

Primary fields in celestial CFT

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ABSTRACT: The basic ingredient of CCFT holography is to regard four-dimensional amplitudes describing conformal wave packets as two-dimensional conformal correlation functions of the operators associated to external particles. By construction, these operators transform as quasi-primary fields under $SL(2, \mathbb{C})$ conformal symmetry group of the celestial sphere. We derive the OPE of the CCFT energy-momentum tensor with the operators representing gauge bosons and show that they transform as Virasoro primaries under diffeomorphisms of the celestial sphere.

KEYWORDS: Conformal Field Theory, Scattering Amplitudes

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1 Introduction

Two-dimensional celestial conformal field theory (CCFT) has been recently considered as a candidate for a holographic description of four-dimensional space-time. The developments leading to CCFT [1, 2] build upon earlier proposals of flat holography [3] and extended BMS symmetries of asymptotically flat spacetime [4]. In this framework, four-dimensional scattering amplitudes are represented as conformal field correlators, with their Lorentz symmetry realized as the $SL(2, \mathbb{C})$ conformal symmetry group of celestial sphere (\mathcal{CS}^2) [5–7]. The origin of this connection is a natural identification of the kinematic variables describing asymptotic directions of external particles with the points on \mathcal{CS}^2 . The energies are dualized by using Mellin transforms into dimensions of conformal field operators [8]. The Mellin transforms of Yang-Mills amplitudes are well-defined but the amplitudes involving gravitons usually diverge in the high energy region. Nevertheless, as demonstrated in [9], this problem does not appear in string amplitudes due to their super-soft ultraviolet behaviour.

At this point, it is rather premature to call CCFT a “theory”. Very few things are known but the emerging picture has some novel and intriguing features. The CCFT correlators describe the scattering processes of conformal wave packets characterized by $SL(2, \mathbb{C})$ conformal weights (h, \bar{h}) . There is a quasi-primary operator associated to each packet. The conformal spin $h - \bar{h}$ is identified with helicity, while the condition of normalizability restricts the dimension $\Delta = h + \bar{h}$ to the principal continuous series with $\Delta = 1 + i\lambda$ [8]. Four-dimensional Lorentz generators are identified with $SL(2, \mathbb{C})$ generators while four-dimensional translations are realized as dimension-shifting operators [10].

The correlators representing four-dimensional Einstein-Yang-Mills scattering amplitudes have a complicated structure. This is quite puzzling because, at least at the tree level, Yang-Mills amplitudes exhibit a remarkable simplicity when written in the helicity basis [11]. There is, however, one limit in which these correlators reveal some interesting features of CCFT. In the so-called conformal soft limit of $\Delta = 1$ [12], the gauge boson operators can be identified with two-dimensional currents representing particles that are soft in the traditional sense (zero energy) [13]. As a result, the correlators involving such

soft insertions are determined by current Ward identities. The conformal soft limit has been subject of many recent studies [13–20].

In a recent work [13], we looked at the soft limit from a different perspective. As Ward identities of CCFT, soft theorems reflect the symmetry algebra — in two dimensions completely determined by the operator product expansion (OPE). We derived the OPE for the operators representing gauge bosons and used standard CFT techniques to derive Ward identities describing the conformally soft limit.

In CCFT, with the scattering amplitudes transformed into conformal correlators, the limit of coinciding points of \mathcal{CS}^2 corresponds to the limit of coinciding momentum directions, that is to the collinear limit. Hence in principle, it should be possible to extract not only the products of gauge boson operators, as it was done in ref. [13], but all OPEs from well-known collinear limits. One particularly important set of OPEs involves the energy momentum tensor. It has been proposed [21, 22] that the energy-momentum tensor of CCFT is the $\Delta = 2$ operator obtained by taking a shadow transform of the $\Delta = 0$ operator associated to the graviton. The analysis of ref. [12] gave further evidence to this proposal. In this work, we put it on solid foundations by studying the products of this $\Delta = 2$ shadow operator with gauge boson operators. We show that the gauge boson operators have the expected OPE with the energy-momentum tensor, appropriate for Virasoro primary fields.

