Non-existence of supersymmetric AdS_5 black rings

J. Grover,[a] **J. Gutowski**[b] **and W. A. Sabra**[c]

[a] *Physics Department University of Aveiro and I3N*
Campus de Santiago, 3810-183 Aveiro, Portugal

[b] *Department of Mathematics University of Surrey,*
Guildford, GU2 7XH, U.K.

[c] *Centre for Advanced Mathematical Sciences and Physics Department,*
American University of Beirut, Lebanon

E-mail: jai@ua.pt, j.gutowski@surrey.ac.uk, ws00@aub.edu.lb

ABSTRACT: It has been proven in arXiv:1303.0853 that all regular supersymmetric near-horizon geometries in minimal five-dimensional gauged supergravity admit automatic supersymmetry enhancement. Using this result, the integrability conditions associated with the existence of the additional supersymmetry are analysed, and the near-horizon geometries are determined explicitly. We show that they all correspond to previously constructed examples. Hence, there are no supersymmetric black ring solutions in minimal five-dimensional gauged supergravity.

Contents

1 Introduction

The study of supersymmetric gravitational solutions has many applications in string theory, in establishing the uniqueness or non-uniqueness of solutions, and in the discovery of new exotic solutions in several supergravity theories. An example of exotic solutions are the black rings of five dimensional ungauged supergravity theories [1–3]. In our present work we will be mainly concerned with minimal $N = 2$, $D = 5$ gauged supergravity. In this theory, supersymmetric solutions can in principle preserve 1/4, 1/2, 3/4 or the maximal proportion of supersymmetry.

Examples of regular asymptotically AdS_5 black holes preserving 1/4 supersymmetry are found in [4, 5]. These solutions were later generalized in [6–8]. 1/4-supersymmetric string solutions have also been constructed in [9, 10]. In [11] a systematic classification of all 1/4-supersymmetric solutions of minimal gauged $N = 2$, $D = 5$ supergravity was performed, this was later extended to more general theories with Abelian vector multiplets [12]. Examples of 1/2-supersymmetric solutions, corresponding to domain walls and black holes without regular horizons, were constructed in [13–16]. In [17], it was shown that all 3/4-supersymmetric solutions are locally AdS_5. However, globally one can have discrete

quotients of AdS_5 as 3/4-supersymmetric configurations [18]. The unique maximally supersymmetric solution preserving all supersymmetries is AdS_5. Systematic classifications of 1/2-supersymmetric solutions were investigated in [19, 20] using spinorial geometry techniques first implemented in the study of higher dimensional solutions in [21–23]. Using these methods, a restricted class of half-supersymmetric near-horizon geometries was analysed in [24].

An interesting phenomenon in supersymmetric black hole physics is the enhancement of supersymmetry for near-horizon geometries. This has been established for many supersymmetric black hole and brane solutions. It is believed that it may hold for all supersymmetric black hole solutions, at least in theories for which there are no higher order curvature corrections. Clearly supersymmetry enhancement puts more restrictions on the topology and geometry of horizons and thus may lead to an explicit determination of the space-time metric. Recently, it was demonstrated in [25] that under some smoothness assumptions, supersymmetric near-horizon black hole geometries of minimal 5-dimensional gauged supergravity preserve at least half of the supersymmetry. Near-horizon geometries with more than half of the supersymmetries preserved are locally isomorphic to AdS_5, with vanishing Maxwell field strengths.

The analysis presented in this paper is more general than that carried out in [24]. This is because in [24] it was assumed that the event horizon corresponds to a null hypersurface for *all* the Killing vectors obtained from Killing spinor bilinears. Here, we shall not make this assumption. We assume that the event horizon is a null hypersurface associated with one of the Killing vectors obtained from one of the Killing spinors, but is not necessarily a null hypersurface with respect to all of the Killing vectors obtained from all of the Killing spinors. In particular, we shall be interested in the case for which one of the Killing spinors exists not only for the near-horizon geometry, but can be extended to give a Killing spinor for the bulk solution, whereas the extra Killing spinor of the near-horizon solution need not extend to a bulk Killing spinor. We assume that in the near-horizon solution, the event horizon corresponds to a null hypersurface associated with the Killing vector obtained from the Killing spinor which extends out to the bulk. However, we shall not assume, as was done previously, that the extra Killing spinor produces a Killing vector which also becomes null at the event horizon.

In this paper, using the results of [25] and employing the integrability conditions associated with the extra supersymmetry, the near-horizon geometries with enhanced supersymmetry are determined explicitly. We shall show that all such solutions correspond to previously known examples. In particular, in [26], all supersymmetric near-horizon geometries were found, assuming the existence of two commuting rotational isometries. Here we shall show that the additional conditions on the geometry obtained from the (automatic) supersymmetry enhancement are sufficient to produce these isometries. Thus it can be established that there are no supersymmetric black ring solutions in minimal five-dimensional gauged supergravity.

We organise our work as follows. In section two, we summarize the results of [4] where conditions for 1/4 supersymmetric near-horizon geometries in minimal gauged $D = 5$ supergravity were obtained. In section three, using the results of [25], we analyse the necessary

conditions for the enhancement of supersymmetry at the horizon. These conditions are sufficient to fix the near-horizon geometry completely, and they all correspond to solutions which are already known. We make use of the the Killing spinor equations, which are analysed using spinorial geometry techniques presented in appendix A, and the associated integrability conditions are presented in appendix B.

2 The 1/4-supersymmetric near-horizon solution

In this section, we briefly summarize the conditions on the event horizon of a regular black hole in minimal gauged $D = 5$ supergravity which are imposed by requiring that the solution should preserve $1/4$ of the supersymmetry [4]. This is the minimal amount of supersymmetry which supersymmetric solutions of this theory can preserve.

The near-horizon metric is given by

$$ds^2 = -r^2\Delta^2 du^2 + 2dudr + 2rhdu + ds_{\mathcal{S}}^2 . \tag{2.1}$$

The horizon is at $r = 0$, $ds_{\mathcal{S}}^2$ is the metric on the (compact) and regular spatial cross-section of the horizon, which does not depend on u or r; h is a 1-form on $\mathcal{S}$ which does not depend on u or r; and Δ is a function which does not depend on u or r.

