partial rules provided an advantage for the king.

The self-evident and natural qualities of symmetry may depend on local and temporary, physical and social conditions. Something similar may be said of the acceptance of artistic trends, i.e., the changes in the perception of beauty, harmony, and symmetry.

Objects and phenomena around us show signs both of symmetry and its lack at the same time. In reality, a thing is symmetrical in one or more aspects. In other words, it conserves one or more of its properties under a particular transformation, (such as a reflection or a rotation), while it is asymmetrical in other aspects: that is, its other properties are not conserved. There is no perfect symmetry (when all properties are preserved) and no perfect asymmetry (when no single property is preserved).

A very asymmetrical world would be ugly, while a very symmetrical world would be boring. This concerns not only our physical environment, but also its artistic representations. Perspective as a symmetry operation that conserves properties during the process of artistic representation, helps us to preserve this beauty in all its classical and modern forms.

Therefore we should also accept the sophisticated forms of combined symmetries that appear in modern art, and which are products of a long development in multiplying and transforming the vanishing points and the artist's viewpoints, as manifestations of perspective representation. Accept it, as it is!

Notes

1. Among these we may mention Lorenzo Ghiberti (*I Commentarii*); Leon Battista Alberti (*De pictura*, *Della pittura*); Piero della Francesca (*De prospectiva pingendi*); Luca Pacioli (*Summa de arithmetica*, *geometria, proportionalita*; *De divina proportione*); Leonardo da Vinci; and Albrecht Dürer (*Undeweysung und Messung*, *Vier Bücher von menschlicher Proportion*).
2. See the Math Forum webpage on perspective drawing: http://mathforum.org/sum95/math_and/perspective/perspect.html.
3. See "Art Studio Chalkboard: The Perspective of Shadows": http://www.saumag.edu/art/studio/chalkboard/lp-shadow.html and also "Art Studio Chalkboard: Drawing Subjects": http://www2.evansville.edu/studiochalkboard/draw.html
4. Multiple-sidedness is meant here first in geometric terms, and secondly figuratively.

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