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$\Delta(54)$ flavor phenomenology and strings

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Abstract: $\Delta(54)$ can serve as a flavor symmetry in particle physics, but remains almost unexplored. We show that in a classification of semi-realistic $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic string orbifolds, $\Delta(54)$ turns out to be the most natural flavor symmetry, providing additional motivation for its study. We revisit its phenomenological potential from a low-energy perspective and subject to the constraints of string models. We find a model with $\Delta(54)$ arising from heterotic orbifolds that leads to the Gatto-Sartori-Tonin relation for quarks and charged-leptons. Additionally, in the neutrino sector, it leads to a normal hierarchy for neutrino masses and a correlation between the reactor and the atmospheric mixing angles, the latter taking values in the second octant and being compatible at three sigmas with experimental data.

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Contents

1 Introduction

The standard model (SM) exhibits features, such as the family repetition and the structure of mixing matrices for quarks and leptons, that suggest an underlying structure. Non-Abelian discrete flavor symmetries appear in many bottom-up models as a promising explanation for these observations [1–3].

A large set of Abelian and non-Abelian discrete symmetries has been successfully investigated in this context [3–26]. Particularly, the groups $\mathbb{Z}_3$ [27–30], S_3 [31–38] and $\Delta(27)$ [22, 39–45] have shed some light on the structure of the quark and neutrino sectors, providing in some cases an explanation of proton stability and dark matter [19, 40, 46–48] or an explanation of the *Dirac-ness* of neutrinos [43]. These symmetries have in common that they are subgroups of $\Delta(54)$, which however has been explored only aiming at a tri-bimaximal neutrino-mixing structure or similar [49–52]. Since θ_{13} is now known to be non-zero, the potential of $\Delta(54)$ as a flavor symmetry must be revisited. To pave the way to a vast revision on this subject is one of the goals of this work.

On the other hand, despite their success, the origin of flavor symmetries remains unexplained in bottom-up model building. Fortunately, non-Abelian flavor symmetries emerge naturally in different compactification schemes of string theory [53–59] that enjoy the properties of the SM or its supersymmetric extension(s), yielding a promising ultraviolet completion of flavor phenomenology.

Toroidal heterotic orbifolds [60, 61] (see e.g. [62] for a comprehensive introduction) lead to models which reproduce the gauge group and matter spectrum of the SM [63], its minimal supersymmetric extension [64–67] and other non-minimal extensions [68], as well as many other observed and/or desirable properties of particle physics [69–75]. As

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we discuss in section 2, following previous findings of [54, 56], a $\Delta(54)$ flavor symmetry can emerge in these constructions as a result of dividing a $\mathbb{T}^2$ torus by $\mathbb{Z}_3$ in the compact dimensions. A paramount difference between the flavor theory emerging in this context and one arbitrarily proposed is that all properties, including the flavor representations and number of fields, are dictated by the string compactification itself, resulting in interesting phenomenological consequences that we aim at studying in this paper.

Due to their geometrical structure, $\mathbb{Z}_3$ or $\mathbb{Z}_3\times\mathbb{Z}_2$ heterotic orbifolds could in principle yield a $\Delta(54)$ flavor symmetry, but it is known that no promising model where this symmetry remains unbroken arises in those cases [76–78]. Therefore, the simplest complete string scenarios with SM-like physics and this flavor symmetry are $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifolds.

In this paper, we explore the phenomenological viability of the $\Delta(54)$ flavor symmetry from a top-down and a bottom-up perspective. After explaining in section 2 how flavor symmetries relate to geometry in heterotic string compactifications, in section 3 we perform a search of semi-realistic $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifold models, which turn out to display $\Delta(54)$ as a flavor symmetry more naturally than other possibilities. In section 4 we inspect the flavor symmetries and spectrum properties of one string sample model. Inspired by the features of the string models, in section 5 we propose a model that reproduces at some level known flavor observations and provides predictions for the neutrino sector. In section 6 we provide our concluding remarks.

2 Origin of flavor symmetries in heterotic orbifolds

We follow here the discussion of [54, 56], stressing some important aspects for our work.

In higher dimensional models, such as the string theories, flavor symmetries result from the geometrical symmetries (and other properties) of the extra dimensions (see e.g. [79] for a field-theoretical proposal). Since in those models the extra dimensions must be compactified in order to justify that we only perceive four dimensions, the compact space adopts geometrical structures which are endowed with symmetries that are passed down, as flavor symmetries, to the fields arising in those constructions.

Among all possibilities, orbifolds are perhaps the simplest compactifications. A d-dimensional orbifold is defined as the quotient of $\mathbb{R}^d$ divided by a discrete group. The resulting space is a compact solid, exhibiting typically some curvature singularities (fixed points of the orbifold), at which matter states may be localized. In the absence of local effects at the singularities, the states attached to all singularities are indistinguishable. The transformations (permutations, reflections, etc.) of those identical states that leave the matter distribution invariant build a (non-Abelian) symmetry of the compactified theory. Note that such transformations are equivalent to field relabelings.

As a first example, let us suppose that an orbifold yields a compact space endowed with two singularities at which two matter generations are chosen to be localized. Since these localized matter generations are indistinguishable, i.e. have identical quantum numbers, excepting of course for their localization properties, a permutation or relabeling of the generations does not alter the system. That is, the system is invariant under an S_2

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permutation symmetry, leading to an effective model with two generations related to each other under the non-trivial (flavor) transformation of that group.

In string theory, the simplest and yet quite promising compactifications of this kind are toroidal heterotic orbifolds [60, 61]. They are achieved by letting first the six extra dimensions of a 10D heterotic string be compact by imposing the quotient $\mathbb{R}^6/\Lambda_G$, where Λ_G can be chosen as a 6D root lattice of a Lie group G. The resulting 6D torus $\mathbb{T}^6 = \mathbb{R}^6/\Lambda_G$ is then divided by a discrete group of its isometries P, yielding the orbifold $\mathbb{O} = \mathbb{T}^6/P$. $\mathbb{O}$ is Abelian when P is Abelian. For simplicity, we shall focus here only on Abelian orbifolds.

Not any arbitrary choice of $\mathbb{T}^6$ and P is admissible. Requiring unbroken supersymmetry in the effective 4D field theory as well as considering topological equivalences between compactifications with different geometries reduce greatly the number of allowed heterotic orbifolds. In fact, all possible 6D orbifolds of this type have been exhaustively classified [80], resulting in a small number of Abelian orbifolds and thus a small number of possible geometrical symmetries to be considered.

In contrast to a bottom-up approach, where matter fields are arbitrarily localized at the singularities or let free in the bulk, in heterotic orbifolds matter localization is restricted by the compactification rules. All fields of the 4D effective field theories emerging from heterotic compactifications arise from the (anomaly, tachyon and ghost free) spectrum of excitations of *closed* strings that are not affected by the action of the orbifold.

In (supersymmetric) heterotic orbifolds, bulk or *untwisted fields* correspond to the orbifold-invariant states arising directly from the 10D closed strings of the uncompactified heterotic string, whose field limit is 10D $\mathcal{N} = 1$ supergravity endowed with an $\mathrm{E}_8 \times \mathrm{E}_8$ or SO(32) Yang-Mills theory. Thus, the 4D gauge (super)fields, generating the unbroken 4D gauge group $\mathcal{G}_{4D} \subset \mathrm{E}_8 \times \mathrm{E}_8$ or SO(32), and some 4D matter states live in the bulk of a heterotic orbifold.