In the notation of [4], a (real) basis for $\mathcal{S}$ is given by Z^1, Z^2, Z^3. The covariant derivatives of the basis elements are fixed by

$$\nabla_A Z_B^i = -\frac{\Delta}{2}(\star Z^i)_{AB} + \gamma_{AB}\big(N \cdot Z^i + \frac{1}{\ell}\delta_{1i}\big) - Z_A^i N_B - \frac{Z_A^i Z_B^1}{\ell} + 2\sqrt{3}\ell^{-1}\epsilon_{1ij}a_A Z_B^j, \tag{2.2}$$

where $\star$ denotes the Hodge dual on $\mathcal{S}$ with positive orientation defined by $Z^1 \wedge Z^2 \wedge Z^3$, and

$$N = h + \frac{2}{\ell}Z^1 , \tag{2.3}$$

and a is a u, r-independent 1-form on $\mathcal{S}$ satisfying

$$da = -\frac{\sqrt{3}}{2} \star N . \tag{2.4}$$

The 2-form flux is given by

$$F = -\frac{\sqrt{3}}{2}\big(du \wedge d(r\Delta) + \star N\big) \tag{2.5}$$

and the Bianchi identity implies that

$$d \star N = 0 , \tag{2.6}$$

and N also satisfies

$$dN = 2\ell^{-1}\Delta \star_3 Z^1 + 2\ell^{-1}N \wedge Z^1 + \Delta \star N + \star d\Delta . \tag{2.7}$$

These conditions constrain the Ricci tensor of $\mathcal{S}$ to be

$$R_{AB} = \gamma_{AB}\left(\frac{\Delta^2}{2} + N^2 + \frac{2}{\ell}(N \cdot Z^1) - \frac{2}{\ell^2}\right) - \nabla_{(A}N_{B)} - \frac{4}{\ell}Z_{(A}^1 N_{B)} - N_A N_B . \tag{2.8}$$

For convenience, we shall set $Z = Z^1$.

2.1 Solutions with $N = 0$

Before proceeding to examine necessary conditions for enhanced supersymmetry, it is useful to consider a special class of solutions for which $N = 0$ in some open set. These have been analysed previously in [4]; we again summarize the analysis here for convenience. If $N = 0$, then (2.8) implies that

$$R_{AB} = (\frac{1}{2}\Delta^2 - 2\ell^{-2})\gamma_{AB} \tag{2.9}$$

and hence the curvature Bianchi identity implies that $d\Delta^2 = 0$, and together with (2.7) this implies that $\Delta = 0$. Then, (2.2) implies that one can introduce local coordinates x, y, z on $\mathcal{S}$ such that

$$Z = dx \tag{2.10}$$

and

$$ds_{\mathcal{S}}^2 = dx^2 + e^{\frac{2x}{\ell}}(dy^2 + dz^2) \tag{2.11}$$

so that the five-dimensional spacetime metric is

$$ds^2 = 2dudr - \frac{4}{\ell}rdxdu + dx^2 + e^{\frac{2x}{\ell}}(dy^2 + dz^2) \ . \tag{2.12}$$

Hence, the spacetime is locally isometric to AdS_5. We have remarked previously that there exist discrete quotients of AdS_5 which are 3/4-supersymmetric. There may also be quotients of AdS_5 which preserve less supersymmetry. In the following analysis, we shall be concerned with geometries other than AdS_5 or its quotients.

3 Analysis of supersymmetry

To proceed, we shall consider some necessary conditions imposed by the enhancement of supersymmetry, which follows as a consequence of the analysis in [25]. We will demonstrate that these conditions can be used to completely determine the near-horizon geometries. The analysis of the Killing spinor equations is carried out in appendix A, using spinorial geometry techniques, and their associated integrability conditions are investigated in appendix B. We exclude the solutions which are (locally) AdS_5 with $F = 0$.

We begin by considering the conditions (B.1)–(B.7). There are two possibilities; either Δ is not identically zero, or $\Delta = 0$ everywhere on $\mathcal{S}$. We shall consider these two cases separately. When Δ is not identically zero, we restrict to a patch on which, without loss of generality, $\Delta > 0$. If Δ is analytic on the horizon then it was shown in [26] that either $\Delta = 0$ everywhere on the horizon, or $\Delta > 0$ everywhere on the horizon. However, in what follows it suffices to assume that Δ is smooth.

3.1 Solutions with $\Delta \neq 0$

We first consider the case for which $\Delta > 0$ on some patch in $\mathcal{S}$. Let P be some point in $\mathcal{S}$; there are two sub-cases to consider. In the first, $N \neq 0$ at P. Then the condition (B.1) is non-trivial. So, for solutions with supersymmetry enhancement, it is clear that the l.h.s. of each of the conditions (B.2)–(B.7) must be proportional to the l.h.s. of the condition (B.1). Hence we obtain six conditions which are necessary and sufficient for the integrability conditions to have a non-trivial solution for λ_-^1, μ, other than that corresponding to AdS_5. Each of these six conditions come from requiring that the determinant of six 2×2 matrices vanish, the first row of the matrices coming from the the l.h.s. of (B.1) and the second rows coming from the l.h.s. of each of (B.2)–(B.7). A detailed analysis of these conditions shows that they are equivalent to imposing

$$\nabla_{(i} N_{j)} = \frac{2\ell^{-1}}{\Delta^2 + N^2}\bigg(\Delta(N_i \star_3 (Z \wedge N)_j + N_j \star_3 (Z \wedge N)_i)$$
$$+ 2(N.Z)N_i N_j - N^2 Z_i N_j - N^2 Z_j N_i\bigg) \tag{3.1}$$

and

$$d\Delta = \frac{4\Delta\ell^{-1}}{\Delta^2 + N^2}\bigg(\Delta \star_3 (Z \wedge N) - N^2 Z + (N.Z)N\bigg) . \tag{3.2}$$

In the second case, $N = 0$ at P. Then in order for there to be enhanced supersymmetry, (B.1) implies that $\lambda_-^1 = 0$, $\mu \neq 0$ at P; (B.2) and (B.3) imply that $d\Delta = 0$ at P, and (B.4)–(B.7) imply that $\nabla_{(i} N_{j)} = 0$ at P. Hence, (3.1) and (3.2) hold at points for which $N = 0$ as well.

