Anomalies and accidental symmetries: charging the scalar leptoquark under $L_\mu - L_\tau$

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ABSTRACT: While the S_3 scalar leptoquark presents a possible tree-level explanation of the $b \to s\ell\ell$ flavour anomalies, it suffers from two conceptual problems which are often disregarded by model-builders. Firstly, the quantum numbers of the S_3 allow for a renormalisable diquark operator that would trigger rapid proton decay unless its coupling were tuned away. Secondly, one expects the leptoquark to have generic couplings to leptons, which require tuning to avoid stringent experimental bounds on lepton flavour violation. By gauging a U(1) current that acts as $L_\mu - L_\tau$ on the Standard Model (SM) fermions, and under which the leptoquark has charge -1, one can remedy both these problems. The additional U(1), which is spontaneously broken at some high scale, is associated with a massive Z' gauge boson and a scalar SM singlet Φ, which play no direct role in mediating the anomalous B meson decays. By computing one- and two-loop mass corrections, we show that this pair of particles can be hidden away at much higher mass scales without destabilising either the Higgs or the leptoquark masses. The only low-energy relic of gauging $L_\mu - L_\tau$ is thus the accidental global symmetry structure of the lagrangian. On the other hand, we find quite generally that an S_3 leptoquark that mediates the $b \to s\ell\ell$ anomalies cannot be much heavier than a few TeV without itself inducing large Higgs mass corrections.

Contents

1 Introduction

It is well known that the collection of anomalies recently observed in neutral current decays of B mesons [1–10] can be explained at tree level by either a leptoquark or a neutral heavy gauge boson, whose mass over coupling is around 30–40 TeV. After integrating out either of these heavy states and matching onto the weak effective theory (WET) one can produce four fermion operators of the form $bs\ell\ell$, $\ell \in \{e, \mu\}$, which can mediate the anomalies. Of the various chirality combinations, global fits to the B-anomaly data favour a non-zero Wilson coefficient for the particular WET operator [11, 12]

$$\mathcal{O}_L = (\overline{s_L}\gamma_\rho b_L)(\overline{\mu_L}\gamma^\rho \mu_L), \tag{1.1}$$

i.e. with left-handed quark and left-handed muon currents. Indeed, turning on the single Wilson coefficient for this operator provides an excellent global fit to the data with a pull of 6.6σ relative to the Standard Model (SM), according to ref. [11].[1]

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[1]In light of an update to LHCb's angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay [13], the statistical pull of this new physics scenario with respect to the SM has increased slightly.

Of the various leptoquarks that can mediate $bs\ell\ell$ processes, only a handful have the right quantum numbers to couple to left-handed quarks and lepton doublets, and so contribute to the operator $\mathcal{O}_L$. Two of these are Lorentz vector states, often called the U_1 and U_3 leptoquarks, which transform in the $(\mathbf{3}, \mathbf{1})_{2/3}$ and $(\mathbf{3}, \mathbf{3})_{2/3}$ representations of $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. A theory in which the SM is supplemented by one of these vector leptoquarks is on its own non-renormalisable, and so must be embedded in an ultraviolet (UV) completion. A variety of suitable UV completions, in which the vector leptoquark typically emerges as a massive gauge boson from some spontaneously broken non-abelian extension of the SM gauge symmetry, have been developed in great detail in recent years [14–19].

If one wishes to avoid this task of building a compelling UV completion, one might prefer to consider scalar leptoquarks, since extending the SM by a scalar leptoquark gives a consistent theory on its own (from the point of view of renormalisability — there remains the separate challenge of explaining why such a scalar should be so light compared to the Planck scale).[2] Only one scalar leptoquark, the one dubbed S_3 that transforms in the $(\bar{\mathbf{3}}, \mathbf{3})_{1/3}$ representation, has couplings to the left-handed quark and lepton doublets of the SM, via terms

$$\mathcal{L} \supset \lambda_{ij}^{QL} \overline{Q_i^c}(i\sigma_2 \sigma_a)L_j S_3^a \qquad (1.2)$$

in the lagrangian (where i,j are family indices and a is an adjoint SU(2) index, and we assume the down-aligned mass basis for quarks). Only this scalar leptoquark can thence give rise to the WET operator (1.1) after being integrating out (see figure 1), provided λ_{22}^{QL} and λ_{32}^{QL} are non-zero, which has the preferred chiral structure to explain the anomalous measurements in the $bs\mu\mu$ system (as was first proposed in [22]).

Extensions of the SM by an S_3 leptoquark suffer from two conceptual difficulties, which are often swept under the rug. Firstly, the quantum numbers of the S_3 leptoquark allow for a gauge-invariant 'diquark' operator in the lagrangian, schematically

$$\mathcal{L} \supset \lambda_{ij}^{QQ} \overline{Q_i^c} S_3 Q_j. \qquad (1.3)$$

Operators of this type violate baryon number, and in particular lead to rapid proton decay (especially the λ_{11}^{QQ} component). In order to pass the stringent experimental constraints on the proton lifetime [23], this diquark term is often set to zero in an *ad hoc* fashion (see e.g. the phenomenological studies in [24, 25]). A more elegant way to outlaw the diquark operator is to embed the S_3 leptoquark in representations of a larger unified gauge group, such as the **126**-dimensional representation of SO(10), in which the diquark operator is forbidden by the SO(10) symmetry [26, 27]. The same can be achieved in an SU(5) GUT [28, 29].

The second troublesome feature of S_3 leptoquark models concerns their couplings to the lepton sector. Based on the symmetries of the theory, one naturally expects the S_3

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[2]One attractive possibility is that the scalar leptoquark is light because it is a pseudo Nambu Goldstone boson (pNGB) associated with global symmetry breaking in a new strong sector, possibly alongside the Higgs boson within the framework of partial compositeness [20, 21].
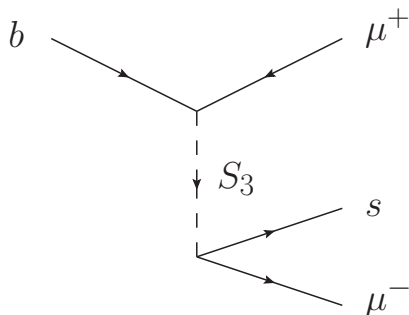
Figure 1. Our model features an S_3 scalar leptoquark that is charged under $L_\mu - L_\tau$, which therefore couples only to second family leptons. This leptoquark mediates the anomalous $b \to s\ell\ell$ transitions via a tree-level contribution that induces $\Delta C_9^\mu = -\Delta C_{10}^\mu$.

leptoquark to couple to all three generations of left-handed lepton doublets, via the lagrangian (1.2). But a generic matrix λ_{ij}^{QL} of couplings is dangerous from the point of view of phenomenology, because it leads to sizeable contributions to a number of rare lepton flavour violating (LFV) processes. For example, decays $\ell \to \ell'\gamma$ are induced at loop level (with the $\mu \to e\gamma$ branching ratio being especially tightly constrained), as are decays such as $Z \to \ell\ell'$ and $\ell \to \ell'\ell'\ell''$. Other LFV processes, such as $B \to K\mu\tau$, would be induced even at tree-level. The experimental bounds are often strongest in LFV processes involving electrons; for example, the branching ratio $B(\mu \to 3e) < 1.0 \times 10^{-12}$ at 90% C.L. [30]. As was the case for the diquark operator discussed above, the couplings of an S_3 leptoquark to electrons must be tuned close to zero by hand in order to pass these pressing constraints from LFV. Even with such aggressive assumptions, LFV measurements in the $\mu - \tau$ system place important bounds on the leptoquark couplings, as discussed for example in [29]. Note that embedding the S_3 in a grand unified theory based on SO(10) or SU(5), along the lines described in the previous paragraph, does nothing to ameliorate this problem.

In this paper, we suggest a simple mechanism for remedying both of these problems within the context of S_3 models for the neutral current B-anomalies. If the S_3 leptoquark is charged under an additional U(1) symmetry, under which the SM fields are assigned appropriate charges, then both the diquark operator (1.3) and the leptoquark couplings to electrons and tauons will be banned at the renormalisable level. In other words, one can achieve the desired structure of leptoquark couplings,

$$\lambda_{ij}^{QL} = \alpha_i \delta_{j2}, \quad \lambda_{ij}^{QQ} = 0 \tag{1.4}$$

with a particular choice of U(1) charges, which we soon discuss, where the α_i are three *a priori* complex couplings. This U(1) symmetry will be spontaneously broken, most likely by some scalar SM singlet field Φ acquiring a vev at some high energy scale above the electroweak scale, leading to a massive Z' gauge boson. (However, it is important to stress that this Z' boson plays no role in mediating the B anomalies, which are mediated solely by the S_3 leptoquark.) In order for the theory to be self-consistent, the U(1) charge assignment must be anomaly-free. The space of family-dependent anomaly-free U(1) extensions of the SM gauge symmetry has been well-studied recently [31–34], and is unhelpfully vast unless further criteria are imposed.

The gauge sector extension. We here explore an especially well-motivated choice, which is to gauge the $L_\mu - L_\tau$ current, under which we assign the S_3 leptoquark a charge of -1. This choice satisfies our desired criteria. Firstly, since all quarks are uncharged under the U(1) while the leptoquark is charged, the diquark operators (1.3) cannot be U(1) invariant, and thus baryon number is restored at the renormalisable level.[3] Secondly, of the couplings in (1.2), only the muonic contributions (λ_{i2}^{QL}) are U(1) invariant and permitted in the renormalisable lagrangian, thereby picking out the muon direction needed to mediate the anomalous B-decays, while simultaneously suppressing LFV processes induced by the leptoquark. Thirdly, the $L_\mu - L_\tau$ charge assignment is anomaly-free. We remark that a similar idea was explored in [35], in which proton decay was prohibited by imposing a global $L_\mu + 2L_e - 3L_\tau$ symmetry (which can be made anomaly-free by including three right-handed neutrinos).

The particular choice of U(1) that we gauge has additional virtues. By gauging what is otherwise an accidental symmetry of the SM we allow for a generic Yukawa sector for the quarks at the renormalisable level, as well as a strictly diagonal Yukawa matrix for the charged leptons. Indeed, if we make a renormalisable Yukawa sector a requirement, then, given only the SM chiral fermion content, the most general anomaly-free $U(1)_X$ charge assignment is of the form $L_i - L_j + aY$ where Y denotes hypercharge, for any pair of lepton numbers L_i and L_j and any rational number a.[4] As discussed in [36], a strictly diagonal charged lepton Yukawa matrix implies that the charged lepton mass eigenstates are aligned with the weak eigenstates. Hence, even though this theory features a Z' boson with non-universal couplings to charged leptons, there is no danger of LFV being induced by the Z'.

Taking a more general view, by gauging an accidental symmetry of the SM we ensure that the global part of this symmetry remains an accidental symmetry of our BSM theory, thereby guaranteeing safety from the LFV constraints. Our leptoquark model shares these virtues with a number of Z' models for the $b \to s\ell\ell$ anomalies in which the Z' also arises from gauging, say, $L_\mu - L_\tau$ symmetry [37–39], or indeed a more general linear combination of the SM's accidental symmetries [36]. By way of comparison, recall that in these Z' models the required flavour-violating couplings to down-type quarks are absent at tree level; rather, they are typically generated by introducing a fourth family of vector-like quarks with somewhat *ad hoc* couplings to the SM fields. This paper explores a different avenue in which we dispense totally with the fourth family of heavy quarks, at the expense of introducing only one extra state, the S_3 leptoquark.[5] The role of the Z' here is only to protect the accidental symmetries of the SM.

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[3]If our goal is to ban the B-violating diquark operator using a gauge symmetry, then charging the S_3 LQ under $U(1)_{L_\mu - L_\tau}$ is arguably a more minimal extension of the SM than embedding the S_3 leptoquark in an **126**-dimensional (**45**-dimensional) representation of a unified SO(10) (SU(5)) theory.

[4]We here choose to set $a = 0$ for simplicity, so that the Z' has no tree-level couplings to quarks (such couplings will, however, be generated radiatively).

[5]Another noteworthy distinction is that our $(L_\mu - L_\tau)$-charged leptoquark model explains the B-anomaly data via the left-handed operator $\mathcal{O}_L$ of (1.1), whereas models in which the corresponding $L_\mu - L_\tau$ heavy gauge boson mediates the anomalies do so via the vector-like operator $(\overline{s_L}\gamma_\rho b_L)(\overline{\mu}\gamma^\rho\mu)$. The global fits of ref. [11] favour the former scenario, largely because the latter does not give a BSM contribution to $BR(B_s \to \mu^+\mu^-)$ in which a $\sim 2\sigma$ discrepancy with the SM persists [3–6].

Finite naturalness of leptoquark models for the B anomalies. In this setup, the Z' and Φ particles associated with the extended gauge sector play no role in mediating the B-physics anomalies, and so their masses and couplings are not tied to the energy scale probed by the anomaly. It would appear that the Z' is phenomenologically inert, and that its effects can be decoupled if we wish, by taking the scale v_Φ of U(1)$_X$ breaking to very high energies, or by taking the gauge coupling g_X to be very small. Things are not quite so simple, at least *a priori*, because taking v_Φ arbitrarily large might destabilise both the Higgs mass and the mass of the S_3 leptoquark through loop corrections. Nonetheless, in section 3 we compute finite loop corrections to the Higgs and leptoquark masses, and find that there exists a limit in which the Z' and Φ masses can be taken parametrically larger than the other mass scales in the theory, while their loop corrections to the Higgs and leptoquark masses remain parametrically small. In other words, we can hide away the particles associated with the extended gauge sector at very high energies, with their only phenomenological signature being the particular symmetry structure (1.4) of the low-energy lagrangian.