Additionally, there are the so-called *twisted fields*, which arise from strings that are closed only due to the action of the orbifold. Twisted fields are always localized at singularities of the orbifold and are thus instrumental in the conception of a flavor theory with non-trivial representations from strings. As long as there are no further compactification ingredients, such as Wilson lines [81] or discrete torsion [82–85], that may lead to differences in the states at the singular points, the *twisted* spectrum is degenerate, i.e. all singularities carry identical twisted string states.

Couplings among string states are subject to a set of constraints called string selection rules [86–92], due to symmetries of the underlying conformal field theory of the compactified string theory. These selection rules establish for which combination of string states there is a non-zero correlation function, and thus a non-zero coupling for the associated effective fields. In the 4D model emerging from an Abelian heterotic orbifold, the selection rules amount to including additional (Abelian $\mathbb{Z}_N \times \mathbb{Z}_M \times \cdots$) symmetries and assign thus appropriate discrete charges to each field in the model.

Thus, we notice that flavor symmetries in Abelian toroidal heterotic orbifolds have two sources: the group of non-Abelian (relabeling) symmetries G_{nA} from the geometrical structure of the compactification space and the group of Abelian symmetries G_A from the string selection rules. In the case that the string selection rules provide a normal subgroup

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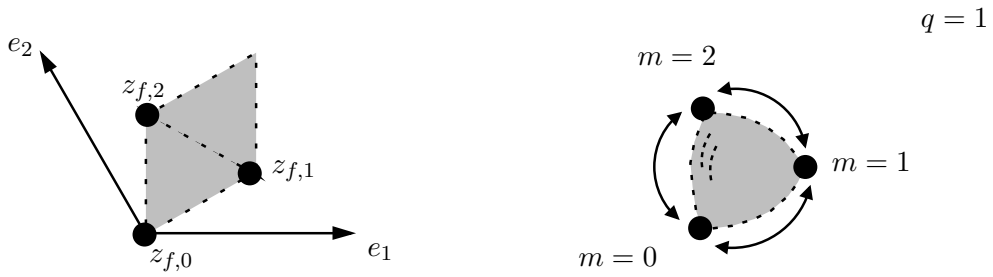

(a) Fixed points. **(b)** Symmetries and charges.

Figure 1. Geometrical origin of a $\Delta(54)$ flavor symmetry in a $\mathbb{T}^2/\mathbb{Z}_3$ orbifold. If the fixed points are not further affected by the compactification, there is an S_3 permutation symmetry. Further, string selection rules impose additional a $\mathbb{Z}_3 \times \mathbb{Z}_3$ symmetry based on the localization charges m and q of twisted states. The resulting symmetry is $S_3 \ltimes \mathbb{Z}_3^2 = \Delta(54)$.

(invariant under conjugation) of the full symmetry group, the resulting flavor symmetry is isomorphic to the semi-direct product $G_{nA} \ltimes G_A$ (see e.g. [93]).

Let us turn now to a relevant example for the present work. Suppose that two extended dimensions are compactified in the orbifold $\mathbb{T}^2/\mathbb{Z}_3$, where we choose the torus to be defined by the root lattice $\Lambda_{\mathrm{SU}(3)}$ which is invariant under the $\mathbb{Z}_3$ generator $\vartheta = e^{2\pi \mathrm{i}/3}$ in complex coordinates. That is, in the orbifold, points z_1 and z_2 of $\mathbb{C}$ are equivalent if they can be related by $z_1 = \vartheta z_2 + \lambda$, $\lambda \in \Lambda_{\mathrm{SU}(3)}$. In this orbifold, there exist three inequivalent fixed points or orbifold singularities[1] $z_{f,m}$, $m = 0, 1, 2$, such that $z_{f,m} = \vartheta z_{f,m} + \lambda_m$ for some lattice vectors λ_m. We can choose the inequivalent fixed points to be $z_{f,0} = 0$, $z_{f,1} = \frac{1}{3}(2e_1 + e_2)$ and $z_{f,2} = \frac{1}{3}(e_1 + 2e_2)$, where $\{e_\alpha\}$ span $\Lambda_{\mathrm{SU}(3)}$, as depicted in figure 1a. The gray region contains all inequivalent points in this orbifold.

Remarking in figure 1a that the upper tip is equivalent to $z_{f,0}$ and that the lines on both sides of $z_{f,i}$, $i \neq 0$, are identified, the orbifold becomes the triangular pillow-like object with three apices displayed in figure 1b. This solid is clearly invariant under all possible apex permutations, as symbolized by the arrows in that figure. Thus, we identify a geometrical S_3 symmetry.

When the current example is applied to heterotic orbifolds, the string selection rules demand additionally that any coupling of the form $\Phi_{m_1}\Phi_{m_2}\Phi_{m_3}\cdots$ among string states Φ_{m_i} (setting $m = 0$ for untwisted states) satisfy first $\sum_i m_i = 0 \mod 3$. Noting that this relation corresponds to a $\mathbb{Z}_3$ symmetry, it can be rewritten as $\prod_i \kappa^{m_i} = \mathbb{1}$ in terms of a $\mathbb{Z}_3$ generator $\kappa = e^{2\pi \mathrm{i}/3}$. Furthermore, assigning a charge $q = 1$ to ϑ-twisted states (and $q = 2$ to ϑ^2-twisted states and $q = 0$ to untwisted states), non-vanishing string couplings require that the couplings themselves be non-twisted, i.e. $\prod_i \vartheta^{q_i} = \mathbb{1}$, which can be rewritten as $\sum_i q_i = 0 \mod 3$. Thus, we identify a $\mathbb{Z}_3 \times \mathbb{Z}_3$ arising from the selection rules. Finally, since the $\mathbb{Z}_3 \times \mathbb{Z}_3$ obtained is a normal subgroup of the group generated by

[1] Analogous results are obtained for the second non-trivial $\mathbb{Z}_3$ group element, ϑ^2.

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S_3 and $\mathbb{Z}_3 \times \mathbb{Z}_3$, then the resulting effective flavor symmetry of a $\mathbb{T}^2/\mathbb{Z}_3$ orbifold can be written as $\Delta(54) = S_3 \ltimes \mathbb{Z}_3^2$.

In the absence of Wilson lines and discrete torsion, twisted string states replicate in all orbifold singularities, thus appearing always with a multiplicity of three and building triplet representations. Since $\vartheta^{-1} = \vartheta^2$, twisted states located at the ϑ^2 fixed points have the opposite geometrical quantum numbers of the ϑ-twisted states. That is, if we label as $\mathbf{3}_{11}$ the ϑ-twisted states,[2] those generated at the ϑ^2 singularities build then the representation $\mathbf{3}_{12}$. Untwisted states and twisted states affected by Wilson lines or discrete torsion are just $\Delta(54)$ trivial singlets $\mathbf{1}_0$. No other $\Delta(54)$ representations appear in this context, yielding a tight and useful string constraint for flavor phenomenology.

