Published for SISSA by Springer

Received: *June 30, 2020*
Revised: *September 11, 2020*
Accepted: *September 26, 2020*
Published: *November 9, 2020*

# Inclusive semileptonic $\Lambda_b$ decays in the Standard Model and beyond

**P. Colangelo,[a] F. De Fazio[a] and F. Loparco[a,b]**

[a] *Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Via Orabona 4, I-70126 Bari, Italy*
[b] *Università degli Studi di Bari, Via Orabona 4, I-70126 Bari, Italy*
*E-mail:* pietro.colangelo@ba.infn.it, fulvia.defazio@ba.infn.it, francesco.loparco1@ba.infn.it

Abstract: Inclusive semileptonic decays of beauty baryons are studied using the heavy quark expansion to $\mathcal{O}(1/m_b^3)$, at leading order in $\alpha_s$. The case of a polarized decaying baryon is examined, with reference to $\Lambda_b$. An extension of the Standard Model effective Hamiltonian inducing $b \to U\ell\bar{\nu}_\ell$ transitions ($U = u$, $c$ and $\ell = e$, $\mu$, $\tau$) is considered, which comprises the full set of D=6 semileptonic operators with left-handed neutrinos. The effects of the new operators in several observables are described.

Keywords: Beyond Standard Model, Heavy Quark Physics

ArXiv ePrint: 2006.13759



https://doi.org/10.1007/JHEP11(2020)032

## Contents



## 1 Introduction

The observations of anomalies in $b \to c$ semileptonic exclusive decays of $B$ mesons, with hints toward possible violation of lepton flavour universality (LFU),[1] require new analyses of related processes involving heavy hadrons with a single $b$ quark, to enlarge the set of observables suitable to test the Standard Model (SM) predictions. The inclusive semileptonic modes are theoretically appealing, since the nonperturbative effects of strong interactions, which necessarily must be taken into account, can be systematically considered by an expansion in the inverse heavy quark mass [2, 3]. The expansion involves a set of long-distance hadronic matrix elements of operators of increasing dimension, which can be classified and parametrized. For each term in the heavy quark expansion perturbative QCD corrections can also be computed at increasing order in $\alpha_s$, therefore a double expansion in $1/m_Q$ and $\alpha_s$ is obtained. Improving the control of QCD effects, in the inclusive as well as in the exclusive processes, is the premise to disentangle the origin of the observed anomalies.

The present study is devoted to the inclusive $b \to c, u$ semileptonic modes of $b$-flavoured baryons, in particular $\Lambda_b \to X_{c,u} \ell^- \bar{\nu}_\ell$. The formalism is developed for a generic baryon, therefore it can also be applied to $\Xi_b$ and $\Omega_b$. In our study the heavy quark mass expansion is considered at $\mathcal{O}(1/m_Q^3)$, and the parametrization of the baryon matrix elements relevant at this order is provided. Moreover, the case of polarized baryon decays is considered at this

[1] For a review see [1].



JHEP11(2020)032

order, the unpolarized case being recovered averaging over the initial baryon polarizations. The semileptonic transitions are analyzed in the Standard Model and in an extension of the SM effective weak Hamiltonian comprising vector, scalar, pseudoscalar, tensor and axial operators. Such Hamiltonian densities have been scrutinized in connection with the flavour anomaly problem, considering $B$ meson exclusive modes, see, e.g., [4–6], but less information is available about their impact on inclusive observables [7–10].

Let us briefly remind the status of the above mentioned flavour anomaly. A small excess in the ratios $R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(B\to D^{(*)}\ell^-\bar{\nu}_\ell)}$ $(\ell = e, \mu)$ with respect to the SM expectations emerges after the BABAR [11, 12], Belle [13–16] and LHCb [17–19] measurements are combined. The tension with SM is presently estimated at $3.1\,\sigma$ level [20–23]. Several interpretations attribute the deviation to the effect of non-SM interactions mainly affecting the third generation. New lepton flavour universality violating interactions could produce at low energies additional operators in the $b \to c\tau^-\bar{\nu}_\tau$ effective weak Hamiltonian, which can be scrutinized using global quantities, namely the decay branching fractions, and also, more efficiently, using observables as the 4d $\bar{B} \to D^*(D\pi, D\gamma)\ell^-\bar{\nu}_\ell$ decay distributions for the three lepton species [5, 24–29]. This kind of analyses are also possible for $B_s$ modes [30].

For $\Lambda_b$ the decay rates and the angular distributions can be considered, although the latter measurements are experimentally challenging. Moreover, the systematic study of New Physics (NP) effects for polarized and unpolarized $\Lambda_b$ would provide a wealth of information. The prime purpose of such investigations is to identify the correlations between different processes induced by the same short-distance transition. If the B anomalies are due to new interactions, correlated pattern of same size effects must be observed in B and $\Lambda_b$ decays: we aim at describing such patterns, starting from the same extended low-energy Hamiltonian scrutinized for $B$ and choosing the same benchmark points studied in that case, so that the size of the possible deviations from SM in the meson and baryon case can be compared. We shall provide the formulae for various observables, they can be used in experimental simulations to assess the sensitivity to the NP operators.

We have to say that measurements of the $\Lambda_b$ polarization at LHC give results compatible with zero [31–34], which means that the $b$ quarks hadronizing in $\Lambda_b$ are mainly produced by QCD processes. However, a sizable longitudinal $\Lambda_b$ polarization is expected for $b$ quarks produced in $Z$ and top quark decays, as shown by the measurements at LEP [35–37]. For this reason, the effects beyond the Standard Model (BSM) in the polarized case have been scrutinized for the exclusive $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$ modes [38–41], in addition to the case of unpolarized baryon [42–47].

The plan of our study is as follows. In section 2 we introduce the semileptonic $b \to c, u$ effective Hamiltonian, which generalizes the SM one by the inclusion of the set of $D = 6$ four-fermion semileptonic operators weighted by complex coefficients. The heavy quark expansion (HQE) to describe the inclusive process $H_b(p, s) \to X_{c,u}\ell^-\bar{\nu}_\ell$ is discussed in section 3 considering the terms up to $\mathcal{O}(1/m_b^3)$. In section 4 we construct the fully differential $\Lambda_b \to X_{c,u}\ell^-\bar{\nu}_\ell$ decay distributions in the case of polarized $\Lambda_b$. In section 5 we analyze several observables in SM and with the extended Hamiltonian density, at a benchmark point in the parameter space of the effective couplings to investigate the sensitivity to the new operators. The last section contains the conclusions.



JHEP11(2020)032

The appendices contain the ingredients developed in our analysis. In appendix A we collect the baryon matrix elements relevant for the OPE at order $1/m_b^3$, considering the spin of the baryon. In appendix B we write the expressions of the structure functions for the hadronic tensor in SM and in the case of the extended Hamiltonian. Appendix C contains the coefficients appearing in the $1/m_b$ expansion the full semileptonic decay widths, for the SM and for the generalized Hamiltonian.

## 2 Effective weak Hamiltonian

We consider the inclusive semileptonic decay of a baryon $H_b$ comprising a single $b$ quark

$$H_b(p,s) \to X_{c,u}(p_X)\ell^-(p_\ell)\bar{\nu}_\ell(p_\nu)\,, \tag{2.1}$$

with $s$ the spin of the decaying baryon. We assume that the process is induced by the low-energy effective Hamiltonian density which extends the SM one,

$$\begin{aligned} H_{\rm eff}^{b\to U\ell\nu} = \frac{G_F}{\sqrt{2}}V_{Ub}\Big[&(1+\epsilon_V^\ell)\left(\bar{U}\gamma_\mu(1-\gamma_5)b\right)\left(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell\right)\\ &+\epsilon_S^\ell\,(\bar{U}b)\left(\bar{\ell}(1-\gamma_5)\nu_\ell\right)+\epsilon_P^\ell\,\left(\bar{U}\gamma_5 b\right)\left(\bar{\ell}(1-\gamma_5)\nu_\ell\right)\\ &+\epsilon_T^\ell\,\left(\bar{U}\sigma_{\mu\nu}(1-\gamma_5)b\right)\left(\bar{\ell}\sigma^{\mu\nu}(1-\gamma_5)\nu_\ell\right)\\ &+\epsilon_R^\ell\,\left(\bar{U}\gamma_\mu(1+\gamma_5)b\right)\left(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell\right)\Big]+h.c.\,. \end{aligned} \tag{2.2}$$

$H_{\rm eff}$ consists of D=6 four-fermion operators with complex lepton-flavour dependent coefficients $\epsilon^\ell_{V,S,P,T,R}$. Only left-handed neutrinos are considered. $U$ can be either the $u$ or the $c$ quark, $V_{Ub}$ is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element. $V_{Ub}$ and $\epsilon_V^\ell$ are independent parameters: the product $V_{Ub}(1+\epsilon_V^\ell)$ is not a mere redefinition of the SM $V_{Ub}$, due to the lepton-flavour dependence of $\epsilon_V^\ell$.

A comment is in order, concerning the operator with right-handed (RH) quark vector current $O_R = (\bar{U}\gamma_\mu(1+\gamma_5)b)(\bar{\ell}\gamma^\mu(1-\gamma_5)\nu)$. This operator is in the set of $D=6$ operators constituting the effective Hamiltonian (2.2), and has been previously considered [48–54]. We shall give analytic formulae that comprise its contribution, providing general expressions for the $\Lambda_b$ fully differential decay distributions and for the integrated distributions. However, in the Standard Model Effective Field Theory the only D=6 operator with a RH quark current, invariant under the SM gauge group, is nonlinear in the Higgs field: $i\left(\bar{U}_R\gamma_\mu b_R\right)(\tilde{H}^\dagger D_\mu H)$, with $D_\mu$ the electroweak (ew) covariant derivative, $H$ the SU(2) Higgs doublet, $\tilde{H}^i=\epsilon^{ij}H^{j*}$, and $\epsilon^{ij}$ the totally antisymmetric tensor [55–57]. At the ew symmetry breaking scale this operator modifies the $WUb$ coupling, but the resulting low energy four-fermion $O_R$ operator in the effective $b\to c,u$ semileptonic Hamiltonian does not violate LFU. Hence, in the framework of this effective field theory it is not involved in $B$ flavour anomalies, and it has been omitted in several analyses [58–60]. Modifications of the $WUb$ vertex are connected to modifications of the quark $Z$ vertices, which are tightly constrained by the electroweak precision observables [57, 61]. Stringent bounds can also be obtained from different processes [62, 63]. For these reasons we shall not include $O_R$

in our phenomenological analysis, since this would require a dedicated study beyond the purposes of the present work.

The Hamiltonian (2.2) can be written as

$$H_{\rm eff}^{b\to U\ell\nu} = \frac{G_F}{\sqrt{2}} V_{Ub} \sum_{i=1}^{5} C_i^\ell \, J_M^{(i)} \, L^{(i)M} + h.c. \,, \tag{2.3}$$

with $C_1^\ell = (1+\epsilon_V^\ell)$ and $C_{2,3,4,5}^\ell = \epsilon_{S,P,T,R}^\ell$. $J_M^{(i)}$ indicates the hadronic and $L^{(i)M}$ the leptonic current in each operator, $M$ generically denotes the set of Lorentz indices contracted between $J$ and $L$. The SM Hamiltonian corresponds to $i=1$ and $\epsilon_{V,S,P,T,R}^\ell = 0$. We keep the mass of the charged lepton $\ell = e, \mu, \tau$ different from zero.

## 3 Inclusive decay width

The decay width of the processes (2.1) is given by

$$d\Gamma = d\Sigma \, \frac{G_F^2 |V_{Ub}|^2}{4 m_H} \sum_{i,j} C_i^* C_j (W^{ij})_{MN} (L^{ij})^{MN}, \tag{3.1}$$

where $G_F$ is the Fermi constant, $q = p_\ell + p_\nu$ the lepton-pair momentum, and $d\Sigma$ the phase space element $d\Sigma = (2\pi) d^4q \, \delta^4(q - p_\ell - p_\nu)[dp_\ell]\,[dp_\nu]$, with $[dp] = \frac{d^3p}{(2\pi)^3 2p^0}$. The leptonic tensor is $(L^{ij})^{MN} = L^{(i)\dagger M} L^{(j)N}$. The hadronic tensor $(W^{ij})_{MN}$ is obtained from the discontinuity of the forward amplitude

$$(T^{ij})_{MN} = i \int d^4x \, e^{-i\,q\cdot x} \langle H_b(p,s)| T[J_M^{(i)\dagger}(x) \, J_N^{(j)}(0)] |H_b(p,s)\rangle \tag{3.2}$$

across the cut corresponding to the process (2.1):

$$(W^{ij})_{MN} = \frac{1}{\pi} {\rm Im} (T^{ij})_{MN}. \tag{3.3}$$

$T^{ij}$ and $W^{ij}$ can be computed exploiting an operator product expansion (OPE) with expansion parameter the inverse $b$ quark mass [2, 3]. To construct the OPE, the hadron momentum $p = m_H v$, with four-velocity $v$, is expressed in terms of the heavy quark mass $m_b$ and of a residual momentum $k$: $p = m_b v + k$. The QCD $b$ quark field is written as $b(x) = e^{-i\, m_b v\cdot x} b_v(x)$, with $b_v(x)$ still defined in QCD and satisfying the equation of motion:

$$b_v(x) = \left( P_+ + \frac{i \not{D}}{2 m_b} \right) b_v(x) \,, \tag{3.4}$$

where $P_+$ is the velocity projector $P_+ = \frac{1+\not{v}}{2}$. In terms of $b_v(x)$ one has:

$$(T^{ij})_{MN} = i \int d^4x \, e^{i\,(m_b v - q)\cdot x} \langle H_b(v,s)| T[\hat{J}_M^{(i)\dagger}(x) \, \hat{J}_N^{(j)}(0)] |H_b(v,s)\rangle \tag{3.5}$$

with $\hat{J}^{(i)}$ containing the field $b_v$. The heavy quark expansion is obtained from

$$(T^{ij})_{MN} = \langle H_b(v,s)| \bar{b}_v(0) \Gamma_M^{(i)\dagger} S_U(p_X) \Gamma_N^{(j)} b_v(0) |H_b(v,s)\rangle \,, \tag{3.6}$$

with $p_X = m_b v + k - q$ and $S_U(p_X)$ the $U$ quark propagator. Replacing $k \to iD$, with $D$ the QCD covariant derivative, the $U$ quark propagator can be expanded:

$$S_U(p_X) = S_U^{(0)} - S_U^{(0)}(i\not{D})S_U^{(0)} + S_U^{(0)}(i\not{D})S_U^{(0)}(i\not{D})S_U^{(0)} + \dots \tag{3.7}$$

where $S_U^{(0)} = \frac{1}{m_b\not{v} - \not{q} - m_U}$. With the definitions $p_U = m_b v - q$, $\mathcal{P} = (\not{p}_U + m_U)$ and $\Delta_0 = p_U^2 - m_U^2$, the expansion at order $1/m_b^3$ is

$$\begin{aligned}
\frac{1}{\pi}\mathrm{Im}(T^{ij})_{MN} &= \frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0}\langle H_b(v,s)|\bar{b}_v[\Gamma_M^{(i)\dagger}\mathcal{P}\Gamma_N^{(j)}]b_v|H_b(v,s)\rangle \\
&- \frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^2}\langle H_b(v,s)|\bar{b}_v[\Gamma_M^{(i)\dagger}\mathcal{P}\gamma^{\mu_1}\mathcal{P}\Gamma_N^{(j)}](iD_{\mu_1})b_v|H_b(v,s)\rangle \\
&+ \frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^3}\langle H_b(v,s)|\bar{b}_v[\Gamma_M^{(i)\dagger}\mathcal{P}\gamma^{\mu_1}\mathcal{P}\gamma^{\mu_2}\mathcal{P}\Gamma_N^{(j)}](iD_{\mu_1})(iD_{\mu_2})b_v|H_b(v,s)\rangle \\
&- \frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^4}\langle H_b(v,s)|\bar{b}_v[\Gamma_M^{(i)\dagger}\mathcal{P}\gamma^{\mu_1}\mathcal{P}\gamma^{\mu_2}\mathcal{P}\gamma^{\mu_3}\mathcal{P}\Gamma_N^{(j)}](iD_{\mu_1})(iD_{\mu_2})(iD_{\mu_3})b_v|H_b(v,s)\rangle\,.
\end{aligned} \tag{3.8}$$

This expression involves $H_b$ matrix elements of QCD operators of increasing dimensions, written as

$$\begin{aligned}
&\langle H_b(v,s)|\bar{b}_v[\Gamma_M^{(i)\dagger}\mathcal{P}\gamma^{\mu_1}\dots\gamma^{\mu_n}\mathcal{P}\Gamma_N^{(j)}](iD_{\mu_1})\dots(iD_{\mu_n})b_v|H_b(v,s)\rangle \\
&\quad = \mathrm{Tr}\left[(\Gamma_M^{(i)\dagger}\mathcal{P}\gamma^{\mu_1}\dots\gamma^{\mu_n}\mathcal{P}\Gamma_N^{(j)})_{ba}\langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle\right]
\end{aligned} \tag{3.9}$$

with $a, b$ Dirac indices. The hadron matrix elements

$$\mathcal{M}_{\mu_1\dots\mu_n} = \langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle \tag{3.10}$$

can be expressed in terms of nonperturbative parameters, the number of which increases with the dimension of the operators. The expansion to order $\mathcal{O}(1/m_b^3)$ requires

$$\langle H_b(v,s)|\bar{b}_v(iD)^2 b_v|H_b(v,s)\rangle = -2m_H\,\hat{\mu}_\pi^2 \tag{3.11}$$
$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|H_b(v,s)\rangle = 2m_H\,\hat{\mu}_G^2 \tag{3.12}$$
$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu)b_v|H_b(v,s)\rangle = 2m_H\,\hat{\rho}_D^3 \tag{3.13}$$
$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})b_v|H_b(v,s)\rangle = 2m_H\,\hat{\rho}_{LS}^3\,. \tag{3.14}$$

A method to compute $\mathcal{M}_{\mu_1\dots\mu_n}$ is exploited in [64] for $B$ meson, and more parameters than those listed in (3.11)–(3.14) are needed for $n = 4$. The order $n = 5$ has also been analyzed [65]. For a heavy baryon, the dependence on the spin four-vector $s_\mu$ must be kept in (3.10). This is important since, for hadrons with spin, considering the hadron polarization leads to interesting observables to analyze.

In appendix A we collect the expressions of the matrix elements needed for the expansion at $\mathcal{O}(1/m_b^3)$ keeping the $s_\mu$ dependence. The computation procedure is described in [64]. One starts from the highest dimension operator, which in our case $n = 3$ has dimension 6, and determines it in the static heavy quark limit, replacing $b_v(x) \to h_v(x)$, the heavy quark field defined in the heavy quark effective theory

(HQET). $h_v(x)$ is related to the QCD $b(x)$ field: $h_v(x) = e^{im_b v\cdot x} P_+ b(x)$. $h_v$ satisfies the equations $P_+ h_v(x) = h_v(x)$ and $v \cdot D\, h_v(x) = 0$. In principle, the matrix element $\mathcal{M}_{\mu_1\mu_2\mu_3} = \langle H_b(v,s)|(\bar{h}_v)_a(iD_{\mu_1})(iD_{\mu_2})(iD_{\mu_3})(h_v)_b|H_b(v,s)\rangle$ can be expanded over the set of 16 independent Dirac matrices. However, in HQET it is given in terms of only two Dirac structures, $P_+$ and $\hat{S}^\mu = P_+\gamma^\mu\gamma_5 P_+$, an observation which simplifies the parametrization [66]. On the other hand, the matrix elements of lower dimension operators are computed in QCD expanding over the full set of Dirac matrices. The coefficients of Dirac structures in the $d$ dimension matrix element are recursively computed from the $d+1$ terms, and eqs. (3.11)–(3.14) are used.

The parameters in eqs. (3.11)–(3.14) are denoted by a hat to distinguish them from the corresponding parameters defined in HQET, with $b_v$ replaced by $h_v$. For $\hat{\mu}_\pi^2$, $\hat{\rho}_D^3$, $\hat{\rho}_{LS}^3$ the difference between the two definitions involves terms appearing at $\mathcal{O}(1/m_b^4)$, hence in our case $\hat{\mu}_\pi^2 = \mu_\pi^2$, $\hat{\rho}_D^3 = \rho_D^3$ and $\hat{\rho}_{LS}^3 = \rho_{LS}^3$. For $\hat{\mu}_G^2$ the relation between the two definitions is $\hat{\mu}_G^2 = \mu_G^2 - \frac{1}{m_b}\left(\rho_D^3 + \rho_{LS}^3\right)$, a combination often present in our expressions.

The formalism is suitable for the analysis in the Standard Model and in NP with the Hamiltonian (2.2). Our results are obtained for non-vanishing charged lepton mass, at order $1/m_b^3$ in the HQE, in the case of a polarized baryon and with all operators in (2.2) taken into account. In the existing literature one or more of the above points are relaxed. For the inclusive semileptonic $B$ decays and non vanishing lepton masses, NP operators have been considered at order $1/m_b^2$ in [10], and we agree with those results at that order after taking the spin-average in our expressions. $V+A$ and $S-P$ operators have been studied at the leading order in the $1/m_b$ expansion in [8], while a $V+A$ operator has been considered for the mode $B \to X\tau\bar{\nu}_\tau$ performing the HQE at $\mathcal{O}(1/m_b^2)$ in [67]. The hadronic tensor has been computed by an OPE in terms of operators comprising the HQET field $h_v$ in [68]. In this analysis the polarized $\Lambda_b$ inclusive semileptonic decay is considered at order $1/m_b^2$ in SM for massless leptons, the case of massive leptons at the same order in $1/m_b$ is studied in [69]. We agree with such results at that order in the $b$ mass expansion. As a last remark, in $b \to u$ semileptonic transition we neglect weak annihilation contributions, which mainly affect the endpoint region of the charged lepton energy spectrum [70].

Using the matrix elements $\mathcal{M}_{\mu_1\ldots\mu_n}$ collected in appendix A the hadronic tensor can be computed. It is expanded in Lorentz structures depending on $v$, $q$ and $s$. The related invariant functions are given in appendix B for the Standard Model and for the effective Hamiltonian eq. (2.2).

## 4 Decay distributions

For the $H_b(v,s) \to X(p_X)\ell^-(p_\ell)\bar{\nu}_\ell(p_\nu)$ transition the four-fold differential decay distribution is given by

$$\frac{d^4\Gamma}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \frac{G_F^2|V_{Ub}|^2}{32(2\pi)^3 m_H}\sum_{i,j} C_i^* C_j \frac{1}{\pi}\mathrm{Im}(T^{ij})_{MN}(L^{ij})^{MN}\,, \tag{4.1}$$

with $p_\ell = (E_\ell, \vec{p}_\ell)$ and $\theta_P$ the angle between $\vec{p}_\ell$ and $\vec{s}$ in the $H_b$ rest frame. The structure functions in which the hadronic tensor is expanded depend on $q^2$ and $v \cdot q$. The various



JHEP11(2020)032

decay distributions are obtained integrating (4.1) over the phase space [71]. To compute the spectrum in $q^2$ and in the charged lepton energy $E_\ell$ the order in the integration must be specified. Integrating first in $E_\ell$, the integration limits are

$$E_1^* \le E_\ell \le E_2^* \,, \qquad E_{1,2}^* = \frac{v\cdot q(q^2+m_\ell^2) \pm \sqrt{(v\cdot q)^2 - q^2}\,(q^2-m_\ell^2)}{2q^2} \,. \tag{4.2}$$

The replacement

$$\frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^n} \to \frac{(-1)^{n-1}}{(n-1)!}\delta^{(n-1)}(\Delta_0) \tag{4.3}$$

in the hadronic tensor can be used to integrate over $v \cdot q$. The last $q^2$ integration is for

$$m_\ell^2 \le q^2 \le (m_b - m_U)^2 \,. \tag{4.4}$$

To compute the charged lepton energy spectrum one integrates in a different order [72]. The first integration over $v \cdot q$ is in the range

$$E_\ell + \frac{(q^2-m_\ell^2)}{2m_\ell^2} E_{\ell-} \le v\cdot q \le E_\ell + \frac{(q^2-m_\ell^2)}{2m_\ell^2} E_{\ell+} \,, \tag{4.5}$$

where $E_{\ell\pm} = E_\ell \pm \sqrt{E_\ell^2 - m_\ell^2}$. Then one integrates over $q^2$ with integration limits

$$\frac{E_{\ell-}}{m_b - E_{\ell-}} \left(m_b^2 - m_U^2 - m_b E_{\ell-}\right) \le q^2 \le \frac{E_{\ell+}}{m_b - E_{\ell+}} \left(m_b^2 - m_U^2 - m_b E_{\ell+}\right) \,. \tag{4.6}$$

The range for the last integration in $E_\ell$ is

$$m_\ell \le E_\ell \le \frac{m_b^2 - m_U^2 + m_\ell^2}{2m_b} \,. \tag{4.7}$$

Keeping the dependence on $\cos\theta_P$, the corresponding double decay distributions are obtained. Notice that the kinematics involves the quark masses, the dependence on the decaying hadron is contained in the matrix elements of the OPE operators. However, the OPE breaks down in the endpoint region of the spectra, as signaled by singularities as the derivatives of the $\delta$ function. Such terms must be resummed in a $H_b$ shape function. Convolution of the distributions with such a function smears the spectra at the endpoint and transforms the phase space boundaries from the partonic to the hadronic kinematics: $q^2_{\max} = (m_{H_b} - m_X)^2$ and $(E_\ell)_{\max} = \frac{m_{H_b}^2 - m_X^2 + m_\ell^2}{2m_{H_b}}$, with $m_X$ the mass of the lightest hadron containing the $U$ quark produced in the decay. We do not include the effects of the shape function, the profile of which is not known in the baryon case, keeping in mind that the OPE results loose reliability in the endpoint region.

Expanding the tensor $T^{ij}$ in invariant functions, as provided in appendix B, the fully differential distribution is obtained upon contraction with the leptonic tensor. We express the distribution as

$$\frac{d^4\Gamma}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \sum_{i,j} \frac{d^4\Gamma^{ij}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} \,. \tag{4.8}$$

In this expression the first term is

$$
\begin{aligned}
&\frac{d^4\Gamma^{11}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}|\,(1+\epsilon_V)\,|^2 \\
&\Big\{ 8(q^2-m_\ell^2)W_1 + 4\left[-(q^2-m_\ell^2)+4E_\ell(v\cdot q-E_\ell)\right]W_2 \\
&+8\left[(q^2+m_\ell^2)v\cdot q-2q^2E_\ell)\right]W_3 + 4m_\ell^2(q^2-m_\ell^2)W_4 + 16m_\ell^2(v\cdot q-E_\ell)W_5 \\
&-2\frac{\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}\left(q^2+m_\ell^2-2(v\cdot q)E_\ell\right)\Big[2G_1(q^2-m_\ell^2)+G_2\big[-(q^2-m_\ell^2) \\
&+4E_\ell(v\cdot q-E_\ell)\big] + 2G_3\left[(q^2+m_\ell^2)v\cdot q-2q^2E_\ell)\right] + 4G_5m_\ell^2(v\cdot q-E_\ell) \\
&-4E_\ell G_6 - 4m_\ell^2G_7 - 4E_\ell G_8 - 2(q^2+m_\ell^2)G_9\Big] \qquad (4.9)\\
&-16\cos\theta_P\sqrt{E_\ell^2-m_\ell^2}\Big[(v\cdot q-2E_\ell)G_6 - m_\ell^2G_7 - v\cdot q\,G_8 - q^2\,G_9\Big]\Big\}
\end{aligned}
$$

where $\mathcal{N}=\frac{G_F^2|V_{Ub}|^2}{32(2\pi)^3m_H}$, $W_a=\frac{1}{\pi}\mathrm{Im}T_a$ and $G_a=\frac{1}{\pi}\mathrm{Im}S_a$ with the index $a=1,2,\ldots$ corresponding to the invariant functions $T_{1-5}$ and $S_{1-9}$ in (B.2)–(B.6) and (B.7)–(B.13). For $\epsilon_V^\ell=0$ this term corresponds to the SM distribution.

Let us consider the other terms in eq. (4.8) for the NP contributions. Considering the scalar and pseudoscalar operators, we have:

$$
\frac{d^4\Gamma^{22(33)}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}|\epsilon_{S(P)}|^2 4(q^2-m_\ell^2)W_{S(P),1} \qquad (4.10)
$$

$$
\frac{d^4\Gamma^{23+32}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}\left(-2\mathrm{Re}[\epsilon_S\epsilon_P^*]\right)\frac{2\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}(q^2-m_\ell^2)(m_\ell^2+q^2-2v\cdot qE_\ell)G_{SP,1} \qquad (4.11)
$$

with $W_{S(P),1}$ and $G_{SP,1}$ obtained from the imaginary parts of the functions $T$ and $S$ in (B.14)–(B.16).