Perhaps more importantly, we also compute loop corrections to the Higgs mass from loops involving the S_3 leptoquark. Because the leptoquark mediates the $b \to s\ell\ell$ anomalies, these loop corrections cannot be tuned away, and their size is unavoidably tied to the magnitude of the $b \to s\ell\ell$ anomalies. We identify the parameter space of S_3 leptoquark models (quite generally) that is finitely natural [40], i.e. for which a mass hierarchy $M_{S_3} \gg M_H$ is radiatively stable. For example, if one prefers a U(2)-like flavour structure in which the leptoquark couples predominantly to the third family, then a finitely natural hierarchy can only be maintained for leptoquark masses $M_{S_3} \lesssim 5\,\text{TeV}$ or so. Moreover, an unavoidable electroweak correction means naturalness can only be maintained for $M_{S_3} \lesssim 5.8\,\text{TeV}$, irrespective of the flavour structure.

A discussion of naturalness, and its interplay with the scale of new physics suggested by the B anomaly data, has thus far been lacking in the model-building literature addressing the B-physics anomalies. In this paper, we find that computing the Higgs mass corrections, and requiring that they are not too big, leads to interesting hints about the mass and couplings of an S_3 leptoquark that can address the B anomalies. In a future work, we will explore in more detail (and in greater generality) the implications of naturalness for models that explain the $b \to s\ell\ell$ anomalies.

The structure of the rest of the paper is as follows. In section 2 we set out the charged leptoquark model. After unpacking the various terms in the lagrangian, we discuss the issue of proton stability in section 2.1, and then describe the Z' sector of the model in section 2.2. We devote section 3 to considerations of finite naturalness, whereby which we establish that a mass hierarchy $M_{Z'} \gg M_{S_3} \gg M_H$ is stable to finite loop corrections, and moreover that a limit exists in which the Z' essentially decouples from the phenomenology. In section 4 we explore the phenomenology of the model in this "decoupling limit", before concluding.

2 Charging the S_3 leptoquark

We consider an extension of the SM by a U(1)$_X$ gauge symmetry, where $X = L_\mu - L_\tau$ for the SM fields. The U(1)$_X$ gauge symmetry will be spontaneously broken by the vacuum

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
Q_L	**3**	**2**	$\frac{1}{6}$	0
u_R	**3**	**1**	$\frac{2}{3}$	0
qd_R	**3**	**1**	$-\frac{1}{3}$	0
L_L^i	**1**	**2**	$-\frac{1}{2}$	$\delta_{i2} - \delta_{i3}$
e_R^i	**1**	**1**	-1	$\delta_{i2} - \delta_{i3}$
H	**1**	**2**	$\frac{1}{2}$	0
S_3	$\bar{\mathbf{3}}$	**3**	$\frac{1}{3}$	-1
Φ	**1**	**1**	0	$\hat{Q}_\Phi$
N_R^i	**1**	**1**	0	$\delta_{i2} - \delta_{i3}$

Table 1. The field content of the charged leptoquark model. In addition to the SM fields, there is a $U(1)_X$ gauge field with flavour non-universal couplings to SM leptons, as well as an S_3 scalar leptoquark and a SM singlet Φ. Finally, including three right-handed neutrino fields N_R^i facilitates a description of neutrino masses and mixing angles, which we comment on briefly in section 2.1.

expectation value (vev) v_Φ of a SM singlet scalar field Φ that is charged under $U(1)_X$, leading to a massive Z' gauge boson. Finally, we suppose there is an S_3 leptoquark with quantum numbers $(\bar{\mathbf{3}}, \mathbf{3})_{1/3}$ under the SM gauge symmetry, and with a charge of -1 under $U(1)_X$. This completes the field content of our model, which is summarised in table 1.

The renormalisable lagrangian contains the following terms

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + [\alpha^i \overline{Q_i^c}(i\sigma_2 \sigma_a) L_2 S_3^a + \text{h.c.}]$$

$$+ |D_\mu S_3|^2 - M_{S_3}^2 |S_3|^2 - \lambda_S |S_3|^4 - \lambda_{HS}|H|^2|S_3|^2 - \lambda'_{HS}|H^\dagger \sigma_a S_3^a|^2$$

$$+ |D_\mu \Phi|^2 + \mu_\Phi^2 |\Phi|^2 - \lambda_\Phi |\Phi|^4 - \lambda_{\Phi S}|\Phi|^2|S_3|^2 - \lambda_{\Phi H}|H|^2|\Phi|^2$$

$$- \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\sin\epsilon}{2}B_{\mu\nu}X^{\mu\nu} - g_X X_\mu \sum_\psi \hat{Q}_\psi \bar{\psi}\gamma^\mu \psi \,, \tag{2.1}$$

where $Q_i = (V_{ji}^* u_L^j, d_L^i)^T$ and $L_i = (U_{ji}^* \nu_L^j, \ell_L^i)^T$ denote the quark and lepton $SU(2)_L$ doublets, and V and U are the CKM and PMNS matrices respectively. The first line of eq. (2.1) contains all the terms from the SM lagrangian, and the all-important leptoquark coupling to quark-muon pairs, which we shall elaborate on shortly. The notation $Q^c = C\overline{Q}^T$ denotes charge conjugation, where as usual $\overline{\psi} = \psi^\dagger \gamma^0$ and the charge conjugation matrix is $C = i\gamma^2 \gamma^0$.

The second and third lines of eq. (2.1) contain the various kinetic and potential terms from the extended scalar sector of the theory, specified by the real parameters M_{S_3}, μ_Φ (both mass dimension one), λ_{HS}, λ'_{HS}, λ_Φ, $\lambda_{\Phi S}$, and $\lambda_{\Phi H}$ (all dimensionless). Note that the sign of the quadratic term for S_3 implies that its vev is at $S_3 = 0$. Finally, the fourth line contains terms associated with the new gauge boson X_μ corresponding to the gauging of $U(1)_X$, where $X_{\mu\nu} = \partial_{[\mu} X_{\nu]}$, g_X is the gauge coupling for $U(1)_X$, and $\hat{Q}_\psi$ denotes the charge of a SM fermion ψ under $U(1)_X$, as given in table 1. The second term on the fourth

line is a tree-level contribution to kinetic mixing between the new $U(1)_X$ gauge boson and B_μ, the gauge field for hypercharge. We shall discuss the effects of this kinetic mixing in section 2.2.

In order to unpack the leptoquark couplings in eq. (2.1), it is helpful to define linear combinations of the components S_3^a which are eigenstates of the electric charge operator, *viz.*

$$S_3^{4/3} = \frac{S_3^1 - iS_3^2}{\sqrt{2}}, \qquad S_3^{1/3} = S_3^3, \qquad S_3^{-2/3} = \frac{S_3^1 + iS_3^2}{\sqrt{2}}, \qquad (2.2)$$

where the superscript $4/3$, $1/3$, and $-2/3$ denote electric charges. Then, after expanding the quark and lepton $SU(2)$ doublets, eq. (2.1) decomposes into the terms

$$\begin{aligned} \mathcal{L} \supset \ & -\alpha^i U_{j2}^* S_3^{1/3} \overline{d_i^c} P_L \nu^j - \sqrt{2}\alpha^i S_3^{4/3} \overline{d_i^c} P_L \mu \\ & + \sqrt{2}\alpha^k V_{ik}^* U_{j2}^* S_3^{-2/3} \overline{u_i^c} P_L \nu^j - \alpha^k V_{ik}^* S_3^{1/3} \overline{u_i^c} P_L \mu + \text{h.c.}, \end{aligned} \qquad (2.3)$$

The second term on the right-hand-side gives the leptoquark coupling to muons and down-type quarks, which shall be responsible for mediating the anomalous B meson decays, which we address in section 4.1.

Importantly, quark flavour violation is linear [41] in our model,[6] with new physics coupling to the particular direction in $SU(3)$ quark flavour space

$$\tilde{Q} \equiv \alpha^i Q_i, \qquad (2.4)$$

where $\alpha_i \in \mathbb{C}^3$ is a 3-component complex vector. This vector could be naturally aligned in some sense with the SM flavour structure, with a plausible ansatz being that $(\alpha_1, \alpha_2, \alpha_3) \propto (V_{ub}, V_{cb}, V_{tb})$ up to order one numbers. This is what one expects in several well-motivated cases, such as the case where the S_3 leptoquark is embedded in a partial compositeness framework [21, 43, 44] (which, as we have already suggested, would offer an attractive explanation for the scalar leptoquark being light). Such an ansatz also follows from the minimal flavour violation (MFV) hypothesis [45, 46], or more generally from imposing a $U(2)^n$ flavour symmetry on the new physics (see e.g. [46–48]). In what follows, we shall refer to such flavour structures in the quark sector as being "U(2)-like", in general.

As we discussed in the Introduction, the lepton sector is very clean. The alignment of new physics with the muon is guaranteed by charging the leptoquark under a lepton family-non-universal gauge symmetry, here $X = L_\mu - L_\tau$ (on the SM fields). This offers a natural, symmetry-based explanation for the observation of lepton flavour universality violation in the measurements of R_K and R_{K^*}, in particular the preferred alignment of new physics with the muon direction, while also preventing dangerous new physics contributions to lepton flavour violation.

At this stage, keeping all the terms generically allowed in the lagrangian (2.1), we have introduced 14 new parameters:

$$\{\alpha_i, g_X, M_{S_3}, \lambda_S, \lambda_{HS}, \lambda'_{HS}, \mu_\Phi, \lambda_\Phi, \lambda_{\Phi S}, \lambda_{\Phi H}, \epsilon, \hat{Q}_\Phi\}. \qquad (2.5)$$

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[6]Consequently, the charged leptoquark framework (as for any single leptoquark model) is also an example of the more general flavour structure known as 'rank-one flavour violation', explored recently in [42].

The mass parameter μ_Φ is related to the other parameters v_Φ, M_X and M_Φ as

$$\mu_\Phi = \sqrt{\lambda_\Phi}\, v_\Phi = \frac{\sqrt{\lambda_\Phi} M_X}{g_X \hat{Q}_\Phi} = \frac{M_\Phi}{\sqrt{2}}. \tag{2.6}$$

Hence we can eliminate $\{\mu_\Phi, \lambda_\Phi\}$ and write everything in terms of the equivalent set

$$\{\alpha_i, g_X, M_{S_3}, M_\Phi, M_X, \lambda_S, \lambda_{HS}, \lambda'_{HS}, \lambda_{\Phi S}, \lambda_{\Phi H}, \epsilon, \hat{Q}_\Phi\}. \tag{2.7}$$

Eventually, we will see how this set of parameters can be trimmed down substantially when we come to discuss phenomenology.

2.1 Proton stability

One of our main motivations for charging the S_3 leptoquark under a $U(1)_X$ gauge symmetry is to preserve baryon number (most pressingly, to prevent proton decay), which is an accidental symmetry of the SM. If the leptoquark had no $U(1)_X$ charge, then nothing forbids one from writing down the renormalisable 'diquark' operators

$$\mathcal{L} \supset \lambda_{ij}^{QQ}\, \overline{Q_i^c}(i\sigma_2 \sigma_a S_3^a)^\dagger Q_j, \tag{2.8}$$

where σ_a are the Pauli matrices, and Q^c denotes charge conjugation as defined above. These operators violates baryon number, and would facilitate proton decay by direct s-channel leptoquark exchange. Because the leptoquark in our theory is charged under $U(1)_X$, while all quark fields are neutral, the operator 2.8 clearly carries net $U(1)_X$ charge and so is banned by the $U(1)_X$ gauge symmetry.

To achieve proton stability, however, it does not suffice to simply check whether the diquark operators are present or not; baryon number (B) can also be violated due to terms in the scalar potential if there are new scalars that couple to SM fermion bilinears. For an example, consider the $\tilde{R}_2$ scalar leptoquark, which has SM quantum numbers $(\bar{\mathbf{3}}, \mathbf{2})_{1/6}$. Even though there is no diquark operator, there is a quartic coupling $\tilde{R}_2 \tilde{R}_2 \tilde{R}_2 H$ which, after integrating out $\tilde{R}_2$ and expanding the Higgs around its vev, leads to the decay $p \to \pi^+ \pi^+ e^- \nu \nu$ at tree-level, and thus to proton instability [49].