This discussion has been explicitly developed for all possible sub-orbifolds (in less than six dimensions) appearing in Abelian toroidal heterotic orbifolds [54], resulting in a reduced number of family symmetries. The findings include, besides $\Delta(54)$, only the symmetries[3] D_4, $(D_4 \times D_4)/\mathbb{Z}_2$, $(D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$, $(D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2$ and $S_7 \ltimes \mathbb{Z}_7^6$. As we shall see, these symmetries are enlarged in the full 6D heterotic orbifold, but can then be finally reduced back to these symmetries in phenomenologically viable models. This may already be considered a phenomenologically relevant observation: not any flavor symmetry is allowed in particle physics if it arises from a compactified string theory.

3 Classification of $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds with $\Delta(54)$

The purpose of this section is to identify string models exhibiting a number of semi-realistic properties and $\Delta(54)$ flavor symmetry in the simplest compactification scheme where such models are present, $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds.

$\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds are characterized by the quotient of a so-called factorizable torus $\mathbb{T}^6 = \mathbb{T}^2_1 \times \mathbb{T}^2_2 \times \mathbb{T}^2_3$ divided by the joint action of two $\mathbb{Z}_3$ isometries of $\mathbb{T}^6$ in the extra dimensions of a heterotic string. In the simplest case,[4] the tori are described by the root lattice of $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2 \times \mathrm{SU}(3)_3$ and the $\mathbb{Z}_3$ generators act diagonally on the tori as

$$\vartheta = \mathrm{diag}\left(e^{2\pi i v_1}, e^{2\pi i v_2}, e^{2\pi i v_3}\right) , \qquad \omega = \mathrm{diag}\left(e^{2\pi i w_1}, e^{2\pi i w_2}, e^{2\pi i w_3}\right) , \tag{3.1}$$

where v and w are the so-called twist vectors

$$v = (1/3, 0, -1/3) , \qquad w = (1/3, -1/3, 0) . \tag{3.2}$$

Consequently, each of the 2-tori are subject to a $\mathbb{Z}_3$ orbifold.

According to our previous discussion, one may conjecture that these constructions lead to a $\Delta(54)^3$ flavor symmetry, but this is wrong. In fact, in this case the relabeling symmetry that naturally appears is $S_3 \times S_3 \times S_3$. Further, concerning the symmetries due to string selection rules, invariance under the two twists, ϑ and ω, leads to two $\mathbb{Z}_3$ symmetries analogous to the one for the q charge in the previous section. In addition,

[2] We follow here the notation of [93] for $\Delta(54)$ representations; see appendix A.

[3] Note though that, under certain conditions, other symmetries may appear, as in [79].

[4] There are 15 $\mathbb{Z}_3 \times \mathbb{Z}_3$ choices, among which many include rototranslations [80].

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localization selection rules introduce one extra $\mathbb{Z}_3$ factor for each 2-torus. That is, the natural flavor symmetry in these heterotic orbifolds is $(S_3 \times S_3 \times S_3) \ltimes \mathbb{Z}_3^5$.

Note, however, that the relabeling symmetry can be further enhanced to S_{27} if the sizes of the tori $\mathbb{T}^2_a$, $a = 1, 2, 3$, are identical and no Wilson lines nor discrete torsion is invoked. As we shall shortly see, phenomenologically viable models only arise if one introduces Wilson lines. In fact, most promising models have two Wilson lines that distinguish the states located at the singularities of two of the tori, retaining only the non-Abelian S_3 relabeling symmetry. Further, there is no reason why all tori should have the same size; their sizes (and also their shapes) are encoded in the values of (untwisted) moduli that can a priori have arbitrary values.

Once the generic geometrical aspects of the compactification have been set, our task is now to apply this compactification to a heterotic string. We restrict ourselves here to the $\mathcal{N} = 1$ $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string, but expect similar results from the $\mathcal{N} = 1$ SO(32) heterotic string.[5] Modular invariance of the partition function demands the orbifold to be embedded into the gauge group $\mathrm{E}_8 \times \mathrm{E}_8$. This gauge embedding consists in choosing a 16D (shift) vector for each of the twists performed in the six compact dimensions and a so-called 16D Wilson-line vector A_α, $\alpha = 1, \ldots, 6$, encoding in the gauge degrees of freedom each e_α of $\mathbb{T}^6$.

The gauge embedding is subject to three constraints. First, modular invariance additionally imposes in $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds that [85]

$$\begin{aligned}
3\,(V^2 - v^2) &= 0 \mod 2\,, & 3\,(V \cdot A_\alpha) &= 0 \mod 2\,, & \alpha &= 1, \ldots, 6\,, \\
3\,(W^2 - w^2) &= 0 \mod 2\,, & 3\,(W \cdot A_\alpha) &= 0 \mod 2\,, & & \\
3\,(V \cdot W - v \cdot w) &= 0 \mod 2\,, & 3\,A_\alpha^2 &= 0 \mod 2\,, & & \\
& & 3\,(A_\alpha \cdot A_\beta) &= 0 \mod 2\,, & \alpha &\neq \beta\,,
\end{aligned} \tag{3.3}$$

where V and W are the 16D vectors that denote respectively the gauge embeddings of the twists v and w of eq. (3.2). Secondly, both V and W must be consistent with a $\mathbb{Z}_3 \times \mathbb{Z}_3$ action. This amounts to requiring that three times these vectors must be a trivial gauge transformation within $\mathrm{E}_8 \times \mathrm{E}_8$, i.e. for the shift vector V (with entries $V^{(i)}$)[6]

$$3\sum_{i=1}^{8} V^{(i)} = 0 \mod 2\,, \qquad 3\sum_{i=9}^{16} V^{(i)} = 0 \mod 2\,, \tag{3.4}$$

demanding that the entries $V^{(i)}$ be all integer or half-integer, independently for $i = 1, \ldots, 8$ and $i = 9, \ldots, 16$. Analogous conditions must then be imposed to W. The final constraint imposes that Wilson-line vectors must be consistent with the choice of $\mathbb{T}^6$ lattice and the action of the orbifold on it. The fact that the lattice vectors e_α are related by the action of ϑ and ω translates to relations among all A_α. For instance, in $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds since $e_2 = \vartheta e_1$ (see e.g. figure 1a, valid in this case), then $A_1 = A_2$ up to a trivial gauge

[5] We also expect promising non-supersymmetric models arising from the $\mathcal{N} = 0$ SO(16)×SO(16) heterotic string, although the presence of tachyons at some level of the theory would still be a worry.

[6] These constraints arise from the fact that the root lattice of each E_8 is even (and self-dual). An arbitrary shift within the lattice does not alter the gauge degrees of freedom.

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transformation in $E_8 \times E_8$. One finds that these geometrical considerations lead to the conditions

$$A_\alpha = A_{\alpha+1}\,, \qquad \alpha = 1,3,5\,, \tag{3.5}$$
$$3\sum_{i=1}^{8} A_\alpha^{(i)} = 0 \mod 2\,, \qquad 3\sum_{i=9}^{16} A_\alpha^{(i)} = 0 \mod 2\,.$$

A comment is in order. Notice that each 2-torus can be affected by up to one inequivalent, non-trivial Wilson line. If one includes the Wilson line $A_{2a-1} = A_{2a}$ associated with the compactification in the $\mathbb{T}^2_a$ torus, $a = 1,2,3$, the relabeling symmetry S_3 of that torus disappears. Thus, with one and two non-vanishing Wilson lines, the non-Abelian relabeling symmetry gets broken down, respectively, to $S_3 \times S_3$ and S_3, while no non-Abelian symmetry is left when all three Wilson lines are non-trivial. Hence, it follows that only models with two non-trivial Wilson lines can lead to a $\Delta(54) = S_3 \ltimes \mathbb{Z}_3^2$ flavor symmetry in $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds.