From the interference terms, we have:

$$
\begin{aligned}
&\frac{d^4\Gamma^{12+21(13+31)}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}\,2\,\mathrm{Re}[(1+\epsilon_V)\epsilon_{S(P)}^*]m_\ell \\
&\Big\{4\Big[2(v\cdot q-E_\ell)W_{SMS(SMP),1}+(q^2-m_\ell^2)W_{SMS(SMP),2}\Big] \\
&-\frac{2\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}\left(q^2+m_\ell^2-2(v\cdot q)E_\ell\right)\Big[2(v\cdot q-E_\ell)G_{SMS(SMP),1} \qquad (4.12)\\
&+(q^2-m_\ell^2)G_{SMS(SMP),2}-2G_{SMS(SMP),3}\Big]+8\cos\theta_P\sqrt{E_\ell^2-m_\ell^2}G_{SMS(SMP),3}\Big\}
\end{aligned}
$$

with $W_{SMS(SMP),i}$ and $G_{SMS(SMP),i}$ obtained from the imaginary parts of the functions $T$ and $S$ in (B.18), (B.19) and (B.20)–(B.23).



JHEP11(2020)032

Continuing with the distributions, we have:

$$
\begin{aligned}
&\frac{d^4\Gamma^{44}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}|\epsilon_T|^2 \\
&\Bigg\{ 16(q^2-m_\ell^2)(q^2+2m_\ell^2)\,(W_{T4}+W_{T9}) \\
&+16\left[-(q^2-m_\ell^2)+8E_\ell(v\cdot q-E_\ell)\right](W_{T2}+W_{T6}-W_{T10}) \\
&+16\left[(q^2-m_\ell^2)v\cdot q+4m_\ell^2(v\cdot q-E_\ell)\right](2W_{T5}+W_{T7}+W_{T8}-W_{T11}-W_{T12}) \\
&+16\left[m_\ell^4+q^2(v\cdot q-2E_\ell)^2-m_\ell^2\left(q^2+v\cdot q(-3v\cdot q+4E_\ell)\right)\right](W_{T14}-W_{T15}) \\
&-8\frac{\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}\,(m_\ell^2+q^2-2E_\ell\, v\cdot q)\,\Big[\left[-(q^2-m_\ell^2)+8E_\ell(v\cdot q-E_\ell)\right](G_{T2}+G_{T6}) \\
&+\left[(q^2-m_\ell^2)v\cdot q+4m_\ell^2(v\cdot q-E_\ell)\right](2G_{T5}+G_{T7}+G_{T8}-G_{T11}-G_{T12}) \\
&+2\left[m_\ell^2(v\cdot q-2E_\ell)+v\cdot q[q^2-4E_\ell(v\cdot q-E_\ell)]\right]G_{T22}\Big] \\
&-4E_\ell\,(2G_{T14}+G_{T23}+v\cdot q\, G_{T24}+G_{T30}+G_{T32}-G_{T34}-G_{T36}) \\
&-(3m_\ell^2+q^2)\,(2G_{T15}+G_{T31}+G_{T33}-G_{T35}-G_{T37}+G_{T27A}+G_{T27B}) \\
&+2[m_\ell^2+E_\ell(v\cdot q-2E_\ell)]\,(G_{T27A}+G_{T27B}+G_{T28}-G_{T29})\Big] \\
&-32\cos\theta_P\sqrt{E_\ell^2-m_\ell^2}\Big[2(v\cdot q-2E_\ell)\,(2G_{T14}+G_{T23}+v\cdot q\, G_{T24}+G_{T30}+G_{T32}-G_{T34}-G_{T36}) \\
&-2m_\ell^2\,(2G_{T15}+G_{T31}+G_{T33}-G_{T35}-G_{T37}+G_{T27A}+G_{T27B}) \\
&+\left[m_\ell^2+v\cdot q(v\cdot q-2E_\ell)\right](G_{T27A}+G_{T27B}+G_{T28}-G_{T29})\Big]\Bigg\}
\end{aligned}
\tag{4.13}
$$

with $W_{Ti}$ and $G_{Ti}$ from the imaginary parts of the functions $T$ and $S$ in (B.28)–(B.60);

$$
\begin{aligned}
&\frac{d^4\Gamma^{14+41}}{dq^2\, d(v\cdot q)\, dE_\ell\, d\cos\theta_P} = \mathcal{N}2\mathrm{Re}[(1+\epsilon_V)\epsilon_T^*]m_\ell \\
&\Bigg\{16(v\cdot q-E_\ell)\left[-3W_{SMT,1}+3W_{SMT,3}-(v\cdot q)(W_{SMT,5}+W_{SMT,7})\right] \\
&+8(q^2-m_\ell^2)\left(-3W_{SMT,2}+3W_{SMT,4}+W_{SMT,5}+W_{SMT,7}\right) \\
&+8m_\ell\left[2q^2E_\ell-(m_\ell^2+q^2)\right]\left(W_{SMT,6}+W_{SMT,8}\right) \\
&+8\frac{\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}\left(q^2+m_\ell^2-2(v\cdot q)E_\ell\right)\Big[(v\cdot q-E_\ell)\left(3G_{SMT,1}-3G_{SMT,3}-G_{SMT,11}+G_{SMT,25}\right) \\
&-3G_{SMT,9}+3G_{SMT,10}-G_{SMT,12}+G_{SMT,16}-v\cdot q\,(G_{SMT,13}-G_{SMT,17}) \\
&+E_\ell\,(-G_{SMT,14}+G_{SMT,18})\Big]-16\cos\theta_P\sqrt{E_\ell^2-m_\ell^2}\Big[3G_{SMT,9}-3G_{SMT,10} \\
&+G_{SMT,12}-G_{SMT,16}+v\cdot q\,(G_{SMT,13}+G_{SMT,14}-G_{SMT,17}-G_{SMT,18})\Big]\Bigg\}
\end{aligned}
\tag{4.14}
$$

with $W_{SMT,i}$ and $G_{SMT,i}$ from the imaginary parts of the functions $T$ and $S$ in (B.62)–(B.70);

$$\frac{d^4\Gamma^{24+42(34+43)}}{dq^2\,d(v\cdot q)\,dE_\ell\,d\cos\theta_P} = \mathcal{N}2\mathrm{Re}[\epsilon_T\epsilon^*_{S(P)}]$$
$$\Bigg\{-8[(q^2+m_\ell^2)(v\cdot q)-2q^2E_\ell]\left(W_{ST(PT),1}+W_{ST(PT),2}\right)$$
$$+4\frac{\cos\theta_P}{\sqrt{E_\ell^2-m_\ell^2}}\left(q^2+m_\ell^2-2(v\cdot q)E_\ell\right)\Bigg[[(q^2+m_\ell^2)(v\cdot q)-2q^2E_\ell]\left(G_{ST(PT),1}+G_{ST(PT),2}\right)$$
$$+(q^2+m_\ell^2)\left(G_{ST(PT),3}-G_{ST(PT),4}\right)+2E_\ell\left(G_{ST(PT),5}+G_{ST(PT),6}\right)\Bigg]$$
$$+16\cos\theta_P\sqrt{E_\ell^2-m_\ell^2}\Big[q^2\left(G_{ST(PT),3}-G_{ST(PT),4}\right)+v\cdot q\left(G_{ST(PT),5}+G_{ST(PT),6}\right)\Big]\Bigg\}. \tag{4.15}$$

In this last case $W_{ST,i}$ and $G_{ST,i}$ are obtained from the imaginary parts of the functions $T$ and $S$ in (B.72)–(B.76).

The distributions related to the $O_R$ operator

$$\frac{d^4\Gamma^{55}}{dq^2\,d(v\cdot q)\,dE_\ell\,d\cos\theta_P} \quad \text{and} \quad \frac{d^4\Gamma^{15+51}}{dq^2\,d(v\cdot q)\,dE_\ell\,d\cos\theta_P} \tag{4.16}$$

have the same form of eq. (4.9) with suitable substitutions: $d^4\Gamma^{55}$ is obtained from eq. (4.9) replacing $|\,(1+\epsilon_V)\,|^2 \to |\epsilon_R|^2$, $W_a \to W_{Ra} = \frac{1}{\pi}\mathrm{Im}T_{Ra}$ and $G_a \to G_{Ra} = \frac{1}{\pi}\mathrm{Im}S_{Ra}$ with the functions $T_R$ and $S_R$ collected in (B.77). In the case of $d^4\Gamma^{15+51}$ the replacements are: $|\,(1+\epsilon_V)\,|^2 \to 2\mathrm{Re}[(1+\epsilon_V)\epsilon_R^*]$, $W_a \to W_{SMRa} = \frac{1}{\pi}\mathrm{Im}T_{SMRa}$ and $G_a \to G_{SMRa} = \frac{1}{\pi}\mathrm{Im}S_{SMRa}$, with the functions $T_{SMR}$ and $S_{SMR}$ collected in (B.78)–(B.86).

Analogously, the distributions

$$\frac{d^4\Gamma^{25+52(35+53)}}{dq^2\,d(v\cdot q)\,dE_\ell\,d\cos\theta_P} \tag{4.17}$$

have the same form of eq. (4.12) substituting $\mathrm{Re}[(1+\epsilon_V)\epsilon^*_{S(P)}] \to \mathrm{Re}[\epsilon_R\epsilon^*_{S(P)}]$, $W_{SMS(SMP)a} \to W_{RS(P)a} = \frac{1}{\pi}\mathrm{Im}T_{RS(RP)a}$, and $G_{SMS(SMP)a} \to G_{RS(RP)a} = \frac{1}{\pi}\mathrm{Im}S_{RS(RP)a}$. The functions $T_{RS(RP)}$ and $S_{RS(RP)}$ are collected in (B.87)–(B.88).

Finally, the distributions

$$\frac{d^4\Gamma^{45+54}}{dq^2\,d(v\cdot q)\,dE_\ell\,d\cos\theta_P} \tag{4.18}$$

have the same form of eq. (4.14) with the substitutions: $\mathrm{Re}[(1+\epsilon_V)\epsilon_T^*] \to \mathrm{Re}[\epsilon_R\epsilon_T^*]$, $W_{SMTa} \to W_{RTa} = \frac{1}{\pi}\mathrm{Im}T_{RTa}$ and $G_{SMTa} \to G_{RS(RP)a} = \frac{1}{\pi}\mathrm{Im}S_{RTa}$, and the functions $T_{RT}$ and $S_{RT}$ collected in (B.89)–(B.109).

The above expressions can be used to compute all double and single decay distributions. We do not present the lengthy formulae here, but only give the full decay width, which can be cast in the form:

$$\Gamma(H_b \to X\ell^-\bar{\nu}_\ell) = \Gamma_b \sum_i \left\{ C_0^{(i)} + \frac{\mu_\pi^2}{m_b^2} C_{\mu_\pi^2}^{(i)} + \frac{\mu_\pi^2}{m_b^2} C_{\mu_G^2}^{(i)} + \frac{\rho_D^3}{m_b^3} C_{\rho_D^3}^{(i)} + \frac{\rho_{LS}^3}{m_b^3} C_{\rho_{LS}^3}^{(i)} \right\}, \qquad (4.19)$$

with $\Gamma_b = \frac{G_F^2 m_b^5 V_{Ub}^2}{192\pi^3}$. The index $i$ indicates the contribution of the various operators and of the interferences: $i = SM, S, P, T, R,\ SP, SMS, SMP,\ SP, ST, SMT$, and $SMR, SR, PR, TR$. The coefficients $C^{(i)}$ are collected in appendix C and contain the couplings $\epsilon^\ell_{V,S,P,T,R}$ in the effective Hamiltonian. All $C^{(i)}_{\rho^3_{LS}}$ vanish.

For the SM terms in eq. (4.19) various perturbative corrections are known. The leading electroweak correction $A_{ew} = 1.014$ is a multiplying factor. QCD corrections are known at $\mathcal{O}(\alpha_s^2)$ for the leading term and at $\mathcal{O}(\alpha_s)$ for the $1/m_b^2$ terms in (4.19), and can be included following, e.g., [73–78]. In ratios of decay widths involving different lepton species such corrections largely cancel out. We do not include QCD corrections in the decay distributions analyzed in the following.

## 5 Numerical results

In our numerical study we use the heavy quark masses in the kinetic scheme $m_b^{kin}(\mu = 0.75\,\text{GeV}) = 4.62\,\text{GeV}$, $m_c^{kin}(\mu = 0.75\,\text{GeV}) = 1.20\,\text{GeV}$, and the up quark mass in the $\overline{\text{MS}}$ scheme $\overline{m}_u(2\,\text{GeV}) = 2.16\pm^{0.49}_{0.26}\,\text{MeV}$ [79]. In the case of $B$ mesons the HQE parameters are constrained fitting the measured lepton energy and the hadronic mass distributions and their moments in $B \to X_c\ell\bar{\nu}_\ell$ decay [78]. For $\Lambda_b$ only few theoretical estimates of $\mu_\pi^2(\Lambda_b)$ exist [80]. A relation between $\mu_\pi^2(\Lambda_b)$ and $\mu_\pi^2(B)$ in terms of the measured mass differences between beauty and charmed mesons and baryons can be exploited [81]:

$$\mu_\pi^2(B) - \mu_\pi^2(\Lambda_b) = \frac{2m_b m_c}{m_b - m_c}\left[(m_{\Lambda_b} - m_{\Lambda_c}) - (\overline{m}_B - \overline{m}_D)\right]\left(1 + \mathcal{O}(1/m_{b,c}^2)\right) \qquad (5.1)$$

($\overline{m}_{B,D}$ is the spin-averaged $B^{(*)}$ and $D^{(*)}$ mass), to obtain $\mu_\pi^2(\Lambda_b)$ from the value of $\mu_\pi^2(B)$. Moreover, the approximation $\rho_D^3(\Lambda_b) \simeq \rho_D^3(B)$ can be adopted, increasing the uncertainty on $\rho_D^3(\Lambda_b)$ with respect to the value for $B$. In our analysis we use: $\mu_\pi^2(\Lambda_b) = (0.50 \pm 0.10)\,\text{GeV}^2$ and $\rho_D^3(\Lambda_b) = (0.17 \pm 0.08)\,\text{GeV}^3$. The HQE parameters $\mu_G^2$ and $\rho_{LS}^3$ are sensitive to the total angular momentum of the light degrees of freedom in the hadron. For $\Lambda_b$ they vanish since the light degrees of freedom have spin zero, but for other baryons they are different from zero. For this reason we include the contributions involving such parameters in the various expressions in the appendices, so that they can be used for different heavy hadrons.

The description of NP effects requires input values for the couplings $\epsilon^\ell_{V,S,P,T,R}$ in the Hamiltonian (2.2). As anticipated, in the phenomenological analysis we do not consider the contribution of the operator $O_R$, hence we set $\epsilon^\ell_R = 0$. For $U = u$, allowed regions for the other couplings have been determined from the analysis of purely leptonic $B$ decays and of semileptonic $B$ transitions to $\pi$ and $\rho(770)$ [82]. Accordingly, for $b \to u\,\mu\,\bar{\nu}_\mu$ we set the benchmark point (BP): $(\text{Re}[\epsilon^\mu_V], \text{Im}[\epsilon^\mu_V]) = (0, 0)$, $(\text{Re}[\epsilon^\mu_P], \text{Im}[\epsilon^\mu_P]) = (-0.03, -0.02)$, $(\text{Re}[\epsilon^\mu_T], \text{Im}[\epsilon^\mu_T]) = (0.12, 0)$ and $(\text{Re}[\epsilon^\mu_S], \text{Im}[\epsilon^\mu_S]) = (-0.04, 0)$. For $b \to u\,\tau\,\bar{\nu}_\tau$ the BP is:



JHEP11(2020)032

| | SM |
|---|---|
| $\mathcal{B}(\Lambda_b \to X_c \mu \bar{\nu}_\mu)$ | $11.0 \times 10^{-2}$ |
| $\mathcal{B}(\Lambda_b \to X_c \tau \bar{\nu}_\tau)$ | $2.4 \times 10^{-2}$ |
| $\mathcal{B}(\Lambda_b \to X_u \mu \bar{\nu}_\mu)$ | $11.65 \times 10^{-4}$ |
| $\mathcal{B}(\Lambda_b \to X_u \tau \bar{\nu}_\tau)$ | $2.75 \times 10^{-4}$ |

**Table 1**. Inclusive semileptonic $\Lambda_b$ branching fractions in SM, obtained for the central values of the parameters.

$\epsilon_V^\tau = 0$, $\epsilon_S^\tau = 0$, $(\text{Re}[\epsilon_P^\tau], \text{Im}[\epsilon_P^\tau]) = (0.01, 0)$ and $(\text{Re}[\epsilon_T^\tau], \text{Im}[\epsilon_T^\tau]) = (0.12, 0)$. For $U = c$ we discuss NP effects i) considering only the tensor operator, with $(\text{Re}(\epsilon_T^\mu), \text{Im}(\epsilon_T^\mu)) = (0.115, -0.005)$ and $(\text{Re}(\epsilon_T^\tau), \text{Im}(\epsilon_T^\tau)) = (-0.067, 0)$, as fixed in [5]. For one observable we also consider ii) non vanishing couplings only for the $\tau$ mode, with $\text{Re}[\epsilon_{\text{V}}^\tau] = 0.16$, $\text{Re}[\epsilon_S^\tau] = -0.235$, $\text{Re}[\epsilon_P^\tau] = -0.095$ and $\text{Re}[\epsilon_T^\tau] = 0.05$ fixed in [59]; the values iii) $\text{Re}[\epsilon_{\text{V}}^\tau] = 0.07$ and iv) $\text{Re}[\epsilon_{\text{S}}^\tau] = 0.025$, $\text{Re}[\epsilon_{\text{P}}^\tau] = 0.535$, also set in [59].

At odds with the $B$ case, at present there is not enough experimental information on $\Lambda_b$ decays to restrict the ranges of the effective couplings. For $b \to c$ modes, semileptonic $\Lambda_b$ transitions to $\Lambda_c^+$, $\Lambda_c^+\pi^+\pi^-$, $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Sigma_c(2455)^0\pi^+$ and $\Sigma_c(2455)^{++}\pi^-$ have been observed. The branching fractions are measured for the modes into $\Lambda_c$ baryons, and the result $\mathcal{B}(\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell + \text{anything}) = (10.9 \pm 2.2) \times 10^{-2}$ is quoted, with $\ell = e, \mu$ [79]. For $b \to u$, the exclusive branching ratio $\mathcal{B}(\Lambda_b \to p \mu^- \bar{\nu}) = (4.1 \pm 1.0) \times 10^{-4}$ is measured [79].

Using $|V_{cb}| = 0.042$ and $|V_{ub}| = 0.0037$, together with $\tau_{\Lambda_b} = (1.471 \pm 0.009) \times 10^{-12}$ s [79], we obtain the inclusive $\Lambda_b$ branching fractions for the two quark transitions and for final $\tau$ and $\mu$ lepton. The results for the Standard Model, for the central value of the parameters and neglecting QCD corrections, are collected in table 1.

A remark about the various sources of uncertainties is in order. In a first-principle computation, such as the one we have described here, the theoretical uncertainties are connected to the quark masses, to the hadronic parameters, to the perturbative corrections and to the size of next-order terms in the heavy quark expansion. All such uncertainties can be reduced in a systematic way. This is the case, in particular, of the values of the hadronic parameters, the knowledge of which can be improved using nonperturbative QCD methods, such as QCD sum rules and lattice QCD. For example, for $\mu_\pi^2(\Lambda_b)$ and $\rho_D^3(\Lambda_b)$ we have quoted an uncertainty of 20% and 50%, respectively, which could be reduced by dedicated QCD analyses. For other baryons, such parameters are even less known and deserve new studies. On the other hand, the sensitivity to NP effects of the observables we have described needs to be assessed in the actual experimental conditions. In this respect, the analytic formulae we have provided can be used, e.g., to scrutinize by appropriate simulations the individual effects of the various low-energy operators in (2.2), when the experimental analyses are planned.



JHEP11(2020)032

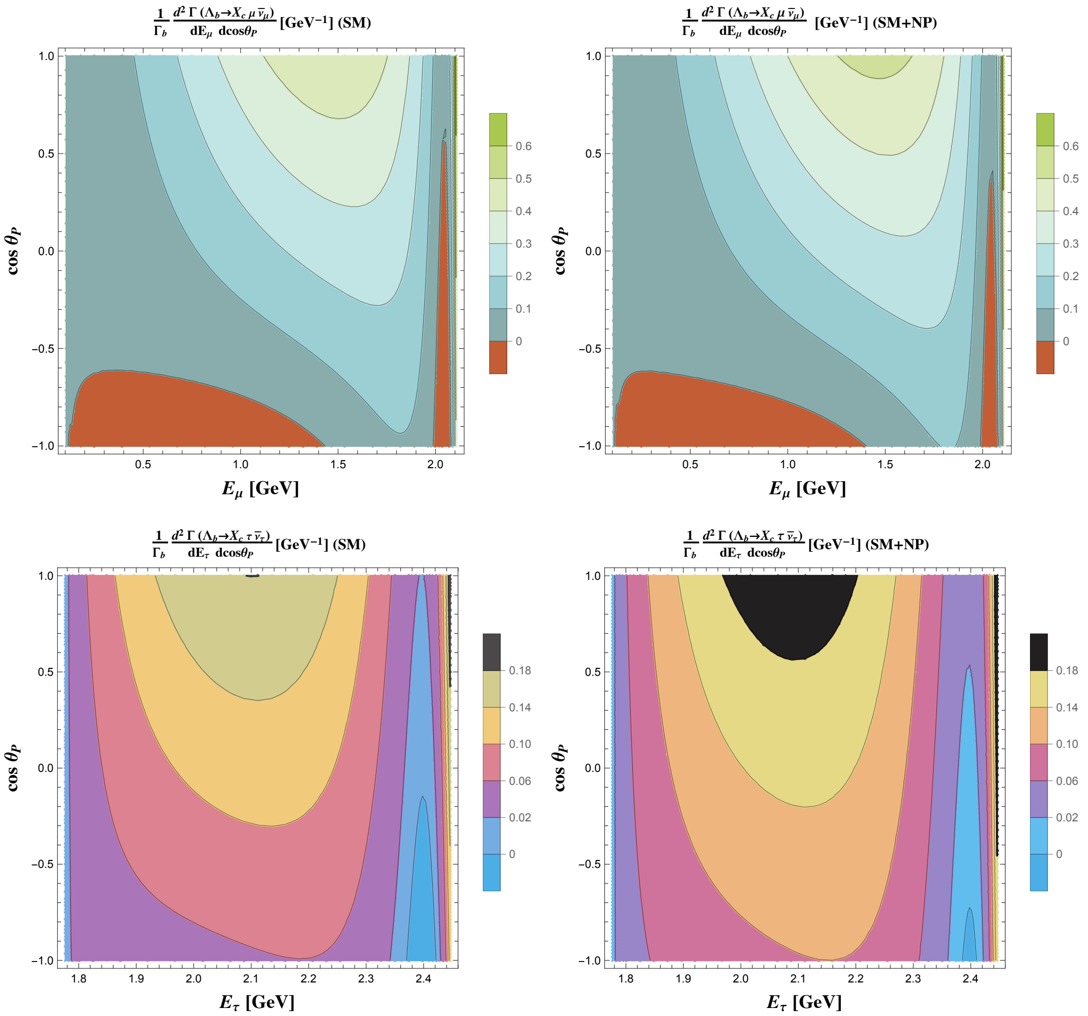


**Figure 1**. Contour plots of the distribution $\frac{1}{\Gamma_b}\frac{d^2\Gamma}{dE_\ell\, d\cos\theta_P}$ for $\Lambda_b \to X_c \ell \bar{\nu}_\ell$. The top and bottom panels refer to $\ell = \mu$ and $\ell = \tau$, respectively, the left and right plots to the Standard Model and to NP at the benchmark point.

### 5.1 Observables in the $\Lambda_b \to X_c \ell \bar{\nu}$ mode

The double differential distribution $\frac{1}{\Gamma_b}\frac{d^2\Gamma}{dE_\ell\, d\cos\theta_P}$ for the SM and NP at the chosen benchmark point is shown in figure 1 for the muon and for the $\tau$ final state. In the case of NP there is an enhancement of the distribution, more pronounced in the $\tau$ case, for charged lepton energy $E_\mu \simeq 1.7\,\mathrm{GeV}$ and $E_\tau \simeq 2.1\,\mathrm{GeV}$.

The charged lepton energy spectrum is useful to assess the role of the various terms in the $1/m_b$ expansion. In figure 2 we show the result for the muon and the $\tau$ case. The impact of the next-to-leading and next-to-next-to-leading corrections in the HQE is higher for large values of $E_\ell$ ($\ell = \mu, \tau$), excluding the end-point region where the expansion breaks down. In the case of $\tau$ the corrections affects a wider energy range. The parametric hierarchy between $1/m_b^2$ and $1/m_b^3$ corrections is numerically confirmed.

JHEP11(2020)032

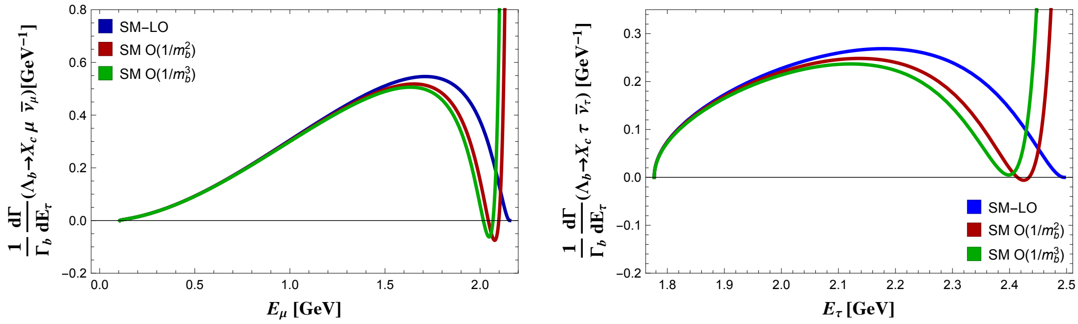


**Figure 2**. Charged lepton energy spectrum in SM for $\Lambda_b \to X_c \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The result at leading order in the HQE (blue line), at $\mathcal{O}(1/m_b^2)$ (red line) and at $\mathcal{O}(1/m_b^3)$ (green line) are displayed.

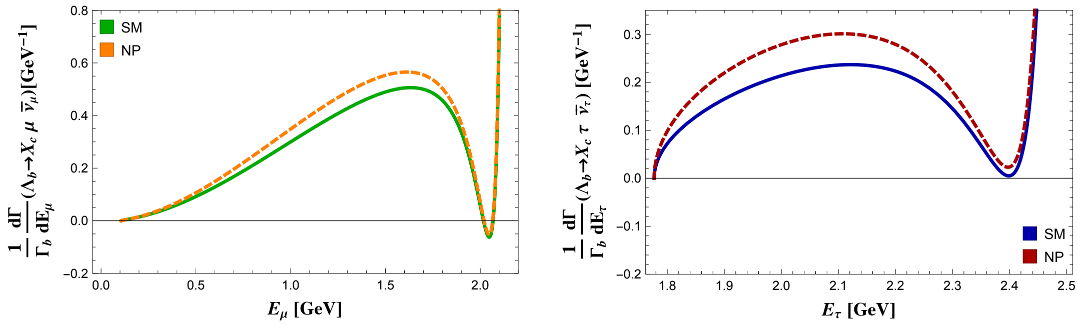


**Figure 3**. Charged lepton energy spectrum for $\Lambda_b \to X_c \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line is the SM result, the dashed line the result for NP at the benchmark point.

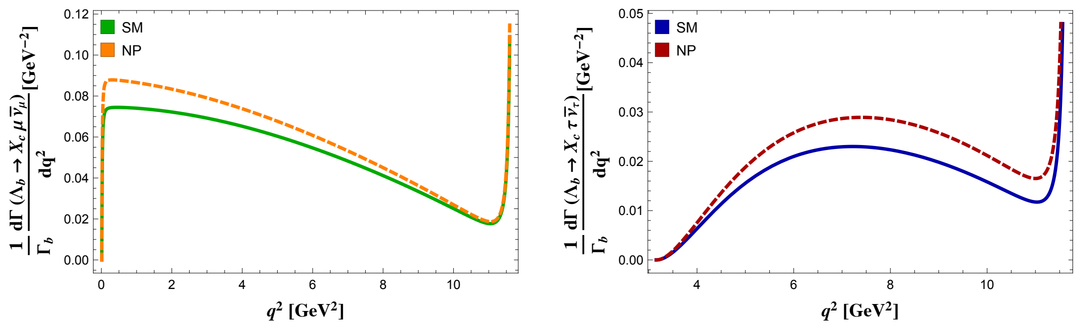


**Figure 4**. Decay distribution in the dilepton invariant mass $q^2$ for $\Lambda_b \to X_c \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line is the SM result, the dashed line the result for NP at the benchmark point.

Comparison of the SM prediction (at $\mathcal{O}(1/m_b^3)$) to NP at the benchmark point is provided in figure 3, where the NP enhancement already observed in the double distribution is evident, in particular for the $\tau$ mode. The enhancement due to NP can also be observed in the $q^2$ spectrum, figure 4: in the muon mode the impact is larger for smaller values of $q^2$, while in the $\tau$ modes the spectrum displays an enhancement in almost all the $q^2$ range.