2 Preliminaries: conformal wave packets of gravitons

A general light-like momentum vector of a massless particle can be written as

$$p^\mu = \omega q^\mu, \quad q^\mu = \frac{1}{2}(1 + |z|^2, z + \bar{z}, -i(z - \bar{z}), 1 - |z|^2), \quad (2.1)$$

where ω is the light-cone energy and q^μ is a null vector — the direction along which the massless state propagates. It is parametrized by complex z which is identified as a point on \mathcal{CS}^2 . On \mathcal{CS}^2 , the $SL(2, \mathbb{C})$ Lorentz group acts as the global conformal symmetry group:

$$z \rightarrow \frac{az + b}{cz + d}, \quad ad - bc = 1. \quad (2.2)$$

The usual spin two graviton plane waves are

$$\epsilon_{\mu\nu}^\ell(p) e^{\mp i|p_0|X^0 \pm i\vec{p}\cdot\vec{X}}, \quad \ell = \pm 2 \quad (2.3)$$

where $\epsilon_{\mu\nu}^\ell(p)$ is the polarization tensor, ℓ is the helicity and the \pm sign in the exponential is used to distinguish between incoming and outgoing solutions.¹ It is convenient to choose a gauge in which

$$\epsilon_{\mu\nu}^\ell(p) = \partial_J q^\mu \partial_J q^\nu, \quad (2.4)$$

where $J = z$ for $l = +2$ and $J = \bar{z}$ for $l = -2$. In order to transform the scattering amplitudes into conformal correlators, the basis of plane waves will be changed to the basis of wave packets transforming as quasi-primary fields under $SL(2, \mathbb{C})$. The chiral weights h and \bar{h} are constrained by $h - \bar{h} = \ell$, but the dimensions $\Delta = h + \bar{h}$ are *a priori* arbitrary.

¹From this point, we will be considering outgoing (+) solutions only.

The quasi-primary wave packets satisfying free spin two wave equations, with $h - \bar{h} = \ell = \pm 2$ and dimension Δ , were constructed in [8]. They are given by

$$h_{\mu\nu}^{\Delta,\ell} = \frac{1}{2} \frac{[(-q \cdot X) \partial_J q_\mu + (\partial_J q \cdot X) q_\mu][(-q \cdot X) \partial_J q_\nu + (\partial_J q \cdot X) q_\nu]}{(-q \cdot X)^{\Delta+2}} \quad (2.5)$$

They are related to Mellin transforms of plane waves in the following way. First, we apply ∂_J^2 to the following identity [8]:

$$\frac{q_\mu q_\nu}{(-q \cdot X)^\Delta} = \frac{1}{2} \frac{1}{\Delta - 1} \left[\partial_\mu \left(\frac{q_\nu}{(-q \cdot X)^{\Delta-1}} \right) + \partial_\nu \frac{q_\mu}{(-q \cdot X)^{\Delta-1}} \right]. \quad (2.6)$$

As a result,

$$\frac{(\partial_J q \cdot X)^2 q_\mu q_\nu}{(-q \cdot X)^{\Delta+2}} = \frac{-2}{\Delta(\Delta+1)} \frac{\partial_J q_\mu \partial_J q_\nu}{(-q \cdot X)^\Delta} - \frac{2}{\Delta+1} \frac{\partial_J q_\mu (\partial_J q \cdot X) q_\nu + \partial_J q_\nu (\partial_J q \cdot X) q_\mu}{(-q \cdot X)^{\Delta+1}} + \text{diff} \quad (2.7)$$

where diff is a “pure gauge” part representing a diffeomorphism. Next, we use

$$\partial_J \frac{(\partial_J q_{\{\mu} q_{\nu\}})}{(-q \cdot X)^\Delta} = \frac{1}{2(\Delta-1)} \partial_J \left[\partial_\mu \frac{(\partial_J q_\nu)}{(-q \cdot X)^{\Delta-1}} + \partial_\nu \frac{(\partial_J q_\mu)}{(-q \cdot X)^{\Delta-1}} \right] \quad (2.8)$$

to obtain

$$\frac{\partial_J q_\mu (\partial_J q \cdot X) q_\nu + \partial_J q_\nu (\partial_J q \cdot X) q_\mu}{(-q \cdot X)^{\Delta+1}} = -\frac{2}{\Delta} \frac{\partial_J q_\mu \partial_J q_\nu}{(-q \cdot X)^\Delta} + \text{diff} \quad (2.9)$$

After combining (2.7) and (2.9), we find that the conformal wave packet can be written as

$$h_{\mu\nu}^{\Delta,\ell} = \frac{1}{2} \frac{\Delta(\Delta-1)}{\Delta+1} \frac{\partial_J q_\mu \partial_J q_\nu}{(-q \cdot X)^\Delta} + \text{diff} \quad (2.10)$$