In particular, the conditions (3.1) and (3.2) imply that

$$\Delta^{-\frac{3}{2}} N^2 + \Delta^{\frac{1}{2}} = \nu_1 \tag{3.3}$$

and

$$\Delta^{-1}\bigg(N.Z - \frac{\ell}{2}(N^2 + \Delta^2)\bigg) = \nu_2 \tag{3.4}$$

for constants ν_1, ν_2.

We shall first consider the case for which $N^2 - (N.Z)^2 \neq 0$ in some patch. Then N and Z are linearly independent in this patch; the case for which N is proportional to Z will be considered later.

It is convenient to define

$$\hat{N} = -\Delta Z + \star(N \wedge Z) \tag{3.5}$$

and note that $N, \hat{N}$ are linearly independent. Consider the 1/4 supersymmetry conditions together with the enhancement conditions (3.1) and (3.2). It is straightforward to see that $\hat{N} = \star dZ$, so $\hat{N}$ is co-closed. Furthermore we find the conditions

$$\nabla_{(i}(\Delta^{-1} N)_{j)} = \nabla_{(i}(\Delta^{-1} \hat{N})_{j)} = 0 \tag{3.6}$$

so $\Delta^{-1}N$ and $\Delta^{-1}\hat{N}$ are Killing vectors, in fact they also commute with each other. Hence, we introduce co-ordinates y^1, y^2 on the horizon such that

$$V = \Delta^{-1}N = \frac{\partial}{\partial y^1}, \qquad W = \Delta^{-1}\hat{N} = \frac{\partial}{\partial y^2} \tag{3.7}$$

and note that

$$\mathcal{L}_V\Delta = \mathcal{L}_W\Delta = 0 \ . \tag{3.8}$$

In addition, as (3.2) implies that $d\Delta \neq 0$, one can take y^1, y^2, Δ as local co-ordinates on $\mathcal{S}$. One then obtains the horizon metric explicitly as:

$$\begin{aligned}
ds_{\mathcal{S}}^2 = &\frac{1}{16}\nu_1\ell^2\Delta^{-2}\big(\nu_1 - (1+\nu_2^2)\Delta^{\frac{1}{2}} - \nu_1\nu_2\ell\Delta - \frac{1}{4}\nu_1^2\ell^2\Delta^{\frac{3}{2}}\big)^{-1}(d\Delta)^2 \\
&+(\nu_1\Delta^{-\frac{1}{2}} - 1)(dy^1)^2 - 2(\nu_2 + \frac{1}{2}\nu_1\ell\Delta^{\frac{1}{2}})dy^1 dy^2 \\
&+\big(\nu_1\Delta^{-\frac{1}{2}} - \nu_2^2 - \nu_1\nu_2\ell\Delta^{\frac{1}{2}} - \frac{1}{4}\nu_1^2\ell^2\Delta\big)(dy^2)^2
\end{aligned} \tag{3.9}$$

with

$$Z = (\nu_2 + \frac{1}{2}\nu_1\ell\Delta^{\frac{1}{2}})dy^1 - dy^2 - \frac{1}{4}\ell\Delta^{-1}d\Delta \,, \tag{3.10}$$

ν_1 and ν_2 are the real constants which appear in the conditions (3.3), (3.4).

It is straightforward to show that this near-horizon data is identical to the near-horizon data for the Chong et al. solution presented in [6, 7, 26], in the case when $\Delta \neq 0$. To establish the correspondence, set

$$\begin{aligned}
\Delta &= \frac{\Delta_0}{\Gamma^2} \\
\nu_1 &= 4\ell^{-2}C^{-2}\Delta_0^{\frac{1}{2}} + C^2\alpha_0^2\Delta_0^{-\frac{3}{2}} \\
\nu_2 &= -\frac{1}{2}C^2\ell\alpha_0\Delta_0^{-1} \\
y^1 &= \frac{1}{4\Delta_0^2 + C^4\ell^2\alpha_0^2}\big(\ell^2 C^4\Delta_0\alpha_0 x^1 + 4\Delta_0^2 x^2\big) \\
y^2 &= \frac{1}{4\Delta_0^2 + C^4\ell^2\alpha_0^2}\big(2C^2\ell\Delta_0^2 x^1 - 2\ell C^2\alpha_0\Delta_0 x^2\big) \ .
\end{aligned} \tag{3.11}$$

Next, consider the special case for which $N^2 - (N.Z)^2 = 0$. Then $N = gZ$ for some function g. It is straightforward to see that (3.2) implies that Δ is constant. On substituting these conditions into (2.2) and (2.7) one finds that

$$N = -\ell^{-1}Z \tag{3.12}$$

and furthermore, an examination of the conditions and (2.2), (2.4) implies that one can without loss of generality choose Z^1, Z^2, Z^3 to satisfy

$$\begin{aligned}
dZ^1 &= -\Delta Z^2 \wedge Z^3 \\
dZ^2 &= \Delta(1 - 3\ell^{-2}\Delta^{-2})Z^1 \wedge Z^3 \\
dZ^3 &= -\Delta(1 - 3\ell^{-2}\Delta^{-2})Z^1 \wedge Z^2 \,,
\end{aligned} \tag{3.13}$$

and hence the metric on $\mathcal{S}$ is either that of the homogeneous metric on the Nil-manifold (when $\Delta = \sqrt{3}\ell^{-1}$), the homogeneous metric on the $\mathrm{SL}(2,\mathrm{R})$ group manifold (when $0 < \Delta < \sqrt{3}\ell^{-1}$), or the homogeneous metric on the $\mathrm{SU}(2)$ group manifold (when $\Delta > \sqrt{3}\ell^{-1}$). We remark that these metrics (and the corresponding spacetime geometries) have been previously derived in [4].