After finding solutions to the constraints (3.3)–(3.5), there are standard techniques, discussed elsewhere in great detail (see e.g. [94, 95]), to determine the spectrum of massless string states, including their gauge quantum numbers, localization, couplings and other properties of the *supersymmetric* effective field theory. Spectra obtained this way must then be inspected from a phenomenological perspective, imposing criteria based on observable particle physics (and/or cosmology) that may discriminate phenomenologically viable models from others.

Clearly, given the number of gauge-embedding parameters, the constraints (3.3)–(3.5) can be satisfied for a large number of shift and Wilson-line vectors, making the task of identifying phenomenologically viable heterotic orbifolds very time-consuming. Fortunately, this task becomes accessible thanks to tools such as the `orbifolder` [96], which automatizes the computation of massless spectra, couplings and other important features of the models.

With the purpose of finding promising models endowed with a $\Delta(54)$ flavor symmetry, we have used the `orbifolder` to randomly construct a large number of inequivalent $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifold models. Models are considered to be equivalent by the software if no differences are found when comparing the full gauge group, the non-Abelian gauge quantum numbers of the resulting states and the number of non-Abelian gauge singlets in the massless spectrum. From the created models, we have then selected the most promising ones. Here, a promising model must yield the SM gauge group, such that the hypercharge generator be non-anomalous and (with normalization) compatible with grand unification, three generations of quarks and leptons, at least a couple of Higgs (super)fields, H_u and H_d, and only vectorlike exotics w.r.t. the SM gauge group.

Our results are as follows. We have obtained over 7×10^6 inequivalent $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifold models,[7] with up to (the maximum of) three inequivalent Wilson lines. After applying our phenomenological constraints, only 789 models exhibit the required properties.

[7] Following the statistical approach of [78, section 2.2], we estimate that the number of generated models represents about 90% of the total of possible models in this scenario.

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We have verified that, considering couplings[8] of the vectorlike exotics with up to six SM singlets, in a large number of these models all exotics decouple once the SM singlets develop vacuum expectation values (VEVs). Other models require higher dimensional operators to yield mass terms for all vectorlike exotics.

An interesting geometrical quality of the promising models regards the effective family symmetry. Among the 789 selected models, most (696) of them have two inequivalent non-vanishing Wilson lines. About 10% of the viable models (81 of them) require one non-trivial Wilson line, and only 12 result from compactifications with three Wilson lines. Therefore, we find that $\Delta(54)$ as a flavor symmetry of (MS)SM-like models is favored in $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifold models.

This outcome is compatible with previous results found in the literature. Particularly, in ref. [97] the authors have found 445 $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifold models with the properties we have required, out of which 369 of them exhibit two non-trivial Wilson lines. In this perspective, our search shows to be more exhaustive.

4 A sample model with stringy $\Delta(54)$ flavor

With the purpose of exploring the flavor phenomenology produced by string compactifications, let us now study the properties of one of the promising models from our $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifold scan, chosen due to its simplicity. The parameters that define the model are the shift vectors

$$3V = \left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};-2,0,0,1,1,1,1,4\right), \tag{4.1a}$$

$$3W = (0,1,1,4,0,0,1,1;1,-1,4,-4,-1,0,0,1), \tag{4.1b}$$

and the Wilson lines

$$3A_1 = 3A_2 = \left(-\frac{7}{2},-\frac{3}{2},\frac{9}{2},\frac{7}{2},-\frac{7}{2},-\frac{3}{2},\frac{5}{2},\frac{7}{2};-3,0,-2,0,-2,-4,3,-2\right), \tag{4.2a}$$

$$3A_3 = 3A_4 = (3,3,-3,-2,-1,2,4,-4;-3,1,-1,-4,1,1,4,1). \tag{4.2b}$$

These parameters yield the unbroken gauge group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times[\mathrm{SU}(2)\times\mathrm{U}(1)^{11}]$, where the additional SU(2) factor is considered hidden because no SM-field carries a charge under that group. However, all fields in the spectrum are charged under the additional U(1) factors.

Due to its two Wilson lines (4.2), the model has the flavor symmetry $S_3\ltimes\mathbb{Z}_3^5\supset\Delta(54)$. The $\Delta(54)$ quantum numbers are associated with the symmetries of the third torus, $\mathbb{T}_3^2$, whose localized states are not affected by any Wilson line. If we allow for the spontaneous breakdown of the three additional $\mathbb{Z}_3$ symmetries by VEVs of appropriate SM singlets transforming as $\mathbf{1}_0$ under $\Delta(54)$, the flavor symmetry in the vacuum is just $\Delta(54)$ and the extra $[\mathrm{SU}(2)\times\mathrm{U}(1)^{11}]$ gauge factors are broken too.

[8]Given the persistent controversy about the selection rules in heterotic orbifolds, we have considered only the so-called rule 4 [90], gauge and space-group invariance, and R-charge conservation [91, 92].

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#	irrep	$\Delta(54)$	label	#	anti-irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i				
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\bar{u}_i$				
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\bar{d}_i$				
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i				
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\bar{e}_i$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\bar{\nu}_i$				
1	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{1}_0$	H_d	1	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\mathbf{1}_0$	H_u
Flavons							
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	ϕ_i^u				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	$\phi_i^{d,e}$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\bar{\phi}_i^{\nu}$				
2	$(\mathbf{1},\mathbf{1})_{0}$	$2\cdot\mathbf{1}_0$	$s^{(d,e)}, s^u$				
128	$(\mathbf{1},\mathbf{1})_{0}$	$77\cdot\mathbf{1}_0+16\cdot\mathbf{3}_{12}+\mathbf{3}_{11}$	N_i				
Exotic states							
16	$(\mathbf{1},\mathbf{2})_{\frac{1}{6}}$	$10\cdot\mathbf{1}_0+2\cdot\mathbf{3}_{12}$	v_i	16	$(\mathbf{1},\mathbf{2})_{-\frac{1}{6}}$	$4\cdot\mathbf{1}_0+4\cdot\mathbf{3}_{12}$	$\bar{v}_i$
3	$(\mathbf{3},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	y_i	3	$(\overline{\mathbf{3}},\mathbf{1})_{0}$	$3\times\mathbf{1}_0$	$\bar{y}_i$
1	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	$\mathbf{1}_0$	z_i	1	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{1}_0$	$\bar{z}_i$
7	$(\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{12}$	x_i	7	$(\mathbf{1},\mathbf{1})_{\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{11}$	$\bar{x}_i$
51	$(\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	$30\cdot\mathbf{1}_0+7\cdot\mathbf{3}_{12}$	w_i	51	$(\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$24\cdot\mathbf{1}_0+9\cdot\mathbf{3}_{12}$	$\bar{w}_i$

Table 1. Massless spectrum. Representations w.r.t. $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L$ are given in bold face, the hypercharge is indicated by the subscript. The 3rd and 7th columns display the $\Delta(54)$ flavor representations.