In general, there are three types of potentially B-violating term that can appear in the (renormalisable) potential of an extended scalar sector [49]. Given at least two new scalars X_1 and X_2, there may be (i) 3-scalar terms of the form $X_1 X_1 X_2$; (ii) 4-scalar terms of the form $X_1 X_1 X_1 X_2$; and (iii) 4-scalar terms of the form $X_1 X_1 X_1 H$ or $X_1 X_1 X_2 H$. In ref. [14] it was shown that *any* of the scalar leptoquarks capable of explaining the B anomaly data lead to dangerous tree-level proton decay, either through such terms in the scalar potential or through diquark operators.[7] In our model there are two new scalars, S_3 and Φ. By

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[7]The vector leptoquarks U_1 and U_3, however, are naturally free of tree-level proton decay. Of the scalar leptoquarks, only the R_2 state (which has SM quantum numbers $(\bar{\mathbf{3}}, \mathbf{2})_{7/6}$) does not induce tree-level proton decay — but this leptoquark predicts $R_{K^{(*)}} > R_{K^{(*)}}^{\text{SM}}$, and so does not provide a good model for the B-physics anomalies (on its own). However, all three of these leptoquarks — R_2, U_1, and U_3 — admit dimension-5 operators that violate baryon number and facilitate proton decay, schematically of the form $QQH\Phi/\Lambda$, where Φ denotes the leptoquark. In the case of the U_1 leptoquark, this dimension-5 operator is forbidden when the U_1 is embedded in, say, a Pati-Salam grand unified theory.

charging the S_3 under $U(1)_X$ one prevents diquark operators as discussed above, and nor are there any permissible terms in the scalar potential that violate B. (Simply by adding up hypercharges, one can rule out operators of all three types listed above.)

Finally, we consider the possibility that baryon number could be violated due to higher-dimension non-renormalisable operators. Schematically, baryon number could be violated by dimension-5 operators involving two quarks and two scalars, one of which is the lepto-quark. If the other scalar is the Higgs then such an operator cannot possibly be $U(1)_X$ invariant. Neither do there exist invariant operators of the form $QQXX^{(\dagger)}$ for $X \in \{S_3, \Phi\}$. However, there is potentially a gauge-invariant dimension-5 operator of the form

$$\frac{c_{ij}}{\Lambda}\overline{Q_i^c}(i\sigma_2\sigma_a S_3^a)^\dagger Q_j \Phi^{(\dagger)}, \tag{2.9}$$

where Λ denotes the cut-off scale of our BSM effective theory (somewhere beyond the scales associated to the S_3, Φ, and X_μ particles). This operator violates baryon number — indeed, it reduces precisely to the diquark operator once Φ acquires its vev.

One might think that this dimension-5 operator could also be forbidden, simply by choosing a charge $\hat{Q}_\Phi \notin \{-1, +1\}$. However, things are not so simple; the charge $\hat{Q}_\Phi$ cannot be freely chosen if our low-energy leptoquark model is to embed in a microscopic theory that can account for neutrino oscillation data, as follows.

As for any model in which $L_\mu - L_\tau$ is gauged, Majorana mass terms are necessary to reproduce the PMNS matrix (see e.g. [50–52]), at least if we assume that a traditional see-saw mechanism is responsible [53]. If we include three right-handed neutrinos with charges 0, $+1$ and -1, which allows strictly diagonal Dirac masses via Yukawa couplings to the Higgs, then a SM singlet scalar field Φ with $U(1)_X$ charge of ± 1 is needed in order to allow off-diagonal Majorana mass terms that involve the first family leptons. The most economical choice is for that scalar field Φ to be the sole BSM scalar field in the model.[8] In that case there are two zero entries in the 3-by-3 right-handed Majorana neutrino mass matrix at the renormalisable level (in the (2,2) and (3,3) positions), which permits a phenomenologically successful description of neutrino oscillations [54, 55]. But this charge assignment $\hat{Q}_\Phi = \pm 1$ immediately allows for the dimension-5 diquark operator (2.9), leading to B-violating effects suppressed by one power of the ratio v_Φ/Λ. The EFT cutoff scale Λ must therefore be pushed high enough to suppress proton decay.

2.2 The $U(1)_X$ sector

The role of the SM singlet scalar field Φ is to spontaneously break $U(1)_X$ by acquiring a non-zero vev, and thus give mass to the corresponding gauge boson, which we denote by the field X_μ (but we will usually refer to the corresponding particle as a Z' in the text). From eq. (2.1), with $\mu_\Phi, \lambda_\Phi > 0$, we find that $\langle\Phi\rangle = v_\Phi/\sqrt{2} = \sqrt{\mu_\Phi^2/2\lambda_\Phi}$. Writing $\Phi(x) = (v_\Phi + \varphi(x))/\sqrt{2}$, one finds that the Z' boson gets a mass of

$$M_X = g_X|\hat{Q}_\Phi|v_\Phi, \tag{2.10}$$

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[8]A richer setup was explored in [50], in which $U(1)_X$-charged scalar fields transforming as electroweak doublets were also considered in the $U(1)_X$-breaking sector, allowing off-diagonal Dirac mass terms for neutrinos.

while the scalar field φ has a mass

$$M_\Phi^2 = 2\mu_\Phi^2 = 2\lambda_\Phi v_\Phi^2 \tag{2.11}$$

Note that there is no Z-Z' mass-mixing in this model, because the Higgs is uncharged under $\mathrm{U}(1)_X$. However, as we will soon see, there is loop-induced kinetic mixing between the Z and Z', which induces couplings of the Z' to quarks.

Of the SM fermions, the Z' boson has tree-level couplings only to leptons, from eq. (2.1). Explicitly, we have the couplings

$$\mathcal{L}_{\ell Z'} = g_X X_\rho \left(\overline{\mu}\gamma^\rho \mu - \overline{\tau}\gamma^\rho \tau + \overline{\nu}_i \gamma^\rho (U\xi U^\dagger)_{ij} P_L \nu_j \right), \tag{2.12}$$

where all fermion fields are written in the physical mass basis, and $\xi = \mathrm{diag}(0, 1, -1)$.

In the lagrangian (2.1) we include a kinetic mixing term $\mathcal{L} \supset \frac{\sin\epsilon}{2} B_{\mu\nu} X^{\mu\nu}$ between the hypercharge and $\mathrm{U}(1)_X$ gauge fields. Even if the tree-level kinetic mixing were small there will be radiative contributions of order

$$\delta \sin \epsilon \sim -\frac{g_X g'}{16\pi^2} \ln \frac{\mu^2}{M_{S_3}^2} \tag{2.13}$$

at one-loop, where μ is the renormalisation scale and M_{S_3} denotes the leptoquark mass. The physical hypercharge and Z' gauge bosons are the eigenstates of the matrix of kinetic terms, $B'_\mu = B_\mu + \sin\epsilon X_\mu$ and $X'_\mu = \cos\epsilon X_\mu$. Substituting the inverse transformation into the lagrangian and expanding in the physical fields B'_μ and X'_μ, one finds that the couplings of the physical hypercharge boson B'_μ are unchanged by this kinetic mixing.[9] The X'_μ couplings acquire a subleading contribution proportional to hypercharge, giving small flavour-universal couplings of the Z' to quarks and thus to protons, proportional to $\tan\epsilon \ll 1$.[10] At a generic point in parameter space, this would give bounds on g_X and M_{S_3} from direct Z' production at the LHC. Ultimately, however, we will identify a finitely natural limit in which the Z' decouples completely from the phenomenology, in which these bounds will be unimportant.

3   Finite naturalness of the scalar sector

There are three heavy BSM particles in our theory; the heavy gauge boson X_μ, the SM singlet Φ, and the S_3 leptoquark. These particles can run in loops which induce corrections to the masses of SM particles. In particular, the Higgs boson, being the only scalar in the SM, will receive corrections to its squared mass that grow quadratically with the heavier masses in our BSM theory. These corrections are calculable within our model, if one assumes that there are no states to be discovered at yet higher energies (or neglects their contributions). Moreover, the additional scalars Φ and S_3 will themselves receive calculable

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[9]This should be contrasted with the effects of mass-mixing between B_μ and X_μ, which occurs for models in which the Higgs is charged under $\mathrm{U}(1)_X$ (see e.g. [56–58]). In that case, the Z boson acquires corrections to its couplings that may be flavour non-universal.

[10]For muon and tau leptons, the kinetic mixing induces only a small correction compared to the direct couplings to the Z', which can be neglected.

mass corrections, and we might expect the lighter of the two to be dragged up by such a loop correction to the higher mass scale. A theory in which the finite loop corrections do not need to be fine-tuned to produce the observed values of couplings and masses is 'finitely natural' [40].

To be concrete, if we denote the one-loop finite correction to the Higgs mass-squared by δM_H^2, then finite naturalness requires that

$$\delta M_H^2 \lesssim M_H^2 \times \Delta, \tag{3.1}$$

where Δ indicates the degree of fine-tuning that one is willing to tolerate. For the SM itself, eq. (3.1) is satisfied with $\Delta = 0.13$ at a renormalisation scale $\mu = m_t$ [40], meaning that the SM is a finitely natural theory. One might view this lack of tuning as merely a coincidence, or, more optimistically, as an attractive feature of the SM that we should seek to preserve when we go beyond the SM.

Because the $b \to s\ell\ell$ anomalies occur in rare processes that are loop and GIM suppressed in the SM, the scale of new physics required is rather heavy, corresponding to a mass over coupling of order 30–40 TeV or so. One might therefore expect the finite corrections to the Higgs mass to be large, and require significant fine-tuning for the observed mass of $M_H \approx 125$ GeV to be finitely natural. We here confront this expectation, focussing on a régime in which there is a strong hierarchy between the three mass scales,

$$M_H \ll M_{S_3} \ll M_\Phi. \tag{3.2}$$

We will show that this mass hierarchy is in fact finitely natural for certain regions of parameter space that can accommodate the B anomalies (at the best fit point). In particular, in section 3.2 we consider the limit where

$$g_X \to 0 \quad \text{and} \quad v_\Phi \to \infty, \quad \text{such that } M_X \text{ remains finite}, \tag{3.3}$$

where recall $M_X = g_X |\hat{Q}_\Phi| v_\Phi$. In this limit the low-energy phenomenology is much simplified, as we discuss in section 4, because the Z' boson decouples.

In this decoupling limit, the Higgs mass (and the hierarchy (3.2)) can be finitely natural provided the leptoquark is not too heavy (but heavy enough to evade the LHC direct search bounds), and provided its coupling α_3 to the top quark is not too large with respect to α_2. These results therefore have interesting implications for model-building frameworks for the B anomalies which invoke a U(2)-like flavour structure, for which couplings of new physics to the third family are very large. The implication is that, for such a flavour structure, the leptoquark should not be much heavier than 5 TeV if we do not wish to destabilize the Higgs mass, which is not too far from the current reach of LHC direct searches. We will discuss the implications of finite naturalness for more general classes of explanations of the B anomalies in a follow up paper.

3.1 Higgs mass stability

There are one-loop corrections to the Higgs mass due to the S_3 leptoquark running in a loop (see figure 2).[11] In the limit that $M_{S_3} \gg M_H$, we find (see appendix B for details of

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[11]We make the simplifying assumption throughout that $\lambda_{\Phi H} = 0$. This assumption is arguably natural because $\lambda_{\Phi H}$ is not generated at one-loop. Moreover, it is pragmatic for two reasons. Firstly, if $\lambda_{\Phi H} \neq 0$
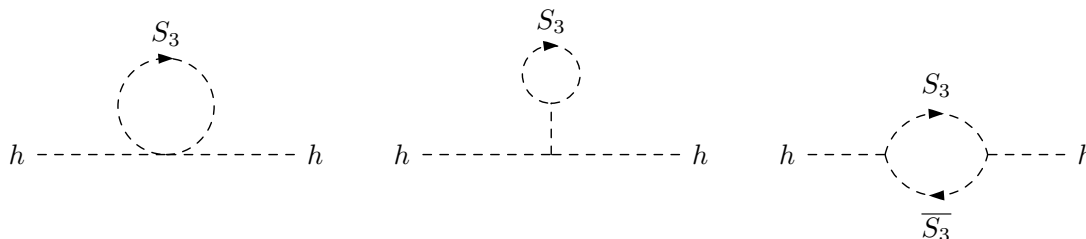
Figure 2. One-loop Feynman diagrams contributing to the Higgs mass, due to the S_3 leptoquark running in the loop.

the computation)

$$(\delta M_H^2)_{HS} = -\frac{9M_{S_3}^2}{16\pi^2}\left[(\lambda_{HS} + \lambda'_{HS})\left(1 + \ln\frac{\mu^2}{M_{S_3}^2}\right) + \mathcal{O}\left(\frac{v^2}{M_{S_3}^2}\right)\right], \qquad (3.4)$$

where μ is the renormalisation scale. It appears that this one-loop correction can be made arbitrarily weak by taking the tree-level couplings $\lambda_{HS}^{(\prime)}$ to be small. However, the coupling λ'_{HS} is generated at one-loop due to the Yukawa-like couplings α_i,[12] whose sizes are fixed by the $b \to s\ell\ell$ anomalies.