JHEP11(2020)032

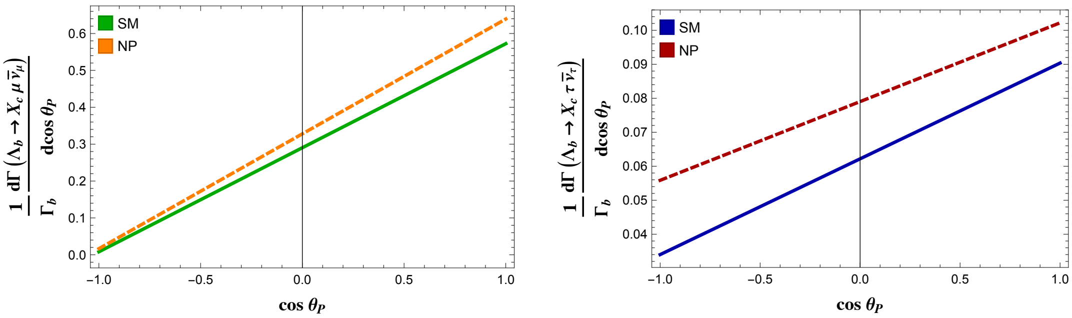


**Figure 5**. $\frac{1}{\Gamma_b}\frac{d\Gamma}{d\cos\theta_P}$ distribution for $\Lambda_b \to X_c \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line is the SM result, the dashed line the NP result at the benchmark point.

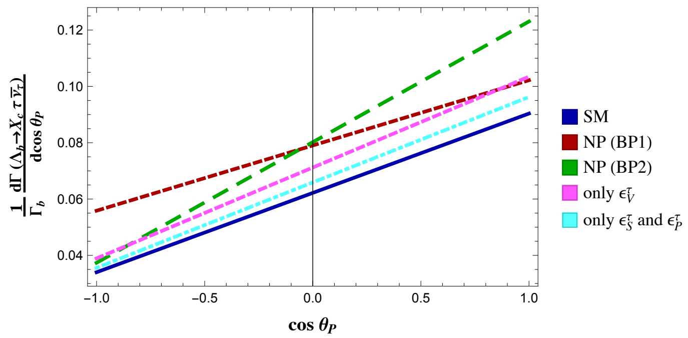


**Figure 6**. $\frac{1}{\Gamma_b}\frac{d\Gamma}{d\cos\theta_P}$ distribution for $\Lambda_b \to X_c \tau \bar{\nu}_\tau$ in SM and in NP scenarios with different values of the effective couplings. The line NP(BP1) corresponds to $\epsilon_T^\tau$ chosen in this paper. The lines NP(BP2), "only $\epsilon_V^\tau$" and "only $\epsilon_S^\tau$ and $\epsilon_P^\tau$" correspond to the values of the couplings fixed in ref. [59], see the text.

A significant sensitivity to NP is found in the $\cos\theta_P$ distribution displayed in figure 5. The dependence of $\frac{d\Gamma}{d\cos\theta_P}$ on $\cos\theta_P$ is linear, and NP contributions modify both the slope and the intercept of the curve. In principle, a measurement of few points in the distribution would allow to access NP. This is confirmed by the comparison of different scenarios, corresponding to different benchmark points. Figure 6 shows how the various operators have a different impact on the intercept and slope of the distribution. In particular, the tensor operator produces a large deviation from SM.

### 5.2 $\Lambda_b \to X_u \ell \bar{\nu}$ mode

$b \to u$ transition displays similar features. The enhancement due to NP appears in the double differential spectra in figure 7, although in this case it is similar in the $\mu$ and $\tau$ mode. The various terms in the HQE alter the lepton energy spectrum for large energy, as shown in figure 8. NP affects a wide $E_\ell$ range, with a similar impact for the muon and $\tau$ modes, figure 9. The enhancement in the $q^2$ spectrum, displayed in figure 10, is lower than in the decay to charm. The distribution in $\cos\theta_P$ is sensitive to NP also in this mode.

JHEP11(2020)032

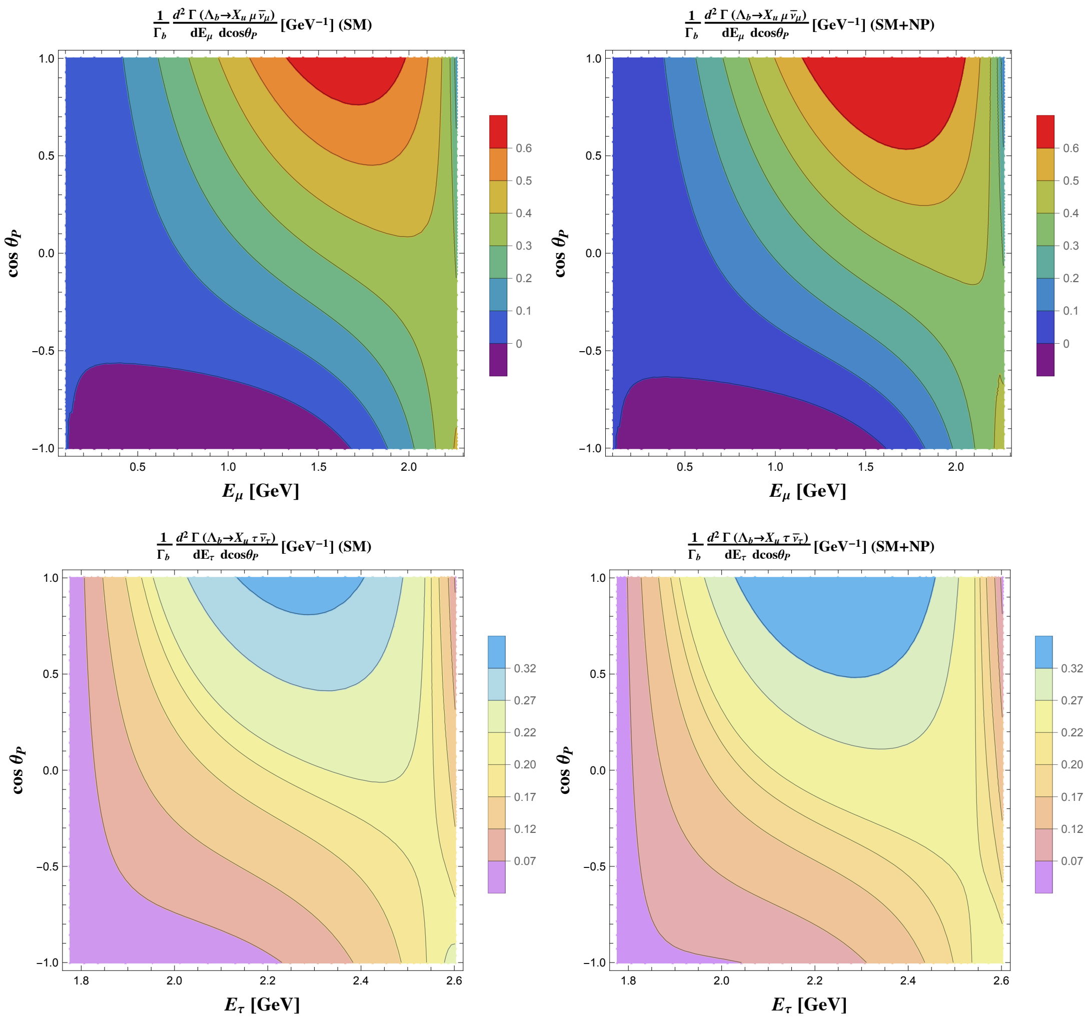


**Figure 7**. Contour plots of the distribution $\frac{1}{\Gamma_b}\frac{d^2\Gamma}{dE_\ell\, d\cos\theta_P}$ for $\Lambda_b \to X_u \ell \bar{\nu}_\ell$. The top and bottom panels refer to $\ell = \mu$ and $\ell = \tau$, respectively, the left and right panels to the Standard Model and to NP at the benchmark point.

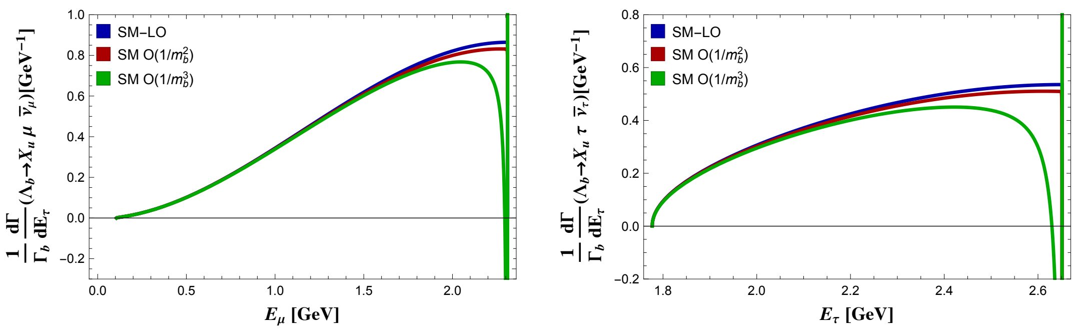


**Figure 8**. Charged lepton energy spectrum in SM for $\Lambda_b \to X_u \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The leading order result in the HQE is the blue line, the $\mathcal{O}(1/m_b^2)$ the red line, the $\mathcal{O}(1/m_b^3)$ the green line.

JHEP11(2020)032

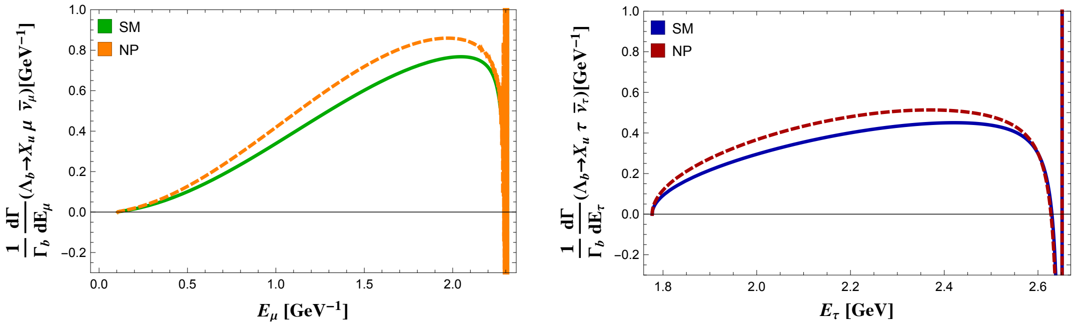


**Figure 9**. Charged lepton energy spectrum for $\Lambda_b \to X_u \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line corresponds to SM, the dashed line to NP at the benchmark point.

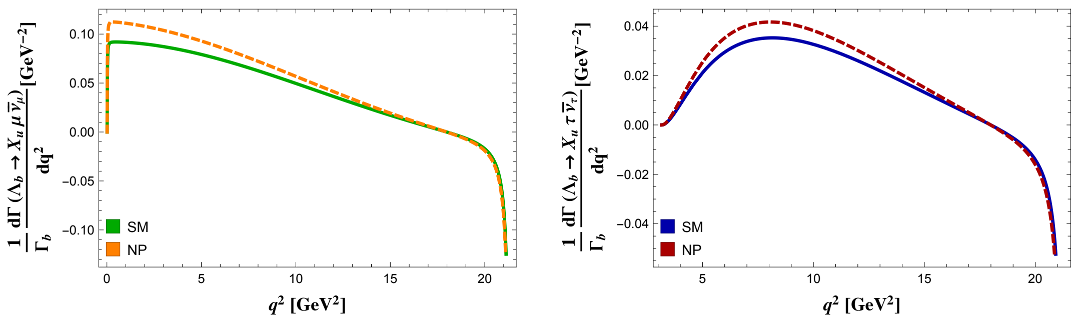


**Figure 10**. $q^2$ distribution for $\Lambda_b \to X_u \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line corresponds to SM, the dashed line to NP at the benchmark point.

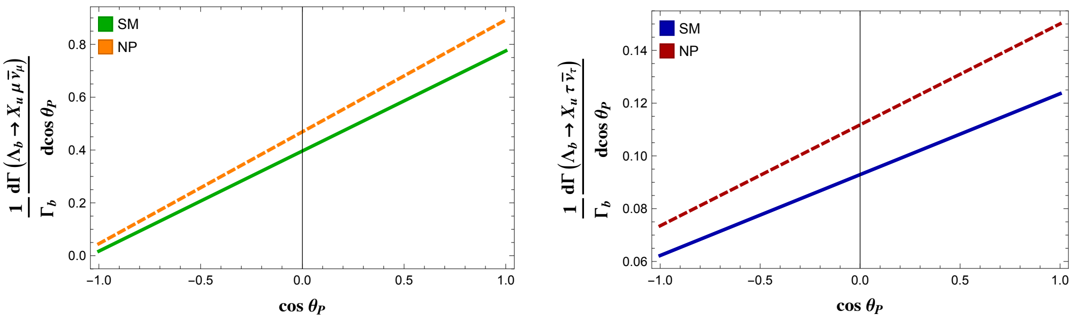


**Figure 11**. Decay distribution $\frac{1}{\Gamma_b}\frac{d\Gamma}{d\cos\theta_P}$ for $\Lambda_b \to X_u \ell \bar{\nu}_\ell$, with $\ell = \mu$ (left) and $\ell = \tau$ (right). The solid line corresponds to SM, the dashed line to NP at the benchmark point.

### 5.3 Ratio $R_{\Lambda_b}(X_U)$

For inclusive semileptonic $\Lambda_b$ decays it is interesting to consider a ratio analogous to $R(D^{(*)})$ for $B$ meson, to compare the $\tau$ and the muon mode using a quantity in which several theoretical uncertainties are canceled:

$$R_{\Lambda_b}(X_U) = \frac{\Gamma(\Lambda_b \to X_U\,\tau\,\bar{\nu}_\tau)}{\Gamma(\Lambda_b \to X_U\,\mu\,\bar{\nu}_\mu)} \qquad (U = u,\, c)\,. \tag{5.2}$$

JHEP11(2020)032

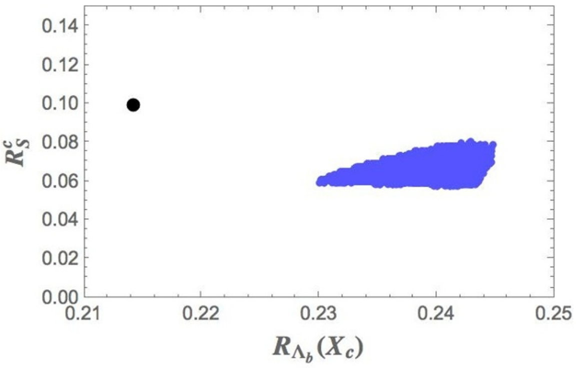


**Figure 12**. $\Lambda_b \to X_c \tau \bar{\nu}_\tau$: correlation between $R_{\Lambda_b}(X_c)$ and the ratio $R_S^c$ of the slopes of the $\frac{d\Gamma}{d\cos\theta_P}$ distribution. The dot corresponds to SM, the broad region to NP with the effective couplings varied as specified in the text.

For this ratio we obtain:

$$R_{\Lambda_b}(X_u)^{SM} = 0.234\,, \qquad R_{\Lambda_b}(X_u)^{NP} = 0.238\,, \tag{5.3}$$

$$R_{\Lambda_b}(X_c)^{SM} = 0.214\,, \qquad R_{\Lambda_b}(X_c)^{NP} = 0.240\,. \tag{5.4}$$

As with the other quantities in this study, the ratios (5.3)–(5.4) are obtained at leading order in $\alpha_s$. $\mathcal{O}(\alpha_s^2)$ corrections have been included in the ratio $R_b(X_c)$ [83], showing that they are small and supporting the expectation that perturbative corrections cancel in the ratios to a large extent. Our results suggest a higher sensitivity of the charm mode to NP. It would be important to observe the correlation of this measurement with the results for $B$ mesons.

As a last observable, we define another ratio sensitive to lepton flavour universality violating NP effects. It can be constructed from the distribution $\frac{d\Gamma(\Lambda_b \to X_U \ell \bar{\nu}_\ell)}{d\cos\theta_P} = A_\ell^U + B_\ell^U \cos\theta_P$. The intercept of the distribution is $A_\ell^U = \frac{1}{2}\Gamma(\Lambda_b \to X_U \ell \bar{\nu}_\ell)$, hence $R_{\Lambda_b}(X_U) = \frac{A_\tau^U}{A_\mu^U}$. The ratio of the slopes $R_S^U = \frac{B_\tau^U}{B_\mu^U}$ has a definite value in SM, and can deviate from it due to NP. In SM we find: $R_S^c = 0.1$ and $R_S^u = 0.08$. A correlation between $R_{\Lambda_b}(X_U)$ and $R_S^U$ can be costructed. As an example, for $\Lambda_b \to X_c \tau \bar{\nu}_\tau$ with the effective Hamiltonian extended including a tensor operator, we vary the couplings $(\mathrm{Re}(\epsilon_T^\mu), \mathrm{Im}(\epsilon_T^\mu))$ and $(\mathrm{Re}(\epsilon_T^\tau), \mathrm{Im}(\epsilon_T^\tau))$ in the regions determined in [5]. The correlation plot in figure 12 shows that the (challenging) measurement of the two ratios would separate SM from NP.

## 6 Conclusions

We have presented a reappraisal of the calculation of the inclusive semileptonic decay width of a heavy hadron, focusing on polarized $\Lambda_b$. We present the expressions for the full differential decay distribution and for the fully integrated width at order $\mathcal{O}(1/m_b^3)$ in the HQE, at leading order in $\alpha_s$ and for non vanishing charged lepton mass. The computation is done extending the SM effective Hamiltonian by the inclusion of the full set of $D = 6$ semileptonic operators, each one weighted by a lepton-flavour dependent coefficient.

Our study improves the SM result, previously known at order $\mathcal{O}(1/m_b^2)$ for a polarized hadron, providing the expressions of the hadronic matrix elements. This allows to analyze other $b$-flavoured baryon modes, as well as other inclusive processes. Moreover, the study supplies the elements for analyzing different operators in the effective weak Hamiltonian density.

Possible NP effects are systematically investigated in various distributions. In particular, in view of the tension in the ratios $R(D^{(*)})$ for $B$ mesons, we have studied the analogous ratios $R_{\Lambda_b}(X_{c,u})$. Among other results, we have found that the $\frac{d\Gamma}{d\cos\theta_P}$ distribution, linear in $\cos\theta_P$, is sensitive to NP. The slope of the distribution, for a hadronic final state $X_c$ or $X_u$, depends on the final lepton species, hence the ratio $R_S^{c,u}$ of the slopes, for $\ell=\tau$ vs $\ell=\mu$, is sensitive to possible lepton flavour universality violation. For $\Lambda_b \to X_c \ell \bar{\nu}_\ell$ when the effective Hamiltonian includes a tensor operator, we have shown that a deviation from SM in $R_{\Lambda_b}(X_c)$ is related to a deviation in $R_S^c$, an interesting, although challenging, correlation to investigate.

## Acknowledgments

We thank J. Aebischer for discussions. FDF thanks A. Buttaro for advice. This study has been carried out within the INFN project (Iniziativa Specifica) QFT-HEP.

## A Hadronic matrix elements

In this appendix we collect the hadronic matrix elements involved in the heavy quark expansion to $\mathcal{O}(1/m_b^3)$. The relations are employed:

$$i\,\epsilon^{\mu\nu\alpha\beta} v_\alpha P_+ \gamma_\beta \gamma_5 P_+ = -P_+(-i\,\sigma^{\mu\nu})P_+\,, \qquad \sigma^{\mu\nu} = -\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}\gamma_5\,. \tag{A.1}$$

The terms independent of the spin four-vector $s_\mu$ agree with ref. [64].

**Dimension 6 operator.** The matrix element is computed in HQET:

$$\langle H_b(v,s)|(\bar{h}_v)_a (iD)^\tau (iD)^\lambda (iD)^\sigma (h_v)_b|H_b(v,s)\rangle = (A^{D6})^{\tau\lambda\sigma}(P_+)_{ba} + (B^{D6})^{\tau\lambda\sigma\mu}\,[P_+\gamma_\mu\gamma_5 P_+]_{ba} \tag{A.2}$$

with $a,b$ Dirac indices. $A^{(D6)}$ and $B^{(D6)}$ are parity-even and parity-odd, respectively. Using the expansion

$$\begin{aligned}(A^{(D6)})^{\tau\lambda\sigma} &= A_1^{(D6)}\Pi^{\tau\sigma}v^\lambda + A_2^{(D6)}\,i\,\epsilon^{\tau\sigma\alpha\beta}v_\alpha s_\beta v^\lambda \\ (B^{(D6)})^{\tau\lambda\sigma\mu} &= B_1^{(D6)}\Pi^{\tau\sigma}v^\lambda s^\mu + B_2^{(D6)}\,i\,\epsilon^{\tau\sigma\alpha\mu}v_\alpha v^\lambda\end{aligned} \tag{A.3}$$

we find:

$$\begin{aligned}A_1^{(D6)} &= -B_1^{(D6)} = \frac{m_H}{3}\hat{\rho}_D^3 \\ A_2^{(D6)} &= \frac{m_H}{2}\hat{\rho}_{LS}^3 \\ B_2^{(D6)} &= \frac{m_H}{6}\hat{\rho}_{LS}^3\,.\end{aligned} \tag{A.4}$$

This gives the expression of the matrix element:

$$\begin{aligned}
&\langle H_b(v,s)|(\bar{h}_v)_a(iD)^\tau(iD)^\lambda(iD)^\sigma(h_v)_b|H_b(v,s)\rangle = \\
&\Big(\frac{m_H}{3}\hat{\rho}_D^3\Pi^{\tau\sigma}v^\lambda + \frac{m_H}{2}\hat{\rho}_{LS}^3\, i\,\epsilon^{\tau\sigma\alpha\beta}v_\alpha s_\beta v^\lambda\Big)\,[P_+]_{ba} \\
&\quad + \Big(-\frac{m_H}{3}\hat{\rho}_D^3\Pi^{\tau\sigma}v^\lambda s^\mu + \frac{m_H}{6}\hat{\rho}_{LS}^3\, i\,\epsilon^{\tau\sigma\alpha\mu}v_\alpha v^\lambda\Big)\,[P_+\gamma_\mu\gamma_5P_+]_{ba}\,. \qquad \text{(A.5)}
\end{aligned}$$

**Dimension 5 operator.** The matrix element, computed in QCD, can be expressed as:

$$\begin{aligned}
\langle H_b(v,s)|(\bar{b}_v)_a(iD)^\tau(iD)^\sigma(b_v)_b|H_b(v,s)\rangle = {}&(A^{(D5)})^{\tau\sigma}g_{ba} + (B^{(D5)})^{\tau\sigma}(\gamma_5)_{ba} + (C^{(D5)})^{\tau\sigma\mu}(\gamma_\mu)_{ba} \\
&+ (D^{(D5)})^{\tau\sigma\mu}(\gamma_\mu\gamma_5)_{ba} + (E^{(D5)})^{\tau\sigma\mu\nu}(-i\sigma_{\mu\nu})_{ba}\,, \qquad \text{(A.6)}
\end{aligned}$$

with $A^{(D5)}, C^{(D5)}, E^{(D5)}$ parity-even and $B^{(D5)}, D^{(D5)}$ parity-odd. They can be expanded as:

$$\begin{aligned}
(A^{(D5)})^{\tau\sigma} &= A_1^{(D5)}g^{\tau\sigma} + A_2^{(D5)}v^\tau v^\sigma + A_3^{(D5)}i\,\epsilon^{\tau\sigma\alpha\beta}v_\alpha s_\beta \\
(B^{(D5)})^{\tau\sigma} &= B_1^{(D5)}v^\tau s^\sigma + B_2^{(D5)}s^\tau v^\sigma \\
(C^{(D5)})^{\tau\sigma\mu} &= C_1^{(D5)}g^{\tau\sigma}v^\mu + C_2^{(D5)}g^{\tau\mu}v^\sigma + C_3^{(D5)}g^{\mu\sigma}v^\tau + C_4^{(D5)}v^\tau v^\sigma v^\mu \\
&\quad + C_5^{(D5)}\,i\,\epsilon^{\tau\sigma\alpha\beta}v_\alpha s_\beta v^\mu + C_6^{(D5)}\,i\,\epsilon^{\sigma\alpha\beta\mu}v_\alpha s_\beta v^\tau + C_7^{(D5)}\,i\,\epsilon^{\tau\alpha\beta\mu}v_\alpha s_\beta v^\sigma \\
(D^{(D5)})^{\tau\sigma\mu} &= D_1^{(D5)}g^{\tau\sigma}s^\mu + D_2^{(D5)}v^\tau v^\sigma s^\mu + D_3^{(D5)}\,i\,\epsilon^{\tau\sigma\theta\mu}v_\theta \\
&\quad + D_4^{(D5)}g^{\sigma\mu}s^\tau + D_5^{(D5)}g^{\tau\mu}s^\sigma + D_6^{(D5)}v^\mu v^\sigma s^\tau + D_7^{(D5)}v^\mu v^\tau s^\sigma \qquad \text{(A.7)} \\
(E^{(D5)})^{\tau\sigma\mu\nu} &= E_1^{(D5)}(g^{\mu\tau}g^{\nu\sigma} - g^{\nu\tau}g^{\mu\sigma}) \\
&\quad + E_2^{(D5)}(g^{\mu\tau}v^\nu - g^{\nu\tau}v^\mu)v^\sigma + E_3^{(D5)}(g^{\mu\sigma}v^\nu - g^{\nu\sigma}v^\mu)v^\tau \\
&\quad + E_4^{(D5)}g^{\tau\sigma}\,i\,\epsilon^{\mu\nu\alpha\beta}v_\alpha s_\beta + E_5^{(D5)}v^\tau v^\sigma\,i\,\epsilon^{\mu\nu\alpha\beta}v_\alpha s_\beta \\
&\quad + E_6^{(D5)}\,i\,\epsilon^{\mu\nu\sigma\alpha}s_\alpha v_\tau + E_7^{(D5)}\,i\,\epsilon^{\mu\nu\tau\alpha}s_\alpha v_\sigma \\
&\quad + E_8^{(D5)}\,i\,\epsilon^{\mu\nu\sigma\alpha}v_\alpha s_\tau + E_9^{(D5)}\,i\,\epsilon^{\mu\nu\tau\alpha}v_\alpha s_\sigma\,.
\end{aligned}$$

We obtain:

$$\begin{aligned}
A_1^{(D5)} &= -A_2^{(D5)} = -\frac{m_H}{6}\hat{\mu}_\pi^2 & A_3^{(D5)} &= \frac{m_H}{4}\Big[\hat{\mu}_G^2 + \frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\Big] \\
B_1^{(D5)} &= -B_2^{(D5)} = \frac{m_H}{12m_b}\hat{\rho}_D^3 \\
C_1^{(D5)} &= -\frac{m_H}{6}\hat{\mu}_\pi^2 & C_2^{(D5)} &= C_3^{(D5)} = \frac{m_H}{12m_b}\left(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3\right) \\
C_4^{(D5)} &= \frac{m_H}{6}\hat{\mu}_\pi^2 - \frac{m_H}{6m_b}\left(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3\right) & C_5^{(D5)} &= \frac{m_H}{4}\Big[\hat{\mu}_G^2 + \frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\Big] \\
C_6^{(D5)} &= -C_7^{(D5)} = -\frac{m_H}{24m_b}\left(2\hat{\rho}_D^3 + 3\hat{\rho}_{LS}^3\right) \\
D_1^{(D5)} &= -D_2^{(D5)} = \frac{m_H}{6}\hat{\mu}_\pi^2 & D_3^{(D5)} &= \frac{m_H}{12}\Big[\hat{\mu}_G^2 + \frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\Big]
\end{aligned}$$



JHEP11(2020)032

$$D_6^{(D5)} = D_7^{(D5)} = \frac{m_H}{12 m_b}\hat{\rho}_D^3$$

$$E_1^{(D5)} = -\frac{m_H}{24}\left[\hat{\mu}_G^2 + \frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\right] \qquad E_2^{(D5)} = -E_3^{(D5)} = \frac{m_H}{24}\hat{\mu}_G^2 + \frac{m_H}{12 m_b}\left(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3\right)$$

$$E_4^{(D5)} = -\frac{m_H}{12}\hat{\mu}_\pi^2 \qquad E_5^{(D5)} = \frac{m_H}{12}\hat{\mu}_\pi^2 - \frac{m_H}{24 m_b}\left(2\hat{\rho}_D^3 + 3\hat{\rho}_{LS}^3\right)$$

$$E_6^{(D5)} = E_7^{(D5)} = \frac{m_H}{48 m_b}\left(2\hat{\rho}_D^3 + 3\hat{\rho}_{LS}^3\right) \qquad D_4^{(D5)} = D_5^{(D5)} = E_8^{(D5)} = E_9^{(D5)} = 0\,.$$