3.2 Solutions with $\Delta = 0$

Next we consider the special case of solutions with $\Delta = 0$. For solutions with $\Delta = 0$ on $\mathcal{S}$, consider some point $P \in \mathcal{S}$. There are again two sub-cases. In the first, $N \neq 0$ at P, and it is straightforward to show that the necessary and sufficient conditions for enhanced supersymmetry is (3.1) with $\Delta = 0$. In the second sub-case, for which $N = 0$ at P, the conditions (B.1)–(B.3) have no content, and (B.4)–(B.7) imply that $\nabla_{(i} N_{j)} = 0$ at P.

So, if $\Delta = 0$, then the necessary and sufficient conditions for enhanced supersymmetry are:

$$\nabla_{(i} N_{j)} = \begin{cases} \frac{2\ell^{-1}}{N^2}\Big(2(N.Z)N_i N_j - N^2 Z_i N_j - N^2 Z_j N_i \Big) & \text{if } N^2 \neq 0 \\ 0 & \text{if } N = 0 \, . \end{cases} \tag{3.14}$$

As we have assumed that N is not identically zero, consider some patch in which $N^2 \neq 0$. Then (3.14), together with the conditions in section 2, implies that

$$dN^2 = \frac{6}{\ell}\Big((N.Z)N - N^2 Z \Big) \tag{3.15}$$

and hence

$$\nabla^i \nabla_i N^2 + \frac{6}{\ell} Z^i \nabla_i N^2 + \frac{12}{\ell}((N.Z) + \ell^{-1})N^2 = 0 \, . \tag{3.16}$$

So, if N^2 is analytic on $\mathcal{S}$, then by the results of [26] it follows that N^2 must be nonzero everywhere on $\mathcal{S}$, because if N^2 were to vanish at some point in $\mathcal{S}$, then N must vanish identically. In what follows, it suffices to assume that $N^2 \neq 0$ in some patch on $\mathcal{S}$.

In addition, (3.14) also implies that

$$(N^2)^{-\frac{2}{3}}(N.Z) - \frac{\ell}{2}(N^2)^{\frac{1}{3}} = \nu \tag{3.17}$$

where ν is constant.

Again, we first consider solutions for which $N^2 - (N.Z)^2 \neq 0$, so that N and Z are linearly independent; the case for which N is proportional to Z will be considered later. It is convenient to define

$$\hat{N} = \star(N \wedge Z) \tag{3.18}$$

and note that N, $\hat{N}$ are linearly independent; and $\hat{N}$ is co-closed. Then it is straightforward to show that the conditions imposed by 1/4-supersymmetry, together with the supersymmetry enhancement condition (3.14) imply that

$$\nabla_{(i}((N^2)^{-\frac{2}{3}}N)_{j)} = \nabla_{(i}((N^2)^{-\frac{2}{3}}\hat{N})_{j)} = 0 \tag{3.19}$$

so $(N^2)^{-\frac{2}{3}}N$, $(N^2)^{-\frac{2}{3}}\hat{N}$ are Killing; and furthermore, the Killing vectors commute.

Hence, we introduce co-ordinates y^1, y^2 on the horizon such that

$$V = (N^2)^{-\frac{2}{3}} N = \frac{\partial}{\partial y^1}, \qquad W = (N^2)^{-\frac{2}{3}} \hat{N} = \frac{\partial}{\partial y^2} \ . \tag{3.20}$$

In addition, (3.14) implies that

$$\mathcal{L}_V N^2 = \mathcal{L}_W N^2 = 0 \tag{3.21}$$

and $dN^2 \neq 0$. We will therefore also use N^2 as a co-ordinate on the horizon.

It is then straightforward to show that, in the co-ordinates y^1, y^2, N^2, the metric on the horizon is

$$ds^2_{\mathcal{S}} = \frac{1}{36} \ell^2 (N^2)^{-2} \big(1 - \frac{1}{4} \ell^2 N^2 - \ell\nu(N^2)^{\frac{2}{3}} - \nu^2(N^2)^{\frac{1}{3}} \big)^{-1} (dN^2)^2$$
$$+ (N^2)^{-\frac{1}{3}} (dy^1)^2 + (N^2)^{-\frac{1}{3}} \big(1 - \frac{1}{4} \ell^2 N^2 - \ell\nu(N^2)^{\frac{2}{3}} - \nu^2(N^2)^{\frac{1}{3}} \big) (dy^2)^2 \tag{3.22}$$

with

$$Z = (\nu + \frac{1}{2} \ell(N^2)^{\frac{1}{3}}) dy^1 - \frac{1}{6} \ell(N^2)^{-1} dN^2 \,, \tag{3.23}$$

ν is the real constant which appears in the condition (3.17).

It is straightforward to show that this near-horizon data is identical to that corresponding to the solution presented in [26], in the case when $\Delta_0 = 0$, and $\alpha_0 \neq 0$. To establish the correspondence, set

$$\Gamma = C^{\frac{2}{3}} \alpha_0^{\frac{2}{3}} (N^2)^{-\frac{1}{3}}$$
$$\nu = -\frac{1}{2} C^{\frac{2}{3}} \ell \alpha_0^{-\frac{1}{3}}$$
$$y^1 = C^{\frac{4}{3}} \alpha_0^{\frac{1}{3}} x^1$$
$$y^2 = -2\ell^{-1} C^{-\frac{2}{3}} \alpha_0^{\frac{1}{3}} x^2 \ . \tag{3.24}$$

Finally, consider the special case when $N^2 - (N.Z)^2 = 0$, so $N = gZ$ for some function g. Note that (3.14) implies that N is a Killing vector. This condition, together with the conditions imposed by (2.2), implies that g is constrained to be constant, and on discarding the case $g = 0$ (which corresponds to AdS_5), one obtains

$$N = -\ell^{-1} Z \ . \tag{3.25}$$

Then the conditions (2.2) and (2.4) imply that one can choose a local co-ordinate x, y, z on $\mathcal{S}$ such that

$$Z = dx, \qquad ds^2_{\mathcal{S}} = dx^2 + \frac{\ell^2}{3y^2} (dy^2 + dz^2) \tag{3.26}$$

so that $ds^2_{\mathcal{S}}$ is the metric on $R \times H^2$. This solution was also previously found in [4].