We estimate the size of a radiatively-generated λ'_{HS} by computing its one-loop beta function (see appendix A), which may be substituted into (3.4) to give a lower estimate of the correction (3.4),

$$(\delta M_H^2)_{\text{anomaly}} \approx \frac{9G_F M_{S_3}^2 m_t^2 |\sum_q \alpha_q V_{tq}|^2}{32\sqrt{2}\pi^4}\left(1 + \ln\frac{\mu^2}{M_{S_3}^2}\right)\ln\frac{\mu^2}{m_t^2} + \dots, \qquad (3.5)$$

where M_t is the top mass, and the subscript 'anomaly' now indicates that this estimated loop correction is tied to the size of the $b \to s\ell\ell$ anomalies. The finite naturalness criterion (3.1) then implies

$$M_{S_3} \lesssim \begin{cases} \dfrac{4.7 \text{ TeV}}{|\alpha_3 + V_{ts}\alpha_2|\sqrt{\ln M_{S_3}^2/m_t^2}}\sqrt{\Delta} & \text{for } \mu = M_{S_3}, \\[4mm] \dfrac{540 \text{ GeV}}{|\alpha_3 + V_{ts}\alpha_2|\sqrt{74 + \ln M_{S_3}^2/m_t^2}}\sqrt{\Delta} & \text{for } \mu = 10^{16}M_{S_3} \sim M_{\text{Planck}}, \end{cases} \qquad (3.6)$$

where in each case μ indicates the renormalisation scale used.[13] Note that this finite naturalness bound depends on the scale of new physics required to explain the B-anomalies, through both the couplings α_i and the leptoquark mass.

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then the symmetry breaking pattern is potentially more complicated, with the vevs of H and Φ being determined by the true minimum of the scalar potential. Secondly, $\lambda_{\Phi H} \neq 0$ would give a potentially large tree-level correction to the Higgs mass.

[12]It turns out that to leading order only λ'_{HS} is generated by one-loop diagrams; λ_{HS} is suppressed relative to it by $m_b^2/m_t^2 \sim 10^{-3}$.

[13]The rationale in presenting two bounds is to estimate how strongly the finite naturalness corrections depend on the scale; the first bound would be appropriate if relevant new physics were to enter just beyond the scale of the leptoquark, while the second bound would be appropriate if there were a 'desert' of no new physics all the way to the Planck scale.

The calculation (3.5) is the combination of two one-loop integrals, and so amounts to a two-loop correction to the Higgs mass. We should therefore consider other corrections at two-loop order. Totally generically, there is a two-loop Higgs mass correction due to electroweak gauge bosons running in the loop, for any BSM theory with an electroweakly-charged scalar [40]. For the S_3 leptoquark, that correction is

$$(\delta M_H^2)_{\text{EW}} = -\frac{\alpha^2 M_{S_3}^2}{32\pi^2}\left(\frac{18}{s_W^4} + \frac{1}{c_W^4}\right)\left(3\ln^2\frac{M_{S_3}^2}{\mu^2} + 4\ln\frac{M_{S_3}^2}{\mu^2} + 7\right), \qquad (3.7)$$

where α is the fine structure constant. For this correction, the finite-naturalness criterion (3.1) implies

$$M_{S_3} \lesssim \begin{cases} 5.8 \text{ TeV}\sqrt{\Delta} & \text{for } \mu = M_{S_3}, \\ 120 \text{ GeV}\sqrt{\Delta} & \text{for } \mu = 10^{16}M_{S_3} \sim M_{\text{Planck}}, \end{cases} \qquad (3.8)$$

for the same pair of renormalisation scales used above. We stress that this second Higgs mass correction is inevitable in *any* model featuring the S_3 leptoquark, simply due to its quantum numbers under the electroweak gauge group. This bound, unlike (3.6), is independent of the Yukawa-like couplings α_i, depending only on the leptoquark mass.

There is also a two-loop correction to the Higgs mass that is very specific to our model, featuring the heavy X boson being exchanged within a muon or tauon loop. However, these corrections scale like $\delta M_H^2 \sim \frac{m_{\mu,\tau}^2}{v^2}\frac{g_X^2 M_X^2}{(4\pi^2)^2}$, and so they vanish in the 'decoupling limit' specified by eq. (3.3).

Typically, one of the two bounds (3.6) or (3.8) will be dominant in a given region of parameter space, for a given choice of Δ.[14] In figure 4, we plot the stronger of the two finite naturalness bounds at any given point in the parameter space. We see that for $\alpha_2/\alpha_3 \ll 1$ and larger leptoquark masses, the bound (3.6) tends to dominate, while for light leptoquarks and/or a larger α_2 contribution to the B-anomalies, the purely electroweak correction (3.8) can dominate.

One should also study the stability of the leptoquark mass, which will receive finite loop corrections that are sensitive to the mass scale of the heavy states associated with the $U(1)_X$ gauge sector. We find that, in the decoupling limit (3.3), finite naturalness of the leptoquark mass gives only very weak constraints on the parameters of the model, that have no impact on the phenomenology. We therefore relegate these calculations to appendix B.2.

3.2 Decoupling the $U(1)_X$ sector

In the limit given by eq. (3.3), in which we take $g_X \to 0$[15] and $v_\Phi \to \infty$ while keeping their product (and thus M_X) finite, all the finite loop corrections to either M_H^2 or $M_{S_3}^2$ that

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[14]A refined estimate of the finite naturalness bound would be obtained by doing a full calculation of the two-loop Higgs mass correction. Here, we are content to provide only a rough estimate of the important two-loop Higgs mass contributions for the parameter space regions of interest.

[15]The reader might worry about taking $g_X \to 0$, because there is growing evidence for the weak gravity conjecture which suggests that one cannot in fact take a gauge coupling to be arbitrarily small [59]. However, the lower limit on g_X is expected to be of order m/M_{Planck} for order-1 charges, where m is the mass of a $U(1)_X$ charged particle in the spectrum. Thus, for all practical purposes we can take g_X as small as we need without running the risk of violating this bound from weak gravity.

involve either the X boson or the heavy scalar Φ running in the loop tend to zero (at least as fast as $g_X^2 M_X^2$). Moreover, even though the Z' mass remains finite, all the physics effects of exchanging the Z' depend only on the parameter v_Φ (as long as the Z' is heavy enough not to be produced on resonance), and so disappear in the limit $v_\Phi \to \infty$, regardless of the coupling. (Note that the mass of the scalar field $M_\Phi = \sqrt{2\lambda_\Phi} v_\Phi \to \infty$ in this limit.) Thus, there is a limit in which the X and Φ are heavy enough that they totally decouple from the low-energy phenomenology, without inducing large finite mass corrections to the leptoquark or the Higgs.

Indeed, this is not all that surprising from the point of view of symmetry. If one takes to zero the various parameters that mix the U(1)$_X$ sector with the SM + S_3 sector, one finds that the lagrangian (2.1) exhibits an enhanced $G = (\text{Poincaré})^2$ spacetime symmetry, which acts by performing independent Poincaré transformations within each sector. Thus, by a famous argument of 't Hooft [60], one expects this limit to be radiatively stable, i.e. one expects the various mass corrections to be small (and indeed to vanish in the limit of complete decoupling).[16]

However, we have seen that the finite corrections (3.5) to the Higgs mass arising from leptoquark loops cannot be made arbitrarily small, because they are tied to the flavour anomalies through the dependence on the couplings $\alpha_{2,3}$. There is also an unavoidable two-loop Higgs mass correction (3.7) due to electroweak gauge bosons running in the loop, which have fixed couplings to the S_3. Both these Higgs mass contributions arise generically for an S_3 leptoquark that can explain the $b \to s\ell\ell$ anomalies. We visualize the 'bounds' from finite naturalness of the Higgs mass, which translate into an upper bounds on M_{S_3}, in figures 3 and 4. For suitably small values of the fine-tuning parameters Δ, the scalar mass hierarchy (3.2) is finitely natural.

4 Phenomenological analysis in the decoupling limit

We now turn to the phenomenology of our charged leptoquark model. At a general point in the parameter space the phenomenology is rich, with constraints associated both to the leptoquark and Z' dynamics. The latter includes constraints from direct Z' production at the LHC, neutrino trident production, and four-lepton production. However, in the previous section we identified a limit in which the Z' decouples from the phenomenology. In this section we analyse the phenomenology in this "decoupling limit" only.

The parameter space is reduced from the set of 14 parameters in (2.7) to the smaller 5-parameter set $\{\alpha_i, M_{S_3}, \lambda_S\}$. Furthermore, for the heavy leptoquark masses of interest,[17] the leptoquark quartic self-coupling λ_S plays little role in the phenomenology, and so we neglect it henceforth. We are thus left with the following set of four parameters,

$$\{\alpha_1, \alpha_2, \alpha_3, M_{S_3}\}, \tag{4.1}$$

which determine the leptoquark phenomenology.[18]

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[16]Note also that in this limit of decoupling the Z' the radiatively-induced kinetic mixing between the Z' and hypercharge gauge bosons also goes to zero, which can again be traced back to the radiative stability of decoupling the U(1)$_X$ sector from the SM + S_3 sector.

[17]The leptoquark mass will be at least 1.7 TeV or so to avoid direct search bounds — see section 4.3.

[18]Note that while most of the parameters we have removed are taken to zero (or infinity) in our decoupling limit, the coupling λ'_{HS} remains finite, but may be written in terms of α_3 using the one-loop estimate of appendix A.

The main experimental constraints on our model in this decoupling limit will be three-fold, coming from (i) the fit to the neutral current B anomaly data (section 4.1), (ii) neutral meson mixing (section 4.2), and (iii) direct leptoquark searches at the LHC (section 4.3). We visualize this combination of constraints in two ways in figures 3 and 4; in the former, we fix benchmark values of the leptoquark mass and plot the constraints in the α_2 *vs.* α_3 plane. Then, to study the mass-dependence, in figure 4 we fix the value of $\Delta C_9^\mu = -\Delta C_{10}^\mu$ using the central value of the global fit [12] to the B anomaly data, and plot the remaining constraints in the α_2/α_3 *vs.* M_{S_3} plane. In both plots we assume for simplicity that the α_i couplings are all real. We also include the 'bounds' (3.6) or (3.8) from finite naturalness of the Higgs mass, calculated with $\mu = M_{S_3}$, which we see have interesting implications on the favoured parameter space.

4.1 Neutral current B anomalies

Assuming that the new physics behind the collection of experimental discrepancies in $b \to s\ell\ell$ occurs only in the muonic channel, we consider the following terms in the WET,

$$\mathcal{L}_{bs\mu\mu}^{\mathrm{NP}} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(\Delta C_9^\mu \mathcal{O}_9^\mu + \Delta C_{10}^\mu \mathcal{O}_{10}^\mu\right), \tag{4.2}$$

where

$$\mathcal{O}_9^\mu = \frac{\alpha}{4\pi} \left(\overline{s}_L \gamma_\mu b_L\right) \left(\overline{\mu}\gamma^\mu \mu\right), \tag{4.3}$$

$$\mathcal{O}_{10}^\mu = \frac{\alpha}{4\pi} \left(\overline{s}_L \gamma_\mu b_L\right) \left(\overline{\mu}\gamma^\mu \gamma_5 \mu\right). \tag{4.4}$$

Integrating out the S_3 leptoquark and matching onto the WET, we obtain

$$\Delta C_9^\mu = -\Delta C_{10}^\mu = \frac{\pi}{\sqrt{2}G_F M_{S_3}^2 \alpha_{\mathrm{EM}}} \left(\frac{\alpha_3 \alpha_2^*}{V_{tb}V_{ts}^*}\right). \tag{4.5}$$

As anticipated (and intended), the leptoquark generates only the operator with left-handed quark current and left-handed lepton current, and so our model predicts $R_K = R_{K^*}$, which gives a good fit to the B-anomaly data [11, 12].

A recent fit for a purely $\Delta C_9^\mu = -\Delta C_{10}^\mu$ scenario [12] (specifically, we use the results of the 2020 addendum, which includes the updated P_5' result [13]) finds $\Delta C_9^\mu = -\Delta C_{10}^\mu = -0.50$ as the best-fit central value, and $-0.69 \leq (\Delta C_9^\mu = -\Delta C_{10}^\mu) \leq -0.32$ as the 2σ range.[19] Assuming that $\alpha_3 \alpha_2^*$ is real and positive, this implies that

$$\frac{M_{S_3}}{\sqrt{\alpha_3 \alpha_2^*}} = 34.7 \text{ TeV} \tag{4.6}$$

as a central value, or

$$30.4 \text{ TeV} \leq \frac{M_{S_3}}{\sqrt{\alpha_3 \alpha_2^*}} \leq 44.6 \text{ TeV} \tag{4.7}$$

as the corresponding 2σ interval. We have used `CKMfitter` (Summer 2019 version) [66, 67] for the CKM input, and neglected the small imaginary part of V_{ts}. For the fixed values $M_{S_3} = 2$, 5 TeV, this 2σ band is plotted (blue) in figure 3.

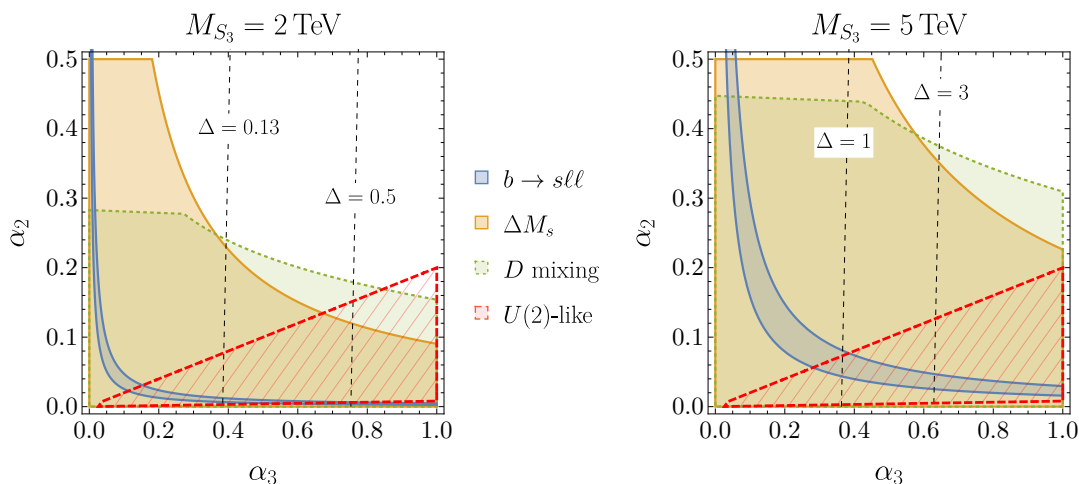[19] Various other fitting methodologies (see [61–65]) yield similar results.