The previous expressions allow us to write the matrix element:

$$\begin{aligned}
&\langle H_b(v,s)|(\bar{b}_v)_a (iD)^\tau (iD)^\sigma (b_v)_b|H_b(v,s)\rangle = -\frac{m_H}{3}\hat{\mu}_\pi^2\,\Pi^{\tau\sigma}\left[P_+ - s_\mu \hat{S}^\mu\right]_{ba} \\
&+\frac{m_H}{2}\left(\hat{\mu}_G^2 + \frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\right) i\,\epsilon^{\tau\sigma\alpha\beta} v_\alpha \left[s_\beta P_+ + \frac{1}{3}\hat{S}_\beta\right]_{ba} + \frac{m_H}{6 m_b}\hat{\rho}_D^3\left[v^\tau s^\sigma P_+\gamma_5 - v^\sigma s^\tau \gamma_5 P_+\right]_{ba} \\
&-\frac{m_H}{6 m_b}\left(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3\right)\left[-v^\tau s^\sigma P_+\gamma_5 - v^\sigma s^\tau\gamma_5 P_+ + (2 v^\tau v^\sigma P_+ - v^\sigma \gamma^\tau P_+ - v^\tau P_+ \gamma^\sigma)(1 - \not{s}\gamma_5)\right]_{ba} \\
&-\frac{m_H}{12 m_b}\hat{\rho}_{LS}^3\left[-v^\tau s^\sigma P_+\gamma_5 - v^\sigma s^\tau \gamma_5 P_+ - (2 v^\tau v^\sigma P_+ - v^\sigma\gamma^\tau P_+ - v^\tau P_+\gamma^\sigma)\not{s}\gamma_5\right]_{ba}. \qquad \text{(A.8)}
\end{aligned}$$

**Dimension 4 operator.** The procedure for computing the matrix element in QCD, using the expansion in Dirac matrices, is analogous to the $D = 5$ case. The results is:

$$\begin{aligned}
\langle H_b(v,s)|(\bar{b}_v)_a (iD)^\tau (b_v)_b|H_b(v,s)\rangle = &\frac{m_H}{2 m_b}\left(\hat{\mu}_\pi^2 - \hat{\mu}_G^2\right)\left[\left(v^\tau P_+ - \frac{1}{3}(\gamma^\tau - v^\tau \not{v})\right)(1 - \not{s}\gamma_5)\right]_{ba} \\
&-\frac{m_H}{3 m_b}\hat{\mu}_\pi^2 s^\tau [P_+\gamma_5]_{ba} \\
&+\frac{m_H}{12 m_b}\hat{\mu}_G^2\left[-(\gamma^\tau - v^\tau\not{v})\not{s}\gamma_5 + 3 s^\tau\gamma_5\right]_{ba} \\
&+\frac{m_H}{12 m_b^2}\left(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3\right)\left[\gamma^\tau - 4 v^\tau \not{v}\right]_{ba} \\
&+\frac{m_H}{12 m_b^2}\hat{\rho}_D^3\left[v^\tau \not{s}\gamma_5 - s^\tau \not{v}\gamma_5\right]_{ba} \qquad \text{(A.9)} \\
&+\frac{m_H}{6 m_b^2}\hat{\rho}_D^3\left[-(\gamma^\tau - 2 v^\tau\not{v})\not{s}\gamma_5 + s^\tau\gamma_5\right]_{ba} \\
&+\frac{m_H}{8 m_b^2}\hat{\rho}_{LS}^3\left[-(\gamma^\tau - 3 v^\tau \not{v})\not{s}\gamma_5 + s^\tau\gamma_5\right]_{ba}.
\end{aligned}$$

**Dimension 3 operator.** The matrix element computed in QCD reads:

$$\begin{aligned}
\langle H_b(v,s)|(\bar{b}_v)_a (b_v)_b|H_b(v,s)\rangle = &\left[\left(m_H P_+ - \frac{m_H}{4 m_b^2}\left(\hat{\mu}_\pi^2 - \hat{\mu}_G^2\right)\right)(1 - \not{s}\gamma_5)\right]_{ba} \qquad \text{(A.10)} \\
&+\frac{m_H}{4 m_b^2}\left(\hat{\mu}_\pi^2 - \hat{\mu}_G^2\right)[P_+\not{s}\gamma_5]_{ba} - \frac{m_H}{6 m_b^2}\left(\hat{\mu}_\pi^2 + \frac{\hat{\rho}_D^3}{m_b}\right)[P_-\not{s}\gamma_5]_{ba}.
\end{aligned}$$

The matrix elements can be related using the equation of motion for $b_v$:

$$\begin{aligned}
\langle H_b(v,s)|\bar{b}_v (iD)^{\mu_1}\dots(iD)^{\mu_n}\Gamma b_v|H_b(v,s)\rangle = &\frac{1}{2}\langle H_b(v,s)|\bar{b}_v (iD)^{\mu_1}\dots(iD)^{\mu_n}\{\Gamma, \not{v}\} b_v|H_b(v,s)\rangle \\
&+\frac{1}{2 m_b}\langle H_b(v,s)|\bar{b}_v\{(i\not{D}), (iD)^{\mu_1}\dots(iD)^{\mu_n}\Gamma\} b_v|H_b(v,s)\rangle \qquad \text{(A.11)}
\end{aligned}$$

for a generic Dirac matrix $\Gamma$. This allows to relate the coefficients of matrix elements of operators of different dimensions, providing a check of the results [84].

## B Hadronic tensor for the Standard Model and for the extended Hamiltonian

We provide the tensor $T^{ij}$ for the $b \to U$ modes ($U = u$, $c$) for the Standard Model and for the effective Hamiltonian in eq. (2.2), expanded in invariant functions. We provide their expressions for the single operators (Standard Model, S, P, T) and for the interferences.

- Standard Model

  This case amounts to choosing $i = j = 1$ in eq. (3.2) and $J^{(1)}_\mu = \bar{U}\gamma_\mu(1-\gamma_5)b$. For a polarized baryon the corresponding tensor $T^{\mu\nu}_{SM}$ can be expanded in terms of the functions $T_{1,\dots,5}$ and $S_{1,\dots,13}$ [68]:

$$
\begin{aligned}
T^{\mu\nu}_{SM} &= -g^{\mu\nu}T_1 + v^\mu v^\nu T_2 - i\,\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta T_3 + q^\mu q^\nu T_4 + (q^\mu v^\nu + q^\nu v^\mu)T_5 \\
&\quad - (q\cdot s)\Big[ - g^{\mu\nu}S_1 + v^\mu v^\nu S_2 - i\,\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta S_3 + q^\mu q^\nu S_4 + (q^\mu v^\nu + q^\nu v^\mu)S_5\Big] \\
&\quad + (s^\mu v^\nu + s^\nu v^\mu)S_6 + (s^\mu q^\nu + s^\nu q^\mu)S_7 + i\,\epsilon^{\mu\nu\alpha\beta}v_\alpha s_\beta\, S_8 + i\,\epsilon^{\mu\nu\alpha\beta}q_\alpha s_\beta\, S_9 \\
&\quad + (s^\mu v^\nu - s^\nu v^\mu)S_{10} + (s^\mu q^\nu - s^\nu q^\mu)S_{11} \\
&\quad + \Big(v^\mu\,\epsilon^{\nu\alpha\beta\delta}q_\alpha v_\beta s_\delta + v^\nu\,\epsilon^{\mu\alpha\beta\delta}q_\alpha v_\beta s_\delta\Big)\, i\,S_{12} \\
&\quad + \Big(q^\mu\,\epsilon^{\nu\alpha\beta\delta}q_\alpha v_\beta s_\delta + q^\nu\,\epsilon^{\mu\alpha\beta\delta}q_\alpha v_\beta s_\delta\Big)\, i\,S_{13}\,.
\end{aligned}
\tag{B.1}
$$

  The $1/m_b$ expansion of the these functions reads:

$$
\begin{aligned}
T_1 &= 2m_H\Bigg\{\frac{1}{\Delta_0}\left[2(m_b - v\cdot q) + \frac{(\hat\mu_\pi^2 - \hat\mu_G^2)}{3m_b} - 2\frac{(\hat\rho_D^3 + \hat\rho_{LS}^3)}{3m_b^2}\right] \\
&\quad + \frac{2}{3m_b\Delta_0^2}\Big[\left(\hat\mu_\pi^2 - \hat\mu_G^2\right)\left[2(q^2 - (v\cdot q)^2)\right. \\
&\quad \left.+ 3(v\cdot q)(m_b - v\cdot q)\right] + \hat\mu_G^2 4m_b(m_b - v\cdot q) \\
&\quad + \frac{(\hat\rho_D^3 + \hat\rho_{LS}^3)}{m_b}\left[6m_b(m_b - v\cdot q) - q^2 + 4(v\cdot q)^2\right] - 4m_b\hat\rho_{LS}^3\Big] \\
&\quad - \frac{8}{3\Delta_0^3}\left[q^2 - (v\cdot q)^2\right](m_b - v\cdot q)\left[\hat\mu_\pi^2 - \frac{\hat\rho_D^3 + \hat\rho_{LS}^3}{m_b}\right] \\
&\quad - \frac{16}{3\Delta_0^4}\hat\rho_D^3\left[q^2 - (v\cdot q)^2\right](m_b - v\cdot q)^2\Bigg\}
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
T_2 &= 2m_H\Bigg\{\frac{2}{\Delta_0}\left[2m_b + \frac{5}{3m_b}\left(\hat\mu_\pi^2 - \hat\mu_G^2\right) - \frac{4}{3m_b^2}\left(\hat\rho_D^3 + \hat\rho_{LS}^3\right)\right] \\
&\quad + \frac{4}{3m_b\Delta_0^2}\Big[7m_b v\cdot q\,\hat\mu_\pi^2 + m_b(2m_b - 5v\cdot q)\,\hat\mu_G^2 \\
&\quad + 6(m_b - v\cdot q)\,\hat\rho_D^3 + 2(2m_b - 3v\cdot q)\,\hat\rho_{LS}^3\Big] \\
&\quad - \frac{8}{3\Delta_0^3}\Big[2m_b\left[q^2 - (v\cdot q)^2\right]\hat\mu_\pi^2 - 2v\cdot q(m_b - v\cdot q)\left(2\hat\rho_D^3 + \hat\rho_{LS}^3\right) + q^2\hat\rho_{LS}^3\Big] \\
&\quad - \frac{32}{3\Delta_0^4}\hat\rho_D^3\left[q^2 - (v\cdot q)^2\right]m_b(m_b - v\cdot q)\Bigg\}
\end{aligned}
\tag{B.3}
$$

$$T_3 = -2m_H\Bigg\{\frac{2}{\Delta_0} + \frac{2}{3m_b^2\Delta_0^2}\Big[5m_b v\cdot q\left(\hat\mu_\pi^2 - \hat\mu_G^2\right) + 6m_b^2\hat\mu_G^2$$
$$+ 2(3m_b - 2v\cdot q)\left(\hat\rho_D^3 + \hat\rho_{LS}^3\right)\Big]$$
$$+ \frac{8}{3\Delta_0^3}\Big[-[q^2-(v\cdot q)^2]\hat\mu_\pi^2 + v\cdot q\,(m_b - v\cdot q)\frac{\hat\rho_D^3}{m_b} - (m_b - v\cdot q)^2\,\frac{\hat\rho_{LS}^3}{m_b}\Big]$$
$$- \frac{16}{3\Delta_0^4}\hat\rho_D^3[q^2-(v\cdot q)^2](m_b - v\cdot q)\Bigg\} \tag{B.4}$$

$$T_4 = 2m_H\Bigg\{\frac{4}{3m_b\Delta_0^2}\Big[2\left(\hat\mu_\pi^2 - \hat\mu_G^2\right) - \frac{\left(\hat\rho_D^3 + \hat\rho_{LS}^3\right)}{m_b}\Big]$$
$$+ \frac{8}{3m_b\Delta_0^3}\Big[2(m_b - v\cdot q)\hat\rho_D^3 + (m_b - 2v\cdot q)\hat\rho_{LS}^3\Big]\Bigg\} \tag{B.5}$$

$$T_5 = 2m_H\Bigg\{-\frac{2}{\Delta_0} + \frac{2}{3m_b\Delta_0^2}\Big[-4m_b\hat\mu_\pi^2 - 5v\cdot q\left(\hat\mu_\pi^2 - \hat\mu_G^2\right) + 4\frac{v\cdot q}{m_b}\left(\hat\rho_D^3 + \hat\rho_{LS}^3\right)\Big]$$
$$+ \frac{8}{3\Delta_0^3}\Bigg[\left[q^2-(v\cdot q)^2\right]\hat\mu_\pi^2 + \left[-2m_b^2 + m_b v\cdot q + (v\cdot q)^2\right]\frac{\hat\rho_D^3}{m_b} \tag{B.6}$$
$$+ \left[-m_b^2 + m_b v\cdot q + (v\cdot q)^2\right]\frac{\hat\rho_{LS}^3}{m_b}\Bigg] + \frac{16}{3\Delta_0^4}\hat\rho_D^3\left[q^2-(v\cdot q)^2\right](m_b - v\cdot q)\Bigg\}$$

$$S_1 = 2m_H\Bigg\{-\frac{2}{\Delta_0}\Big[1 - \frac{7\hat\mu_\pi^2 - 9\hat\mu_G^2}{12m_b^2} + \frac{\hat\rho_D^3}{6m_b^3}\Big]$$
$$+ \frac{2}{3m_b\Delta_0^2}\left[-5v\cdot q\,\hat\mu_\pi^2 + 3(v\cdot q - 2m_b)\hat\mu_G^2 - 4\hat\rho_D^3 - 3\hat\rho_{LS}^3\right]$$
$$+ \frac{8}{3\Delta_0^3}\Big[\left[q^2-(v\cdot q)^2\right]\hat\mu_\pi^2 - v\cdot q\,(m_b - v\cdot q)\frac{\hat\rho_D^3}{m_b}\Big] \tag{B.7}$$
$$+ \frac{16}{3\Delta_0^4}\hat\rho_D^3(m_b - v\cdot q)\left[q^2-(v\cdot q)^2\right]\Bigg\}$$

$$S_2 = 2m_H\Bigg\{\frac{2}{3\Delta_0^2}\left[4m_b\hat\mu_\pi^2 - 6m_b\hat\mu_G^2 - 8\hat\rho_D^3 - 9\hat\rho_{LS}^3\right]$$
$$+ \frac{8}{3\Delta_0^3}\left[2(m_b - v\cdot q)\hat\rho_D^3 - 3(v\cdot q)\,\hat\rho_{LS}^3\right]\Bigg\} \tag{B.8}$$

$$S_3 = -2m_H\Bigg\{\frac{2}{3m_b\Delta_0^2}\Big[2\hat\mu_\pi^2 + \frac{\hat\rho_D^3}{m_b}\Big] + \frac{4}{3m_b\Delta_0^3}\Big[2(m_b - v\cdot q)\hat\rho_D^3 - 3m_b\hat\rho_{LS}^3\Big]\Bigg\} \tag{B.9}$$

with $S_4 = 0$ and $S_5 = S_3$,

$$S_6 = 2m_H\Bigg\{\frac{1}{\Delta_0}\Big[-2m_b - \frac{1}{2m_b}\left(\hat\mu_\pi^2 + \hat\mu_G^2\right) - \frac{\hat\rho_D^3}{3m_b^2}\Big]$$
$$+ \frac{1}{3\Delta_0^2}\Big[-10v\cdot q\,\hat\mu_\pi^2 - 4(m_b + v\cdot q)\frac{\hat\rho_D^3}{m_b} - 9v\cdot q\frac{\hat\rho_{LS}^3}{m_b}\Big]$$
$$+ \frac{4}{3\Delta_0^3}\Big[2m_b[q^2-(v\cdot q)^2]\,\hat\mu_\pi^2 - 2v\cdot q\,(m_b - v\cdot q)\hat\rho_D^3 + 3[q^2-(v\cdot q)^2]\,\hat\rho_{LS}^3\Big]$$
$$+ \frac{16m_b}{3\Delta_0^4}[q^2-(v\cdot q)^2](m_b - v\cdot q)\hat\rho_D^3\Bigg\} \tag{B.10}$$

$$
\begin{aligned}
S_7 = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\left[1 - \frac{7\hat{\mu}_\pi^2 - 9\hat{\mu}_G^2}{12m_b^2} + \frac{\hat{\rho}_D^3}{6m_b^3}\right] \\
& + \frac{1}{3m_b\Delta_0^2}\left[2(2m_b + 3v\cdot q)\hat{\mu}_\pi^2 + 6(m_b - v\cdot q)\hat{\mu}_G^2 + 2(2m_b - v\cdot q)\frac{\hat{\rho}_D^3}{m_b} + 3\hat{\rho}_{LS}^3\right] \\
& + \frac{8}{3\Delta_0^3}\left[-[q^2 - (v\cdot q)^2]\hat{\mu}_\pi^2 + (m_b - v\cdot q)\hat{\rho}_D^3\right] \\
& - \frac{16}{3\Delta_0^4}[q^2 - (v\cdot q)^2](m_b - v\cdot q)\hat{\rho}_D^3 \Bigg\}
\end{aligned} \tag{B.11}
$$

$$
\begin{aligned}
S_8 = -2m_H \Bigg\{ & -\frac{2m_b}{\Delta_0}\left[1 - \frac{5\hat{\mu}_\pi^2 - 3\hat{\mu}_G^2}{12m_b^2} - \frac{\hat{\rho}_D^3}{6m_b^3}\right] \\
& + \frac{1}{3\Delta_0^2}\left[-10v\cdot q\,\hat{\mu}_\pi^2 - 12(m_b - v\cdot q)\,\hat{\mu}_G^2 - 4(3m_b - 2v\cdot q)\frac{\hat{\rho}_D^3}{m_b} + 9v\cdot q\,\frac{\hat{\rho}_{LS}^3}{m_b}\right] \\
& + \frac{4}{3\Delta_0^3}\left[[q^2 - (v\cdot q)^2]\;\left[2m_b\hat{\mu}_\pi^2 - 3\hat{\rho}_{LS}^3\right] - 2v\cdot q\,(m_b - v\cdot q)\hat{\rho}_D^3\right] \\
& + \frac{16}{3\Delta_0^4}\hat{\rho}_D^3 m_b(m_b - v\cdot q)[q^2 - (v\cdot q)^2] \Bigg\}
\end{aligned} \tag{B.12}
$$

$$
\begin{aligned}
S_9 = -2m_H \Bigg\{ & \frac{2}{\Delta_0}\left[1 - \frac{7\hat{\mu}_\pi^2 - 9\hat{\mu}_G^2}{12m_b^2} + \frac{\hat{\rho}_D^3}{6m_b^3}\right] \\
& + \frac{1}{3m_b\Delta_0^2}\left[2(2m_b + 3v\cdot q)\,\hat{\mu}_\pi^2 + 6(m_b - v\cdot q)\,\hat{\mu}_G^2 - 2v\cdot q\,\frac{\hat{\rho}_D^3}{m_b} - 3\hat{\rho}_{LS}^3\right] \\
& + \frac{8}{3\Delta_0^3}\left[-[q^2 - (v\cdot q)^2]\,\hat{\mu}_\pi^2 + (m_b - v\cdot q)\hat{\rho}_D^3\right] \\
& - \frac{16}{3\Delta_0^4}\hat{\rho}_D^3(m_b - v\cdot q)[q^2 - (v\cdot q)^2] \Bigg\}
\end{aligned} \tag{B.13}
$$

and $S_{10,11,12,13} = 0$.

- Scalar operator in $H_{\rm eff}$

  This case amounts to choosing $i = j = 2$ in eq. (3.2). $T_S$ is expanded as

$$
T_S = T_{S1} + (q\cdot s)S_{S1} \tag{B.14}
$$

with

$$
\begin{aligned}
T_{S1} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\left[(m_b + m_U - v\cdot q) - \frac{m_b + m_U}{2m_b^2}\left(\hat{\mu}_\pi^2 - \hat{\mu}_G^2\right)\right] \\
& + \frac{1}{3m_b\Delta_0^2}\bigg[\Big(2[q^2 - (v\cdot q)^2] + 3v\cdot q\,(m_b + m_U - v\cdot q)\Big)\left(\hat{\mu}_\pi^2 - \hat{\mu}_G^2\right) \\
& - [q^2 - 4(v\cdot q)^2]\frac{\hat{\rho}_D^3 + \hat{\rho}_{LS}^3}{m_b}\bigg] \\
& + \frac{4}{3\Delta_0^3}[q^2 - (v\cdot q)^2]\left[-(m_b + m_U - v\cdot q)\hat{\mu}_\pi^2 + (m_b - v\cdot q)\frac{\hat{\rho}_D^3}{m_b} - v\cdot q\,\frac{\hat{\rho}_{LS}^3}{m_b}\right] \\
& - \frac{8}{3\Delta_0^4}[q^2 - (v\cdot q)^2](m_b - v\cdot q)(m_b + m_U - v\cdot q)\hat{\rho}_D^3 \Bigg\}
\end{aligned} \tag{B.15}
$$

and $S_{S1} = 0$.

- Pseudoscalar operator in $H_{\rm eff}$

  The tensor is obtained choosing $i = j = 3$ in eq. (3.2), and is expanded as

$$T_P = T_{P1} + (q \cdot s)\, S_{P1}\,. \tag{B.16}$$

  The two functions in (B.16) are given by the corresponding ones in (B.14) replacing $m_U \to -m_U$.

- Interference between the SM and the scalar operator in $H_{\rm eff}$

  The tensor is obtained when $(i,j) = (1,2)$ and $(2,1)$ in eq. (3.2). We denote the two contributions as $T_{SMS}$ and $T_{SSM}$, respectively. Using the expansion

$$\begin{aligned} T^\mu_{SMS} &= T_{SMS,1}\, v^\mu + T_{SMS,2}\, q^\mu \\ &\quad - (q\cdot s)\Big[S_{SMS,1}\, v^\mu + S_{SMS,2}\, q^\mu\Big] + S_{SMS,3}\, s^\mu + S_{SMS,4}\, i\, \epsilon^{\mu\alpha\beta\delta} q_\alpha v_\beta s_\delta \end{aligned} \tag{B.17}$$

  and the analogous one for $T_{SSM}$, we find:

$$\begin{aligned} T_{SMS,1} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}(m_b+m_U) \\ & - \frac{v\cdot q}{3m_b^2\Delta_0^2}\Big[-5m_b(m_b+m_U)\hat\mu_\pi^2 + m_b(m_b+5m_U)\hat\mu_G^2 - 4(m_b-m_U)\left(\hat\rho_D^3+\hat\rho_{LS}^3\right)\Big] \\ & - \frac{4}{3m_b\Delta_0^3}\Big[m_b(m_b+m_U)\left[q^2-(v\cdot q)^2\right]\hat\mu_\pi^2 - (m_b+m_U)v\cdot q\,(m_b-v\cdot q)\hat\rho_D^3 \\ & + \left[m_b\left[q^2-(v\cdot q)^2\right] + m_U(v\cdot q)^2\right]\hat\rho_{LS}^3\Big] \\ & - \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b+m_U)(m_b-v\cdot q)\left[q^2-(v\cdot q)^2\right] \Bigg\} \end{aligned} \tag{B.18}$$

$$\begin{aligned} T_{SMS,2} = 2m_H \Bigg\{ & -\frac{1}{\Delta_0}\left[1-\frac{(\hat\mu_\pi^2-\hat\mu_G^2)}{2m_b^2}\right] \\ & - \frac{1}{3m_b^2\Delta_0^2}\Big[m_b(2m_U+3v\cdot q)\left(\hat\mu_\pi^2-\hat\mu_G^2\right) + 2m_b^2\left(\hat\mu_\pi^2+\hat\mu_G^2\right) + (m_b-m_U)\left(\hat\rho_D^3+\hat\rho_{LS}^3\right)\Big] \\ & - \frac{4}{3m_b\Delta_0^3}\Big[-m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2 + \hat\rho_D^3(m_b+m_U)(m_b-v\cdot q) - m_U v\cdot q\hat\rho_{LS}^3\Big] \\ & + \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b-v\cdot q)\left[q^2-(v\cdot q)^2\right] \Bigg\} \end{aligned} \tag{B.19}$$

$$\begin{aligned} S_{SMS,1} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\left[1-\frac{(5\hat\mu_\pi^2-3\hat\mu_G^2)}{12m_b^2} - \frac{\hat\rho_D^3}{6m_b^3}\right] \\ & + \frac{1}{3m_b^2\Delta_0^2}\Big[m_b(2m_b+2m_U+5v\cdot q)\hat\mu_\pi^2 - 6m_b v\cdot q\,\hat\mu_G^2 \\ & + (-m_b+m_U-4v\cdot q)\hat\rho_D^3 - \frac{9}{2}(v\cdot q)\,\rho_{LS}^3\Big] \\ & + \frac{2}{3m_b\Delta_0^3}\Big[-2m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2 + 2(m_b-v\cdot q)(m_b+m_U+v\cdot q)\hat\rho_D^3 - 3(v\cdot q)^2\hat\rho_{LS}^3\Big] \\ & - \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b-v\cdot q)\left[q^2-(v\cdot q)^2\right] \Bigg\} \end{aligned} \tag{B.20}$$

$$\begin{aligned} S_{SMS,2} = 2m_H \Bigg\{ & \frac{1}{6m_b^2\Delta_0^2}\Big[-4m_b\hat\mu_\pi^2 + 6m_b\hat\mu_G^2 + 4\hat\rho_D^3 + 3\hat\rho_{LS}^3\Big] \\ & + \frac{2}{3m_b\Delta_0^3}\Big[-2(m_b-v\cdot q)\hat\rho_D^3 + 3v\cdot q\hat\rho_{LS}^3\Big] \Bigg\} \end{aligned} \tag{B.21}$$

$$
\begin{aligned}
S_{SMS,3} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\Big[-m_b - m_U + v\cdot q + \frac{7m_b+7m_U-5v\cdot q}{12m_b^2}\hat\mu_\pi^2 \\
& - \frac{3m_b+3m_U-v\cdot q}{4m_b^2}\hat\mu_G^2 - \frac{m_b+m_U+v\cdot q}{6m_b^3}\hat\rho_D^3\Big] \\
& + \frac{1}{3m_b\Delta_0^2}\Big[-\left(2q^2+v\cdot q(3m_b+3m_U-5v\cdot q)\right)\hat\mu_\pi^2 \\
& + 3\left(q^2+v\cdot q(m_b+m_U-2v\cdot q)\right)\hat\mu_G^2 + \left(2q^2+v\cdot q(-m_b+m_U-4v\cdot q)\right)\frac{\hat\rho_D^3}{m_b} \\
& + 3[q^2-3(v\cdot q)^2]\frac{\hat\rho_{LS}^3}{2m_b}\Big] \\
& + \frac{2}{3m_b\Delta_0^3}[q^2-(v\cdot q)^2]\Big[2m_b(m_b+m_U-v\cdot q)\hat\mu_\pi^2 - 2(m_b-v\cdot q)\hat\rho_D^3 + 3v\cdot q\hat\rho_{LS}^3\Big] \\
& + \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b-v\cdot q)[q^2-(v\cdot q)^2](m_b+m_U-v\cdot q)\Bigg\}
\end{aligned} \tag{B.22}
$$

$$
\begin{aligned}
S_{SMS,4} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\Big[1 - \frac{(5\hat\mu_\pi^2-3\hat\mu_G^2)}{12m_b^2} - \frac{\hat\rho_D^3}{6m_b^3}\Big] \\
& + \frac{1}{3m_b^2\Delta_0^2}\Big[5m_b v\cdot q\hat\mu_\pi^2 + 6m_b(m_b-v\cdot q)\hat\mu_G^2 + 2(3m_b-2v\cdot q)\hat\rho_D^3 + \frac{3}{2}(4m_b-3v\cdot q)\hat\rho_{LS}^3\Big] \\
& - \frac{2}{3m_b\Delta_0^3}\Big[2m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2 - 2v\cdot q(m_b-v\cdot q)\hat\rho_D^3 + 3\left[(m_b-v\cdot q)^2+m_b m_U\right]\hat\rho_{LS}^3\Big] \\
& - \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b-v\cdot q)[q^2-(v\cdot q)^2]\Bigg\}
\end{aligned} \tag{B.23}
$$

and $T_{SSM,i} = T_{SMS,i}$ (for $i = 1,2$), $S_{SSM,i} = S_{SMS,i}$ (for $i = 1,2,3$), $S_{SSM,4} = -S_{SMS,4}$.

- Interference between the SM and the pseudoscalar operators in $H_{\rm eff}$

  The tensor is obtained for $(i,j) = (1,3)$ and $(i,j) = (3,1)$ in eq. (3.2). We denote the two contributions as $T_{SMP}$ and $T_{PSM}$, respectively, with the expansion

$$
\begin{aligned}
T^\mu_{SMP} &= T_{SMP,1}\, v^\mu + T_{SMP,2}\, q^\mu \\
& - (q\cdot s)\Big[S_{SMP,1}\, v^\mu + S_{SMP,2}\, q^\mu\Big] + S_{SMP,3}\, s^\mu + S_{SMP,4}\; i\,\epsilon^{\mu\alpha\beta\delta} q_\alpha v_\beta s_\delta\,,
\end{aligned} \tag{B.24}
$$

  and the analogous one for $T_{PSM}$. The functions in (B.24) are given by the corresponding ones in (B.17) replacing $m_U \to -m_U$.