	Q_i	$\bar{d}_i^c$	$\bar{u}_i^c$	L_i	$\bar{e}_i^c$	$\bar{\nu}_i$	H_u	H_d	ϕ_i^u	$\phi_i^{(d,e)}$	$\bar{\phi}_i^{\nu}$	s^u	$s^{(d,e)}$
$\Delta(54)$	$\mathbf{3}_{11}$	$\mathbf{3}_{11}$	$\mathbf{3}_{11}$	$\mathbf{3}_{11}$	$\mathbf{3}_{11}$	$\mathbf{3}_{12}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}_{11}$	$\mathbf{3}_{11}$	$\mathbf{3}_{12}$	$\mathbf{1}_0$	$\mathbf{1}_0$
$\mathbb{Z}_3^{(1)}$	ω	1	ω	1	ω	1	1	1	1	1	1	ω	ω^2
$\mathbb{Z}_3^{(2)}$	1	ω^2	1	ω^2	1	ω	1	1	1	ω	ω	1	1
$\mathbb{Z}_3^{(3)}$	ω	1	ω	1	ω	1	1	1	1	1	1	ω	ω^2

Table 2. Flavor representations for the SM matter and flavon fields in a $\mathbb{Z}_3\times\mathbb{Z}_3$ sample model. The $\mathbb{Z}_3$ charges are defined in terms of the twist and field localizations in (4.3) with $\omega=e^{2\pi\mathrm{i}/3}$.

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The gauge and $\Delta(54)$ representations of the massless matter spectrum of our sample model are provided in table 1. As explained before, the only possible $\Delta(54)$ representations are the trivial singlet $\mathbf{1}_0$ and the two triplets $\mathbf{3}_{11}$ and $\mathbf{3}_{12}$. One particular feature of the observable sector is that, directly from the string computation, only three SM generations that build non-trivial flavor representations arise, while the Higgs states are untwisted fields and thus uncharged under the flavor symmetry. On the other hand, the exotic particles are vectorlike w.r.t. the SM gauge group, but not necessarily under the flavor group. Despite this hurdle, there exist SM singlets N_i in the appropriate flavor representations, so that all exotics and the singlets N_i themselves can acquire masses when $\langle N_i \rangle \neq 0$.

To understand better the flavor phenomenology of the observable sector of this model, we display in table 2 all flavor charges of the SM superfields and some gauge singlets that shall serve as flavons. The $\mathbb{Z}_3$ charges are given in terms of

$$\omega^q \qquad \text{and} \qquad \kappa_a{}^{m_a} \equiv (e^{2\pi \mathrm{i}/3})^{m_a} \qquad \text{with} \quad a = 1,2;\, q, m_a = 0,1,2\,, \tag{4.3}$$

where ω is the (eigenvalue of the) second twist in eq. (3.1), κ_a correspond to the $\mathbb{Z}_3$ generators associated with the localization labels m_a in the (first or second) torus $\mathbb{T}^2_a$, as described in section 2, and q is the power of the twist that yields the corresponding twisted states. Note that $\omega = \kappa_a = e^{2\pi \mathrm{i}/3}$.

Since the SM matter fields are charged under flavor symmetries, the presence of the properly charged s and ϕ flavon fields allows for Yukawa couplings in the (non-renormalizable) superpotential, which in this case can be written as follows

$$\begin{aligned} W_Y = {} & y^u_{ijk} Q_i H_u \bar{u}_j \phi^u_k s_u + y^d_{ijk} Q_i H_d \bar{d}_j \phi^{(d,e)}_k s^{(d,e)} + y^e_{ijk} L_i H_d \bar{e}_j \phi^{(d,e)}_k s^{(d,e)} \\ & + y^\nu_{ijkl} L_i H_u \bar{\nu}_j + \lambda_{ijk} \bar{\nu}_i \bar{\nu}_j \bar{\phi}^\nu_k\,, \qquad\qquad i,j,k = 1,2,3, \end{aligned} \tag{4.4}$$

where the summation over repeated indices must follow the rules of the product of $\Delta(54)$ representations that lead to invariant singlets (cf. appendix A),

$$\mathbf{1}_0 \subset \mathbf{3}_{11} \times \mathbf{3}_{12}, \quad \mathbf{1}_0 \subset \mathbf{3}_{11} \times \mathbf{3}_{11} \times \mathbf{3}_{11}\,, \quad \mathbf{1}_0 \subset \mathbf{3}_{12} \times \mathbf{3}_{12} \times \mathbf{3}_{12}.$$

In principle, all Yukawa-coupling coefficients, y and λ, are computable by applying CFT techniques for the string model. However, it is known that there are still some challenges to be solved for non-renormalizable couplings. The best we can do here is to estimate that y are order one (but with a suppression due non-renormalizability) because they include the untwisted Higgs fields, whereas λ must be somewhat suppressed because all involved fields are twisted. We observe that the second row of W_Y admits neutrino masses from a type I see-saw mechanism with three right-handed (RH) neutrinos with proper ϕ flavon VEVs. Similarly, the Dirac masses of charged leptons and quarks are determined by the VEVs of other flavons ϕ and s. We point out that the structure of masses for down-quarks and charged leptons is predicted in this model to be identical because the flavons involved in the corresponding couplings are unavoidably the same. As we shall see, this enforces a more stringent sort of $b - \tau$ unification.

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5 Fermion masses from a $\Delta(54)$ flavor symmetry

The properties of the string-derived model presented before can be now studied from a bottom-up perspective. Although our string sample model is supersymmetric and all couplings are determined at the compactification scale, the general structure of the Yukawa Lagrangian at low energies can be determined from W_Y, if we insist on retaining the $\Delta(54)$ flavor symmetry in the soft-breaking sector. Besides, it is known that Yukawa couplings do not receive large contributions through the renormalization running [98]. Similarly, threshold corrections shall not alter the mass and mixing structure of quarks and leptons, since it depends mainly on mass ratios. Therefore, we can safely study the viability of the model by restricting ourselves to the behavior of the appropriate non-supersymmetric fields.

In a compact notation, the effective Yukawa Lagrangian for quarks and charged leptons that is obtained from W_Y reads

$$\begin{aligned}\mathcal{L}_Y^f &= y_1^f \left[F_1 H \bar{f}_1 \phi_1 + F_2 H \bar{f}_2 \phi_2 + F_3 H \bar{f}_3 \phi_3\right] \\ &\quad + y_2^f \left[(F_1 H \bar{f}_2 + F_2 H \bar{f}_1)\phi_3 + (F_3 H \bar{f}_1 + F_1 H \bar{f}_3)\phi_2 + (F_2 H \bar{f}_3 + F_3 H \bar{f}_2)\phi_1\right] + h.c.\,,\end{aligned} \tag{5.1}$$

where generically F and $\bar{f}$ denote respectively the left-chiral and right-chiral components of SM fermions, H labels the Higgs associated with $\bar{f}$, and ϕ stands for flavon fields. Further, we have let the VEVs of the s flavons be absorbed in the Yukawas y, as they do not alter the structure of the couplings.