Figure 3. Constraints on the leptoquark couplings to $s\mu$ (α_2) and $b\mu$ (α_3), for fixed leptoquark masses of 2 TeV (left) and 5 TeV (right), both of which are above the mass range excluded by LHC direct searches. In addition to the 2σ contours from a global fit to the $b \to s\ell\ell$ data (blue), from B_s mixing constraints (orange), and from D mixing constraints (green, dotted), we plot contours from finite naturalness of the Higgs mass, in this case coming from eq. (3.6). Finally, the red hatched regions are compatible with a U(2)-like flavour structure, for which $\alpha_2/\alpha_3 \in [0.2, 5]V_{ts}$. From the left plot, we see that a lighter mass leptoquark requires significantly less Higgs-mass tuning; the contour with $\Delta = 0.13$ indicates comparable tuning to that in the SM. For the 5 TeV leptoquark, we see that a U(2)-like structure can only accommodate the 2σ fit to the B-anomalies with $\Delta \gtrsim 1$, meaning the finite loop correction to the Higgs mass is greater than 125 GeV.

4.2 Neutral meson mixing

As for any model that fits the $b \to s\ell\ell$ anomalies, an important constraint comes from B meson mixing amplitudes, in particular from the B_s meson mixing observable ΔM_s [68, 69]. In our model, as a consequence of gauging the leptoquark under $L_\mu - L_\tau$, we have forced a direct connection between B_s mixing and the flavour anomalies, both of which are governed by the combination of couplings $\alpha_3\alpha_2^*$. (For generic leptoquark couplings to leptons, one finds a sum over all lepton generations in the box diagram that contributes to the mixing, thus breaking this direct connection.)

Using the most recent SM predictions for ΔM_s [69], which is

$$\Delta M_s^{\text{SM}} = \left(18.4^{+0.7}_{-1.2}\right) \text{ ps}^{-1}, \tag{4.8}$$

we obtain the following constraint on the parameters of our model,

$$0.88 \leq \left| 1 + \left(\frac{\alpha_3\alpha_2^*}{M_{S_3}/6.3 \text{ TeV}}\right)^2 \right| \leq 1.08 \tag{4.9}$$

at 2σ. Assuming that $\alpha_3\alpha_2^*$ is real and positive, this reduces to the bound

$$\frac{M_{S_3}}{\alpha_3\alpha_2^*} \geq 22 \text{ TeV}. \tag{4.10}$$

For the fixed values $M_{S_3} = 2, 5$ TeV, this bound is plotted (orange) in figure 3, and is consistent with the whole region plotted in figure 4.
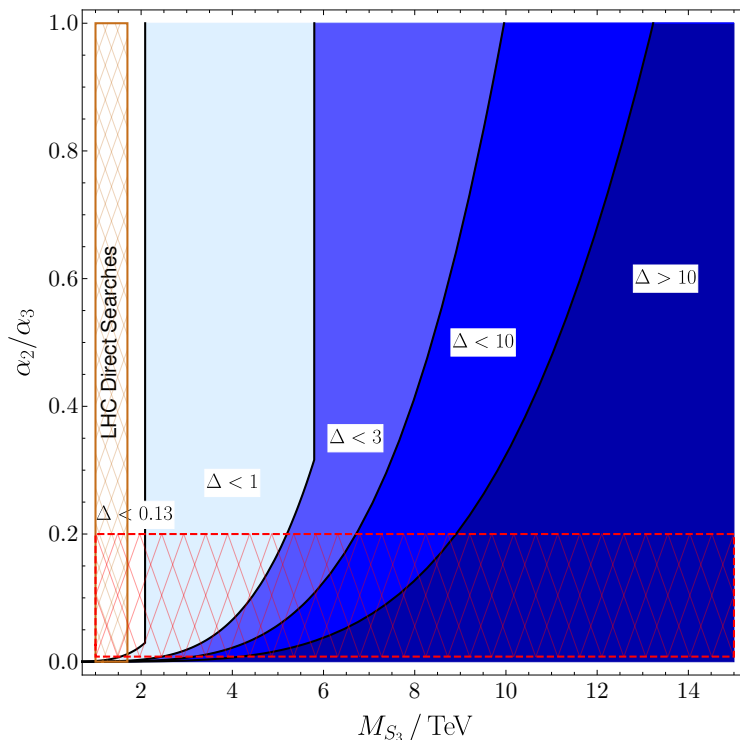
Figure 4. Constraints on the ratio α_2/α_3 *vs.* the leptoquark mass, with $\Delta C_9^\mu = -\Delta C_{10}^\mu$ everywhere fixed to the central value of the global fit [12] to B anomaly data. The brown hatched region ($M_{S_3} \leq 1.7\,\text{TeV}$) is excluded by ATLAS direct searches [70]. The constraint from B_s meson mixing is not visible on the plot, since at the best fit point for the anomalies it only imposes an upper bound of $M_{S_3} \leq 55\,\text{TeV}$. The blue shaded regions indicate the degree of fine-tuning of the finite Higgs mass corrections, with $\Delta > 1$ indicating that the finite loop corrections exceed $125\,\text{GeV}$. The region with $\Delta \leq 0.13$ exhibits a similar degree of naturalness to the SM, for which only a sliver of parameter space remains compatible with direct searches. Finally, the red hatched region shows the U(2)-like region where $\alpha_2/\alpha_3 \in [0.2, 5]V_{ts}$, which requires a tuning $\Delta > 1$ for $M_{S_3} > 5.2\,\text{TeV}$.

There are similar bounds arising from other neutral meson mixing amplitudes, which constrain different combinations of the leptoquark couplings α_i, depending on the quark content of the meson. Constraints from B_d [69] and kaon[20] mixing translate to the bounds

$$\frac{M_{S_3}}{\alpha_3 \alpha_1^*} \geq 91 \text{ TeV} \qquad \text{and} \qquad \frac{M_{S_3}}{\alpha_2 \alpha_1^*} \geq 72 \text{ TeV} \tag{4.11}$$

respectively, again assuming that the corresponding products $\alpha_i \alpha_j^*$ are real and positive. If we are in a region of parameter space where the neutral current anomaly is explained (4.7), these two observables can place an upper limit $|\alpha_1| \leq 0.55$. In general however, since α_1 is not connected to the flavour anomalies, it can be taken much smaller, making both these bounds from B_d and kaon mixing essentially unimportant.

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[20]For kaon mixing, we use the results from ref. [71] as updated at 'La Thuile 2018' by L. Silvestrini, for which slides are available at https://agenda.infn.it/event/14377/contributions/24434/attachments/17481/19830/silvestriniLaThuile.pdf.

More importantly, there are contributions to D meson mixing which cannot be avoided by taking α_1 to be small, since we chose a basis in which the CKM mixing is in the up sector, and so the up quark couples to S_3 through $\sum_k \alpha_k V_{1k}^*$. The BSM contribution to the effective $\Delta C = 2$ Hamiltonian is

$$C_D^1 = \frac{5}{128\pi^2} \frac{1}{M_{S_3}^2} \left(\sum_j \alpha_j V_{2j}^* \right)^2 \left(\sum_k \alpha_k^* V_{1k} \right)^2 . \tag{4.12}$$

We take our bound from the recent UTfit update (see footnote 20). For the limiting value $\alpha_1 = 0$, the D mixing bound translates to

$$\frac{M_{S_3}}{|(\alpha_2(\alpha_2 + 0.04\alpha_3)|} \gtrsim 25 \, \text{TeV} , \quad \frac{M_{S_3}}{\sqrt{\alpha_3\alpha_2}|\alpha_2 + 0.05\alpha_3|} \gtrsim 25 \, \text{TeV}, \tag{4.13}$$

for the real and imaginary parts of C_D^1 respectively. However, we remark that non-zero α_1 can lead to cancellations that weaken this constraint, particularly in the U(2)-like region with $\alpha_1/\alpha_2 \sim V_{td}/V_{ts}$. It is also the case that there is significant uncertainty in the value of the SM prediction for D mixing and how much is contributed from short and long distance effects, and so the green shaded regions in figure 3 should be considered subject to these caveats.

To obtain the B_s mixing bound (4.10) we assumed that the leptoquark coupling product $\alpha_3 \alpha_2^*$ was real. Of course, these couplings are in general complex numbers, and any relative phases between the components α_i are constrained by CP-violating observables, such as the CP asymmetry $A_{\text{CP}}^{\text{mix}}$ [68]. For a leptoquark mass of $5 \, \text{TeV}$ or lower, for which a U(2)-like flavour structure remains finitely natural (see figure 5 and section 4.4), the experimental constraint from $A_{\text{CP}}^{\text{mix}}$ is very weak, and the relative phase between α_2 and α_3 is essentially unconstrained. (The relative phases with respect to α_1 are, however, more strongly constrained, by both phases in B_d meson mixing and also by decays sensitive to the imaginary parts of Wilson coefficients, e.g. $K_L \to \pi^0 \mu^+ \mu^-$. But, since α_1 is essentially a free parameter that plays no role in mediating the B anomalies, we shall not consider these phases further in this paper.)

4.3 Leptoquark pair production at the LHC

The third important constraint on our reduced parameter space (4.1) comes from direct leptoquark production at the LHC.

The particularities of our flavour structure, which arise from charging the leptoquark under $L_\mu - L_\tau$, make its collider phenomenology rather specific, and a little different from that discussed for other S_3 leptoquark solutions to B anomaly data; in particular, decays to third-family leptons are often considered as a dominant channel (see e.g. [29, 72]), whereas our charged leptoquark decays only to second-family leptons. The most up-to-date constraints in muonic decay channels come from searches by ATLAS [70] for pair-produced scalar leptoquarks decaying into two muons plus two or more jets, which we use here.

The cleanest (and tightest) constraints come from pair production of the $S_3^{4/3}$ component, each of which decays exclusively to $\overline{d_i}\mu^+$, for some down-type quark d_i. (The other

components of the S_3 have tree-level decays to neutrinos, which generally result in weaker limits.) The specific branching ratios for each final state are determined by the direction of the vector α_i, as

$$\mathrm{BR}(S_3^{4/3} \to \overline{d_i}\mu^+) \approx \frac{|\alpha_i|^2}{\sum_j |\alpha_j|^2}, \tag{4.14}$$

in the limit that all down-type quarks and the muon are massless. In ref. [70], limits on the S_3 mass are computed for each channel, in each case assuming that the leptoquark decays exclusively into one specific combination of quark flavour and lepton flavour. For our model this is not generically the case, but in the region of parameter space where $\alpha_3 \gg \alpha_{1,2}$, the $S_3^{4/3}$ component decays exclusively to $\bar{b}\mu^+$, and the ATLAS limit is $M_{S_3^{4/3}} \gtrsim 1.7\,\mathrm{TeV}$ [70]. Conversely, if $\alpha_3 \ll \alpha_{1,2}$, the $S_3^{4/3}$ component decays to $\bar{q}\mu^+$, where q is a light quark ($\bar{d}$ or $\bar{s}$), for which the limit is again $M_{S_3^{4/3}} \gtrsim 1.7\,\mathrm{TeV}$ [70].[21] Since the mass exclusion is roughly the same in both these extreme cases, it seems reasonable to interpolate that the 2σ bound on the leptoquark mass for some generic value of couplings α_i should be around

$$M_{S_3^{4/3}} \gtrsim 1.7\,\mathrm{TeV}. \tag{4.15}$$

This is the direct search bound that we plot in figure 4.

4.4 Other constraints

There are other constraints on the direction $(\alpha_1, \alpha_2, \alpha_3)$ of flavour violation in quark space, coming from precision measurements of correlated flavour observables. Since our charged leptoquark setup exhibits rank-one flavour violation, with couplings only to muons, the results from ref. [42] are directly transferable. In figure 5 we reproduce the constraints from ref. [42], relevant to the particular case of the S_3 leptoquark couplings, subject to the simplifying assumption that $\alpha_i \in \mathbb{R}$. The constraints are plotted in the space of polar angles θ and ϕ that define the direction of the α_i vector, *viz.* $\alpha_i = \sqrt{\sum_i \alpha_i^2}\ \hat{n}_i$, where $\hat{n}_i^T = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.

Figure 5 includes constraints, as calculated in ref. [42], principally from observables probing $d_i \to d_j\mu^+\mu^-$ transitions, specifically from $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$, $\mathrm{BR}(B^+ \to \pi^+\mu^+\mu^-)$, $\mathrm{BR}(K_S \to \mu^+\mu^-)$, and $\mathrm{BR}(K_L \to \mu^+\mu^-)$. Observables sensitive to $d_i \to d_j\overline{\nu}\nu$ transitions are also included, the most important being $\mathrm{BR}(K^+ \to \pi^+\nu_\mu\overline{\nu_\mu})$, remembering that, for simplicity, we set the imaginary parts of Wilson coefficients to zero and so have no sensitivity to e.g. $\mathrm{BR}(K_L \to \pi^0\nu_\mu\overline{\nu_\mu})$.