- Interference between the scalar and pseudoscalar operators in $H_{\rm eff}$

  This case amounts to choosing $(i,j) = (2,3)$ and $(i,j) = (3,2)$ in eq. (3.2). We denote the two terms as $T_{SP}$ and $T_{PS}$, respectively. Writing

$$
T_{SP} = T_{SP,1} - (q\cdot s)\, S_{SP,1} \tag{B.25}
$$

  and analogously for $T_{PS}$, we have $T_{SP,1} = T_{PS,1} = 0$ and

$$
\begin{aligned}
S_{SP,1} = S_{PS,1} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\left[1 - m_b\frac{(7\hat\mu_\pi^2 - 9\hat\mu_G^2)}{12m_b^2} + \frac{\hat\rho_D^3}{6m_b^3}\right] + \frac{v\cdot q}{3m_b\Delta_0^2}\,(5\hat\mu_\pi^2 - 3\hat\mu_G^2) \\
& + \frac{4}{3m_b\Delta_0^3}\,[-m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2 + v\cdot q(m_b - v\cdot q)\hat\rho_D^3] \\
& - \frac{8}{3\Delta_0^4}\hat\rho_D^3(m_b - v\cdot q)[q^2 - (v\cdot q)^2]\Bigg\}.
\end{aligned} \tag{B.26}
$$

- Tensor operator in $H_{\rm eff}$

  This case amounts to choosing $i = j = 4$ in eq. (3.2). The corresponding tensor $T_T$ can be expanded as:

$$
\begin{aligned}
&T_T^{\mu'\nu'\mu\nu} = i\,\epsilon^{\mu\nu\mu'\nu'}\left[T_{T0}-(q\cdot s)S_{T0}\right]+\left[g^{\mu\mu'}g^{\nu\nu'}-g^{\mu\nu'}g^{\nu\mu'}\right]\left[T_{T1}-(q\cdot s)S_{T1}\right]\\
&+\Big\{-g^{\mu\mu'}\Big[v^{\nu}v^{\nu'}\left[T_{T2}-(q\cdot s)S_{T2}\right]\\
&-i\,\epsilon^{\nu\nu'\alpha\beta}v_\alpha q_\beta\left[T_{T3}-(q\cdot s)S_{T3}\right]+q^{\nu}q^{\nu'}\left[T_{T4}-(q\cdot s)S_{T4}\right]\\
&+(q^{\nu}v^{\nu'}+q^{\nu'}v^{\nu})\left[T_{T5}-(q\cdot s)S_{T5}\right]\Big]+\left(\mu\leftrightarrow\nu\wedge\mu'\leftrightarrow\nu'\right)-\left(\mu\leftrightarrow\nu\right)-\left(\mu'\leftrightarrow\nu'\right)\Big\}\\
&+\Big\{i\,v^{\mu}\epsilon^{\alpha\nu\mu'\nu'}v_\alpha\left[T_{T6}-(q\cdot s)S_{T6}\right]+i\,q^{\mu}\epsilon^{\alpha\nu\mu'\nu'}v_\alpha\left[T_{T7}-(q\cdot s)S_{T7}\right]\\
&+i\,v^{\mu}\epsilon^{\alpha\nu\mu'\nu'}q_\alpha\left[T_{T8}-(q\cdot s)S_{T8}\right]+i\,q^{\mu}\epsilon^{\alpha\nu\mu'\nu'}q_\alpha\left[T_{T9}-(q\cdot s)S_{T9}\right]-\left(\mu\leftrightarrow\nu\right)\Big\}\\
&+\Big\{i\,v^{\mu'}\epsilon^{\alpha\nu'\mu\nu}v_\alpha\left[T_{T10}-(q\cdot s)S_{T10}\right]+i\,q^{\mu'}\epsilon^{\alpha\nu'\mu\nu}v_\alpha\left[T_{T11}-(q\cdot s)S_{T11}\right]\\
&+i\,v^{\mu'}\epsilon^{\alpha\nu'\mu\nu}q_\alpha\left[T_{T12}-(q\cdot s)S_{T12}\right]+i\,q^{\mu'}\epsilon^{\alpha\nu'\mu\nu}q_\alpha\left[T_{T13}-(q\cdot s)S_{T13}\right]-\left(\mu'\leftrightarrow\nu'\right)\Big\}\\
&+\Big\{-g^{\mu\mu'}\Big[(s^{\nu}v^{\nu'}+s^{\nu'}v^{\nu})\,S_{T14}+(q^{\nu}s^{\nu'}+q^{\nu'}s^{\nu})\,S_{T15}+i\,\epsilon^{\nu\nu'\alpha\beta}v_\alpha s_\beta\,S_{T16}\\
&+i\,\epsilon^{\nu\nu'\alpha\beta}q_\alpha s_\beta\,S_{T17}+(s^{\nu}v^{\nu'}-s^{\nu'}v^{\nu})\,S_{T18}+(q^{\nu}s^{\nu'}-q^{\nu'}s^{\nu})\,S_{T19}\\
&+iv^{\nu}\,\epsilon^{\nu'\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T20A}+iv^{\nu'}\,\epsilon^{\nu\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T20B}\\
&+iq^{\nu}\,\epsilon^{\nu'\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T21A}+iq^{\nu'}\,\epsilon^{\nu\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T21B}\Big]\\
&+v^{\mu}v^{\mu'}\Big[(q\cdot s)\,i\,\epsilon^{\nu\nu'\alpha\beta}q_\alpha v_\beta\,S_{T22}+i\,\epsilon^{\nu\nu'\alpha\beta}v_\alpha s_\beta\,S_{T23}+i\,\epsilon^{\nu\nu'\alpha\beta}q_\alpha s_\beta\,S_{T24}\\
&+iq^{\nu}\,\epsilon^{\nu'\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T25A}+iq^{\nu'}\,\epsilon^{\nu\alpha\beta\delta}q_\alpha v_\beta s_\delta\,S_{T25B}\Big]\\
&-i\,\epsilon^{\mu\mu'\alpha\beta}v_\alpha q_\beta\,v^{\nu}s^{\nu'}\,S_{T26}+q^{\mu}v^{\mu'}\left[i\,\epsilon^{\nu\nu'\alpha\beta}v_\alpha s_\beta\,S_{T27A}\right]+q^{\mu'}v^{\mu}\left[i\,\epsilon^{\nu\nu'\alpha\beta}v_\alpha s_\beta\,S_{T27B}\right]\\
&+\left(\mu\leftrightarrow\nu\wedge\mu'\leftrightarrow\nu'\right)-\left(\mu\leftrightarrow\nu\right)-\left(\mu'\leftrightarrow\nu'\right)\Big\}\\
&+i\,(q^{\mu}v^{\nu}-q^{\nu}v^{\mu})\epsilon^{\mu'\nu'\alpha\beta}v_\alpha q_\beta\,T_{T14}+i\,(q^{\mu'}v^{\nu'}-q^{\nu'}v^{\mu'})\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta\,T_{T15}\\
&+i\,(q^{\mu}v^{\nu}-q^{\nu}v^{\mu})\epsilon^{\mu'\nu'\alpha\beta}v_\alpha s_\beta\,S_{T28}+i\,(q^{\mu'}v^{\nu'}-q^{\nu'}v^{\mu'})\epsilon^{\mu\nu\alpha\beta}v_\alpha s_\beta\,S_{T29}\\
&+\Big\{i\,s^{\mu}\epsilon^{\alpha\nu\mu'\nu'}v_\alpha\,S_{T30}+i\,s^{\mu}\epsilon^{\alpha\nu\mu'\nu'}q_\alpha\,S_{T31}+i\,v^{\mu}\epsilon^{\alpha\nu\mu'\nu'}s_\alpha\,S_{T32}+i\,q^{\mu}\epsilon^{\alpha\nu\mu'\nu'}s_\alpha\,S_{T33}\\
&-\left(\mu\leftrightarrow\nu\right)\Big\}\\
&+\Big\{i\,s^{\mu'}\epsilon^{\alpha\nu'\mu\nu}v_\alpha\,S_{T34}+i\,s^{\mu'}\epsilon^{\alpha\nu'\mu\nu}q_\alpha\,S_{T35}+i\,v^{\mu'}\epsilon^{\alpha\nu'\mu\nu}s_\alpha\,S_{T36}\\
&+i\,q^{\mu'}\epsilon^{\alpha\nu'\mu\nu}s_\alpha\,S_{T37}-\left(\mu'\leftrightarrow\nu'\right)\Big\}
\end{aligned}
\tag{B.27}
$$

JHEP11(2020)032

The various functions are given by:

$$\begin{aligned}
T_{T0} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\Big[-2(m_b - v\cdot q) - \frac{5}{3m_b}\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right) + \frac{4}{3m_b^2}\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&+ \frac{2}{3m_b\Delta_0^2}\Big[[-2q^2-3m_b v\cdot q+5(v\cdot q)^2]\hat{\mu}_\pi^2 + [-3m_b^2+2q^2+6m_b v\cdot q-5(v\cdot q)^2]\hat{\mu}_G^2 \\
&+ \frac{-7m_b(m_b-v\cdot q)+q^2-4(v\cdot q)^2}{m_b}\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)+3m_b\hat{\rho}_{LS}^3\Big] \\
&+ \frac{8}{3m_b\Delta_0^3}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\Big[m_b\hat{\mu}_\pi^2-\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&+ \frac{16}{3\Delta_0^4}(m_b-v\cdot q)^2[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned} \tag{B.28}$$

$$\begin{aligned}
S_{T0} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\Big[1-\frac{(7\hat{\mu}_\pi^2-9\hat{\mu}_G^2)}{12m_b^2}+\frac{\hat{\rho}_D^3}{6m_b^3}\Big] + \frac{2}{3m_b\Delta_0^2}\Big[5v\cdot q\,\hat{\mu}_\pi^2+3(m_b-v\cdot q)\hat{\mu}_G^2+\hat{\rho}_D^3\Big] \\
&+ \frac{8}{3m_b\Delta_0^3}\Big[-m_b[q^2-(v\cdot q)^2]\,\hat{\mu}_\pi^2+v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3\Big] \\
&- \frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned} \tag{B.29}$$

$$\begin{aligned}
T_{T1} = 2m_H \Bigg\{ & \frac{1}{\Delta_0}\Big[2(m_b - v\cdot q) + \frac{5}{3m_b}\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right) - \frac{4}{3m_b^2}\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&+ \frac{2}{3m_b\Delta_0^2}\Big[[2q^2+(3m_b-5v\cdot q)v\cdot q]\hat{\mu}_\pi^2 + [4m_b^2-2q^2-7m_b v\cdot q+5(v\cdot q)^2]\hat{\mu}_G^2 \\
&+4m_b\hat{\rho}_D^3+\frac{4(m_b-v\cdot q)^2-q^2}{m_b}\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&- \frac{8}{3m_b\Delta_0^3}[q^2-(v\cdot q)^2]\Big[(m_b-v\cdot q)\left(m_b\hat{\mu}_\pi^2-\hat{\rho}_D^3\right)-(2m_b-v\cdot q)\hat{\rho}_{LS}^3\Big] \\
&- \frac{16}{3\Delta_0^4}(m_b-v\cdot q)^2[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned} \tag{B.30}$$

$$\begin{aligned}
T_{T2} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\Big[2m_b + \frac{5}{3m_b}\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right) - \frac{4}{3m_b^2}\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&+ \frac{4}{3m_b\Delta_0^2}\Big[7m_b(v\cdot q)\hat{\mu}_\pi^2+m_b(4m_b-5v\cdot q)\hat{\mu}_G^2 \\
&+2(4m_b-3v\cdot q)\hat{\rho}_D^3+2(2m_b-3v\cdot q)\hat{\rho}_{LS}^3\Big] \\
&+ \frac{8}{3\Delta_0^3}\Big[-2m_b[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2+4v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3 \\
&+[q^2+2v\cdot q(m_b-v\cdot q)]\rho_{LS}^3\Big] \\
&- \frac{32}{3\Delta_0^4}m_b(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned} \tag{B.31}$$

$$\begin{aligned}
T_{T3} = 2m_H \Bigg\{ & -\frac{2}{\Delta_0} - \frac{2}{3m_b^2\Delta_0^2}\Big[5m_b v\cdot q\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right)+6m_b^2\hat{\mu}_G^2 \\
&+2(3m_b-2v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
&+ \frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2-v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3+(m_b-v\cdot q)^2\rho_{LS}^3\Big] \\
&+ \frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned} \tag{B.32}$$

JHEP11(2020)032

$$T_{T4}=2m_H\bigg\{\frac{4}{3m_b^2\Delta_0^2}\Big[2m_b\left(\hat\mu_\pi^2-\hat\mu_G^2\right)-\left(\hat\rho_D^3+\hat\rho_{LS}^3\right)\Big]$$

$$+\frac{8}{3m_b\Delta_0^3}\Big[2(m_b-v\cdot q)\hat\rho_D^3+(3m_b-2v\cdot q)\rho_{LS}^3\Big]\bigg\} \tag{B.33}$$

$$T_{T5}=2m_H\bigg\{-\frac{2}{\Delta_0}-\frac{2}{3m_b^2\Delta_0^2}\Big[m_b(4m_b+5v\cdot q)\hat\mu_\pi^2+m_b(2m_b-5v\cdot q)\hat\mu_G^2$$

$$+2(m_b-2v\cdot q)\left(\hat\rho_D^3+\hat\rho_{LS}^3\right)\Big]$$

$$+\frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2+[-2m_b^2+m_bv\cdot q+(v\cdot q)^2]\hat\rho_D^3$$

$$+[-m_b^2-m_bv\cdot q+(v\cdot q)^2]\hat\rho_{LS}^3\Big] \tag{B.34}$$

$$+\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat\rho_D^3\bigg\}$$

and

$$S_{T1}=2m_H\bigg\{-\frac{2}{\Delta_0}\Big[1-\frac{(7\hat\mu_\pi^2-9\hat\mu_G^2)}{12m_b^2}+\frac{\hat\rho_D^3}{6m_b^3}\Big]$$

$$+\frac{2}{3m_b\Delta_0^2}\Big[-v\cdot q\left(5\hat\mu_\pi^2-3\hat\mu_G^2\right)+2\left(2\hat\rho_D^3+3\hat\rho_{LS}^3\right)\Big]$$

$$+\frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat\mu_\pi^2-v\cdot q(m_b-v\cdot q)\hat\rho_D^3\Big] \tag{B.35}$$

$$+\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat\rho_D^3\bigg\}$$

$$S_{T2}=2m_H\bigg\{\frac{2}{3m_b\Delta_0^2}\Big[2m_b\left(2\hat\mu_\pi^2+3\hat\mu_G^2\right)+8\hat\rho_D^3+9\hat\rho_{LS}^3\Big]$$

$$+\frac{8}{3\Delta_0^3}\Big[2(m_b-v\cdot q)\hat\rho_D^3+3v\cdot q\hat\rho_{LS}^3\Big]\bigg\} \tag{B.36}$$

$$S_{T3}=2m_H\bigg\{-\frac{2}{3m_b^2\Delta_0^2}\left[2m_b\hat\mu_\pi^2+\hat\rho_D^3\right]-\frac{4}{3m_b\Delta_0^3}\Big[2(m_b-v\cdot q)\hat\rho_D^3-3m_b\hat\rho_{LS}^3\Big]\bigg\} \tag{B.37}$$

with $S_{T4}=0$,

$$S_{T5}=2m_H\left\{-\frac{2}{3m_b^2\Delta_0^2}\left[2m_b\hat\mu_\pi^2+\hat\rho_D^3\right]-\frac{4}{3m_b\Delta_0^3}\left[2(m_b-v\cdot q)\hat\rho_D^3+3m_b\hat\rho_{LS}^3\right]\right\} \tag{B.38}$$

$$T_{T6}=2m_H\bigg\{\frac{4m_b}{\Delta_0}\Big[1+\frac{5}{6m_b^2}\left(\hat\mu_\pi^2-\hat\mu_G^2\right)-\frac{2}{3m_b^3}\left(\hat\rho_D^3+\hat\rho_{LS}^3\right)$$

$$+\frac{2}{3m_b\Delta_0^2}\Big[14m_b(v\cdot q)\hat\mu_\pi^2+m_b(7m_b-10v\cdot q)\hat\mu_G^2$$

$$+3(5m_b-4v\cdot q)\hat\rho_D^3+4(2m_b-3v\cdot q)\hat\rho_{LS}^3\Big] \tag{B.39}$$

$$-\frac{16}{3\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\,\hat\mu_\pi^2-v\cdot q\,(m_b-v\cdot q)\left(2\hat\rho_D^3+\hat\rho_{LS}^3\right)\Big]$$

$$-\frac{32}{3\Delta_0^4}m_b(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat\rho_D^3\bigg\}$$

$$S_{T6}=2m_H\left\{\frac{8}{3\Delta_0^2}\hat{\mu}_\pi^2+\frac{16}{3\Delta_0^3}(m_b-v\cdot q)\hat{\rho}_D^3\right\} \tag{B.40}$$

$$\begin{aligned}T_{T7}=2m_H\Bigg\{&-\frac{2}{\Delta_0}\\&-\frac{2}{3m_b^2\Delta_0^2}\Big[m_b(4m_b+5v\cdot q)\hat{\mu}_\pi^2+m_b(2m_b-5v\cdot q)\hat{\mu}_G^2+2(m_b-2v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big]\\&+\frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2+[-2m_b^2+m_bv\cdot q+(v\cdot q)^2]\hat{\rho}_D^3-[m_b^2-(v\cdot q)^2]\hat{\rho}_{LS}^3\Big]\\&+\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}\end{aligned} \tag{B.41}$$

$$S_{T7}=S_{T8}=-S_{T11}=S_{T12}=2m_H\left\{-\frac{2}{3m_b^2\Delta_0^2}\left(2m_b\hat{\mu}_\pi^2+\hat{\rho}_D^3\right)-\frac{8}{3m_b\Delta_0^3}(m_b-v\cdot q)\hat{\rho}_D^3\right\} \tag{B.42}$$

$$\begin{aligned}T_{T8}=2m_H\Bigg\{&-\frac{2}{\Delta_0}\\&-\frac{2}{3m_b^2\Delta_0^2}\Big[m_b(4m_b+5v\cdot q)\hat{\mu}_\pi^2+m_b(m_b-5v\cdot q)\hat{\mu}_G^2+(m_b-4v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big]\\&+\frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2+[-2m_b^2+m_bv\cdot q+(v\cdot q)^2]\hat{\rho}_D^3-[m_b^2-(v\cdot q)^2]\hat{\rho}_{LS}^3\Big]\\&+\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}\end{aligned} \tag{B.43}$$

$$T_{T9}=2m_H\left\{\frac{4}{3m_b^2\Delta_0^2}\Big[2m_b\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right)-\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big]+\frac{16}{3m_b\Delta_0^3}(m_b-v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\right\} \tag{B.44}$$

$$T_{T10}=2m_H\left\{-\frac{2}{3\Delta_0^2}\left(m_b\hat{\mu}_G^2+\hat{\rho}_D^3\right)\right\} \tag{B.45}$$

$$\begin{aligned}T_{T11}=2m_H\Bigg\{&\frac{2}{\Delta_0}+\frac{2}{3m_b^2\Delta_0^2}\Big[5m_b(v\cdot q)\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right)+6m_b^2\hat{\mu}_G^2\\&+2(3m_b-2v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big]\\&-\frac{8}{3\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2-v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3+(m_b-v\cdot q)^2\hat{\rho}_{LS}^3\Big]\\&-\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}\end{aligned} \tag{B.46}$$

$$
\begin{aligned}
T_{T12} = 2m_H \Bigg\{ & -\frac{2}{\Delta_0} \\
& -\frac{2}{3m_b^2\Delta_0^2}\Big[5m_b v\cdot q\left(\hat{\mu}_\pi^2-\hat{\mu}_G^2\right)+5m_b^2\hat{\mu}_G^2+(5m_b-4v\cdot q)\left(\hat{\rho}_D^3+\hat{\rho}_{LS}^3\right)\Big] \\
& +\frac{8}{3m_b\Delta_0^3}\Big[m_b[q^2-(v\cdot q)^2]\,\hat{\mu}_\pi^2-v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3+(m_b-v\cdot q)^2\,\hat{\rho}_{LS}^3\Big] \\
& +\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\} \qquad \text{(B.47)}
\end{aligned}
$$

with $T_{T13} = S_{T9} = S_{T10} = S_{T13} = 0$,

$$
T_{T14} = -T_{T15} = -\frac{16m_H}{3\Delta_0^3}\hat{\rho}_{LS}^3 \qquad \text{(B.48)}
$$

$$
\begin{aligned}
S_{T14} = 2m_H \Bigg\{ & -\frac{2m_b}{\Delta_0}\Big[1+\frac{\left(\hat{\mu}_\pi^2+\hat{\mu}_G^2\right)}{4m_b^2}+\frac{\hat{\rho}_D^3}{6m_b^3}\Big] \\
& -\frac{1}{3\Delta_0^2}\Big[10v\cdot q\,\hat{\mu}_\pi^2+12(m_b-v\cdot q)\hat{\mu}_G^2+4\frac{(4m_b-3v\cdot q)}{m_b}\hat{\rho}_D^3-9\frac{v\cdot q}{m_b}\hat{\rho}_{LS}^3\Big] \\
& +\frac{4}{3\Delta_0^3}\Big[[q^2-(v\cdot q)^2](2m_b\hat{\mu}_\pi^2-3\hat{\rho}_{LS}^3)-2v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3\Big] \\
& +\frac{16}{3\Delta_0^4}m_b(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\} \qquad \text{(B.49)}
\end{aligned}
$$

$$
\begin{aligned}
S_{T15} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\Big[1-\frac{7\hat{\mu}_\pi^2-9\hat{\mu}_G^2}{12m_b^2}+\frac{\hat{\rho}_D^3}{6m_b^3}\Big] \\
& +\frac{1}{3m_b^2\Delta_0^2}\Big[2m_b(2m_b+3v\cdot q)\hat{\mu}_\pi^2+6m_b(m_b-v\cdot q)\hat{\mu}_G^2-2v\cdot q\,\hat{\rho}_D^3-3m_b\hat{\rho}_{LS}^3\Big] \\
& +\frac{8}{3\Delta_0^3}\Big[-[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2+(m_b-v\cdot q)\hat{\rho}_D^3\Big] \qquad \text{(B.50)} \\
& -\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
S_{T16} = 2m_H \Bigg\{ & -\frac{2m_b}{\Delta_0}\Big[1-\frac{\left(5\hat{\mu}_\pi^2-3\hat{\mu}_G^2\right)}{12m_b^2}-\frac{\hat{\rho}_D^3}{6m_b^3}\Big] \\
& +\frac{2}{3m_b\Delta_0^2}\Big[-5m_b v\cdot q\hat{\mu}_\pi^2-6m_b(m_b-v\cdot q)\hat{\mu}_G^2-2(3m_b-2v\cdot q)\hat{\rho}_D^3+\frac{9}{2}v\cdot q\,\hat{\rho}_{LS}^3\Big] \\
& +\frac{4}{3\Delta_0^3}\Big[[q^2-(v\cdot q)^2](2m_b\hat{\mu}_\pi^2-3\hat{\rho}_{LS}^3)-2v\cdot q(m_b-v\cdot q)\hat{\rho}_D^3\Big] \\
& +\frac{16}{3\Delta_0^4}m_b(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\Bigg\} \qquad \text{(B.51)}
\end{aligned}
$$



JHEP11(2020)032

$$S_{T17}=2m_H\left\{\frac{2}{\Delta_0}\left[1-\frac{\left(7\hat{\mu}_\pi^2-9\hat{\mu}_G^2\right)}{12m_b^2}+\frac{\hat{\rho}_D^3}{6m_b^3}\right]\right.$$
$$+\frac{2}{3m_b^2\Delta_0^2}\left[m_b(2m_b+3v\cdot q)\hat{\mu}_\pi^2+3m_b(m_b-v\cdot q)\hat{\mu}_G^2-v\cdot q\hat{\rho}_D^3-\frac{3}{2}m_b\hat{\rho}_{LS}^3\right]$$
$$\left.-\frac{8}{3\Delta_0^3}\left[[q^2-(v\cdot q)^2]\hat{\mu}_\pi^2-(m_b-v\cdot q)\hat{\rho}_D^3\right]-\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\right\} \tag{B.52}$$

$$S_{T22}=-16m_H\frac{1}{\Delta_0^3}\hat{\rho}_{LS}^3 \tag{B.53}$$

$$S_{T23}=2m_H\left\{-\frac{4}{\Delta_0^2}\left(m_b\hat{\mu}_G^2+\hat{\rho}_D^3\right)-\frac{8}{\Delta_0^3}q^2\hat{\rho}_{LS}^3\right\} \tag{B.54}$$

$$S_{T24}=16m_H\frac{1}{\Delta_0^3}(v\cdot q)\hat{\rho}_{LS}^3 \tag{B.55}$$

with $S_{T18} = S_{T19} = S_{T20A} = S_{T20B} = S_{T21A} = S_{T21B} = S_{T25A} = S_{T25B} = S_{T26} = 0$,

$$S_{T27A} = S_{T27B} = \frac{4m_H}{\Delta_0^2}\left\{\hat{\mu}_G^2 + \frac{(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3)}{m_b}\right\} \tag{B.56}$$

$$S_{T28} = -S_{T29} = \frac{4m_H}{3m_b\Delta_0^2}\left\{3m_b\hat{\mu}_G^2 + 5\hat{\rho}_D^3 + 6\hat{\rho}_{LS}^3\right\} \tag{B.57}$$

$$S_{T30} = S_{T32} = 2m_H\left\{-\frac{2m_b}{\Delta_0}\left[1+\frac{1}{4m_b^2}\left(\hat{\mu}_\pi^2+\hat{\mu}_G^2\right)+\frac{1}{6m_b^3}\hat{\rho}_D^3\right]\right.$$
$$-\frac{2}{3m_b\Delta_0^2}\left[5m_b v\cdot q\hat{\mu}_\pi^2 + 3m_b(m_b - v\cdot q)\hat{\mu}_G^2 + (5m_b - 2v\cdot q)\hat{\rho}_D^3\right]$$
$$+\frac{8}{3\Delta_0^3}\left[m_b[q^2-(v\cdot q)^2]\,\hat{\mu}_\pi^2 - v\cdot q(m_b - v\cdot q)\hat{\rho}_D^3\right] \tag{B.58}$$
$$\left.+\frac{16}{3\Delta_0^4}m_b(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\right\}$$

$$S_{T31} = S_{T33} = S_{T35} = -S_{T37} = 2m_H\left\{\frac{2}{\Delta_0}\left[1-\frac{1}{12m_b^2}\left(7\hat{\mu}_\pi^2-9\hat{\mu}_G^2\right)+\frac{1}{6m_b^3}\hat{\rho}_D^3\right]\right.$$
$$+\frac{2}{3m_b^2\Delta_0^2}\left[m_b(2m_b+3v\cdot q)\hat{\mu}_\pi^2+3m_b(m_b-v\cdot q)\hat{\mu}_G^2+(m_b-v\cdot q)\hat{\rho}_D^3\right]$$
$$-\frac{8}{3\Delta_0^3}\left[[q^2-(v\cdot q)^2]\,\hat{\mu}_\pi^2-(m_b-v\cdot q)\hat{\rho}_D^3\right] \tag{B.59}$$
$$\left.-\frac{16}{3\Delta_0^4}(m_b-v\cdot q)[q^2-(v\cdot q)^2]\hat{\rho}_D^3\right\}$$

$$S_{T34} = -S_{T36} = 2m_H \Bigg\{ -\frac{2m_b}{\Delta_0}\Big[1 - \frac{1}{12m_b^2}\left(5\hat{\mu}_\pi^2 - 3\hat{\mu}_G^2\right) - \frac{1}{6m_b^3}\hat{\rho}_D^3\Big]$$
$$- \frac{2}{3m_b\Delta_0^2}\Big[5m_b v\cdot q\hat{\mu}_\pi^2 + 3m_b(m_b - v\cdot q)\hat{\mu}_G^2 + 3m_b\hat{\rho}_D^3\Big]$$
$$+ \frac{8}{3\Delta_0^3}\Big[m_b[q^2 - (v\cdot q)^2]\,\hat{\mu}_\pi^2 - v\cdot q(m_b - v\cdot q)\hat{\rho}_D^3\Big] \tag{B.60}$$
$$+ \frac{16}{3\Delta_0^4} m_b(m_b - v\cdot q)[q^2 - (v\cdot q)^2]\hat{\rho}_D^3 \Bigg\}$$