From the Yukawa Lagrangian (5.1), the Dirac mass matrices for the charged fermions (namely, up and down quarks, and charged leptons) generically take the form

$$M_f^D = \begin{pmatrix} y_1^f \phi_1^f & y_2^f \phi_3^f & y_2^f \phi_2^f \\ y_2^f \phi_3^f & y_1^f \phi_2^f & y_2^f \phi_1^f \\ y_2^f \phi_2^f & y_2^f \phi_1^f & y_1^f \phi_3^f \end{pmatrix} . \tag{5.2}$$

Let us now make a phenomenological assumption on the flavon VEVs. Suppose the possibility of a VEV alignment of the form $\langle \phi^f \rangle = v_\phi^f (0, r^f, 1)$, with $f = u, d, e$, for some real values v_ϕ^f and r^f. This greatly simplifies the mass matrices to

$$M_f^D = \begin{pmatrix} 0 & a^f & a^f r^f \\ a^f & b^f r^f & 0 \\ a^f r^f & 0 & b^f \end{pmatrix} , \tag{5.3}$$

where we define $a^f \equiv y_2^f v_\phi^f$ and $b^f \equiv y_1^f v_\phi^f$. Using now the invariant traces and determinant of M_f^D (we take a negative m_1^f to compensate the minus sign in the determinant),

$$\begin{aligned} \mathrm{tr}\, M_f^D &= b^f (1 + r^f) && \stackrel{!}{=} -m_1^f + m_2^f + m_3^f \,, \\ \mathrm{tr}(M_f^D)^2 &= [2(a^f)^2 + (b^f)^2][1 + (r^f)^2] && \stackrel{!}{=} (m_1^f)^2 + (m_2^f)^2 + (m_3^f)^2 \,, \\ \det M_f^D &= -(a^f)^2 b^f \,[1 + (r^f)^3] && \stackrel{!}{=} -m_1^f m_2^f m_3^f \,, \end{aligned} \tag{5.4}$$

it is straightforward to write down the Dirac mass matrices in terms of its eigenvalues, i.e. the three (observable) fermion masses of type f, m_i^f.

Clearly, any solution to the invariants (5.4) provides the right masses for quarks and charged leptons. If we take e.g. the hierarchical solution, i.e. $r^f \ll 1$ and $a^f \ll b^f$, the mass matrices take the form

$$M_f^D \approx \begin{pmatrix} 0 & \sqrt{m_1^f m_2^f} & \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} \\ \sqrt{m_1^f m_2^f} & m_2^f - m_1^f & 0 \\ \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} & 0 & m_3^f \end{pmatrix}, \tag{5.5}$$

which corresponds to

$$r^f \approx (m_2^f - m_1^f)/m_3^f\,, \qquad (a^f)^2 \approx m_1^f m_2^f\,, \qquad b^f \approx m_3^f\,. \tag{5.6}$$

We notice that the hierarchical solution is compatible with the hierarchy of observed fermion masses.

In the down-quark sector, this structure gives the Gatto-Sartori-Tonin formula for the Cabibbo angle, which is approximately the ratio $(M_d^D)_{12}/(M_d^D)_{22}$,

$$\lambda_C \approx \sqrt{\frac{m_d}{m_s}}\,, \tag{5.7}$$

where we additionally used that $m_d/m_s \ll 1$.[9] The other two mixing angles are very small at leading order, but could be generated if some of the vectorlike quarks mix with the SM quarks, see for instance [99].

For charged leptons, on the other hand, the same flavon VEV alignment must be imposed because down-quarks and charged leptons share the same flavons. It follows that the corresponding mass matrix is diagonalized by a rotation in the 1–2 entries with the mixing angle of the order $\sqrt{m_e/m_\mu}$.

There is another consequence of the parallelism between the down-quarks and charged leptons. Since $r^d = r^e$, it follows from eq. (5.6) that the following mass relation in our model is required

$$\frac{m_s - m_d}{m_b} \stackrel{!}{=} \frac{m_\mu - m_e}{m_\tau}. \tag{5.8}$$

This relation does not match observations. We find that some possibilities to amend eq. (5.8) include either to abandon the flavor structure in the soft-terms of the supersymmetry breaking sector or that some (colored and uncolored) exotics acquire masses after the breakdown of $\Delta(54)$, providing different suppression factors for down-quarks and charged leptons. The latter can be achieved by allowed couplings as those represented in figure 2, which yield effective contributions to Yukawa couplings, such as

$$\frac{1}{m_v m_x m_z} Q H_d \bar{d} \langle N_i N_j N_k \rangle + \frac{1}{m_v m_x m_w} L H_d \bar{e} \langle N_i' N_j' N_k' \rangle\,,$$

[9] Eq. (5.7) has a small correction of order $\sqrt{m_u/m_c}$ from the up-quark sector.

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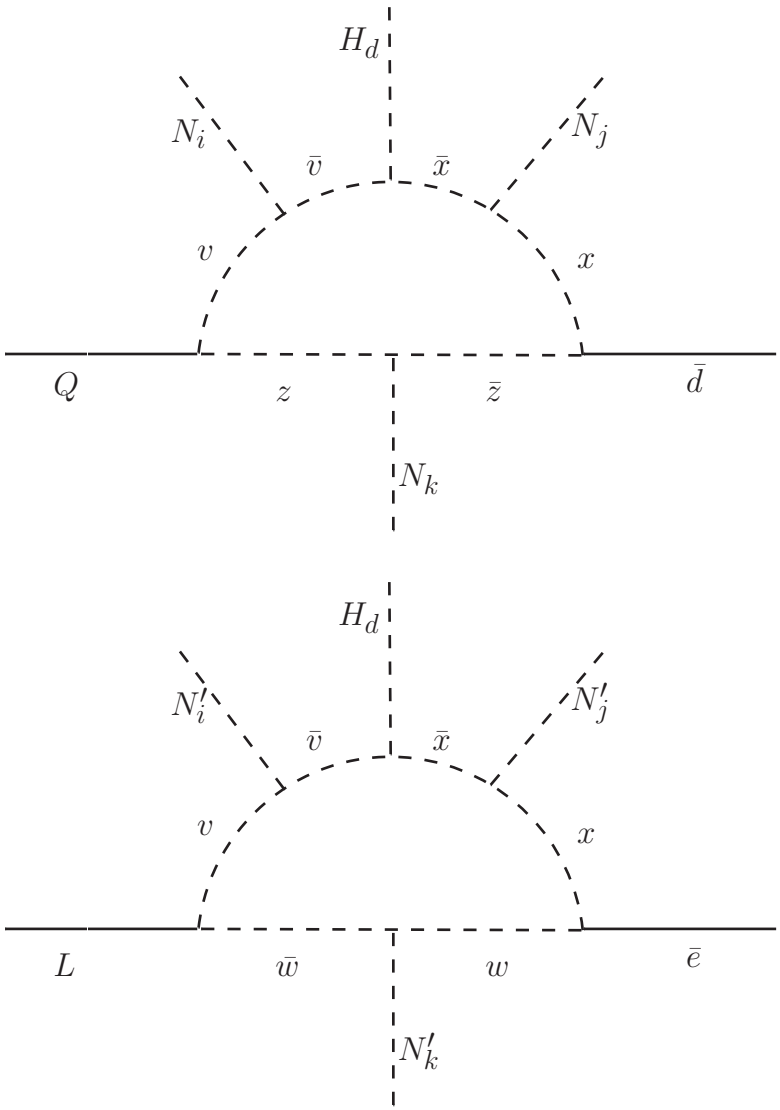

Figure 2. Phenomenologically viable operators in the model presented that may alleviate the tension observed by the predicted relation (5.8).

where both $N_{i,j,k}$ and $N'_{i,j,k}$ denote some of the 128 flavons of table 1, and m_χ denotes the effective mass of a given exotic field χ. Realizing particularly that m_z and m_w differ in general and, moreover, that the flavons in the couplings may be different, we find that the issue underlined by the constraint (5.8) may be alleviated. Unfortunately, even if this hurdle is tackled, we do not expect these effects to alter the smallness of the remaining two quark mixing angles since that depends on the hierarchical structure of the fermion masses.