We overlay the constraints coming from finite naturalness of the Higgs mass, for leptoquark masses equal to 4, 5, and 5.5 TeV. For each mass, the region above the black line has a tuning $\Delta > 1$. For heavy leptoquark masses, we can see that the Higgs mass becomes destablized (in the sense that Δ exceeds one) when the new physics is aligned with the third family, i.e. when $\alpha_3 \gg \alpha_2$. Specifically, we find that a U(2)-like flavour structure, here defined by $\alpha_2/\alpha_3 \in [0.2, 5]V_{ts}$ as indicated by the hatched red regions in figures 3, 4,

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[21]This bound assumes that α_1 is not so large that leptoquark pair production is no longer dominated by gluon fusion, which anyway seems ruled out by the combined meson mixing and neutral current anomaly bound on α_1.
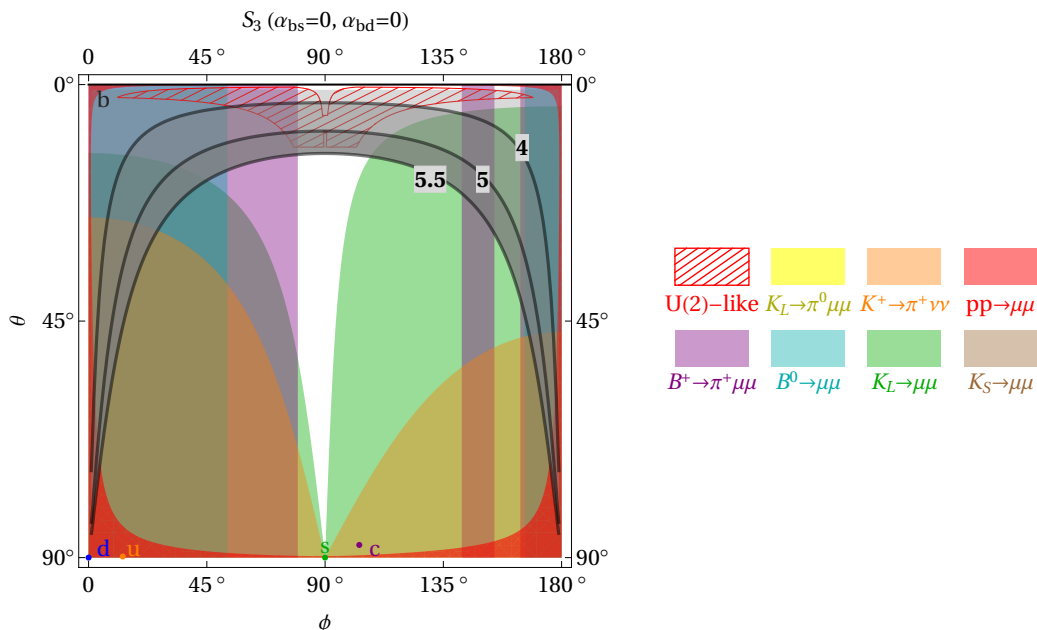
Figure 5. Constraints on the polar angles θ and ϕ, which specify the direction of the 3-vector α_i (assumed real) of leptoquark couplings, from a variety of correlated flavour observables (updated from [42] with the latest $b \to s\ell\ell$ fit). The white region is allowed at 2σ. The black lines are finite naturalness contours with $\Delta = 1$, for given values of the leptoquark mass below 5.8 TeV (for which the anomaly-tied correction (3.5) is the dominant one). Above each line, the leptoquark induces a Higgs mass correction greater than 125 GeV or so, and so the model requires fine-tuning. We find that a U(2)-like flavour structure is compatible with finite naturalness (i.e. $\Delta \lesssim 1$) for $M_{S_3} \lesssim 5$ TeV or so. When $M_{S_3} \geq 5.8$ TeV there is an inevitable tuning of $\Delta \geq 1$ coming from the purely electroweak 2-loop correction (3.7), which is independent of (θ, ϕ) (i.e. independent of the flavour structure).

& 5, becomes disfavoured by finite naturalness for leptoquark masses greater than 5 TeV or so, as can be seen also in figure 4.

The decays $B \to K^{(*)} \nu \bar{\nu}$, while in principal important, are in fact constrained by precisely the same Wilson coefficient, $\Delta C_9^\mu (= -\Delta C_{10}^\mu)$ that mediates the B anomalies. Again, this is a consequence of our gauging $L_\mu - L_\tau$, which means the leptoquark only has decays to muon neutrinos. The theory prediction for $R_{\nu\nu}^{(*)} \equiv \mathrm{BR}\left(B \to K^{(*)} \nu \nu\right) / \mathrm{BR}\left(B \to K^{(*)} \nu \nu\right)_{\mathrm{SM}}$ is [73]

$$R_{\nu\nu}^{(*)} \approx 1 + \frac{1}{3} \frac{\mathrm{Re}\{\Delta C_9^\mu\}}{C_L^{\mathrm{SM}}}, \tag{4.16}$$

where $C_L^{\mathrm{SM}} = -6.38 \pm 0.06$ [74]. The tighter experimental bound comes from the Belle measurement $R_{\nu\nu}^* < 2.7$ at 2σ [75], which gives only a very weak constraint $M_{S_3}/\sqrt{\alpha_3 \alpha_2^*} \geq 4.4$ TeV, which is safely satisfied for the entire 2σ range that fits the B anomaly data. Turning this around, one might rather interpret the BSM correction in eq. (4.16) as a prediction of our model; assuming the best fit value of ΔC_9^μ, we expect $R_{\nu\nu}^{(*)} \approx 1.026$, i.e. only a 2.6% correction to the SM prediction (which is significantly smaller than for typical

leptoquark models, which often feature large branching ratios to tau neutrinos). Indeed, taking only the 2σ range of the fit to B anomaly data, we expect the correction to $R_{\nu\nu}^{(*)}$ to be no greater than 3.6%.

Finally, we checked a number of other observables which gave only very weak constraints. This includes constraints from $b\bar{b} \to \mu^+\mu^-$, which implies $M_{S_3}/|\alpha_3| \geq 1.8$ TeV; constraints from Higgs production and decay as well as the oblique parameters S and T, which are modified by the loop-induced coupling λ'_{HS}, but which give even weaker constraints; and constraints from the lifetime ratio $\tau(B_s)/\tau(B_d)$.

5 Discussion

In this paper we put forward a scenario in which the SM gauge group is extended by a $U(1)_X$ gauge symmetry, where $X = L_\mu - L_\tau$ on the SM fermions, and its matter content is augmented by a scalar leptoquark that has charge -1 under $U(1)_X$. By charging the leptoquark under $L_\mu - L_\tau$ we resolve two conceptual problems with S_3 models, both of which result from the fact that an uncharged leptoquark extension does not respect the SM's accidental symmetries. These accidental symmetries, specifically baryon number and the three individual charged lepton numbers, are very good symmetries of Nature, at least for energy scales probed by contemporary colliders.

Specifically, charging the S_3 leptoquark bans the presence of renormalisable diquark operators in the lagrangian, and in fact pushes back baryon number-violating effects to dimension-6 in the effective field theory. Thus, we expect the proton to decay no faster in our model than in the Standard Model effective field theory. On the leptonic side, by gauging $L_\mu - L_\tau$ there is no danger of lepton flavour violation, and the S_3-mediated FCNCs appear exclusively in muons, which is in good agreement with global fits to the $b \to s\ell\ell$ anomaly data.

By gauging $L_\mu - L_\tau$, which we assume is spontaneously broken somewhere above the weak scale, a new heavy gauge boson and heavy scalar appear in the spectrum of the theory. This pair of particles play no role in mediating the B anomalies, and we show that they may be hidden away at very high mass scales without destabilizing either the Higgs mass or the leptoquark mass through loop corrections. However, we do find that the Higgs mass inevitably receives significant loop corrections due to the leptoquark itself, whose couplings to the top quark must be big enough to explain the B anomalies. This is the case for any S_3 leptoquark that mediates the anomalies in $b \to s\ell\ell$, a fact that seems to have escaped attention in previous literature. By requiring the Higgs mass not be fine-tuned against its finite one-loop corrections, we obtain bounds on the mass and couplings of the leptoquark.

Because the Higgs mass correction is largest due to a loop involving top-quarks, this finite-naturalness argument requires the coupling α_3 of the leptoquark to the third family quark doublet to be not too big. Intriguingly, for leptoquark masses greater than about 5.2 TeV or so, one struggles to accommodate a $U(2)$-like flavour structure without introducing significant tuning of the Higgs mass. For such masses, naturalness therefore favours a larger degree of 'strange-alignment' in the leptoquark couplings. For leptoquarks heavier than 5.8 TeV, the Higgs mass requires significant tuning regardless of the flavour structure, because of an unavoidable electroweak two-loop contribution.

To summarise the results of our finite naturalness study, there is a wide (but narrowing) window of U(2)-compatible S_3 leptoquark masses, $M_{S_3} \in [1.7, 5.2]$ TeV, which are heavy enough to evade direct searches at the LHC but light enough to remain (finitely) natural. It will be interesting to chart the fate of this U(2)-compatible mass window in the near future, as LHC searches push M_{S_3} higher, and as correlated flavour observables are measured with increasingly good precision.

Acknowledgments

A   Loop induced couplings

To estimate the size of the radiatively-generated coupling between the leptoquarks and the Higgs (or the Φ field), we take the one-loop beta functions $\beta \equiv d\lambda/d\ln\mu^2$, and make the estimate

$$\delta\lambda(\mu) \approx (\lim_{\lambda\to 0} \beta_\lambda) \ln\frac{\mu^2}{M^2}, \tag{A.1}$$

where M is the mass of the virtual particle in the loop, and we take the tree-level scalar coupling to zero as part of our assumption that the one-loop effect is dominant.

Computing the β functions for scalar quartic coupling are standard computations [76–78], and we extract the results for λ_{HS}, λ'_{HS}, and $\lambda_{\Phi S}$ as follows:

$$\lim_{\lambda'_{HS}, g\to 0} \beta_{\lambda'_{HS}} = -\frac{G_F m_t^2 |\sum_q \alpha_q V_{tq}|^2}{2\sqrt{2}\pi^2}, \tag{A.2}$$

$$\lim_{\lambda_{HS}, g\to 0} \beta_{\lambda_{HS}} \approx 0, \tag{A.3}$$

(where for these first two we have dropped the terms in the beta function proportional to the electroweak coupling g to avoid double counting against the 2-loop result eq. 3.7)

$$\lim_{\lambda_{\Phi S}\to 0} \beta_{\lambda_{\Phi S}} = -\frac{g_X^4 \hat{Q}_\Phi^2}{4\pi^2} + \mathcal{O}\left(\frac{M_{S_3}^4}{M_X^4}\right). \tag{A.4}$$

The contributions to λ_{HS} occur due to diagrams with bottom quarks running in loops, and so $\beta_{\lambda_{HS}}$ is suppressed with respect to $\beta_{\lambda'_{HS}}$ by a factor of $m_b^2/m_t^2 \sim 10^{-3}$, which we safely neglect throughout. Most importantly for the discussion in the main text, we thus estimate

$$\lambda'_{HS}(\mu) \approx -\frac{G_F m_t^2 |\sum_q \alpha_q V_{tq}|^2}{2\sqrt{2}\pi^2} \ln\frac{\mu^2}{m_t^2} \approx -0.013|\alpha_3 + V_{ts}\alpha_2|^2 \ln\frac{\mu^2}{m_t^2} \tag{A.5}$$

and $\lambda_{HS}, \lambda_{\Phi S} \approx 0$.

B   Scalar one-loop mass corrections

Firstly, define the following two loop functions,

$$A_0(M) = M^2 \left(1 + \ln \frac{\mu^2}{M^2}\right) \equiv \texttt{PVA[0,M]} , \tag{B.1}$$

$$B_0(M_1, M_2, M_3) = 2 + \Lambda(M_1^2, M_2, M_3) - \frac{(M_1^2 + M_2^2 - M_3^2) \ln \frac{M_2^2}{M_3^2}}{2M_1^2} + \ln \frac{\mu^2}{M_3^2} \tag{B.2}$$

$$\equiv \texttt{PVB[0,0,M1\^{}2,M2,M3]}, \tag{B.3}$$

where the notation $\texttt{PVA}$ and $\texttt{PVB}$ are borrowed from $\texttt{Package-X}$ syntax [79–81], where

$$\Lambda(M_1^2, M_2, M_3) = \frac{\sqrt{\lambda(M_1^2, M_2^2, M_3^2)}}{M_1^2} \ln \left(\frac{\sqrt{\lambda(M_1^2, M_2^2, M_3^2)} - M_1^2 + M_2^2 + M_3^2}{2M_2 M_3} \right), \tag{B.4}$$

and where λ is the Källén function

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \tag{B.5}$$

These loop functions appear in the various mass corrections we compute in the rest of this appendix.