- Interference between the SM and the tensor operators in $H_{\rm eff}$

  The tensor is obtained for $(i,j) = (1,4)$ and $(i,j) = (4,1)$ in eq. (3.2). We denote the two contributions with $T_{SMT}$ and $T_{TSM}$, respectively. We write:

$$T^{\alpha\mu\nu}_{\rm SMT} = i\,(g^{\alpha\mu}\,v^\nu - g^{\alpha\nu}\,v^\mu)\,T_{{\rm SMT},1} + i\,(g^{\alpha\mu}\,q^\nu - g^{\alpha\nu}\,q^\mu)\,T_{{\rm SMT},2} + \epsilon^{\alpha\mu\nu\beta}\,v_\beta\,T_{{\rm SMT},3}$$
$$+ \epsilon^{\alpha\mu\nu\beta}\,q_\beta\,T_{{\rm SMT},4} + i\,v^\alpha\,(q^\mu\,v^\nu - q^\nu\,v^\mu)\,T_{{\rm SMT},5} + i\,q^\alpha\,(q^\mu\,v^\nu - q^\nu\,v^\mu)\,T_{{\rm SMT},6}$$
$$+ v^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,q_\delta\,T_{{\rm SMT},7} + q^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,q_\delta\,T_{{\rm SMT},8}$$
$$-(q\cdot s)\Big[i\,(g^{\alpha\mu}\,v^\nu - g^{\alpha\mu}\,v^\mu)\,S_{{\rm SMT},1} + i\,(g^{\alpha\mu}\,q^\nu - g^{\alpha\mu}\,q^\mu)\,S_{{\rm SMT},2} + \epsilon^{\alpha\mu\nu\beta}\,v_\beta\,S_{{\rm SMT},3}$$
$$+ \epsilon^{\alpha\mu\nu\beta}\,q_\beta\,S_{{\rm SMT},4} + i\,v^\alpha\,(q^\mu\,v^\nu - q^\nu\,v^\mu)\,S_{{\rm SMT},5} + i\,q^\alpha\,(q^\mu\,v^\nu - q^\nu\,v^\mu)\,S_{{\rm SMT},6}$$
$$+ v^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,q_\delta\,S_{{\rm SMT},7} + q^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,q_\delta\,S_{{\rm SMT},8}\Big]$$
$$+ i\,(g^{\alpha\mu}\,s^\nu - g^{\alpha\mu}\,s^\mu)\,S_{{\rm SMT},9} + \epsilon^{\alpha\mu\nu\beta}\,s_\beta\,S_{{\rm SMT},10} + i\,s^\alpha\,(q^\mu\,v^\nu - q^\nu\,v^\mu)\,S_{{\rm SMT},11}$$
$$+ i\,v^\alpha\,(v^\mu\,s^\nu - v^\nu\,s^\mu)\,S_{{\rm SMT},12} + i\,v^\alpha\,(q^\mu\,s^\nu - q^\nu\,s^\mu)\,S_{{\rm SMT},13}$$
$$+ i\,q^\alpha\,(v^\mu\,s^\nu - v^\nu\,s^\mu)\,S_{{\rm SMT},14} + i\,q^\alpha\,(q^\mu\,s^\nu - q^\nu\,s^\mu)\,S_{{\rm SMT},15}$$
$$+ v^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,s_\delta\,S_{{\rm SMT},16} + v^\alpha\,\epsilon^{\mu\nu\beta\delta}\,q_\beta\,s_\delta\,S_{{\rm SMT},17}$$
$$+ q^\alpha\,\epsilon^{\mu\nu\beta\delta}\,v_\beta\,s_\delta\,S_{{\rm SMT},18} + q^\alpha\,\epsilon^{\mu\nu\beta\delta}\,q_\beta\,s_\delta\,S_{{\rm SMT},19}$$
$$+ (g^{\alpha\mu}\,\epsilon^{\nu\beta\delta\tau}\,q_\beta\,v_\delta\,s_\tau - g^{\alpha\nu}\,\epsilon^{\mu\beta\delta\tau}\,q_\beta\,v_\delta\,s_\tau)\,S_{{\rm SMT},20}$$
$$+ (v^\mu\,\epsilon^{\alpha\nu\beta\delta}\,v_\beta\,s_\delta - v^\nu\,\epsilon^{\alpha\mu\beta\delta}\,v_\beta\,s_\delta)\,S_{{\rm SMT},21} + (v^\mu\,\epsilon^{\alpha\nu\beta\delta}\,q_\beta\,s_\delta - v^\nu\,\epsilon^{\alpha\mu\beta\delta}\,q_\beta\,s_\delta)\,S_{{\rm SMT},22}$$
$$+ (q^\mu\,\epsilon^{\alpha\nu\beta\delta}\,v_\beta\,s_\delta - q^\nu\,\epsilon^{\alpha\mu\beta\delta}\,v_\beta\,s_\delta)\,S_{{\rm SMT},23} + (q^\mu\,\epsilon^{\alpha\nu\beta\delta}\,q_\beta\,s_\delta - q^\nu\,\epsilon^{\alpha\mu\beta\delta}\,q_\beta\,s_\delta)\,S_{{\rm SMT},24}$$
$$+ s^\alpha\,\epsilon^{\mu\nu\beta\delta}\,q_\beta\,v_\delta\,S_{{\rm SMT},25} + (q^\mu v^\nu - q^\nu v^\mu)\,\epsilon_{\alpha\beta\delta\theta} q^\beta v^\delta s^\theta\,S_{SMT,26} \tag{B.61}$$

  The results for the various functions read:

$$T_{{\rm SMT},1} = -T_{{\rm SMT},3}$$
$$= 2m_H \Bigg\{\frac{2m_U}{\Delta_0} + \frac{2m_U}{3m_b^2\,\Delta_0^2}\Big[5m_b\,v\cdot q\,(\hat{\mu}_\pi^2 - \hat{\mu}_G^2)$$
$$+ 4\,m_b^2\,\hat{\mu}_G^2 + 4\,(m_b - v\cdot q)\,(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3)\Big]$$
$$- \frac{8m_U}{3m_b\Delta_0^3}\Big[m_b\,[q^2 - (q\cdot v)^2]\,\hat{\mu}_\pi^2 - (m_b - v\cdot q)\,v\cdot q\,(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3)\Big] \tag{B.62}$$
$$- \frac{16m_U}{3\Delta_0^4}\,(m_b - v\cdot q)\,[q^2 - (v\cdot q)^2]\,\hat{\rho}_D^3\Bigg\}$$

$$T_{\mathrm{SMT},2} = -T_{\mathrm{SMT},4} = 2m_H\Bigg\{ -\frac{2m_U}{3m_b^2\,\Delta_0^2}\left[2m_b\,(\hat\mu_\pi^2-\hat\mu_G^2)-(\hat\rho_D^3+\hat\rho_{LS}^3)\right] - \frac{8m_U}{3m_b\Delta_0^3}\,(m_b - v\cdot q)\,(\hat\rho_D^3+\hat\rho_{LS}^3)\Bigg\} \tag{B.63}$$

$$T_{\mathrm{SMT},5} = T_{\mathrm{SMT},7} = 2m_H\left\{\frac{8m_U}{3\Delta_0^3}\,\hat\rho_{LS}^3\right\} \tag{B.64}$$

with $T_{\mathrm{SMT},6} = T_{\mathrm{SMT},8} = 0$,

$$S_{\mathrm{SMT},1} = -S_{\mathrm{SMT},3} = 2m_H\left\{\frac{2m_U}{3m_b^2\,\Delta_0^2}\,(2\,m_b\,\hat\mu_\pi^2+\hat\rho_D^3) + \frac{8m_U}{3m_b\Delta_0^3}\,(m_b - q\cdot v)\,\hat\rho_D^3\right\} \tag{B.65}$$

$$S_{\mathrm{SMT},9} = -S_{\mathrm{SMT},10} = 2m_H\Bigg\{ -\frac{2m_U}{\Delta_0}\left[1-\frac{7\hat\mu_\pi^2-9\hat\mu_G^2}{12m_b^2}+\frac{\hat\rho_D^3}{6m_b^3}\right] - \frac{2m_U}{3m_b^2\,\Delta_0^2}\left[3\,m_b\,v\cdot q\,(\hat\mu_\pi^2-\hat\mu_G^2)+6\,m_b^2\,\hat\mu_G^2+(4m_b-v\cdot q)\,\hat\rho_D^3+3\,m_b\,\hat\rho_{LS}^3\right] + \frac{8m_U}{3\Delta_0^3}\,[q^2-(v\cdot q)^2]\,\hat\mu_\pi^2 + \frac{16m_U}{3\Delta_0^4}\,(m_b-v\cdot q)\,[q^2-(v\cdot q)^2]\,\hat\rho_D^3\Bigg\} \tag{B.66}$$

$$S_{\mathrm{SMT},11} = 2m_H\left\{-\frac{4m_U}{\Delta_0^3}\,\hat\rho_{LS}^3\right\} \tag{B.67}$$

$$S_{\mathrm{SMT},12} = 2m_H\left\{\frac{4m_U}{3m_b\,\Delta_0^2}\left[3\,m_b\,\hat\mu_G^2+4\,\hat\rho_D^3+\frac{9}{2}\,\hat\rho_{LS}^3\right]+\frac{8m_U}{\Delta_0^3}\,v\cdot q\,\hat\rho_{LS}^3\right\} \tag{B.68}$$

$$S_{\mathrm{SMT},16} = 2m_H\left\{-\frac{2m_U}{3m_b\,\Delta_0^2}\left[6m_b\,\hat\mu_G^2+8\,\hat\rho_D^3+9\,\hat\rho_{LS}^3\right]-\frac{8m_U}{\Delta_0^3}\,v\cdot q\,\hat\rho_{LS}^3\right\}. \tag{B.69}$$

Several functions vanish: $S_{\mathrm{SMT},(2,4,5,6,7,8)} = 0$ and $S_{\mathrm{SMT},(15,19,20,21,22,23,24,26)} = 0$. The relations hold: $S_{\mathrm{SMT},13} = S_{\mathrm{SMT},14} = -\frac{1}{3}S_{\mathrm{SMT},17} = -S_{\mathrm{SMT},18} = S_{\mathrm{SMT},25} = S_{\mathrm{SMT},11}$ and

$$\begin{aligned} T_{\mathrm{TSM},i} &= -T_{\mathrm{SMT},i} && (i = 1,2,5)\\ T_{\mathrm{TSM},i} &= T_{\mathrm{SMT},i} && (i = 3,4,7)\\ S_{\mathrm{TSM},i} &= -S_{\mathrm{SMT},i} && (i = 1,9,11,12,13,14)\\ S_{\mathrm{TSM},i} &= S_{\mathrm{SMT},i} && (i = 3,10,16,17,18,25)\,. \end{aligned} \tag{B.70}$$

- Interference between the scalar and tensor operators in $H_{\mathrm{eff}}$

The tensor is obtained for $(i,j) = (2,4)$ and $(i,j) = (4,2)$ in eq. (3.2). We denote the two contributions as $T^{\mu\nu}_{ST}$ and $T^{\mu\nu}_{TS}$, respectively. Writing

$$
\begin{aligned}
T^{\mu\nu}_{ST} = {} & \epsilon^{\mu\nu\alpha\beta}\, v_\alpha\, q_\beta\, T_{ST,1} + i\,(q^\mu\, v^\nu - q^\nu\, v^\mu)\, T_{ST,2} \\
& - (q\cdot s)\left[\epsilon^{\mu\nu\alpha\beta}\, v_\alpha\, q_\beta\, S_{ST,1} + i\,(q^\mu\, v^\nu - q^\nu\, v^\mu)\, S_{ST,2}\right] \\
& + \epsilon^{\mu\nu\alpha\beta}\, q_\alpha\, s_\beta\, S_{ST,3} + i\,(q^\mu\, s^\nu - q^\nu\, s^\mu)\, S_{ST,4} \\
& + \epsilon^{\mu\nu\alpha\beta}\, v_\alpha\, s_\beta\, S_{ST,5} + i\,(s^\mu\, v^\nu - v^\nu\, s^\mu)\, S_{ST,6} \\
& + \left[q^\mu\, \epsilon^{\nu\alpha\beta\delta} q_\alpha v_\beta s_\delta - q^\nu\, \epsilon^{\mu\alpha\beta\delta} q_\alpha v_\beta s_\delta\right] S_{ST,7} \\
& + \left[v^\mu\, \epsilon^{\nu\alpha\beta\delta} q_\alpha v_\beta s_\delta - v^\nu\, \epsilon^{\mu\alpha\beta\delta} q_\alpha v_\beta s_\delta\right] S_{ST,8}
\end{aligned}
\tag{B.71}
$$

we obtain:

$$
\begin{aligned}
T_{ST,1} = T_{ST,2} = 2\, m_H \Bigg\{ & -\frac{1}{\Delta_0} - \frac{1}{3m_b^2\,\Delta_0^2}\Big[5\, m_b\, v\cdot q\,(\hat\mu_\pi^2 - \hat\mu_G^2) + 4\, m_b^2 \hat\mu_G^2 \\
& + 4\,(m_b - v\cdot q)\,(\hat\rho_D^3 + \hat\rho_{LS}^3)\Big] \\
& + \frac{4}{3m_b\Delta_0^3}\Big[[q^2 - (v\cdot q)^2]\,\hat\mu_\pi^2 - (m_b - v\cdot q)\, v\cdot q\,\hat\rho_D^3 \\
& + [(m_b - v\cdot q)^2 + m_b\, m_U]\,\hat\rho_{LS}^3\Big] + \frac{8}{3\Delta_0^4}\,(m_b - v\cdot q)\,[q^2 - (v\cdot q)^2]\,\hat\rho_D^3 \Bigg\}
\end{aligned}
\tag{B.72}
$$

$$
S_{ST,1} = S_{ST,2} = 2\, m_H \Bigg\{ -\frac{1}{3m_b^2\,\Delta_0^2}\,(2\, m_b\, \hat\mu_\pi^2 + \hat\rho_D^3) - \frac{2}{3m_b\Delta_0^3}\Big[2(m_b - v\cdot q)\,\hat\rho_D^3 + 3\, m_b\,\hat\rho_{LS}^3\Big]\Bigg\}
\tag{B.73}
$$

$$
\begin{aligned}
S_{ST,3} = -S_{ST,4} = 2\, m_H \Bigg\{ & -\frac{1}{\Delta_0}\Big[1 - \frac{7\,\hat\mu_\pi^2 - 9\hat\mu_G^2}{12 m_b^2} + \frac{\hat\rho_D^3}{6 m_b^3}\Big] \\
& - \frac{1}{3m_b^2\,\Delta_0^2}\Big[m_b\,[2(m_b + m_U) + 3 v\cdot q]\,\hat\mu_\pi^2 + 3\, m_b\,(m_b - m_U - v\cdot q)\,\hat\mu_G^2 \\
& + [2(m_b - m_U) - v\cdot q]\,\hat\rho_D^3 + \frac{3}{2}(m_b - m_U)\,\hat\rho_{LS}^3\Big] \\
& + \frac{4}{3m_b\Delta_0^3}\Big[m_b\,[q^2 - (v\cdot q)^2]\,\hat\mu_\pi^2 - (m_b - v\cdot q)\,(m_b + m_U)\,\hat\rho_D^3 + \frac{3}{2}\, m_U\, v\cdot q\,\hat\rho_{LS}^3\Big] \\
& + \frac{8}{3\Delta_0^4}\,(m_b - v\cdot q)\,[q^2 - (v\cdot q)^2]\,\hat\rho_D^3 \Bigg\}
\end{aligned}
\tag{B.74}
$$

$$
\begin{aligned}
S_{ST,5} = S_{ST,6} = 2\, m_H \Bigg\{ & \frac{m_b + m_U}{\Delta_0}\Big[1 - \frac{5\,\hat\mu_\pi^2 - 3\hat\mu_G^2}{12 m_b^2} - \frac{\hat\rho_D^3}{6 m_b^3}\Big] \\
& + \frac{1}{3m_b^2\,\Delta_0^2}\Big[5\, m_b\,(m_b + m_U)\, v\cdot q\,\hat\mu_\pi^2 - 6\, m_b\, m_U\, v\cdot q\,\hat\mu_G^2 \\
& + (m_b - m_U)\, v\cdot q\,(4\,\hat\rho_D^3 + \frac{9}{2}\,\hat\rho_{LS}^3)\Big] \\
& - \frac{4}{3m_b\Delta_0^3}\Big(m_b\,(m_b + m_U)\,[q^2 - (v\cdot q)^2]\,\hat\mu_\pi^2 - (m_b - v\cdot q)\,(m_b + m_U)\, q\cdot v\,\hat\rho_D^3 \\
& + \frac{3}{2}\,(m_b\, q^2 - (m_b - m_U)\,(v\cdot q)^2)\,\hat\rho_{LS}^3\Big] \\
& - \frac{8}{3\Delta_0^4}\,(m_b + m_U)\,(m_b - v\cdot q)\,[q^2 - (q\cdot v)^2]\,\hat\rho_D^3 \Bigg\}
\end{aligned}
\tag{B.75}
$$

and $S_{ST,7} = S_{ST,8} = 0$. The relations hold:

$$\begin{aligned} T_{ST,i} &= T_{TS,i} \quad \text{and} \quad S_{ST,i} = S_{TS,i} \quad (i = 1,3,5,7,8) \\ T_{ST,i} &= -T_{TS,i} \quad \text{and} \quad S_{ST,i} = -S_{TS,i} \quad (i = 2,4,6). \end{aligned} \tag{B.76}$$

- Interference between the pseudoscalar and tensor operators in $H_{\text{eff}}$

  This case amounts to choosing $(i, j) = (3, 4)$ and $(4, 3)$ in eq. (3.2). We denote the two contributions with $T^{\mu\nu}_{PT}$ and $T^{\mu\nu}_{TP}$, respectively. We use the same expansion as for the scalar-tensor interference in eq. (B.71). The functions $T_{TP,i}$ and $S_{TP,i}$ are obtained from the corresponding ones in (B.71) substituting $T_{TP,i}(m_U) = -T_{TS,i}(-m_U)$ and $S_{TP,i}(m_U) = -S_{TS,i}(-m_U)$. Analogous relations hold between $T_{PT,i}$, $S_{PT,i}$, and $T_{TS,i}$, $S_{ST,i}$.

- Right-handed operator $O_R$ in $H_{\text{eff}}$

  This case amounts to choosing $i = j = 5$ in eq. (3.2). The corresponding tensor $T^{\mu\nu}_R$ can be expanded as in eq. (B.1), substituting $T_i \to T_{Ri}$ and $S_i \to S_{Ri}$. The following relations hold:

$$\begin{aligned} &T_{Ri} = T_i \quad (i = 1,2,4,5) && T_{R3} = -T_3 \\ &S_{Ri} = S_i \quad (i = 3,4,8,9,10,11,12,13) && S_{Ri} = -S_i \quad (i = 1,2,5,6,7) \end{aligned} \tag{B.77}$$

- Interference between the SM and the $O_R$ operators in $H_{\text{eff}}$

$$\begin{aligned} T_{SMR,1} = 2\, m_H\, m_U \Bigg\{ &-\frac{2}{\Delta_0}\left(1 - \frac{\hat\mu_\pi^2 - \hat\mu_G^2}{2m_b^2}\right) \\ &+ \frac{2}{3m_b\,\Delta_0^2}\Big[-3\, v\cdot q\, (\hat\mu_\pi^2 - \hat\mu_G^2) - 4\, m_b \hat\mu_G^2 - 2\,(\hat\rho_D^3 + \hat\rho_{LS}^3)\Big] \\ &+ \frac{8}{3\Delta_0^3}\,[q^2 - (v\cdot q)^2]\,\hat\mu_\pi^2 + \frac{16}{3\Delta_0^4}\,(m_b - v\cdot q)\,[q^2 - (v\cdot q)^2]\,\hat\rho_D^3 \Bigg\} \end{aligned} \tag{B.78}$$

$$T_{SMR,2} = -2\, m_H\, m_U \left\{ \frac{8}{3\,\Delta_0^2}\left(\hat\mu_G^2 + 2\,\frac{(\hat\rho_D^3 + \hat\rho_{LS}^3)}{m_b}\right) + \frac{16}{3\Delta_0^3}\, v\cdot q\, \hat\rho_{LS}^3 \right\} \tag{B.79}$$

$$T_{SMR,3} = T_{SMR,4} = 0 \tag{B.80}$$

$$T_{SMR,5} = \frac{16\, m_H\, m_U\, \hat\rho_{LS}^3}{3\,\Delta_0^3} \tag{B.81}$$

$$
\begin{aligned}
S_{SMR,8} = 2\, m_H\, m_U \Bigg\{ & \frac{2}{\Delta_0}\left(1 - \frac{5\hat{\mu}_\pi^2}{12 m_b^2} + \frac{\hat{\mu}_G^2}{4 m_b^2} - \frac{\hat{\rho}_D^3}{6 m_b^3}\right) \\
& + \frac{1}{3 m_b\, \Delta_0^2}\Bigg[10\, v\cdot q\, \hat{\mu}_\pi^2 + 12(m_b - v\cdot q)\, \hat{\mu}_G^2 \\
& + 4\,(3 m_b - 2 v\cdot q)\, \frac{\hat{\rho}_D^3}{m_b} + 3(4 m_b - 3 v\cdot q)\frac{\hat{\rho}_{LS}^3}{m_b}\Bigg] \\
& - \frac{8}{3\Delta_0^3}\left[[q^2 - (v\cdot q)^2]\, \hat{\mu}_\pi^2 - v\cdot q(m_b - v\cdot q)\frac{\hat{\rho}_D^3}{m_b} - 3\, v\cdot q(2 m_b - v\cdot q)\frac{\hat{\rho}_{LS}^3}{2 m_b}\right] \\
& - \frac{16}{3\Delta_0^4}\,(m_b - v\cdot q)\, [q^2 - (v\cdot q)^2]\, \hat{\rho}_D^3 \Bigg\}
\end{aligned}
\tag{B.82}
$$

$$
\begin{aligned}
S_{SMR,9} = -2\, m_H\, m_U \Bigg\{ & \frac{1}{3 m_b\, \Delta_0^2}\left(4\, \hat{\mu}_\pi^2 - 6\, \hat{\mu}_G^2 - 4\, \frac{\hat{\rho}_D^3}{m_b} - 3\, \frac{\hat{\rho}_{LS}^3}{m_b}\right) \\
& + \frac{8}{3 m_b \Delta_0^3}\left((m_b - v\cdot q)\, \hat{\rho}_D^3 + \frac{3}{2}\,(2 m_b - v\cdot q)\, \hat{\rho}_{LS}^3\right)\Bigg\}
\end{aligned}
\tag{B.83}
$$

$$
\begin{aligned}
S_{SMR,10} = 2\, m_H\, m_U \Bigg\{ & \frac{1}{\Delta_0}\left(2 - \frac{5\hat{\mu}_\pi^2}{6 m_b^2} + \frac{\hat{\mu}_G^2}{2 m_b^2} - \frac{\hat{\rho}_D^3}{3 m_b^3}\right) \\
& + \frac{1}{3 m_b\, \Delta_0^2}\left(10\, v\cdot q\, \hat{\mu}_\pi^2 - 12\, v\cdot q\, \hat{\mu}_G^2 - 4\,(m_b + 2 v\cdot q)\, \frac{\hat{\rho}_D^3}{m_b} - 9 v\cdot q\, \frac{\hat{\rho}_{LS}^3}{m_b}\right) \\
& - \frac{8}{3\Delta_0^3}\left([q^2 - (v\cdot q)^2]\, \hat{\mu}_\pi^2 - v\cdot q\,(m_b - v\cdot q)\frac{\hat{\rho}_D^3}{m_b} + 3\,(v\cdot q)^2\, \frac{\hat{\rho}_{LS}^3}{2 m_b}\right) \\
& - \frac{16}{3\Delta_0^4}\,(m_b - v\cdot q)\, [q^2 - (v\cdot q)^2]\, \hat{\rho}_D^3 \Bigg\}
\end{aligned}
\tag{B.84}
$$

$$
\begin{aligned}
S_{SMR,11} = 2\, m_H\, m_U \Bigg\{ & \frac{1}{3 m_b\, \Delta_0^2}\left(-4\, \hat{\mu}_\pi^2 + 6\, \hat{\mu}_G^2 + 4\, \frac{\hat{\rho}_D^3}{m_b} + 3\, \frac{\hat{\rho}_{LS}^3}{m_b}\right) \\
& - \frac{8}{3 m_b \Delta_0^3}\left((m_b - v\cdot q)\, \hat{\rho}_D^3 - \frac{3}{2}\, v\cdot q\, \hat{\rho}_{LS}^3\right)\Bigg\}
\end{aligned}
\tag{B.85}
$$

and $S_{SMR,(1,2,3,4,5,6,7,12,13)} = 0$. In addition we have:

$$
\begin{aligned}
T_{RSM,i} &= T_{SMR,i} && (i = 1,2,3,4,5) \\
S_{RSM,i} &= S_{SMR,i} && (i = 1,2,3,4,5,6,7,8,9,12,13) \\
S_{RSM,i} &= -S_{SMR,i} && (i = 10,11).
\end{aligned}
\tag{B.86}
$$

- Interference between the right-handed and the scalar operators in $H_{\text{eff}}$

  The tensor is obtained when $(i,j) = (5,2)$ and $(2,5)$ in eq. (3.2). We denote the two contributions as $T_{RS}$ and $T_{SR}$, respectively. Using the expansion as in eq. (B.17) changing $T_{SMSi} \to T_{RSi}$ and $S_{SMSi} \to S_{RSi}$ we find:

$$\begin{aligned} T_{RSi} &= T_{SMSi} \qquad (i=1,2) \\ S_{RSi} &= -S_{SMSi} \qquad (i=1,2,3) \qquad S_{RS4} = S_{SMS4} \end{aligned} \tag{B.87}$$

and analogous relations in the case of the structures in $T_{SR}$.

- Interference between the right-handed and the pseudoscalar operators in $H_{\text{eff}}$

  The tensor is obtained when $(i,j) = (5,3)$ and $(3,5)$ in eq. (3.2). We denote the two contributions as $T_{RP}$ and $T_{PR}$, respectively. Using an expansion analogous to (B.24), substituting $T_{SMPi} \to T_{RPi}$ and $S_{SMPi} \to S_{RPi}$, we find:

$$\begin{aligned} T_{RPi} &= -T_{SMPi} \quad (i=1,2) \\ S_{RPi} &= S_{SMPi} \quad (i=1,2,3) \qquad S_{RP4} = -S_{SMP4} \end{aligned} \tag{B.88}$$

  and analogous relations in the case of the structures in $T_{PR}$.