4   Conclusions

We have analysed the integrability conditions obtained from the Killing spinor equations for near-horizon geometries in minimal five-dimensional gauged supergravity. By making use of the fact that near-horizon solutions other than AdS_5 must be exactly half-supersymmetric [25], we show that these integrability conditions are strong enough to completely determine the near-horizon geometries. We demonstrate that all of the near-horizon solutions correspond to examples which have already been obtained in [4] and [26]. It should be noted that although we have explicitly given the co-ordinate transformations required in order to recover the black hole near-horizon geometries obtained from [26] for the sake of completeness, this is not strictly necessary. This is because once one has obtained the two commuting isometries $\Delta^{-1}N, \Delta^{-1}\hat{N}$ and $(N^2)^{-\frac{2}{3}}N, (N^2)^{-\frac{2}{3}}\hat{N}$ in sections 3.1 and 3.2 respectively, the near-horizon geometries are then fixed using the results of [26].

As none of the near-horizon geometries contains a horizon section $\mathcal{S}$ which is topologically $S^1 \times S^2$, this implies that there are no regular supersymmetric asymptotically AdS_5 black rings in minimal five dimensional supergravity. The analogous result for pseudo-supersymmetric asymptotically dS_5 black rings in minimal five dimensional supergravity has already been established in [27]. In this case, all (pseudo)-supersymmetric solutions automatically preserve at least half of the supersymmetry in contrast to the theory with a negative cosmological constant, so the analysis was more straightforward.

The status of supersymmetric asymptotically AdS_5 or dS_5 black rings in non-minimal gauged supergravity, for example coupled to some abelian vector multiplets, remains undetermined. As we have proven that such solutions do not exist in the minimal theories, they must lie in a sector of the theory which does not admit a reduction to the minimal theory. It is known that there exist supersymmetric near-horizon geometries with horizon section $S^1 \times S^2$ in gauged supergravity with a negative cosmological constant, which cannot be reduced to a solution of the minimal theory, [28]. However, it is not yet known if a full black ring solution in this theory exists. Work on such solutions is in progress.

Acknowledgments

A   The Killing spinor equations

The Killing spinor equations adapted to a null basis have been computed in appendix B of [20], using spinorial geometry techniques. These were originally developed to classify solutions in ten and eleven dimensional supergravity theories, [21–23], and can also be adapted to the classification of supersymmetric black holes. We use the same conventions

for the Dirac Killing spinors, which are elements of the set of all complexified forms on R^2, spanned by $1, e_1, e_2$ and $e_{12} = e_1 \wedge e_2$. A generic Killing spinor is then written as

$$\epsilon = \lambda_+^1(1 + e_1) + \lambda_-^1(1 - e_1) + \lambda_+^{\bar{1}}(e_{12} - e_2) + \lambda_-^{\bar{1}}(e_{12} + e_2) \, . \tag{A.1}$$

We work with a basis in which the metric is:

$$ds^2 = -2\mathbf{e}^+\mathbf{e}^- + (\mathbf{e}^1)^2 + 2\mathbf{e}^2\mathbf{e}^{\bar{2}} \tag{A.2}$$

where $\mathbf{e}^+, \mathbf{e}^-, \mathbf{e}^1$ are real, and $\mathbf{e}^2$, $\mathbf{e}^{\bar{2}}$ are complex conjugate. We shall investigate the supersymmetry of regular near-horizon geometries using the conventions set out in [4]. In particular, note that in order to rewrite the Killing spinor equations computed in [20] in terms of the conventions adopted in [4], we make the following replacements:

$$\chi \to \frac{2\sqrt{3}}{\ell}, \qquad \chi V_I X^I \to \frac{1}{\ell}, \qquad H \to \frac{2}{\sqrt{3}}F, \qquad \chi A \to \frac{2}{\sqrt{3}\ell}A \tag{A.3}$$

and we also re-label the basis indices as

$$1 \to 2, \qquad \bar{1} \to \bar{2}, \qquad 2 \to 1 \tag{A.4}$$

and make a sign change to the spin connection

$$\omega_{\mu_1,\mu_2\mu_3} \to -\omega_{\mu_1,\mu_2\mu_3} \tag{A.5}$$

due to the signature difference between [20] and [4].

In order to evaluate the Killing spinor equations acting on a generic spinor (A.1) in the background of a 1/4-supersymmetric near-horizon geometry, we define the following null basis:

$$\begin{aligned}
\mathbf{e}^+ &= -du \\
\mathbf{e}^- &= dr + rh - \frac{1}{2}r^2\Delta^2 du \\
\mathbf{e}^1 &= Z \\
\mathbf{e}^2 &= \frac{1}{\sqrt{2}}(Z^2 + iZ^3) \\
\mathbf{e}^{\bar{2}} &= \frac{1}{\sqrt{2}}(Z^2 - iZ^3) \, .
\end{aligned} \tag{A.6}$$

It is then straightforward to evaluate all the components of the Killing spinor equation, making use of the conditions on the near-horizon solutions found in [4] and summarized in section 2. We analyse the Killing spinor equation, first by integrating the + and the − components to solve for the Killing spinor, and then by evaluating the integrability conditions associated with the remaining three components.

A.1 Analysis of $-$ and $+$ components

We first analyse the $-$ component. This component implies that

$$\partial_-\lambda^1_+ = 0, \qquad \partial_-\lambda^{\bar{1}}_+ = 0 \tag{A.7}$$

and

$$\partial_-\lambda^1_- + (-\frac{1}{\sqrt{2}}\Delta - \sqrt{2}i\ell^{-1})\lambda^1_+ = 0$$
$$\partial_-\lambda^{\bar{1}}_- - \frac{1}{\sqrt{2}}\Delta\lambda^{\bar{1}}_+ = 0 . \tag{A.8}$$

Hence λ^1_+, $\lambda^{\bar{1}}_+$ are independent of r, and

$$\lambda^1_- = r(\frac{1}{\sqrt{2}}\Delta + \sqrt{2}i\ell^{-1})\lambda^1_+ + \mu^1_-$$
$$\lambda^{\bar{1}}_- = \frac{r\Delta}{\sqrt{2}}\lambda^{\bar{1}}_+ + \mu^{\bar{1}}_- \tag{A.9}$$

where μ^1_-, $\mu^{\bar{1}}_-$ are also independent of r.