5.1 The neutrino sector

For neutrinos, the major difference w.r.t. the other sectors is that, besides the presence of Majorana mass terms, neutrinos build a conjugate $\Delta(54)$ triplet, $\mathbf{3}_{12}$. Therefore, renormalizable Yukawa couplings become possible.

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As stated before, the neutrino masses arise from a type I see-saw according to the second row of the superpotential (4.4). From there, we can read off the Yukawa Lagrangian for neutrinos:

$$\begin{aligned}\mathcal{L}_Y^\nu \;=\; & y_1^\nu \left[L_1 H_u \bar{\nu}_1 + L_2 H_u \bar{\nu}_2 + L_3 H_u \bar{\nu}_3\right] \\ & + \lambda_1 \left[\bar{\nu}_1 \bar{\nu}_1 \bar{\phi}_1^\nu + \bar{\nu}_2 \bar{\nu}_2 \bar{\phi}_2^\nu + \bar{\nu}_3 \bar{\nu}_3 \bar{\phi}_3^\nu\right] \\ & + \lambda_2 \left[2\bar{\nu}_1 \bar{\nu}_2 \bar{\phi}_3^\nu + 2\bar{\nu}_1 \bar{\nu}_3 \bar{\phi}_2^\nu + 2\bar{\nu}_2 \bar{\nu}_3 \bar{\phi}_1^\nu\right] .\end{aligned} \tag{5.9}$$

Hence, the Dirac neutrino mass matrix is proportional to the identity matrix, while RH neutrino masses are governed by a structure similar to the one in eq. (5.2), that is,

$$M_{RH} = \begin{pmatrix} \lambda_1 \bar{\phi}_1^\nu & \lambda_2 \bar{\phi}_3^\nu & \lambda_2 \bar{\phi}_2^\nu \\ \lambda_2 \bar{\phi}_3^\nu & \lambda_1 \bar{\phi}_2^\nu & \lambda_2 \bar{\phi}_1^\nu \\ \lambda_2 \bar{\phi}_2^\nu & \lambda_2 \bar{\phi}_1^\nu & \lambda_1 \bar{\phi}_3^\nu \end{pmatrix} . \tag{5.10}$$

We can now make a working assumption about the VEV of the neutrino flavon $\bar{\phi}^\nu$. Considering the alignment $\langle \bar{\phi}^\nu \rangle = v_{\nu_3} (R_1, \delta, 1)$, the light neutrino mass matrix becomes

$$M_\nu \;=\; \lambda \begin{pmatrix} \delta - R^2 R_1^2 & R(-1 + R R_1 \delta) & R(-\delta^2 + R R_1) \\ R(-1 + R R_1 \delta) & R_1 - R^2 \delta^2 & R(R\delta - R_1^2) \\ R(-\delta^2 + R R_1) & R(R\delta - R_1^2) & R_1 \delta - R^2 \end{pmatrix} , \tag{5.11}$$

where we used the definitions

$$R = \lambda_2/\lambda_1 , \qquad \lambda = y_1^2 \langle H_u \rangle^2 \Big/ \left[\lambda_1 v_{\nu_3} \left(R_1 \delta + 2R^3 R_1 \delta - R^2 (1 + R_1^3 + \delta^3)\right)\right] . \tag{5.12}$$

After performing a scan of our parameters, restricting the values of the computed Δm_{12}^2, Δm_{13}^2 and neutrino mixing angles to lie within the 3σ region of the global fits [100], we find that the mass matrix in eq. (5.11) is compatible only with a normal hierarchy of neutrino masses, i.e. an inverted hierarchy is disfavored, coinciding with recent preliminary results from the T2K collaboration [101].

Furthermore, we observe that our model leads to a correlation between the atmospheric and the reactor mixing angles in normal ordering, as displayed by the blue region in figure 3. Comparing with the precision intervals, we see that the atmospheric mixing angle lies in the second octant, approximately between 51.3 and 53.1 degrees, while the reactor mixing angle has values between 7.8 and 8.9 degrees, in agreement with the oscillation global fits within 3σ. These values are crucial for the model since a better measurement of the neutrino mixing angles could falsify it.

A final result from our parameter scan is that the lightest neutrino mass, m_{ν_1}, takes values in the region between 6 meV and 6.8 meV, and the sum of the light neutrino masses, $\sum m_\nu$, lies in the interval between 65 meV and 70 meV, in consistency with data.

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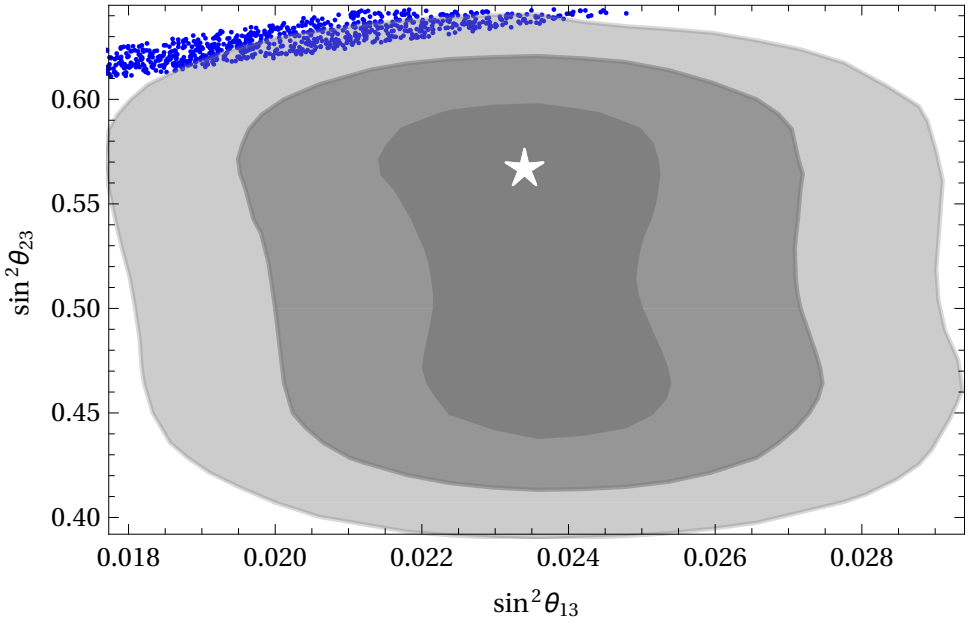

Figure 3. Correlation between the atmospheric and reactor mixing angles for normal mass ordering in a string-inspired $\Delta(54)$ flavor model. The correlation (blue) points in the upper-left part of the plot result from a scan of our parameters λ, δ, R, R_1, imposing consistency within 3σ with measured values of Δm^2_{12}, Δm^2_{13} and θ_{12}. The dark/light/lighter gray areas correspond to $1\sigma/2\sigma/3\sigma$ experimental precision around the best fit value (denoted by the star) for the neutrino mixing angles [100].