- Interference between the right handed and the tensor operators in $H_{\text{eff}}$

  The tensor is obtained for $(i,j) = (5,4)$ and $(i,j) = (4,5)$ in eq. (3.2), with the two contributions denoted as $T_{RT}$ and $T_{TR}$, respectively. Using an expansion as in eq. (B.61), with $T_{SMTi} \to T_{RTi}$ and $S_{SMTi} \to S_{RTi}$, we find:

$$\begin{aligned} T_{RT,1} = -T_{RT,3} = 2m_H \Bigg\{ & -\frac{2m_b}{\Delta_0} - \frac{2}{3\,\Delta_0^2}\left[5v\cdot q\,\hat{\mu}_\pi^2 + (4m_b - 3v\cdot q)\hat{\mu}_G^2 + 4\hat{\rho}_D^3\right] \\ & + \frac{8}{3\Delta_0^3}\left[m_b\,[q^2 - (q\cdot v)^2]\,\hat{\mu}_\pi^2 - (m_b - v\cdot q)\,v\cdot q\,\hat{\rho}_D^3\right] \\ & + \frac{16m_b}{3\Delta_0^4}\,(m_b - v\cdot q)\,[q^2 - (v\cdot q)^2]\,\hat{\rho}_D^3 \Bigg\} \end{aligned} \tag{B.89}$$

$$\begin{aligned} T_{RT,2} = -T_{RT,4} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\left[1 - \frac{\hat{\mu}_\pi^2 - \hat{\mu}_G^2}{2m_b^2}\right] \\ & + \frac{1}{\Delta_0^2}\left[\frac{2(2m_b + 3v\cdot q)}{3m_b}\mu_\pi^2 + \frac{2(4m_b - 3v\cdot q)}{3m_b}\mu_G^2 + \frac{2}{3m_b}(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3)\right] \\ & - \frac{8}{3\Delta_0^3}\left[[q^2 - (v\cdot q)^2]\,\mu_\pi^2 - (m_b - v\cdot q)\hat{\rho}_D^3\right] \\ & - \frac{16}{3\Delta_0^4}\,(m_b - v\cdot q)[q^2 - (v\cdot q)^2]\,\hat{\rho}_D^3 \Bigg\} \end{aligned} \tag{B.90}$$

$$T_{RT,5} = T_{RT,7} = 2m_H \left\{ \frac{4}{3\Delta_0^2}\left[\mu_G^2 + \frac{2}{m_b}(\hat{\rho}_D^3 + \hat{\rho}_{LS}^3)\right] + \frac{8}{3\Delta_0^3} v\cdot q\hat{\rho}_{LS}^3 \right\} \tag{B.91}$$

$$T_{RT,6} = T_{RT,8} = -2m_H \left\{ \frac{8}{3\Delta_0^3}\hat{\rho}_{LS}^3 \right\} \tag{B.92}$$

$$
\begin{aligned}
S_{RT,1} = -2m_H \Bigg\{ & \frac{1}{\Delta_0}\left[2-\frac{5\mu_\pi^2}{6m_b^2}+\frac{\mu_G^2}{2m_b^2}-\frac{\hat\rho_D^3}{3m_b^3}\right] \\
& + \frac{1}{\Delta_0^2}\left[\frac{2m_b(2m_b+5v\cdot q)}{3m_b^2}\mu_\pi^2+\frac{4m_b(m_b-v\cdot q)}{m_b^2}\mu_G^2\right. \\
& \left. + \frac{2(5m_b-4v\cdot q)}{3m_b^2}\hat\rho_D^3+\frac{4m_b-3v\cdot q}{m_b^2}\hat\rho_{LS}^3\right] \\
& + \frac{1}{\Delta_0^3}\left[-\frac{8}{3}\left[q^2-(q\cdot v)^2\right]\mu_\pi^2+\frac{8}{3m_b}(m_b^2-v\cdot q^2)\hat\rho_D^3+4v\cdot q\frac{(2m_b-v\cdot q)}{m_b}\hat\rho_{LS}^3\right] \\
& - \frac{16}{3\Delta_0^4}(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\Bigg\}
\end{aligned}
\tag{B.93}
$$

$$
\begin{aligned}
S_{RT,2} = 2m_H \Bigg\{ & \frac{1}{\Delta_0^2}\left[\frac{(4\mu_\pi^2-6\mu_G^2)}{3m_b}-\frac{4\hat\rho_D^3+3\hat\rho_{LS}^3}{3m_b^2}\right] \\
& + \frac{1}{\Delta_0^3}\left[\frac{8(m_b-v\cdot q)}{3m_b}\hat\rho_D^3+\frac{4(2m_b-v\cdot q)}{m_b}\hat\rho_{LS}^3\right]\Bigg\}
\end{aligned}
\tag{B.94}
$$

$$
\begin{aligned}
S_{RT,3} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\left[1-\frac{5\mu_\pi^2-3\mu_G^2}{12m_b^2}-\frac{\hat\rho_D^3}{6m_b^3}\right] \\
& + \frac{1}{3\Delta_0^2}\left[\frac{2(2m_b+5v\cdot q)}{m_b}\mu_\pi^2+\frac{12(m_b-v\cdot q)}{m_b}\mu_G^2+\frac{2(5m_b-4v\cdot q)}{m_b^2}\hat\rho_D^3\right. \\
& \left. + \frac{3(4m_b-3v\cdot q)}{m_b^2}\hat\rho_{LS}^3\right] \\
& - \frac{4}{3\Delta_0^3}\left[2\left[q^2-(v\cdot q)^2\right]\mu_\pi^2-\frac{2[m_b^2-(v\cdot q)^2]}{m_b}\hat\rho_D^3-\frac{3v\cdot q(m_b-v\cdot q)}{m_b}\hat\rho_{LS}^3\right] \\
& - \frac{16}{3\Delta_0^4}(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\Bigg\}
\end{aligned}
\tag{B.95}
$$

$$
\begin{aligned}
S_{RT,4} = 2m_H \Bigg\{ & \frac{1}{3m_b\Delta_0^2}\left[-4\mu_\pi^2+6\mu_G^2+\frac{4\hat\rho_D^3+3\hat\rho_{LS}^3}{m_b}\right] \\
& - \frac{4}{3m_b\Delta_0^3}(m_b-v\cdot q)(2\hat\rho_D^3+3\hat\rho_{LS}^3)\Bigg\}
\end{aligned}
\tag{B.96}
$$

$$
S_{RT,7} = S_{RT,26} = -2m_H\frac{4}{\Delta_0^3}\hat\rho_{LS}^3 \tag{B.97}
$$

$$
\begin{aligned}
S_{RT,9} = 2m_H \Bigg\{ & \frac{2m_b}{\Delta_0}\left[\frac{m_b-v\cdot q}{m_b}+\frac{m_b+5v\cdot q}{12m_b^3}\mu_\pi^2-\frac{m_b+v\cdot q}{4m_b^3}\mu_G^2\right. \\
& \left. + \frac{-3m_b+v\cdot q}{6m_b^4}\hat\rho_D^3-\frac{1}{2m_b^3}\hat\rho_{LS}^3\right] \\
& + \frac{1}{\Delta_0^2}\left[\frac{2[2q^2+(3m_b-5v\cdot q)v\cdot q]}{3m_b}\mu_\pi^2+\frac{4m_b^2-6m_bv\cdot q+4(v\cdot q)^2-2q^2}{m_b}\mu_G^2\right. \\
& \left. + \frac{2(8m_b^2-2q^2-7m_bv\cdot q+4(v\cdot q)^2)}{3m_b^2}\hat\rho_D^3+\frac{2m_b^2-q^2-6m_bv\cdot q+3(v\cdot q)^2}{m_b^2}\hat\rho_{LS}^3\right] \\
& - \frac{4}{3\Delta_0^3}\left[q^2-(q\cdot v)^2\right]\left[2(m_b-v\cdot q)\mu_\pi^2-2\frac{(m_b-v\cdot q)}{m_b}\hat\rho_D^3-3\frac{2m_b-v\cdot q}{m_b}\hat\rho_{LS}^3\right] \\
& - \frac{16}{3\Delta_0^4}(m_b-v\cdot q)^2\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\Bigg\}
\end{aligned}
\tag{B.98}
$$

$$
\begin{aligned}
S_{RT,10} = -2m_H \Bigg\{ & \frac{2m_b}{\Delta_0}\Bigg[\frac{m_b - v\cdot q}{m_b} + \frac{m_b + 5v\cdot q}{12m_b^3}\mu_\pi^2 - \frac{m_b + v\cdot q}{4m_b^3}\mu_G^2 \\
& + \frac{-3m_b + v\cdot q}{6m_b^4}\hat{\rho}_D^3 - \frac{1}{2m_b^3}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{1}{\Delta_0^2}\Bigg[\frac{2[2q^2 + (3m_b - 5v\cdot q)v\cdot q]}{3m_b}\mu_\pi^2 + \frac{4m_b^2 - 6m_b v\cdot q + 4(v\cdot q)^2 - 2q^2}{m_b}\mu_G^2 \\
& + \frac{2(8m_b^2 - 2q^2 - 7m_b v\cdot q + 4(v\cdot q)^2)}{3m_b^2}\hat{\rho}_D^3 + \frac{2m_b^2 - q^2 - 6m_b v\cdot q + 3(v\cdot q)^2}{m_b^2}\hat{\rho}_{LS}^3\Bigg] \\
& - \frac{4}{3\Delta_0^3}\left[q^2 - (q\cdot v)^2\right]\Bigg[2(m_b - v\cdot q)\mu_\pi^2 - 2\frac{(m_b - v\cdot q)}{m_b}\hat{\rho}_D^3 - 3\frac{m_b - v\cdot q}{m_b}\hat{\rho}_{LS}^3\Bigg] \\
& - \frac{16}{3\Delta_0^4}(m_b - v\cdot q)^2\left[q^2 - (q\cdot v)^2\right]\hat{\rho}_D^3\Bigg\}
\end{aligned}
\tag{B.99}
$$

$$
\begin{aligned}
S_{RT,11} = 2m_H \Bigg\{ & -\frac{2}{\Delta_0}\Bigg[1 - \frac{5}{12m_b^2}\mu_\pi^2 + \frac{1}{4m_b^2}\mu_G^2 - \frac{1}{6m_b^3}\hat{\rho}_D^3\Bigg] \\
& - \frac{1}{\Delta_0^2}\Bigg[\frac{10v\cdot q}{3m_b}\mu_\pi^2 + \frac{4(m_b - v\cdot q)}{m_b}\mu_G^2 + \frac{8(m_b - v\cdot q)}{3m_b^2}\hat{\rho}_D^3 + \frac{4m_b - 3v\cdot q}{m_b^2}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{4}{3\Delta_0^3}\Bigg[2\left[q^2 - (q\cdot v)^2\right]\mu_\pi^2 - \frac{2v\cdot q(m_b - v\cdot q)}{m_b}\hat{\rho}_D^3 + \frac{3(m_b - v\cdot q)^2}{m_b}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{16}{3\Delta_0^4}(m_b - v\cdot q)\left[q^2 - (q\cdot v)^2\right]\hat{\rho}_D^3\Bigg\}
\end{aligned}
\tag{B.100}
$$

$$
\begin{aligned}
S_{RT,12} = 2m_H \Bigg\{ & -\frac{4m_b}{\Delta_0}\Bigg[1 + \frac{5\mu_\pi^2 - 9\mu_G^2}{12m_b^2} - \frac{10\hat{\rho}_D^3 + 9\hat{\rho}_{LS}^3}{12m_b^3}\Bigg] \\
& - \frac{2}{3\Delta_0^2}\Bigg[14v\cdot q\,\mu_\pi^2 + 6(m_b - 2v\cdot q)\mu_G^2 + \frac{12(m_b - v\cdot q)}{m_b}\hat{\rho}_D^3 + \frac{3(3m_b - 5v\cdot q)}{m_b}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{8}{3\Delta_0^3}\Bigg[2m_b\left[q^2 - (q\cdot v)^2\right]\mu_\pi^2 - v\cdot q(m_b - v\cdot q)(4\hat{\rho}_D^3 + 3\hat{\rho}_{LS}^3)\Bigg] \\
& + \frac{32}{3\Delta_0^4}m_b(m_b - v\cdot q)\left[q^2 - (q\cdot v)^2\right]\hat{\rho}_D^3\Bigg\}
\end{aligned}
\tag{B.101}
$$

$$
\begin{aligned}
S_{RT,13} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\Bigg[1 - \frac{5}{12m_b^2}\mu_\pi^2 + \frac{1}{4m_b^2}\mu_G^2 - \frac{1}{6m_b^3}\hat{\rho}_D^3\Bigg] \\
& + \frac{1}{\Delta_0^2}\Bigg[\frac{2(4m_b + 5v\cdot q)}{3m_b}\mu_\pi^2 - \frac{4v\cdot q}{m_b}\mu_G^2 - \frac{4(m_b + 2v\cdot q)}{3m_b^2}\hat{\rho}_D^3 - \frac{3v\cdot q}{m_b^2}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{8}{3\Delta_0^3}\Bigg[-\left[q^2 - (q\cdot v)^2\right]\mu_\pi^2 + \frac{2m_b^2 - m_b v\cdot q - (v\cdot q)^2}{m_b}\hat{\rho}_D^3 \\
& + 3\frac{m_b^2 - (v\cdot q)^2}{2m_b}\hat{\rho}_{LS}^3\Bigg] \\
& - \frac{16}{3\Delta_0^4}(m_b - v\cdot q)\left[q^2 - (q\cdot v)^2\right]\hat{\rho}_D^3\Bigg\}
\end{aligned}
\tag{B.102}
$$

$$
\begin{aligned}
S_{RT,14} = 2m_H \Bigg\{ & \frac{2}{\Delta_0}\Bigg[1 - \frac{5}{12m_b^2}\mu_\pi^2 + \frac{1}{4m_b^2}\mu_G^2 - \frac{1}{6m_b^3}\hat{\rho}_D^3\Bigg] \\
& + \frac{1}{\Delta_0^2}\Bigg[\frac{2(4m_b + 5v\cdot q)}{3m_b}\mu_\pi^2 - \frac{4v\cdot q}{m_b}\mu_G^2 - \frac{8v\cdot q}{3m_b^2}\hat{\rho}_D^3 - \frac{3v\cdot q}{m_b^2}\hat{\rho}_{LS}^3\Bigg] \\
& + \frac{8}{3\Delta_0^3}\Bigg[-\left[q^2 - (q\cdot v)^2\right]\mu_\pi^2 + \frac{2m_b^2 - m_b v\cdot q - (v\cdot q)^2}{m_b}\hat{\rho}_D^3 + \frac{m_b^2 - (v\cdot q)^2}{2m_b}\hat{\rho}_{LS}^3\Bigg] \\
& - \frac{16}{3\Delta_0^4}(m_b - v\cdot q)\left[q^2 - (q\cdot v)^2\right]\hat{\rho}_D^3\Bigg\}
\end{aligned}
\tag{B.103}
$$



JHEP11(2020)032

$$S_{RT,15} = 2m_H \left\{ -\frac{2}{\Delta_0^2}\left[\frac{(4\mu_\pi^2-6\mu_G^2)}{3m_b} - \frac{4\hat\rho_D^3+3\hat\rho_{LS}^3}{3m_b^2}\right] - \frac{8(m_b-v\cdot q)}{3m_b\Delta_0^3}(2\hat\rho_D^3+3\hat\rho_{LS}^3)\right\} \tag{B.104}$$

$$\begin{aligned} S_{RT,16} = 2m_H &\left\{\frac{4m_b}{\Delta_0}\left[1+\frac{5\mu_\pi^2-9\mu_G^2}{12m_b^2}-\frac{10\hat\rho_D^3+9\hat\rho_{LS}^3}{12m_b^3}\right]\right.\\ &+\frac{2}{3\Delta_0^2}\left[14v\cdot q\,\mu_\pi^2+6(m_b-2v\cdot q)\mu_G^2+\frac{12(m_b-v\cdot q)}{m_b}\hat\rho_D^3+\frac{3(3m_b-5v\cdot q)}{m_b}\hat\rho_{LS}^3\right]\\ &-\frac{8}{3\Delta_0^3}\left[2m_b\left[q^2-(q\cdot v)^2\right]\mu_\pi^2-4v\cdot q(m_b-v\cdot q)\hat\rho_D^3\right.\\ &\left.+\frac{3[q^2-2v\cdot q(m_b-v\cdot q)]}{2}\hat\rho_{LS}^3\right]\\ &\left.-\frac{32}{3\Delta_0^4}m_b\,(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\right\} \end{aligned} \tag{B.105}$$

$$\begin{aligned} S_{RT,17} = 2m_H &\left\{-\frac{2}{\Delta_0}\left[1-\frac{5}{12m_b^2}\mu_\pi^2+\frac{1}{4m_b^2}\mu_G^2-\frac{1}{6m_b^3}\hat\rho_D^3\right]\right.\\ &-\frac{1}{\Delta_0^2}\left[\frac{2(4m_b+5v\cdot q)}{3m_b}\mu_\pi^2-\frac{4v\cdot q}{m_b}\mu_G^2-\frac{4(m_b+2v\cdot q)}{3m_b^2}\hat\rho_D^3-\frac{3v\cdot q}{m_b^2}\hat\rho_{LS}^3\right]\\ &-\frac{8}{3\Delta_0^3}\left[-\left[q^2-(q\cdot v)^2\right]\mu_\pi^2+\frac{2m_b^2-m_bv\cdot q-(v\cdot q)^2}{m_b}\hat\rho_D^3\right.\\ &\left.\left.+\frac{3(m_b^2-m_bv\cdot q-(v\cdot q)^2)}{2m_b}\hat\rho_{LS}^3\right]+\frac{16}{3\Delta_0^4}\,(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\right\} \end{aligned} \tag{B.106}$$

$$\begin{aligned} S_{RT,18} = 2m_H &\left\{-\frac{2}{\Delta_0}\left[1-\frac{5}{12m_b^2}\mu_\pi^2+\frac{1}{4m_b^2}\mu_G^2-\frac{1}{6m_b^3}\hat\rho_D^3\right]\right.\\ &-\frac{1}{\Delta_0^2}\left[\frac{2(4m_b+5v\cdot q)}{3m_b}\mu_\pi^2-\frac{4v\cdot q}{m_b}\mu_G^2-\frac{8v\cdot q}{3m_b^2}\hat\rho_D^3-\frac{3v\cdot q}{m_b^2}\hat\rho_{LS}^3\right]\\ &-\frac{8}{3\Delta_0^3}\left[-\left[q^2-(q\cdot v)^2\right]\mu_\pi^2+\frac{2m_b^2-m_bv\cdot q-(v\cdot q)^2}{m_b}\hat\rho_D^3\right.\\ &\left.\left.+\frac{3(m_b^2-m_bv\cdot q-(v\cdot q)^2)}{2m_b}\hat\rho_{LS}^3\right]+\frac{16}{3\Delta_0^4}\,(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\right\} \end{aligned} \tag{B.107}$$

$$\begin{aligned} S_{RT,19} = 2m_H &\left\{\frac{2}{\Delta_0^2}\left[\frac{(4\mu_\pi^2-6\mu_G^2)}{3m_b}-\frac{4\hat\rho_D^3+3\hat\rho_{LS}^3}{3m_b^2}\right]\right.\\ &\left.+\frac{1}{\Delta_0^3}\left[\frac{16(m_b-v\cdot q)}{3m_b}\hat\rho_D^3+\frac{4(m_b-2v\cdot q)}{m_b}\hat\rho_{LS}^3\right]\right\} \end{aligned} \tag{B.108}$$

$$\begin{aligned} S_{RT,25} = 2m_H &\left\{-\frac{2}{\Delta_0}\left[1-\frac{5}{12m_b^2}\mu_\pi^2+\frac{1}{4m_b^2}\mu_G^2-\frac{1}{6m_b^3}\hat\rho_D^3\right]\right.\\ &-\frac{1}{\Delta_0^2}\left[\frac{10v\cdot q}{3m_b}\mu_\pi^2+\frac{4(m_b-v\cdot q)}{m_b}\mu_G^2+\frac{8(2m_b-v\cdot q)}{3m_b^2}\hat\rho_D^3\right.\\ &\left.+\frac{4m_b-3v\cdot q}{m_b^2}\hat\rho_{LS}^3\right]\\ &+\frac{4}{3\Delta_0^3}\left[2\left[q^2-(q\cdot v)^2\right]\mu_\pi^2-\frac{2v\cdot q(m_b-v\cdot q)}{m_b}\hat\rho_D^3+\frac{3(m_b-v\cdot q)^2}{m_b}\hat\rho_{LS}^3\right]\\ &\left.+\frac{16}{3\Delta_0^4}\,(m_b-v\cdot q)\left[q^2-(q\cdot v)^2\right]\hat\rho_D^3\right\} \end{aligned} \tag{B.109}$$

and $S_{RT,(5,6,8,20,21,22,23,24)} = 0$. Moreover, we have:

$$\begin{aligned} T_{RT,i} &= -T_{TR,i} && (i = 1,2,5,6) \\ S_{RT,i} &= -S_{TR,i} && (i = 1,2,9,11,12,13,14,15) \\ T_{RT,i} &= T_{TR,i} && (i = 3,4,7,8) \\ S_{RT,i} &= S_{TR,i} && (i = 3,4,7,10,16,17,18,19,25,26). \end{aligned} \tag{B.110}$$

## C Coefficients in the $1/m_b$ expansion of the inclusive semileptonic decay width

To provide the coefficients in eq. (4.19) we define the variables

$$\rho = \frac{m_U^2}{m_b^2}\,, \qquad \rho_\ell = \frac{m_\ell^2}{m_b^2}\,. \tag{C.1}$$

In the formulae $\sqrt{\lambda}$ stays for $\sqrt{\lambda(1,\rho,\rho_\ell)}$. Factorizing the effective couplings in the Hamiltonian eq. (2.3), we define for $A = 0,\ \mu_\pi^2,\ \mu_G^2, \rho_D^3$: $C_A^{(SM)} = |1+\epsilon_V|^2\,\mathcal{C}_A^{(SM)}$, $C_A^{(i)} = |\epsilon_i|^2\,\mathcal{C}_A^{(i)}$ for i=S, P, T, R, and $C_A^{(ij)} = 2\mathrm{Re}(\epsilon_i\,\epsilon_j^*)\,\mathcal{C}_A^{(ij)}$ for (i,j)=(S,P), (SM,S), (SM,P), (SM,T), (SMR), (S,T), (P,T), (RS), (RP), (RT). We also define:

$$\mathcal{L}_1 = \log\Big[\frac{(1+\sqrt{\lambda}-\rho+\rho_\ell)^2}{4\rho_\ell}\Big]\,, \qquad \mathcal{L}_2 = \log\Big[\frac{(1+\sqrt{\lambda}+\rho-\rho_\ell)^2}{4\rho}\Big]\,. \tag{C.2}$$

With appropriate manipulations our results for SM agree with [8].

- Standard Model:

$$\begin{aligned} \mathcal{C}_0^{(SM)} &= -2\,\mathcal{C}_{\mu_\pi^2}^{(SM)} = \sqrt{\lambda}\Big[1-7\rho-7\rho^2+\rho^3-(7-12\rho+7\rho^2)\rho_\ell \\ &\quad - 7(1+\rho)\rho_\ell^2+\rho_\ell^3\Big] \\ &\quad + 12\Big\{(1-\rho^2)\rho_\ell^2\mathcal{L}_1+(1-\rho_\ell^2)\rho^2\mathcal{L}_2\Big\} \end{aligned} \tag{C.3}$$

$$\begin{aligned} \mathcal{C}_{\mu_G^2}^{(SM)} &= \frac{\sqrt{\lambda}}{2}\Big[-3+5\rho-19\rho^2+5\rho^3+(5+28\rho-35\rho^2)\rho_\ell \\ &\quad -(19+35\rho)\rho_\ell^2+5\rho_\ell^3\Big] \\ &\quad + 6\Big\{(1-5\rho^2)\rho_\ell^2\mathcal{L}_1+(1-5\rho_\ell^2)\rho^2\mathcal{L}_2\Big\} \end{aligned} \tag{C.4}$$

$$\begin{aligned} \mathcal{C}_{\rho_D^3}^{(SM)} &= \frac{2}{3}\sqrt{\lambda}\Big[17+\rho-11\rho^2+5\rho^3+\rho_\ell(4+18\rho-32\rho^2) \\ &\quad +\rho_\ell^2(-23-35\rho)+2\rho_\ell^3\Big] \\ &\quad - 8\Big\{\rho_\ell^2(-1+5\rho^2+\rho_\ell)\mathcal{L}_1+[1-\rho_\ell+\rho_\ell^2(-1+5\rho^2+\rho_\ell)]\mathcal{L}_2\Big\} \end{aligned} \tag{C.5}$$

- S and P:

$$\begin{aligned}
\mathcal{C}_0^{(S)} &= -2\mathcal{C}_{\mu_\pi^2}^{(S)} = \frac{\sqrt{\lambda}}{8}\Big[1+4\sqrt{\rho}-7\rho+40\rho^{3/2}-7\rho^2+4\rho^{5/2}+\rho^3 \\
&\quad +\rho_\ell(-7-20\sqrt{\rho}+12\rho-20\rho^{3/2}-7\rho^2)+\rho_\ell^2(-7-8\sqrt{\rho}-7\rho)+\rho_\ell^3\Big] \\
&\quad -\frac{3}{2}\Big\{(-1+\sqrt{\rho})(1+\sqrt{\rho})^3\rho_\ell^2\mathcal{L}_1 \\
&\quad +\rho^{3/2}\left[2\rho+2(-1+\rho_\ell)^2+\sqrt{\rho}(-1+\rho_\ell^2)\right]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.6}$$

$$\begin{aligned}
\mathcal{C}_{\mu_G^2}^{(S)} &= \frac{\sqrt{\lambda}}{16}\Big[13-132\sqrt{\rho}+45\rho-24\rho^{3/2}-27\rho^2+12\rho^{5/2}+5\rho^3 \\
&\quad +\rho_\ell(-27+84\sqrt{\rho}+68\rho-60\rho^{3/2}-35\rho^2)+\rho_\ell^2(-3-24\sqrt{\rho}-35\rho)+5\rho_\ell^3\Big] \\
&\quad +\frac{3}{4}\Big\{(1+\sqrt{\rho})^2(1+4\sqrt{\rho}-5\rho)\rho_\ell^2\,\mathcal{L}_1 \\
&\quad +\rho^{1/2}\big[-2\rho^2+4(-1+\rho_\ell)^2+\rho(10+4\rho_\ell-6\rho_\ell^2) \\
&\quad +\rho^{3/2}(1-5\rho_\ell^2)+4\sqrt{\rho}(-1+\rho_\ell^2)\big]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.7}$$

$$\begin{aligned}
\mathcal{C}_{\rho_D^3}^{(S)} &= \frac{\sqrt{\lambda}}{12}\Big[59+44\sqrt{\rho}+37\rho-28\rho^{3/2}-17\rho^2+8\rho^{5/2}+5\rho^3 \\
&\quad +\rho_\ell(-53+44\sqrt{\rho}+54\rho-40\rho^{3/2}-35\rho^2)+\rho_\ell^2(13-16\sqrt{\rho}-35\rho)+5\rho_\ell^3\Big] \\
&\quad +\Big\{-(1+\sqrt{\rho})^2(2-6\sqrt{\rho}+5\rho)\rho_\ell^2\mathcal{L}_1 \\
&\quad +\left[-(2+2\sqrt{\rho}+5\rho)+4\rho_\ell-(1+\sqrt{\rho})^2(2-6\sqrt{\rho}+5\rho)\rho_\ell^2\right]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.8}$$

In the pseudoscalar case the coefficients are obtained from the corresponding ones in the scalar case changing the sign of $m_U$ and of the odd powers of $\sqrt{\rho}$.

- T:

$$\begin{aligned}
\mathcal{C}_0^{(T)} &= -2\mathcal{C}_{\mu_\pi^2}^{(T)} = 12\sqrt{\lambda}\Big[1-7\rho-7\rho^2+\rho^3-(7-12\rho+7\rho^2)\rho_\ell \\
&\quad -7(1+\rho)\rho_\ell^2+\rho_\ell^3\Big] \\
&\quad +144\Big\{(1-\rho^2)\rho_\ell^2\mathcal{L}_1+(1-\rho_\ell^2)\rho^2\mathcal{L}_2\Big\}
\end{aligned} \tag{C.9}$$

$$\begin{aligned}
\mathcal{C}_{\mu_G^2}^{(T)} &= 2\sqrt{\lambda}\Big[-25-25\rho-49\rho^2+15\rho^3+\rho_\ell(47+44\rho-105\rho^2) \\
&\quad -\rho_\ell^2(73+105\rho)+15\rho_\ell^3\Big] \\
&\quad +24\Big\{\rho_\ell^2(1-3\rho)(3+5\rho)\mathcal{L}_1+\rho(4+3\rho-(4+15\rho)\rho_\ell^2)\mathcal{L}_2\Big\}
\end{aligned} \tag{C.10}$$

$$\begin{aligned}
\mathcal{C}_{\rho_D^3}^{(T)} &= 8\sqrt{\lambda}\Big[3-11\rho-9\rho^2+5\rho^3+\rho_\ell(23+6\rho-31\rho^2)-39\rho_\ell^2(1+\rho)+5\rho_\ell^3 \\
&\quad +4(1-\rho)(1+\rho-\rho_\ell)\rho_\ell\Big] \\
&\quad -32\Big\{\rho_\ell^2[-6+5\rho(1+3\rho)+4\rho_\ell]\mathcal{L}_1 \\
&\quad +\left[2-5\rho+[-6+5\rho(1+3\rho)]\rho_\ell^2+4\rho_\ell^3\right]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.11}$$

- S - P interference: The coefficients vanish.

- SM - S and SM - P interference:

$$\begin{aligned}
\mathcal{C}_0^{(SMS)} &= -2\mathcal{C}_{\mu_\pi^2}^{(SMS)} \\
&= \frac{\sqrt{\lambda}}{2}(1-\sqrt{\rho})\sqrt{\rho_\ell}\Big[1+3\sqrt{\rho}-2\rho+3\rho^{3/2}+\rho^2 \\
&\quad +\rho_\ell(10+15\sqrt{\rho}+10\rho)+\rho_\ell^2\Big] \\
&\quad -3\sqrt{\rho_\ell}\Big\{\rho_\ell(1+\sqrt{\rho})\left[(1-\rho)^2+(1-\sqrt{\rho}+\rho)\rho_\ell\right]\mathcal{L}_1 \qquad \text{(C.12)} \\
&\quad +\rho^{3/2}\left[\sqrt{\rho}(-1+\rho_\ell)+(1-\rho_\ell)^2+\rho\rho_\ell\right]\mathcal{L}_2\Big\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{\mu_G^2}^{(SMS)} &= \frac{\sqrt{\lambda}}{4}(1-\sqrt{\rho})\sqrt{\rho_\ell}\Big[5-15\sqrt{\rho}-10\rho+9\rho^{3/2} \\
&\quad +5\rho^2+\rho_\ell(2+45\sqrt{\rho}+50\rho)+5\rho_\ell^2\Big] \\
&\quad -\frac{3}{2}\sqrt{\rho_\ell}\Big\{\rho_\ell\Big[(1+5\sqrt{\rho})(1-\rho)^2+(1-2\sqrt{\rho}-2\rho+5\rho^{3/2})\rho_\ell\Big]\mathcal{L}_1 \\
&\quad +\sqrt{\rho}\big[-2+2\sqrt{\rho}+\rho-\rho^{3/2}+\rho_\ell(4-10\rho+\rho^{3/2}+5\rho^2) \qquad \text{(C.13)} \\
&\quad +\rho_\ell^2(-2-2\sqrt{\rho}+5\rho)\big]\mathcal{L}_2\Big\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{\rho_D^3}^{(SMS)} &= -\frac{\sqrt{\rho_\ell}}{6}\sqrt{\lambda}\Big[-16+28\sqrt{\rho}+2\rho-26\rho^{3/2}+2\rho^2+10\rho^{5/2} \\
&\quad +\rho_\ell(41-95\sqrt{\rho}-37\rho+103\rho^{3/2})-\rho_\ell^2(13-7\sqrt{\rho}) \\
&\quad -3(1+\sqrt{\rho})(1+\rho-\rho_\ell)\rho_\ell\Big] \qquad \text{(C.14)} \\
&\quad +2\sqrt{\rho_\ell}(1-\sqrt{\rho})\Big[\rho_\ell\big[2-2\sqrt{\rho}-5\rho+4\rho^{3/2}+5\rho^2+\rho_\ell(-1+2\sqrt{\rho}+5\rho)\big]\mathcal{L}_1 \\
&\quad +\big[-1+\rho_\ell(1+\sqrt{\rho})^2(2-6\sqrt{\rho}+5\rho)+\rho_\ell^2(-1+2\sqrt{\rho}+5\rho)\big]\mathcal{L}_2\Big\}
\end{aligned}$$

The coefficients in the SM-P case are obtained from the corresponding ones in the SM-S case changing the sign of $m_U$ and of the odd powers of $\sqrt{\rho}$.