Next, we analyse the $+$ component. On substituting the above conditions into this component, and expanding out the resulting expressions in powers of r we find:

$$-\partial_u\lambda^1_+ + (-\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta - \sqrt{2}i\ell^{-1})\mu^1_- + ih_2\mu^{\bar{1}}_- = 0$$
$$-\partial_u\lambda^{\bar{1}}_+ + ih_{\bar{2}}\mu^1_- + (\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta)\mu^{\bar{1}}_- = 0 \tag{A.10}$$

together with the algebraic conditions

$$(\ell^{-1}h_1 + \frac{3i\Delta}{\ell} + 2\ell^{-2})\lambda^1_+ = 0, \qquad h_{\bar{2}}\lambda^1_+ = 0 . \tag{A.11}$$

If $\lambda^1_+ \neq 0$, then the above algebraic conditions imply that $h_2 = 0$, $\Delta = 0$ and $h_1 = -2\ell^{-1}$. The solution is then simply AdS_5 with $F = 0$. As we are interested in solutions other than this, we shall henceforth set $\lambda^1_+ = 0$. Also note that this implies that λ^1_- is independent of r.

On substituting these conditions back into the $+$ component of the Killing spinor equations we then find the conditions

$$\partial_u\lambda^1_- = 0, \qquad \partial_u\mu^{\bar{1}}_- = 0 \tag{A.12}$$

together with

$$(i\partial_1\Delta - \frac{i}{2}\Delta h_1 + \frac{3i\Delta}{\ell} + \frac{1}{2}\Delta^2)\lambda^1_- + (-\sqrt{2}i\partial_2\Delta + \frac{i}{\sqrt{2}}\Delta h_2)\mu^{\bar{1}}_- = 0$$
$$\sqrt{2}\partial_{\bar{2}}\Delta\lambda^1_- + \partial_1\Delta\mu^{\bar{1}}_- = 0 . \tag{A.13}$$

A.2 Analysis of remaining components

Combining all of the previous conditions, and integrating them up, we find that either the solution is AdS_5, or

$$\lambda^1_+ = 0$$

$$\lambda^{\bar{1}}_+ = u\big(ih_{\bar{2}}\lambda^1_- + (\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta)\mu\big) + \sigma$$

$$\lambda^{\bar{1}}_- = \frac{r\Delta}{\sqrt{2}}\left(u\big(ih_{\bar{2}}\lambda^1_- + (\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta)\mu\big) + \sigma\right) + \mu \tag{A.14}$$

where λ^1_-, μ, σ are independent of r and u. We also find the following algebraic conditions on λ^1_-, μ:

$$(-\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta - \sqrt{2}i\ell^{-1})\lambda^1_- + ih_2\mu = 0 \tag{A.15}$$

$$(i\partial_1\Delta - \frac{i}{2}\Delta h_1 + \frac{3i\Delta}{\ell} + \frac{1}{2}\Delta^2)\lambda^1_- + (-\sqrt{2}i\partial_2\Delta + \frac{i}{\sqrt{2}}\Delta h_2)\mu = 0 \tag{A.16}$$

$$\sqrt{2}\partial_{\bar{2}}\Delta\lambda^1_- + \partial_1\Delta\mu = 0 . \tag{A.17}$$

Note in particular, that if $\lambda^1_- = \mu = 0$ then the set of solutions in this class can be at most 1/4-supersymmetric. As we are interested in solutions preserving exactly 1/2 of the supersymmetry, we discard this case.

A.2.1 Analysis of 1 component

This component is equivalent to

$$\partial_1\lambda^{\bar{1}}_+ = 0 \tag{A.18}$$

and

$$\partial_1\lambda^1_- = \sqrt{2}h_2\mu + (\frac{2\sqrt{3}i}{\ell}a_1 + \frac{1}{\ell})\lambda^1_-$$

$$\partial_1\mu = -\sqrt{2}h_{\bar{2}}\lambda^1_- . \tag{A.19}$$

On substituting (A.19) into (A.18), and making use of the previous conditions, we find

$$\partial_1\sigma = 0 \tag{A.20}$$

together with

$$\big(i\nabla_1 h_{\bar{2}} + (-\frac{1}{2}\Delta + \frac{i}{\ell})h_{\bar{2}}\big)\lambda^1_- + \big(\frac{i}{\sqrt{2}}\nabla_1 h_1 + \frac{1}{\sqrt{2}}\partial_1\Delta\big)\mu = 0 \tag{A.21}$$

where here ∇ denotes the Levi-Civita connection on the horizon.

A.2.2 Analysis of 2 and $\bar{2}$ components

These components are equivalent to:

$$\partial_2 \lambda_+^{\bar{1}} = 0, \qquad \partial_{\bar{2}} \lambda_+^{\bar{1}} = 0 \tag{A.22}$$

and

$$\begin{aligned}
\partial_2 \lambda_-^1 &= (h_2 + \frac{2\sqrt{3}i}{\ell} a_2)\lambda_-^1 \\
\partial_2 \mu &= (\sqrt{2} h_1 + 2\sqrt{2}\ell^{-1})\lambda_-^1 - h_2 \mu \\
\partial_{\bar{2}} \lambda_-^1 &= (-h_{\bar{2}} + \frac{2\sqrt{3}i}{\ell} a_{\bar{2}})\lambda_-^1 - (\sqrt{2} h_1 + 3\sqrt{2}\ell^{-1})\mu \\
\partial_{\bar{2}} \mu &= h_{\bar{2}} \mu \, .
\end{aligned} \tag{A.23}$$