B.1   Higgs mass corrections

There are three Feynman diagrams that contribute to the Higgs mass at one-loop, as shown in figure 2 (in the main text), all of which involve a leptoquark running in a loop. Considering the diagrams in order from left to right in figure 2, there is firstly a diagram in which the leptoquark loop attaches to the Higgs propagator at a four-point vertex, giving

$$\delta M_H^2 = \frac{9}{32\pi^2}(\lambda_{HS} + \lambda'_{HS})A_0(M_{S_3}). \tag{B.6}$$

Secondly, there is a tadpole diagram giving

$$\delta M_H^2 = \frac{-27}{32\pi^2}(\lambda_{HS} + \lambda'_{HS})A_0(M_{S_3}). \tag{B.7}$$

Finally, there is a diagram involving two insertions of the Higgs-S_3-S_3 vertex, which gives

$$\delta M_H^2 = \frac{3v^2}{16\pi^2}(3\lambda_{HS}^2 + 6\lambda_{HS}\lambda'_{HS} + 5\lambda'^2_{HS})B_0(M_H, M_{S_3}, M_{S_3}), \tag{B.8}$$

In the limit of interest, $M_{S_3} \gg M_H$, the first two diagrams give the leading contributions and we neglect the third, resulting in eq. (3.4) from the main text.

B.2   Finite naturalness of the leptoquark mass

In this appendix we discuss the finite naturalness of the leptoquark mass, which receives a multitude of one-loop corrections involving both SM particles and the heavy fields associated with the U(1)$_X$ symmetry. In the U(1)$_X$ decoupling limit (3.3) that we explore in the main text, we find that the relative size of these loop corrections is small, giving only weak
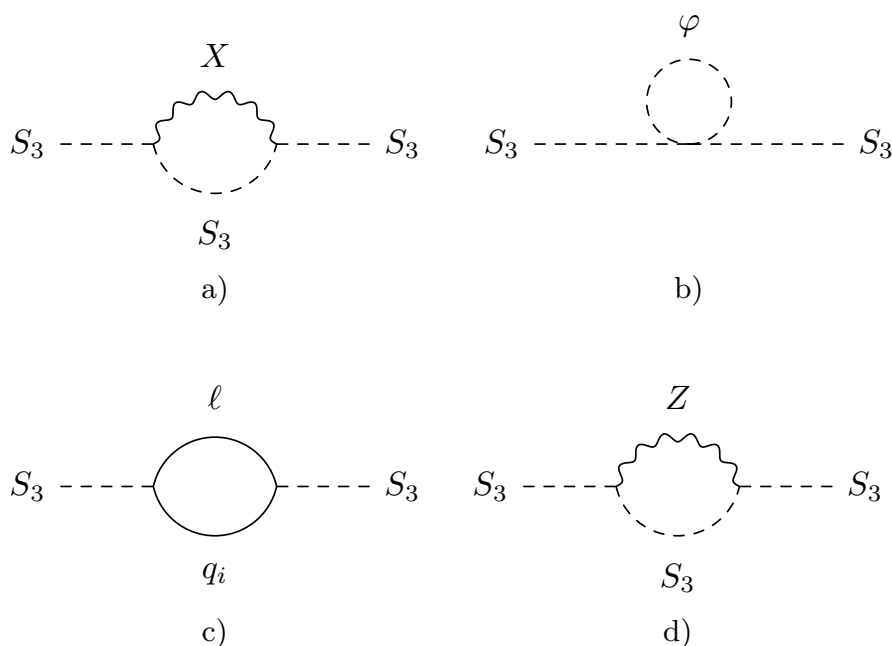
Figure 6. Example one-loop Feynman diagrams contributing to radiative corrections to the leptoquark mass, from a) the X gauge boson, b) the Φ scalar, c) SM fermions, and d) EW gauge bosons.

constraints on the parameter space of our charged leptoquark model. These constraints do not appear in any of figures 3, 4, or 5, where we plot only the finite naturalness contours coming from the Higgs mass tuning.

Before we discuss radiative corrections, we remark that there are several 'tree-level' contributions to the leptoquark mass encoded in the lagrangian (2.1). In addition to the bare mass term, the potential terms $-\lambda_{HS}|H|^2|S_3|^2$ and $-\lambda'_{HS}|H^\dagger \sigma_a S_3^a|^2$ give leptoquark mass contributions once the Higgs is expanded around its vev. Moreover, the contributions from the λ'_{HS} term are different for each of the three SU(2) components of S_3, giving a mass splitting

$$\Delta_M = M_{S_3^{1/3}} - M_{S_3^{4/3}} = M_{S_3^{-2/3}} - M_{S_3^{1/3}} = \frac{\lambda'_{HS}}{4}\frac{v^2}{M_{S_3^{4/3}}} + \mathcal{O}\left(\frac{v^4}{M_{S_3}^3}\right), \qquad \text{(B.9)}$$

which is small for the multi-TeV mass leptoquarks of interest here. There is also a tree-level mass contribution from the potential term $-\lambda_{\Phi S}|\Phi|^2|S_3|^2$ upon expanding Φ about its vev v_Φ, which could be large given that we expect U(1)$_X$ to be broken at some higher energy scale. Indeed the coupling $\lambda_{\Phi S}$, like the couplings $\lambda_{HS}^{(\prime)}$ discussed above, is generated at one-loop. From eq. (A.4) we expect this mass contribution to scale like $\delta M_{S_3^{4/3}}^2 \sim g_X^2 M_X^2/4\pi^2$, which is comparable to the one-loop corrections that we discuss next.

We now discuss radiative corrections, and finite naturalness of the leptoquark mass. Within our assumed mass hierarchy, the largest corrections are naïvely those proportional to M_Φ and M_X, coming from these heavier particles running in loops. Neglecting small

electroweak-nonuniversal contributions, the dominant mass corrections are (see figures [6]a and [6]b for the responsible Feynman diagrams)

$$(\delta M_{S_3}^2)_X = \frac{g_X^2 M_X^2}{4\pi^2} - \frac{\lambda_{\Phi S} M_X^2}{4\pi^2}\frac{M_X^2}{M_\Phi^2} - \frac{\lambda_{\Phi S} M_\Phi^2}{8\pi^2} \tag{B.10}$$
$$+ \left(\frac{g_X^2 M_X^2}{2\pi^2} - \frac{\lambda_{\Phi S} M_X^2}{2\pi^2}\frac{M_X^2}{M_\Phi^2}\right)\ln\frac{\mu^2}{M_X^2} - \frac{\lambda_{\Phi S} M_\Phi^2}{8\pi^2}\ln\frac{\mu^2}{M_\Phi^2} + \ldots,$$

where the subscript 'X' indicates that these corrections are due to particles in the $\mathrm{U}(1)_X$ sector. In the decoupling limit specified by eq. (3.3) these contributions become vanishingly small.

In that limit, the leading one-loop corrections to the leptoquark masses are due to various SM particles running in the loop. Again ignoring the small electroweak-nonuniversal corrections,[22] we have (see figure [6]c)

$$(\delta M_{S_3}^2)_{\text{fermion}} = -\frac{M_{S_3}^2}{8\pi^2}\left(\sum_i |\alpha_i|^2\right)\left(2 + \ln\frac{\mu^2}{M_{S_3}^2}\right), \tag{B.11}$$

coming from the quark-lepton loops, which gives only very weak finite naturalness bounds

$$\sum_i |\alpha_i|^2 \lesssim \begin{cases} 40\Delta & \text{for } \mu = M_{S_3} \\ \Delta & \text{for } \mu = 10^{16} M_{S_3} \sim M_{\text{Planck}}. \end{cases} \tag{B.12}$$

The contribution from EW gauge bosons is (figure [6]d)

$$(\delta M_{S_3}^2)_{\text{EW}} = -\frac{M_{S_3}^2}{144\pi^2}\left(16e^2 + 9g^2\left\{1 + \frac{(s_W^2 - 3c_W^2)^2}{9c_W^2}\right\}\right)\left(7 + 3\ln\frac{\mu^2}{M_{S_3}^2}\right)$$
$$= \begin{cases} -0.03 M_{S_3}^2 & \text{for } \mu = M_{S_3} \\ -1.2 M_{S_3}^2 & \text{for } \mu = 10^{16} M_{S_3} \sim M_{\text{Planck}}. \end{cases} \tag{B.13}$$

Thus, if we take the "Planck scale" condition then a degree of fine tuning is unavoidable (assuming there is no cancellation between different mass corrections). This follows simply from the existence of an S_3 leptoquark charged under the SM gauge group, even without any fermion couplings.

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References

[1] LHCb collaboration, *Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays*, *JHEP* **08** (2017) 055 [`arXiv:1705.05802`] [INSPIRE].

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[22]We here also ignore terms arising from the $\lambda_{HS}^{(\prime)}$ couplings, which we assume are suppressed by a further loop factor (as in section 3.1).

[2] LHCb collaboration, *Search for lepton-universality violation in $B^+ \to K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **122** (2019) 191801 [`arXiv:1903.09252`] [ɪɴSPIRE].

[3] ATLAS collaboration, *Study of the rare decays of B_s^0 and B^0 mesons into muon pairs using data collected during 2015 and 2016 with the ATLAS detector*, *JHEP* **04** (2019) 098 [`arXiv:1812.03017`] [ɪɴSPIRE].

[4] CMS collaboration, *Measurement of the $B_s^0 \to \mu^+ \mu^-$ Branching Fraction and Search for $B^0 \to \mu^+ \mu^-$ with the CMS Experiment*, *Phys. Rev. Lett.* **111** (2013) 101804 [`arXiv:1307.5025`] [ɪɴSPIRE].

[5] CMS, LHCB collaboration, *Observation of the rare $B_s^0 \to \mu^+ \mu^-$ decay from the combined analysis of CMS and LHCb data*, *Nature* **522** (2015) 68 [`arXiv:1411.4413`] [ɪɴSPIRE].

[6] LHCb collaboration, *Measurement of the $B_s^0 \to \mu^+ \mu^-$ branching fraction and effective lifetime and search for $B^0 \to \mu^+ \mu^-$ decays*, *Phys. Rev. Lett.* **118** (2017) 191801 [`arXiv:1703.05747`] [ɪɴSPIRE].

[7] LHCb collaboration, *Measurement of form-factor-independent observables in the decay $B^0 \to K^{*0} \mu^+ \mu^-$*, *Phys. Rev. Lett.* **111** (2013) 191801 [`arXiv:1308.1707`] [ɪɴSPIRE].

[8] LHCb collaboration, *Angular analysis of the $B^0 \to K^{*0} \mu^+ \mu^-$ decay using 3 fb^{-1} of integrated luminosity*, *JHEP* **02** (2016) 104 [`arXiv:1512.04442`] [ɪɴSPIRE].

[9] CMS collaboration, *Measurement of angular parameters from the decay $B^0 \to K^{*0} \mu^+ \mu^-$ in proton-proton collisions at $\sqrt{s} = 8$ TeV*, *Phys. Lett. B* **781** (2018) 517 [`arXiv:1710.02846`] [ɪɴSPIRE].

[10] ATLAS collaboration, *Angular analysis of $B_d^0 \to K^* \mu^+ \mu^-$ decays in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *JHEP* **10** (2018) 047 [`arXiv:1805.04000`] [ɪɴSPIRE].

[11] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D.M. Straub, *B-decay discrepancies after Moriond 2019*, *Eur. Phys. J. C* **80** (2020) 252 [`arXiv:1903.10434`] [ɪɴSPIRE].

[12] M. Algueró et al., *Emerging patterns of new physics with and without lepton flavour universal contributions*, *Eur. Phys. J. C* **79** (2019) 714 [*Addendum ibid.* **80** (2020) 511] [`arXiv:1903.09578`] [ɪɴSPIRE].

[13] LHCb collaboration, *Measurement of CP-averaged observables in the $B^0 \to K^{*0} \mu^+ \mu^-$ decay*, *Phys. Rev. Lett.* **125** (2020) 011802 [`arXiv:2003.04831`] [ɪɴSPIRE].

[14] N. Assad, B. Fornal and B. Grinstein, *Baryon number and lepton universality violation in leptoquark and diquark models*, *Phys. Lett. B* **777** (2018) 324 [`arXiv:1708.06350`] [ɪɴSPIRE].

[15] L. Di Luzio, A. Greljo and M. Nardecchia, *Gauge leptoquark as the origin of B-physics anomalies*, *Phys. Rev. D* **96** (2017) 115011 [`arXiv:1708.08450`] [ɪɴSPIRE].

[16] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett. B* **779** (2018) 317 [`arXiv:1712.01368`] [ɪɴSPIRE].

[17] L. Calibbi, A. Crivellin and T. Li, *Model of vector leptoquarks in view of the B-physics anomalies*, *Phys. Rev. D* **98** (2018) 115002 [`arXiv:1709.00692`] [ɪɴSPIRE].

[18] R. Barbieri and A. Tesi, *B-decay anomalies in Pati-Salam* SU(4), *Eur. Phys. J. C* **78** (2018) 193 [`arXiv:1712.06844`] [ɪɴSPIRE].

[19] A. Greljo and B.A. Stefanek, *Third family quark-lepton unification at the TeV scale*, *Phys. Lett. B* **782** (2018) 131 [`arXiv:1802.04274`] [ɪɴSPIRE].