6 Final remarks

Flavor symmetries arise naturally in string compactifications, which provide a promising ultraviolet completion of usual bottom-up setups. Particularly, we have shown that $\Delta(54)$, as a flavor symmetry, appears most naturally in semi-realistic $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifold compactifications. We have identified almost 700 models with that flavor symmetry and other promising particle-physics features, such as SM gauge group and three generations of matter fields. By their nature, these constructions reduce the arbitrariness of low-energy models by constraining the fields and their (flavor and gauge) transformation properties and thereby providing useful guidelines to inspect flavor phenomenology.

To test the viability of $\Delta(54)$ flavor scenarios arising from strings, we have studied the phenomenology of one simple string model from our classification, whose properties may differ from the other identified models. In this model, SM fermion fields transform as triplets of the flavor symmetry while the Higgs fields do not transform. As a result of the flavor quantum numbers, the quarks and charged leptons acquire masses through dimension-6 operators, and the Dirac neutrino masses as well as the RH Majorana neutrino masses are generated at renormalizable level. Furthermore, we observe that choosing some special flavon-VEV alignments results in the following flavor phenomenology features:

- correct masses for quarks and charged leptons;
- proper Gatto-Sartori-Tonin relation in the quark sector (although the other two mixing angles are very small);

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- a mass relation between the down-quark sector and the charged leptonic sector (see eq. (5.8));
- compatibility (only) with normal hierarchy of neutrino masses;
- smallest neutrino mass of order $6-7$ meV; and
- PMNS matrix compatible with current constraints (atmospheric and reactor mixing angles are in the 3σ region of the global best fit), with the atmospheric mixing angle greater than 45 degrees.

Interestingly, an inverted hierarchy being disfavored as well as the atmospheric mixing angle lying in the second octant, are features compatible with recent preliminary findings of the T2K collaboration [101]. This outcome lets us assert that $\mathbb{Z}_3\times\mathbb{Z}_3$ heterotic orbifolds and $\Delta(54)$ as a flavor symmetry provide a fertile playground for useful phenomenology which should be further investigated.

The particular model we have studied here was chosen due to its neat simplicity: it has only three SM generations, the extra gauge sector includes only a hidden SU(2) and Abelian symmetries, and all SM fields build $\Delta(54)$ triplets. These properties are only shared by three more models in the set of promising $\mathbb{Z}_3\times\mathbb{Z}_3$ compactifications. Other models include additional (exotic) vectorlike pairs of quarks and leptons, larger Abelian and non-Abelian hidden gauge symmetries, and some SM fields may build only trivial representations of $\Delta(54)$. This does not imply that other models are more or less promising, but their analysis is somewhat more involved and shall be the purpose of future studies.

Despite these encouraging features, there are still some challenges to overcome. First, in heterotic orbifolds it is challenging to obtain the VEV alignments chosen in section 5 because VEVs must be settled by a moduli stabilization mechanism that is not fully understood. Secondly, we found that two of the quark mixing angles in our model are too small and the mass relation eq. (5.8) is incorrect. To attempt to alleviate these issues, one should study in detail the soft-terms and other corrections in this kind of models. Another potential hurdle is the absence of a symmetry that forbids rapid proton decay. However, it is conceivable that such symmetry does appear as one of the extra $\mathbb{Z}_3$ symmetries of another model where matter fields have the correct charges. Finally, as in most flavor models, flavor-changing neutral currents pose a challenge that must and shall be studied elsewhere in the context of our proposal.

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A $\Delta(54)$ tensor product for triplet representations

In this appendix, we provide the features of $\Delta(54)$ that are relevant for our proposal, following the notation of ref. [93]. The $\Delta(54)$ symmetry group has two one-dimensional, four two-dimensional and four three-dimensional irreducible representations. These representations are denoted as $\mathbf{1}_0$ (invariant under the group), $\mathbf{1}_1$, $\mathbf{2}_1$, $\mathbf{2}_2$, $\mathbf{2}_3$, $\mathbf{2}_4$, $\mathbf{3}_{11}$, $\mathbf{3}_{12}$, $\mathbf{3}_{21}$ and $\mathbf{3}_{22}$.

Due to the matter content of our model, the only tensor products that are relevant in this work are those among the three-dimensional representations $\mathbf{3}_{11}$ and $\mathbf{3}_{12}$, which are obtained as

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}_{11}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}_{11}} = \begin{pmatrix} x_1 y_1 \\ x_2 y_2 \\ x_3 y_3 \end{pmatrix}_{\mathbf{3}_{12}} \oplus \begin{pmatrix} x_2 y_3 + x_3 y_2 \\ x_3 y_1 + x_1 y_3 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{3}_{12}} \oplus \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}_{\mathbf{3}_{22}}, \tag{A.1}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}_{12}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}_{12}} = \begin{pmatrix} x_1 y_1 \\ x_2 y_2 \\ x_3 y_3 \end{pmatrix}_{\mathbf{3}_{11}} \oplus \begin{pmatrix} x_2 y_3 + x_3 y_2 \\ x_3 y_1 + x_1 y_3 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{3}_{11}} \oplus \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}_{\mathbf{3}_{21}}, \tag{A.2}$$

and finally

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}_{11}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}_{12}} &= \left(x_1 y_1 + x_2 y_2 + x_3 y_3\right)_{\mathbf{1}_0} \oplus \begin{pmatrix} x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3 \\ \omega x_1 y_1 + \omega^2 x_2 y_2 + x_3 y_3 \end{pmatrix}_{\mathbf{2}_1} \\ &\oplus \begin{pmatrix} x_1 y_2 + \omega^2 x_2 y_3 + \omega x_3 y_1 \\ \omega x_1 y_3 + \omega^2 x_2 y_1 + x_3 y_2 \end{pmatrix}_{\mathbf{2}_2} \oplus \begin{pmatrix} x_1 y_3 + \omega^2 x_2 y_1 + \omega x_3 y_2 \\ \omega x_1 y_2 + \omega^2 x_2 y_3 + x_3 y_1 \end{pmatrix}_{\mathbf{2}_3} \\ &\oplus \begin{pmatrix} x_1 y_3 + x_2 y_1 + x_3 y_2 \\ x_1 y_2 + x_2 y_3 + x_3 y_1 \end{pmatrix}_{\mathbf{2}_4}, \end{aligned} \tag{A.3}$$

where $\omega = e^{2\pi \mathrm{i}/3}$. It follows that the only products of $\Delta(54)$ triplets up to trilinear order that yield invariant combinations are $\mathbf{3}_{11} \otimes \mathbf{3}_{12}$, $\mathbf{3}_{11} \otimes \mathbf{3}_{11} \otimes \mathbf{3}_{11}$ and $\mathbf{3}_{12} \otimes \mathbf{3}_{12} \otimes \mathbf{3}_{12}$. The latter two products lead to two invariant singlets $\mathbf{1}_0$ each.

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