On substituting (A.23) into (A.22) and making use of the previous conditions, we find

$$\partial_2 \sigma = 0, \qquad \partial_{\bar{2}} \sigma = 0 \tag{A.24}$$

and

$$\begin{aligned}
&\left(i \nabla_2 h_{\bar{2}} + \frac{1}{2}\Delta h_1 - \frac{i}{\ell} h_1 + 2\Delta \ell^{-1}\right)\lambda_-^1 \\
&+ \left(\frac{i}{\sqrt{2}} \nabla_2 h_1 + h_2\left(-\frac{1}{2\sqrt{2}}\Delta + \frac{3i}{\sqrt{2}\ell}\right) + \frac{1}{\sqrt{2}}\partial_2 \Delta\right)\mu = 0
\end{aligned} \tag{A.25}$$

and

$$i \nabla_{\bar{2}} h_{\bar{2}} \lambda_-^1 + \left(\frac{i}{\sqrt{2}} \nabla_{\bar{2}} h_1 + \frac{1}{\sqrt{2}}\partial_{\bar{2}}\Delta + \frac{1}{2\sqrt{2}}\Delta h_{\bar{2}} - \frac{3}{\sqrt{2}} i\ell^{-1} h_{\bar{2}}\right)\mu = 0 \, . \tag{A.26}$$

We remark that it is straightforward to show that the solutions preserve at least $1/4$ of the supersymmetry, because all of the conditions are satisfied by setting $\lambda_-^1 = \mu = 0$ and taking σ to be a non-zero complex constant. For the simplest, supersymmetric AdS_5 regular black hole solution first constructed in [4], it is also straightforward to see explicitly how the supersymmetry is enhanced from $1/4$ to $1/2$. An additional spinor is obtained by setting $\lambda_-^1 = \sigma = 0$ and taking μ to be a complex constant, together with Δ constant and $h = -\frac{3}{\ell} Z$.

B The integrability conditions

It is then straightforward to evaluate the integrability conditions associated with the conditions (A.19) and (A.23). On combining these with the algebraic conditions (A.15), (A.16), (A.17), (A.21), (A.25), (A.26), and removing those conditions which are linearly dependent,

one obtains the following conditions:

$$\left(-\frac{i}{\sqrt{2}}h_1 + \frac{1}{\sqrt{2}}\Delta - \sqrt{2}i\ell^{-1}\right)\lambda_-^1 + ih_2\mu = 0 \tag{B.1}$$

$$(\partial_1\Delta + 4\ell^{-1}\Delta)\lambda_-^1 - \sqrt{2}\partial_2\Delta\mu = 0 \tag{B.2}$$

$$\sqrt{2}\partial_{\bar{2}}\Delta\lambda_-^1 + \partial_1\Delta\mu = 0 \tag{B.3}$$

$$(i\psi_{1\bar{2}} + i\ell^{-1}h_{\bar{2}})\lambda_-^1 + \left(\frac{i}{\sqrt{2}}\psi_{11} + \frac{1}{2\sqrt{2}}\partial_1\Delta\right)\mu = 0 \tag{B.4}$$

$$(i\psi_{2\bar{2}} + i\ell^{-1}h_1 + \frac{1}{4}\partial_1\Delta + 4i\ell^{-2})\lambda_-^1 + \left(\frac{i}{\sqrt{2}}\psi_{12} - \frac{i}{\sqrt{2}}\ell^{-1}h_2\right)\mu = 0 \tag{B.5}$$

$$i\psi_{\bar{2}\bar{2}}\lambda_-^1 + \left(\frac{i}{\sqrt{2}}\psi_{1\bar{2}} + \frac{1}{2\sqrt{2}}\partial_{\bar{2}}\Delta - \frac{3}{\sqrt{2}}i\ell^{-1}h_{\bar{2}}\right)\mu = 0 \tag{B.6}$$

$$(\psi_{12} + \frac{i}{2}\partial_2\Delta + \ell^{-1}h_2)\lambda_-^1 - \sqrt{2}\psi_{22}\mu = 0 \tag{B.7}$$

where

$$\psi_{ij} = \nabla_{(i}h_{j)} \ . \tag{B.8}$$

C Chong et al. solution

The near-horizon geometry of the cohomogenity-2 BPS black holes of Chong et al. [6, 7] has near-horizon data [26]:

$$ds_{\mathcal{S}}^2 = \frac{\ell^2\Gamma d\Gamma^2}{4P(\Gamma)} + \left(C^2\Gamma - \frac{\Delta_0^2}{\Gamma^2}\right)\left(dx^1 + \frac{\Delta_0(\alpha_0 - \Gamma)}{C^2\Gamma^3 - \Delta_0^2}dx^2\right)^2 + \frac{4\Gamma P(\Gamma)}{\ell^2(C^2\Gamma^3 - \Delta_0^2)}(dx^2)^2 \ , \tag{C.1}$$

where

$$P(\Gamma) = \Gamma^3 - \frac{C^2\ell^2}{4}(\Gamma - \alpha_0)^2 - \frac{\Delta_0^2}{C^2} \tag{C.2}$$

with C and Δ_0 positive constants and α_0 an arbitrary constant. Furthermore,

$$\Delta = \frac{\Delta_0}{\Gamma^2} \tag{C.3}$$

and

$$h = \Gamma^{-1}\left(\left(C^2\Gamma - \frac{\Delta_0^2}{\Gamma^2}\right)\left(dx^1 + \frac{\Delta_0(\alpha_0 - \Gamma)}{C^2\Gamma^3 - \Delta_0^2}dx^2\right) - d\Gamma\right) \tag{C.4}$$

and

$$Z = \frac{\ell(\alpha_0 - \Gamma)C^2}{2\Gamma}dx^1 + \frac{2\Delta_0}{\ell C^2\Gamma}dx^2 + \frac{\ell}{2\Gamma}d\Gamma \ . \tag{C.5}$$

References

[1] R. Emparan and H.S. Reall, *A Rotating black ring solution in five-dimensions*, *Phys. Rev. Lett.* **88** (2002) 101101 [`hep-th/0110260`] [INSPIRE].