[20] B. Gripaios, *Composite leptoquarks at the LHC*, *JHEP* **02** (2010) 045 [`arXiv:0910.1789`] [ɪɴSPIRE].

[21] B. Gripaios, M. Nardecchia and S.A. Renner, *Composite leptoquarks and anomalies in B-meson decays*, *JHEP* **05** (2015) 006 [`arXiv:1412.1791`] [ɪɴSPIRE].

[22] G. Hiller and M. Schmaltz, R_K *and future* $b \to s\ell\ell$ *physics beyond the standard model opportunities*, *Phys. Rev. D* **90** (2014) 054014 [`arXiv:1408.1627`] [ɪɴSPIRE].

[23] PARTICLE DATA GROUP, *Review of particle physics*, *Phys. Rev. D* **98** (2018) 030001.

[24] A. Angelescu, D. Bečirević, D.A. Faroughy and O. Sumensari, *Closing the window on single leptoquark solutions to the B-physics anomalies*, *JHEP* **10** (2018) 183 [`arXiv:1808.08179`] [ɪɴSPIRE].

[25] B.C. Allanach, T. Corbett and M. Madigan, *Sensitivity of future hadron colliders to leptoquark pair production in the di-muon di-jets channel*, *Eur. Phys. J. C* **80** (2020) 170 [`arXiv:1911.04455`] [ɪɴSPIRE].

[26] G. Senjanović and A. Sokorac, *Light leptoquarks in* SO(10), *Z. Phys. C* **20** (1983) 255 [ɪɴSPIRE].

[27] I. Doršner, S. Fajfer, A. Greljo, J.F. Kamenik and N. Košnik, *Physics of leptoquarks in precision experiments and at particle colliders*, *Phys. Rept.* **641** (2016) 1 [`arXiv:1603.04993`] [ɪɴSPIRE].

[28] I. Doršner, S. Fajfer and N. Košnik, *Leptoquark mechanism of neutrino masses within the grand unification framework*, *Eur. Phys. J. C* **77** (2017) 417 [`arXiv:1701.08322`] [ɪɴSPIRE].

[29] I. Doršner, S. Fajfer, D.A. Faroughy and N. Košnik, *The role of the* S_3 *GUT leptoquark in flavor universality and collider searches*, *JHEP* **10** (2017) 188 [`arXiv:1706.07779`] [ɪɴSPIRE].

[30] SINDRUM collaboration, *Search for the decay* $\mu^+ \to e^+e^+e^-$, *Nucl. Phys. B* **299** (1988) 1 [ɪɴSPIRE].

[31] B.C. Allanach, J. Davighi and S. Melville, *An anomaly-free ATLAS: charting the space of flavour-dependent gauged* U(1) *extensions of the Standard Model*, *JHEP* **02** (2019) 082 [*Erratum ibid.* **08** (2019) 064] [`arXiv:1812.04602`] [ɪɴSPIRE].

[32] B.C. Allanach, B. Gripaios and J. Tooby-Smith, *Solving local anomaly equations in gauge-rank extensions of the Standard Model*, *Phys. Rev. D* **101** (2020) 075015 [`arXiv:1912.10022`] [ɪɴSPIRE].

[33] B.C. Allanach, B. Gripaios and J. Tooby-Smith, *Anomaly cancellation with an extra gauge boson*, *Phys. Rev. Lett.* **125** (2020) 161601 [`arXiv:2006.03588`] [ɪɴSPIRE].

[34] D.B. Costa, *Anomaly-free* $U(1)^m$ *extensions of the Standard Model*, *Phys. Rev. D* **102** (2020) 115006 [`arXiv:2007.08733`] [ɪɴSPIRE].

[35] T. Hambye and J. Heeck, *Proton decay into charged leptons*, *Phys. Rev. Lett.* **120** (2018) 171801 [`arXiv:1712.04871`] [ɪɴSPIRE].

[36] W. Altmannshofer, J. Davighi and M. Nardecchia, *Gauging the accidental symmetries of the standard model, and implications for the flavor anomalies*, *Phys. Rev. D* **101** (2020) 015004 [`arXiv:1909.02021`] [ɪɴSPIRE].

[37] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Quark flavor transitions in* $L_\mu - L_\tau$ *models*, *Phys. Rev. D* **89** (2014) 095033 [`arXiv:1403.1269`] [ɪɴSPIRE].

[38] W. Altmannshofer and I. Yavin, *Predictions for lepton flavor universality violation in rare B decays in models with gauged* $L_\mu - L_\tau$, *Phys. Rev. D* **92** (2015) 075022 [`arXiv:1508.07009`] [ɪɴSPIRE].

[39] W. Altmannshofer, S. Gori, S. Profumo and F.S. Queiroz, *Explaining dark matter and B decay anomalies with an $L_\mu - L_\tau$ model*, *JHEP* **12** (2016) 106 [arXiv:1609.04026] [INSPIRE].

[40] M. Farina, D. Pappadopulo and A. Strumia, *A modified naturalness principle and its experimental tests*, *JHEP* **08** (2013) 022 [arXiv:1303.7244] [INSPIRE].

[41] B. Gripaios, M. Nardecchia and S.A. Renner, *Linear flavour violation and anomalies in B physics*, *JHEP* **06** (2016) 083 [arXiv:1509.05020] [INSPIRE].

[42] V. Gherardi, D. Marzocca, M. Nardecchia and A. Romanino, *Rank-one flavor violation and B-meson anomalies*, *JHEP* **10** (2019) 112 [arXiv:1903.10954] [INSPIRE].

[43] D.B. Kaplan, *Flavor at SSC energies: a new mechanism for dynamically generated fermion masses*, *Nucl. Phys. B* **365** (1991) 259 [INSPIRE].

[44] B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi and L. Vecchi, *On partial compositeness and the CP asymmetry in charm decays*, *Nucl. Phys. B* **867** (2013) 394 [arXiv:1205.5803] [INSPIRE].

[45] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: an effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [hep-ph/0207036] [INSPIRE].

[46] R. Barbieri, G. Isidori, J. Jones-Perez, P. Lodone and D.M. Straub, U(2) *and minimal flavour violation in supersymmetry*, *Eur. Phys. J. C* **71** (2011) 1725 [arXiv:1105.2296] [INSPIRE].

[47] R. Barbieri, G. Isidori, A. Pattori and F. Senia, *Anomalies in B-decays and* U(2) *flavour symmetry*, *Eur. Phys. J. C* **76** (2016) 67 [arXiv:1512.01560] [INSPIRE].

[48] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B-physics anomalies: a guide to combined explanations*, *JHEP* **11** (2017) 044 [arXiv:1706.07808] [INSPIRE].

[49] J.M. Arnold, B. Fornal and M.B. Wise, *Simplified models with baryon number violation but no proton decay*, *Phys. Rev. D* **87** (2013) 075004 [arXiv:1212.4556] [INSPIRE].

[50] J. Heeck and W. Rodejohann, *Gauged $L_\mu - L_\tau$ symmetry at the electroweak scale*, *Phys. Rev. D* **84** (2011) 075007 [arXiv:1107.5238] [INSPIRE].

[51] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [arXiv:1503.03477] [INSPIRE].

[52] M. Bauer, P. Foldenauer and J. Jaeckel, *Hunting all the hidden photons*, *JHEP* **18** (2020) 094 [arXiv:1803.05466] [INSPIRE].

[53] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of* 10^9 *muon decays*, *Phys. Lett. B* **67** (1977) 421.

[54] L. Lavoura, *Zeros of the inverted neutrino mass matrix*, *Phys. Lett. B* **609** (2005) 317 [hep-ph/0411232] [INSPIRE].

[55] E.I. Lashin and N. Chamoun, *Zero minors of the neutrino mass matrix*, *Phys. Rev. D* **78** (2008) 073002 [arXiv:0708.2423] [INSPIRE].

[56] T. Bandyopadhyay, G. Bhattacharyya, D. Das and A. Raychaudhuri, *Reappraisal of constraints on Z′ models from unitarity and direct searches at the LHC*, *Phys. Rev. D* **98** (2018) 035027 [arXiv:1803.07989] [INSPIRE].

[57] B.C. Allanach and J. Davighi, *Third family hypercharge model for $R_{K^{(*)}}$ and aspects of the fermion mass problem*, *JHEP* **12** (2018) 075 [arXiv:1809.01158] [INSPIRE].

[58] B.C. Allanach and J. Davighi, *Naturalising the third family hypercharge model for neutral current B-anomalies*, *Eur. Phys. J. C* **79** (2019) 908 [arXiv:1905.10327] [INSPIRE].

[59] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The string landscape, black holes and gravity as the weakest force*, *JHEP* **06** (2007) 060 [hep-th/0601001] [INSPIRE].

[60] G. 't Hooft, *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, *NATO Sci. Ser. B* **59** (1980) 135.

[61] K. Kowalska, D. Kumar and E.M. Sessolo, *Implications for new physics in $b \to s\mu\mu$ transitions after recent measurements by Belle and LHCb*, *Eur. Phys. J. C* **79** (2019) 840 [arXiv:1903.10932] [INSPIRE].

[62] M. Ciuchini et al., *New Physics in $b \to s\ell^+\ell^-$ confronts new data on Lepton Universality*, *Eur. Phys. J. C* **79** (2019) 719 [arXiv:1903.09632] [INSPIRE].

[63] P. Arnan, A. Crivellin, M. Fedele and F. Mescia, *Generic loop effects of new scalars and fermions in $b \to s\ell^+\ell^-$ and a vector-like 4th generation*, *JHEP* **06** (2019) 118 [arXiv:1904.05890] [INSPIRE].

[64] G. D'Amico et al., *Flavour anomalies after the R_{K^*} measurement*, *JHEP* **09** (2017) 010 [arXiv:1704.05438] [INSPIRE].

[65] A.K. Alok, A. Dighe, S. Gangal and D. Kumar, *Continuing search for new physics in $b \to s\mu\mu$ decays: two operators at a time*, *JHEP* **06** (2019) 089 [arXiv:1903.09617] [INSPIRE].

[66] CKMFITTER GROUP collaboration, *CP violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J. C* **41** (2005) 1 [hep-ph/0406184] [INSPIRE].

[67] CKMFITTER collaboration, *Summer 2019 results*, http://ckmfitter.in2p3.fr/www/results/plots_summer19/ckm_res_summer19.html.

[68] L. Di Luzio, M. Kirk and A. Lenz, *Updated B_s-mixing constraints on new physics models for $b \to s\ell^+\ell^-$ anomalies*, *Phys. Rev. D* **97** (2018) 095035 [arXiv:1712.06572] [INSPIRE].

[69] L. Di Luzio, M. Kirk, A. Lenz and T. Rauh, *ΔM_s theory precision confronts flavour anomalies*, *JHEP* **12** (2019) 009 [arXiv:1909.11087] [INSPIRE].

[70] ATLAS collaboration, *Search for pairs of scalar leptoquarks decaying into quarks and electrons or muons in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector*, *JHEP* **10** (2020) 112 [arXiv:2006.05872] [INSPIRE].

[71] UTFIT collaboration, *Model-independent constraints on $\Delta F = 2$ operators and the scale of new physics*, *JHEP* **03** (2008) 049 [arXiv:0707.0636] [INSPIRE].

[72] D.A. Faroughy, A. Greljo and J.F. Kamenik, *Confronting lepton flavor universality violation in B decays with high-p_T tau lepton searches at LHC*, *Phys. Lett. B* **764** (2017) 126 [arXiv:1609.07138] [INSPIRE].

[73] A.J. Buras, J. Girrbach-Noe, C. Niehoff and D.M. Straub, *$B \to K^{(*)}\nu\bar{\nu}$ decays in the standard model and beyond*, *JHEP* **02** (2015) 184 [arXiv:1409.4557] [INSPIRE].

[74] W. Altmannshofer, A.J. Buras, D.M. Straub and M. Wick, *New strategies for new physics search in $B \to K^*\nu\bar{\nu}$, $B \to K\nu\bar{\nu}$ and $B \to X_s\nu\bar{\nu}$ decays*, *JHEP* **04** (2009) 022 [arXiv:0902.0160] [INSPIRE].

[75] BELLE collaboration, *Search for $B \to h\nu\bar{\nu}$ decays with semileptonic tagging at Belle*, *Phys. Rev. D* **96** (2017) 091101 [*Addendum ibid.* **97** (2018) 099902] [arXiv:1702.03224] [INSPIRE].

[76] T.P. Cheng, E. Eichten and L.-F. Li, *Higgs phenomena in asymptotically free gauge theories*, *Phys. Rev. D* **9** (1974) 2259 [INSPIRE].

[77] M.E. Machacek and M.T. Vaughn, *Two loop renormalization group equations in a general quantum field theory. 3. Scalar quartic couplings*, *Nucl. Phys. B* **249** (1985) 70 [INSPIRE].

[78] I. Schienbein, F. Staub, T. Steudtner and K. Svirina, *Revisiting RGEs for general gauge theories*, *Nucl. Phys. B* **939** (2019) 1 [arXiv:1809.06797] [INSPIRE].

[79] Package-X, https://packagex.hepforge.org/.

[80] H.H. Patel, *Package-X: a Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **197** (2015) 276 [arXiv:1503.01469] [INSPIRE].

[81] H.H. Patel, *Package-X 2.0: a Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **218** (2017) 66 [arXiv:1612.00009] [INSPIRE].