- SM – T interference:

$$\begin{aligned}
\mathcal{C}_0^{(SMT)} &= -2\mathcal{C}_{\mu_\pi^2}^{(SMT)} = 12\sqrt{\lambda}\sqrt{\rho\rho_\ell}\Big[-2-5\rho+\rho^2-5\rho_\ell(1-2\rho)+\rho_\ell^2\Big] \qquad \text{(C.15)} \\
&\qquad +72\sqrt{\rho\rho_\ell}\left\{\rho_\ell[(1-\rho)^2+\rho\rho_\ell]\mathcal{L}_1+\rho[(1-\rho_\ell)^2+\rho\rho_\ell]\mathcal{L}_2\right\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{\mu_G^2}^{(SMT)} &= 6\sqrt{\lambda}\sqrt{\rho\rho_\ell}\Big[-4-11\rho+5\rho^2+\rho_\ell(-3+50\rho)+5\rho_\ell^2\Big] \\
&\quad +12\sqrt{\rho\rho_\ell}\Big\{\rho_\ell[-1-14\rho+15\rho^2+(2+15\rho)\rho_\ell]\mathcal{L}_1 \qquad \text{(C.16)} \\
&\quad +[2+3\rho+\rho_\ell(-4-14\rho+15\rho^2)+\rho_\ell^2(2+15\rho)]\mathcal{L}_2\Big\}
\end{aligned}$$

JHEP11(2020)032

$$\begin{aligned}
\mathcal{C}^{(SMT)}_{\rho_D^3} &= 4\sqrt{\lambda}\sqrt{\rho\rho_\ell}\Big[-8-14\rho+10\rho^2+\rho_\ell(-35+103\rho)+7\rho_\ell^2-3\rho_\ell(1+\rho-\rho_\ell)\Big] \\
&\quad + 48\sqrt{\rho\rho_\ell}\Big\{\rho_\ell\left[-5\rho(1-\rho)-\rho_\ell(1-5\rho)\right]\mathcal{L}_1 \\
&\quad + \left[1-5\rho\rho_\ell(1-\rho)-\rho_\ell^2(1-5\rho)\right]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.17}$$

- T − S and T − P interference: The coefficients vanish.

- R:

$$\begin{aligned}
\mathcal{C}^{(R)}_0 &= -2\,\mathcal{C}^{(R)}_{\mu_\pi^2} = \mathcal{C}^{(SM)}_0 \\
\mathcal{C}^{(R)}_{\mu_G^2} &= \mathcal{C}^{(SM)}_{\mu_G^2} \\
\mathcal{C}^{(R)}_{\hat\rho_D^3} &= \mathcal{C}^{(SM)}_{\hat\rho_D^3}
\end{aligned} \tag{C.18}$$

- SM − R interference:

$$\begin{aligned}
\mathcal{C}^{(SMR)}_0 &= -2\mathcal{C}^{(SMR)}_{\mu_\pi^2} = -2\sqrt{\lambda}\sqrt{\rho}\left(1+10\rho+\rho^2-5\rho_\ell-5\rho\rho_\ell-2\rho_\ell^2\right) \\
&\quad + 12\sqrt{\rho}\Big\{-\rho_\ell^2(1-\rho)\mathcal{L}_1+\rho\big[\rho+(1-\rho_\ell)^2\big]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.19}$$

$$\begin{aligned}
\mathcal{C}^{(SMR)}_{\mu_G^2} &= -\frac{1}{3}\sqrt{\lambda}\sqrt{\rho}\left(13-14\rho+13\rho^2+43\rho_\ell-77\rho\rho_\ell-86\rho_\ell^2\right) \\
&\quad + 2\sqrt{\rho}\Big\{\rho_\ell^2(-9+21\rho+4\rho_\ell)\mathcal{L}_1 \\
&\quad + (2-3\rho+3\rho^2-6\rho\rho_\ell-6\rho_\ell^2+21\rho\rho_\ell^2+4\rho_\ell^3)\mathcal{L}_2\Big\}
\end{aligned} \tag{C.20}$$

$$\begin{aligned}
\mathcal{C}^{(SMR)}_{\hat\rho_D^3} &= -\frac{8}{3}\sqrt{\lambda}\sqrt{\rho}\left(11-7\rho+2\rho^2+14\rho_\ell-13\rho\rho_\ell-19\rho_\ell^2\right) \\
&\quad + 16\sqrt{\rho}\Big\{\rho_\ell^2(-2+4\rho+\rho_\ell)\mathcal{L}_1+\big[1+\rho_\ell^2(-2+4\rho+\rho_\ell)\big]\mathcal{L}_2\Big\}
\end{aligned} \tag{C.21}$$

- R − S interference:

$$\begin{aligned}
\mathcal{C}^{(RS)}_0 &= -2\,\mathcal{C}^{(RS)}_{\mu_\pi^2} = \mathcal{C}^{(SMS)}_0 \\
\mathcal{C}^{(RS)}_{\mu_G^2} &= \mathcal{C}^{(SMS)}_{\mu_G^2} \\
\mathcal{C}^{(RS)}_{\hat\rho_D^3} &= \mathcal{C}^{(SMS)}_{\hat\rho_D^3}
\end{aligned} \tag{C.22}$$

- R − P interference:

$$\begin{aligned}
\mathcal{C}^{(RP)}_0 &= -2\,\mathcal{C}^{(RP)}_{\mu_\pi^2} = \mathcal{C}^{(SMP)}_0 \\
\mathcal{C}^{(RP)}_{\mu_G^2} &= \mathcal{C}^{(SMP)}_{\mu_G^2} \\
\mathcal{C}^{(RP)}_{\hat\rho_D^3} &= \mathcal{C}^{(SMP)}_{\hat\rho_D^3}
\end{aligned} \tag{C.23}$$



JHEP11(2020)032

- R − T interference:

$$\mathcal{C}_0^{(RT)} = -2\mathcal{C}_{\mu_\pi^2}^{(RT)} = 12\sqrt{\lambda}\sqrt{\rho_\ell}\left(1-5\rho-2\rho^2+10\rho_\ell-5\rho\rho_\ell+\rho_\ell^2\right) - 72\sqrt{\rho_\ell}\Big\{\rho_\ell\big[(1-\rho)^2+\rho_\ell\big]\mathcal{L}_1-\rho^2(1-\rho_\ell)\mathcal{L}_2\Big\} \tag{C.24}$$

$$\mathcal{C}_{\mu_G^2}^{(RT)} = -2\sqrt{\lambda}\sqrt{\rho_\ell}\left(17-7\rho+20\rho^2+2\rho_\ell+65\rho\rho_\ell-7\rho_\ell^2\right) + 12\sqrt{\rho_\ell}\Big\{\rho_\ell(5+6\rho-11\rho^2-3\rho_\ell-2\rho\rho_\ell)\mathcal{L}_1 + \rho\left(2+3\rho-11\rho\rho_\ell-2\rho_\ell^2\right)\mathcal{L}_2\Big\} \tag{C.25}$$

$$\mathcal{C}_{\hat\rho_D^3}^{(RT)} = 4\sqrt{\lambda}\sqrt{\rho_\ell}\big[8+14\rho-10\rho^2-25\rho_\ell-43\rho\rho_\ell+5\rho_\ell^2+3\rho_\ell(1+\rho-\rho_\ell)\big] + 48\sqrt{\rho_\ell}\Big\{\rho_\ell\left(2+\rho-3\rho^2-\rho_\ell-\rho\rho_\ell\right)\mathcal{L}_1 - \left(1-2\rho_\ell-\rho\rho_\ell+3\rho^2\rho_\ell+\rho_\ell^2+\rho\rho_\ell^2\right)\mathcal{L}_2\Big\}. \tag{C.26}$$



## References

[1] S. Bifani, S. Descotes-Genon, A. Romero Vidal and M.-H. Schune, *Review of Lepton Universality tests in B decays*, *J. Phys. G* **46** (2019) 023001 [arXiv:1809.06229] [INSPIRE].

[2] J. Chay, H. Georgi and B. Grinstein, *Lepton energy distributions in heavy meson decays from QCD*, *Phys. Lett. B* **247** (1990) 399 [INSPIRE].

[3] I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *QCD predictions for lepton spectra in inclusive heavy flavor decays*, *Phys. Rev. Lett.* **71** (1993) 496 [hep-ph/9304225] [INSPIRE].

[4] P. Biancofiore, P. Colangelo and F. De Fazio, *On the anomalous enhancement observed in $B \to D^{(*)}\tau\bar\nu_\tau$ decays*, *Phys. Rev. D* **87** (2013) 074010 [arXiv:1302.1042] [INSPIRE].

[5] P. Colangelo and F. De Fazio, *Scrutinizing $\overline{B} \to D^*(D\pi)\,\ell^-\overline{\nu}_\ell$ and $\overline{B} \to D^*(D\gamma)\,\ell^-\overline{\nu}_\ell$ in search of new physics footprints*, *JHEP* **06** (2018) 082 [arXiv:1801.10468] [INSPIRE].

[6] S. Bhattacharya, S. Nandi and S. Kumar Patra, *$b \to c\tau\nu_\tau$ decays: a catalogue to compare, constrain, and correlate new physics effects*, *Eur. Phys. J. C* **79** (2019) 268 [arXiv:1805.08222] [INSPIRE].

[7] P. Colangelo and F. De Fazio, *Tension in the inclusive versus exclusive determinations of $|V_{cb}|$: a possible role of new physics*, *Phys. Rev. D* **95** (2017) 011701 [arXiv:1611.07387] [INSPIRE].

[8] T. Mannel, A.V. Rusov and F. Shahriaran, *Inclusive semitauonic B decays to order $\mathcal{O}(\Lambda^3_{QCD}/m_b^3)$*, *Nucl. Phys. B* **921** (2017) 211 [arXiv:1702.01089] [INSPIRE].

[9] S. Kamali, A. Rashed and A. Datta, *New physics in inclusive $B \to X_c\ell\bar\nu$ decay in light of $R(D^{(*)})$ measurements*, *Phys. Rev. D* **97** (2018) 095034 [arXiv:1801.08259] [INSPIRE].

JHEP11(2020)032

[10] S. Kamali, *New physics in inclusive semileptonic B decays including nonperturbative corrections*, Int. J. Mod. Phys. A **34** (2019) 1950036 [arXiv:1811.07393] [INSPIRE].

[11] BABAR collaboration, *Evidence for an excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays*, Phys. Rev. Lett. **109** (2012) 101802 [arXiv:1205.5442] [INSPIRE].

[12] BABAR collaboration, *Measurement of an Excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ Decays and Implications for Charged Higgs Bosons*, Phys. Rev. D **88** (2013) 072012 [arXiv:1303.0571] [INSPIRE].

[13] BELLE collaboration, *Measurement of the branching ratio of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ relative to $\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell$ decays with hadronic tagging at Belle*, Phys. Rev. D **92** (2015) 072014 [arXiv:1507.03233] [INSPIRE].

[14] BELLE collaboration, *Measurement of the branching ratio of $\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau$ relative to $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ decays with a semileptonic tagging method*, Phys. Rev. D **94** (2016) 072007 [arXiv:1607.07923] [INSPIRE].

[15] BELLE collaboration, *Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$*, Phys. Rev. Lett. **118** (2017) 211801 [arXiv:1612.00529] [INSPIRE].

[16] BELLE collaboration, *Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$ with one-prong hadronic $\tau$ decays at Belle*, Phys. Rev. D **97** (2018) 012004 [arXiv:1709.00129] [INSPIRE].

[17] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu)$*, Phys. Rev. Lett. **115** (2015) 111803 [*Erratum ibid.* **115** (2015) 159901] [arXiv:1506.08614] [INSPIRE].

[18] LHCb collaboration, *Test of Lepton Flavor Universality by the measurement of the $B^0 \to D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong $\tau$ decays*, Phys. Rev. D **97** (2018) 072013 [arXiv:1711.02505] [INSPIRE].

[19] LHCb collaboration, *Measurement of the ratio of the $B^0 \to D^{*-}\tau^+\nu_\tau$ and $B^0 \to D^{*-}\mu^+\nu_\mu$ branching fractions using three-prong $\tau$-lepton decays*, Phys. Rev. Lett. **120** (2018) 171802 [arXiv:1708.08856] [INSPIRE].

[20] S. Fajfer, J.F. Kamenik and I. Nisandzic, *On the $B \to D^*\tau\bar{\nu}_\tau$ Sensitivity to New Physics*, Phys. Rev. D **85** (2012) 094025 [arXiv:1203.2654] [INSPIRE].

[21] D. Bigi, P. Gambino and S. Schacht, *$R(D^*)$, $|V_{cb}|$, and the Heavy Quark Symmetry relations between form factors*, JHEP **11** (2017) 061 [arXiv:1707.09509] [INSPIRE].

[22] F.U. Bernlochner, Z. Ligeti, M. Papucci and D.J. Robinson, *Combined analysis of semileptonic B decays to D and $D^*$: $R(D^{(*)})$, $|V_{cb}|$, and new physics*, Phys. Rev. D **95** (2017) 115008 [*Erratum ibid.* **97** (2018) 059902] [arXiv:1703.05330] [INSPIRE].

[23] S. Jaiswal, S. Nandi and S.K. Patra, *Updates on SM predictions of $|V_{cb}|$ and $R(D^*)$ in $B \to D^*\ell\nu_\ell$ decays*, JHEP **06** (2020) 165 [arXiv:2002.05726] [INSPIRE].

[24] R. Alonso, A. Kobach and J. Martin Camalich, *New physics in the kinematic distributions of $\bar{B} \to D^{(*)}\tau^-(\to \ell^-\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau$*, Phys. Rev. D **94** (2016) 094021 [arXiv:1602.07671] [INSPIRE].

[25] Z. Ligeti, M. Papucci and D.J. Robinson, *New Physics in the Visible Final States of $B \to D^{(*)}\tau\nu$*, JHEP **01** (2017) 083 [arXiv:1610.02045] [INSPIRE].

[26] A.K. Alok, D. Kumar, S. Kumbhakar and S.U. Sankar, *$D^*$ polarization as a probe to discriminate new physics in $\bar{B} \to D^*\tau\bar{\nu}$*, Phys. Rev. D **95** (2017) 115038 [arXiv:1606.03164] [INSPIRE].

[27] M.A. Ivanov, J.G. Körner and C.-T. Tran, *Probing new physics in $\bar{B}^0 \to D^{(*)}\tau^-\bar{\nu}_\tau$ using the longitudinal, transverse, and normal polarization components of the tau lepton*, Phys. Rev. D **95** (2017) 036021 [arXiv:1701.02937] [INSPIRE].

[28] D. Bečirević, M. Fedele, I. Nišandžić and A. Tayduganov, *Lepton Flavor Universality tests through angular observables of $\overline{B} \to D^{(*)}\ell\overline{\nu}$ decay modes*, arXiv:1907.02257 [INSPIRE].

[29] M. Algueró, S. Descotes-Genon, J. Matias and M. Novoa-Brunet, *Symmetries in $B \to D^*\ell\nu$ angular observables*, JHEP **06** (2020) 156 [arXiv:2003.02533] [INSPIRE].

[30] LHCb collaboration, *Measurement of the shape of the $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ differential decay rate*, arXiv:2003.08453 [INSPIRE].

[31] LHCb collaboration, *Measurements of the $\Lambda_b^0 \to J/\psi\Lambda$ decay amplitudes and the $\Lambda_b^0$ polarisation in pp collisions at $\sqrt{s} = 7$ TeV*, Phys. Lett. B **724** (2013) 27 [arXiv:1302.5578] [INSPIRE].

[32] ATLAS collaboration, *Measurement of the parity-violating asymmetry parameter $\alpha_b$ and the helicity amplitudes for the decay $\Lambda_b^0 \to J/\psi + \Lambda^0$ with the ATLAS detector*, Phys. Rev. D **89** (2014) 092009 [arXiv:1404.1071] [INSPIRE].

[33] CMS collaboration, *Measurement of the $\Lambda_b$ polarization and angular parameters in $\Lambda_b \to J/\psi\,\Lambda$ decays from pp collisions at $\sqrt{s} = 7$ and 8 TeV*, Phys. Rev. D **97** (2018) 072010 [arXiv:1802.04867] [INSPIRE].

[34] LHCb collaboration, *Measurement of the $\Lambda_b^0 \to J/\psi\Lambda$ angular distribution and the $\Lambda_b^0$ polarisation in pp collisions*, JHEP **06** (2020) 110 [arXiv:2004.10563] [INSPIRE].

[35] ALEPH collaboration, *Measurement of the $\Lambda_b$ polarization in Z decays*, Phys. Lett. B **365** (1996) 437 [INSPIRE].

[36] OPAL collaboration, *Measurement of the average polarization of b baryons in hadronic $Z^0$ decays*, Phys. Lett. B **444** (1998) 539 [hep-ex/9808006] [INSPIRE].

[37] DELPHI collaboration, *$\Lambda_b$ polarization in $Z^0$ decays at LEP*, Phys. Lett. B **474** (2000) 205 [INSPIRE].

[38] F.U. Bernlochner, Z. Ligeti, D.J. Robinson and W.L. Sutcliffe, *Precise predictions for $\Lambda_b \to \Lambda_c$ semileptonic decays*, Phys. Rev. D **99** (2019) 055008 [arXiv:1812.07593] [INSPIRE].

[39] P. Böer, A. Kokulu, J.-N. Toelstede and D. van Dyk, *Angular Analysis of $\boldsymbol{\Lambda_b \to \Lambda_c(\to \Lambda\pi)\ell\bar{\nu}}$*, JHEP **12** (2019) 082 [arXiv:1907.12554] [INSPIRE].

[40] N. Penalva, E. Hernández and J. Nieves, *Further tests of lepton flavour universality from the charged lepton energy distribution in $b \to c$ semileptonic decays: The case of $\Lambda_b \to \Lambda_c\ell\bar{\nu}_\ell$*, Phys. Rev. D **100** (2019) 113007 [arXiv:1908.02328] [INSPIRE].

[41] M. Ferrillo, A. Mathad, P. Owen and N. Serra, *Probing effects of new physics in $\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu}_\mu$ decays*, JHEP **12** (2019) 148 [arXiv:1909.04608] [INSPIRE].

[42] R. Dutta, *$\Lambda_b \to (\Lambda_c, p)\,\tau\,\nu$ decays within standard model and beyond*, Phys. Rev. D **93** (2016) 054003 [arXiv:1512.04034] [INSPIRE].

[43] S. Shivashankara, W. Wu and A. Datta, *$\Lambda_b \to \Lambda_c\tau\bar{\nu}_\tau$ Decay in the Standard Model and with New Physics*, Phys. Rev. D **91** (2015) 115003 [arXiv:1502.07230] [INSPIRE].

[44] X.-Q. Li, Y.-D. Yang and X. Zhang, *$\mathit{\Lambda}_b \to \mathit{\Lambda}_c\tau\bar{\nu}_\tau$ decay in scalar and vector leptoquark scenarios*, JHEP **02** (2017) 068 [arXiv:1611.01635] [INSPIRE].




JHEP11(2020)032

[45] A. Datta, S. Kamali, S. Meinel and A. Rashed, *Phenomenology of $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ using lattice QCD calculations*, *JHEP* **08** (2017) 131 [arXiv:1702.02243] [INSPIRE].

[46] E. Di Salvo, F. Fontanelli and Z.J. Ajaltouni, *Detailed Study of the Decay $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$*, *Int. J. Mod. Phys. A* **33** (2018) 1850169 [arXiv:1804.05592] [INSPIRE].

[47] A. Ray, S. Sahoo and R. Mohanta, *Probing new physics in semileptonic $\Lambda_b$ decays*, *Phys. Rev. D* **99** (2019) 015015 [arXiv:1812.08314] [INSPIRE].

[48] M.B. Voloshin, *Bound on $V + A$ admixture in the $b \to c$ current from inclusive versus exclusive semileptonic decays of B mesons*, *Mod. Phys. Lett. A* **12** (1997) 1823 [hep-ph/9704278] [INSPIRE].

[49] B.M. Dassinger, R. Feger and T. Mannel, *Testing the left-handedness of the $b \to c$ transition*, *Phys. Rev. D* **75** (2007) 095007 [hep-ph/0701054] [INSPIRE].

[50] B. Dassinger, R. Feger and T. Mannel, *Complete Michel Parameter Analysis of inclusive semileptonic $b \to c$ transition*, *Phys. Rev. D* **79** (2009) 075015 [arXiv:0803.3561] [INSPIRE].

[51] A. Crivellin, *Effects of right-handed charged currents on the determinations of $|V(ub)|$ and $|V(cb)|$*, *Phys. Rev. D* **81** (2010) 031301 [arXiv:0907.2461] [INSPIRE].

[52] A.J. Buras, K. Gemmler and G. Isidori, *Quark flavour mixing with right-handed currents: an effective theory approach*, *Nucl. Phys. B* **843** (2011) 107 [arXiv:1007.1993] [INSPIRE].

[53] R. Feger, T. Mannel, V. Klose, H. Lacker and T. Luck, *Limit on a Right-Handed Admixture to the Weak $b \to c$ Current from Semileptonic Decays*, *Phys. Rev. D* **82** (2010) 073002 [arXiv:1003.4022] [INSPIRE].

[54] F.U. Bernlochner, Z. Ligeti and S. Turczyk, *New ways to search for right-handed current in $B \to \rho \ell \bar{\nu}$ decay*, *Phys. Rev. D* **90** (2014) 094003 [arXiv:1408.2516] [INSPIRE].

[55] W. Buchmüller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, *Nucl. Phys. B* **268** (1986) 621 [INSPIRE].

[56] V. Cirigliano, J. Jenkins and M. Gonzalez-Alonso, *Semileptonic decays of light quarks beyond the Standard Model*, *Nucl. Phys. B* **830** (2010) 95 [arXiv:0908.1754] [INSPIRE].

[57] J. Aebischer and J. Kumar, *Flavour Violating Effects of Yukawa Running in SMEFT*, *JHEP* **09** (2020) 187 [arXiv:2005.12283] [INSPIRE].

[58] R. Alonso, B. Grinstein and J. Martin Camalich, *Lepton universality violation and lepton flavor conservation in B-meson decays*, *JHEP* **10** (2015) 184 [arXiv:1505.05164] [INSPIRE].

[59] R.-X. Shi, L.-S. Geng, B. Grinstein, S. Jäger and J. Martin Camalich, *Revisiting the new-physics interpretation of the $b \to c\tau\nu$ data*, *JHEP* **12** (2019) 065 [arXiv:1905.08498] [INSPIRE].

[60] F. Feruglio, P. Paradisi and O. Sumensari, *Implications of scalar and tensor explanations of $R_{D^{(*)}}$*, *JHEP* **11** (2018) 191 [arXiv:1806.10155] [INSPIRE].

[61] J. Aebischer, J. Kumar, P. Stangl and D.M. Straub, *A Global Likelihood for Precision Constraints and Flavour Anomalies*, *Eur. Phys. J. C* **79** (2019) 509 [arXiv:1810.07698] [INSPIRE].

[62] V. Bernard, M. Oertel, E. Passemar and J. Stern, *$K(\mu 3)^L$ decay: A stringent test of right-handed quark currents*, *Phys. Lett. B* **638** (2006) 480 [hep-ph/0603202] [INSPIRE].



JHEP11(2020)032

[63] S. Alioli, V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, *Right-handed charged currents in the era of the Large Hadron Collider*, JHEP **05** (2017) 086 [arXiv:1703.04751] [INSPIRE].

[64] B.M. Dassinger, T. Mannel and S. Turczyk, *Inclusive semi-leptonic B decays to order $1/m_b^4$*, JHEP **03** (2007) 087 [hep-ph/0611168] [INSPIRE].

[65] T. Mannel, S. Turczyk and N. Uraltsev, *Higher Order Power Corrections in Inclusive B Decays*, JHEP **11** (2010) 109 [arXiv:1009.4622] [INSPIRE].

[66] T. Mannel, *Higher order 1/m corrections at zero recoil*, Phys. Rev. D **50** (1994) 428 [hep-ph/9403249] [INSPIRE].

[67] Y. Grossman and Z. Ligeti, *The inclusive $\bar{B} \to \tau\bar{\nu}X$ decay in two Higgs doublet models*, Phys. Lett. B **332** (1994) 373 [hep-ph/9403376] [INSPIRE].

[68] A.V. Manohar and M.B. Wise, *Inclusive semileptonic B and polarized $\Lambda_b$ decays from QCD*, Phys. Rev. D **49** (1994) 1310 [hep-ph/9308246] [INSPIRE].

[69] S. Balk, J.G. Korner and D. Pirjol, *Inclusive semileptonic decays of polarized $\Lambda_b$ baryons into polarized $\tau$ leptons*, Eur. Phys. J. C **1** (1998) 221 [hep-ph/9703344] [INSPIRE].

[70] I.I.Y. Bigi and N.G. Uraltsev, *Weak annihilation and the endpoint spectrum in semileptonic B decays*, Nucl. Phys. B **423** (1994) 33 [hep-ph/9310285] [INSPIRE].

[71] M. Jezabek and L. Motyka, *Perturbative QCD corrections to inclusive lepton distributions from semileptonic $b \to c\tau\bar{\nu}_\tau$ decays*, Acta Phys. Polon. B **27** (1996) 3603 [hep-ph/9609352] [INSPIRE].

[72] M. Gremm, G. Kopp and L.M. Sehgal, *$\tau$ polarization in $\Lambda_b \to X_c\tau\bar{\nu}$ and $B \to X_c\tau\bar{\nu}$*, Phys. Rev. D **52** (1995) 1588 [hep-ph/9502207] [INSPIRE].

[73] A. Czarnecki, M. Jezabek and J.H. Kühn, *Radiative corrections to $b \to c\tau\bar{\nu}_\tau$*, Phys. Lett. B **346** (1995) 335 [hep-ph/9411282] [INSPIRE].

[74] M. Jezabek and L. Motyka, *Tau lepton distributions in semileptonic B decays*, Nucl. Phys. B **501** (1997) 207 [hep-ph/9701358] [INSPIRE].

[75] F. De Fazio and M. Neubert, *$B \to X_u\ell\bar{\nu}_\ell$ decay distributions to order $\alpha_s$*, JHEP **06** (1999) 017 [hep-ph/9905351] [INSPIRE].

[76] M. Trott, *Improving extractions of $|V(cb)|$ and $m(b)$ from the hadronic invariant mass moments of semileptonic inclusive B decay*, Phys. Rev. D **70** (2004) 073003 [hep-ph/0402120] [INSPIRE].

[77] V. Aquila, P. Gambino, G. Ridolfi and N. Uraltsev, *Perturbative corrections to semileptonic b decay distributions*, Nucl. Phys. B **719** (2005) 77 [hep-ph/0503083] [INSPIRE].

[78] A. Alberti, P. Gambino, K.J. Healey and S. Nandi, *Precision Determination of the Cabibbo-Kobayashi-Maskawa Element $V_{cb}$*, Phys. Rev. Lett. **114** (2015) 061802 [arXiv:1411.6560] [INSPIRE].

[79] Particle Data Group collaboration, *Review of Particle Physics*, Prog. Theor. Exp. Phys. **2020** (2020) 083501.

[80] P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver, *On the b quark kinetic energy in $\Lambda_b$*, Phys. Rev. D **54** (1996) 4622 [hep-ph/9512334] [INSPIRE].

[81] I.I.Y. Bigi, *The QCD perspective on lifetimes of heavy flavor hadrons*, hep-ph/9508408 [INSPIRE].

[82] P. Colangelo, F. De Fazio and F. Loparco, *Probing New Physics with $\bar{B} \to \rho(770)\,\ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(1260)\,\ell^- \bar{\nu}_\ell$*, Phys. Rev. D **100** (2019) 075037 [arXiv:1906.07068] [INSPIRE].

[83] S. Biswas and K. Melnikov, *Second order QCD corrections to inclusive semileptonic $b \to X(c) l \bar{\nu}_l$ decays with massless and massive lepton*, JHEP **02** (2010) 089 [arXiv:0911.4142] [INSPIRE].

[84] T. Mannel and K.K. Vos, *Reparametrization Invariance and Partial Re-Summations of the Heavy Quark Expansion*, JHEP **06** (2018) 115 [arXiv:1802.09409] [INSPIRE].

JHEP11(2020)032