[2] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric black rings and three-charge supertubes*, *Phys. Rev.* **D 71** (2005) 024033 [`hep-th/0408120`] [INSPIRE].

[3] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *A Supersymmetric black ring*, *Phys. Rev. Lett.* **93** (2004) 211302 [`hep-th/0407065`] [INSPIRE].

[4] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS$_5$ black holes*, *JHEP* **02** (2004) 006 [`hep-th/0401042`] [INSPIRE].

[5] J.B. Gutowski and H.S. Reall, *General supersymmetric AdS$_5$ black holes*, *JHEP* **04** (2004) 048 [`hep-th/0401129`] [INSPIRE].

[6] Z.-W. Chong, M. Cvetič, H. Lü and C.N. Pope, *General non-extremal rotating black holes in minimal five-dimensional gauged supergravity*, *Phys. Rev. Lett.* **95** (2005) 161301 [`hep-th/0506029`] [INSPIRE].

[7] Z.W. Chong, M. Cvetič, H. Lü and C.N. Pope, *Five-dimensional gauged supergravity black holes with independent rotation parameters*, *Phys. Rev.* **D 72** (2005) 041901 [`hep-th/0505112`] [INSPIRE].

[8] H.K. Kunduri, J. Lucietti and H.S. Reall, *Supersymmetric multi-charge AdS$_5$ black holes*, *JHEP* **04** (2006) 036 [`hep-th/0601156`] [INSPIRE].

[9] A.H. Chamseddine and W.A. Sabra, *Magnetic strings in five-dimensional gauged supergravity theories*, *Phys. Lett.* **B 477** (2000) 329 [`hep-th/9911195`] [INSPIRE].

[10] D. Klemm and W.A. Sabra, *Supersymmetry of black strings in $D = 5$ gauged supergravities*, *Phys. Rev.* **D 62** (2000) 024003 [`hep-th/0001131`] [INSPIRE].

[11] J.P. Gauntlett and J.B. Gutowski, *All supersymmetric solutions of minimal gauged supergravity in five-dimensions*, *Phys. Rev.* **D 68** (2003) 105009 [*Erratum ibid.* **D 70** (2004) 089901] [`hep-th/0304064`] [INSPIRE].

[12] J.B. Gutowski and W. Sabra, *General supersymmetric solutions of five-dimensional supergravity*, *JHEP* **10** (2005) 039 [`hep-th/0505185`] [INSPIRE].

[13] S.L. Cacciatori, D. Klemm and W.A. Sabra, *Supersymmetric domain walls and strings in $D = 5$ gauged supergravity coupled to vector multiplets*, *JHEP* **03** (2003) 023 [`hep-th/0302218`] [INSPIRE].

[14] K. Behrndt, A.H. Chamseddine and W.A. Sabra, *BPS black holes in $N = 2$ five-dimensional AdS supergravity*, *Phys. Lett.* **B 442** (1998) 97 [`hep-th/9807187`] [INSPIRE].

[15] D. Klemm and W.A. Sabra, *Charged rotating black holes in $5 - D$ Einstein-Maxwell (A)dS gravity*, *Phys. Lett.* **B 503** (2001) 147 [`hep-th/0010200`] [INSPIRE].

[16] D. Klemm and W.A. Sabra, *General (anti-)de Sitter black holes in five-dimensions*, *JHEP* **02** (2001) 031 [`hep-th/0011016`] [INSPIRE].

[17] J. Grover, J.B. Gutowski and W. Sabra, *Vanishing preons in the fifth dimension*, *Class. Quant. Grav.* **24** (2007) 417 [`hep-th/0608187`] [INSPIRE].

[18] J.M. Figueroa-O'Farrill, J. Gutowski and W. Sabra, *The Return of the four- and five-dimensional preons*, *Class. Quant. Grav.* **24** (2007) 4429 [`arXiv:0705.2778`] [INSPIRE].

[19] J.B. Gutowski and W.A. Sabra, *Half-Supersymmetric Solutions in Five-Dimensional Supergravity*, *JHEP* **12** (2007) 025 [*Erratum ibid.* **1004** (2010) 042] [arXiv:0706.3147] [INSPIRE].

[20] J. Grover, J.B. Gutowski and W. Sabra, *Null Half-Supersymmetric Solutions in Five-Dimensional Supergravity*, *JHEP* **10** (2008) 103 [arXiv:0802.0231] [INSPIRE].

[21] J. Gillard, U. Gran and G. Papadopoulos, *The Spinorial geometry of supersymmetric backgrounds*, *Class. Quant. Grav.* **22** (2005) 1033 [hep-th/0410155] [INSPIRE].

[22] U. Gran, J. Gutowski and G. Papadopoulos, *The Spinorial geometry of supersymmetric IIB backgrounds*, *Class. Quant. Grav.* **22** (2005) 2453 [hep-th/0501177] [INSPIRE].

[23] U. Gran, P. Lohrmann and G. Papadopoulos, *The Spinorial geometry of supersymmetric heterotic string backgrounds*, *JHEP* **02** (2006) 063 [hep-th/0510176] [INSPIRE].

[24] J.B. Gutowski and W.A. Sabra, *Enhanced Horizons*, *Class. Quant. Grav.* **27** (2010) 235011 [arXiv:0807.4714] [INSPIRE].

[25] J. Grover, J.B. Gutowski, G. Papadopoulos and W.A. Sabra, *Index Theory and Supersymmetry of 5D Horizons*, *JHEP* **06** (2014) 020 [arXiv:1303.0853] [INSPIRE].

[26] H.K. Kunduri, J. Lucietti and H.S. Reall, *Do supersymmetric anti-de Sitter black rings exist?*, *JHEP* **02** (2007) 026 [hep-th/0611351] [INSPIRE].

[27] J. Grover and J. Gutowski, *Horizons in de-Sitter Supergravity*, *JHEP* **04** (2010) 009 [arXiv:1001.2460] [INSPIRE].

[28] H.K. Kunduri and J. Lucietti, *Near-horizon geometries of supersymmetric AdS$_5$ black holes*, *JHEP* **12** (2007) 015 [arXiv:0708.3695] [INSPIRE].