Note that in two special cases, of $\Delta = 0$ and $\Delta = 1$, the wave functions (2.10) represent diffeomorphisms. Pasterski and Shao [8] showed that for a generic Δ , the wave functions (2.10) are normalizable only if $\Delta = 1 + i\lambda$, i.e. when the fields belong to the principal continuous series. The amplitudes involving gravitons of the principal series have been discussed in [9] and their conformally soft limit of $\Delta = 1$ in [16, 17]. In the next section, we focus on $\Delta = 0$, which is another case of pure diffeomorphism.

The connection to Mellin-transformed plane waves is established in the following way. The Mellin transform of the plane wave (2.3) is

$$G_{\mu\nu}^{\Delta,\ell}(X^\mu, z, \bar{z}) \equiv \partial_J q_\mu \partial_J q_\nu \int_0^\infty d\omega \omega^{\Delta-1} e^{\mp i\omega q \cdot X - \epsilon\omega}. \quad (2.11)$$

The conformal wave function (2.10) can be written as²

$$h_{\mu\nu}^{\Delta,\ell} = f(\Delta) G_{\mu\nu}^{\Delta,\ell} + \text{diff} \quad (2.12)$$

where

$$f(\Delta) = \frac{1}{2} \frac{\Delta(\Delta-1)}{\Gamma(\Delta+2)} \quad (2.13)$$

²We skip some irrelevant i^Δ factors.

In the next section, we will discuss the shadow transforms of amplitudes involving $\Delta = 0$ gravitons,³ which should correspond to the insertions of the energy-momentum tensor. Note that the respective Mellin transforms are accompanied by the normalization factors $f(\Delta)$ that vanish in the $\Delta = 0$ limit, therefore in order to obtain finite results, the amplitudes must be singular. Indeed, finite OPE coefficients will be extracted from such singular limits.

3 OPE of the energy-momentum tensor with primary fields

We wish to discuss two statements. The first statement is that, as anticipated in [21, 22], the CCFT energy-momentum tensor $T(z)$ is given by a shadow transform [23] of the operator $\mathcal{G}_{\Delta=0}(z, \bar{z})$ associated to the $\Delta = 0$ limit ($h = -1, \bar{h} = 1$) of the quasi-primary graviton wave packet (2.5) with helicity $\ell = -2$, hence associated to a non-trivial effect of a pure diffeomorphism, see eq. (2.10). More precisely,

$$T(z) = \frac{3!}{2\pi} \int d^2 z' \frac{1}{(z' - z)^4} \mathcal{G}_{\Delta=0}(z', \bar{z}') \tag{3.1}$$

The second statement is that the gauge boson operators $\mathcal{O}(z, \bar{z})$ associated to quasi-primary spin one wave functions transform as conformal primaries under \mathcal{CS}^2 diffeomorphisms. This is equivalent to the OPE

$$T(z)\mathcal{O}(w, \bar{w}) = \frac{h}{(z - w)^2} \mathcal{O}(w, \bar{w}) + \frac{1}{z - w} \partial_w \mathcal{O}(w, \bar{w}) + \dots, \tag{3.2}$$

where h is the chiral weight of \mathcal{O} . In this section, we prove the second statement, assuming that the first one is true. To that end, we will be considering tree-level Einstein-Yang-Mills amplitudes involving one graviton and an arbitrary number N of gauge bosons. In ref. [24] it was shown that such amplitudes can be expressed as linear combinations of pure Yang-Mills amplitudes involving $N+1$ gauge bosons, but our discussion will not rely on this relation.

We will proceed in the following steps. We will start from the Mellin transformation converting the standard amplitude into a correlator of the graviton operator $\mathcal{G}_{\Delta}(z', \bar{z}')$ with N gauge boson operators \mathcal{O} inserted at arbitrary points z_n . Next, we will convert \mathcal{G}_{Δ} into its shadow $\tilde{\mathcal{G}}_{\Delta}(z, \bar{z})$ and extract collinear singularities in the limit of z coinciding with one of z_n . After taking the $\Delta = 0$ limit, we will extract the OPE coefficients.

The CCFT correlator of N gauge bosons and one helicity $\ell = -2$ graviton is given by

$$\begin{aligned} \left\langle \prod_{n=1}^N \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{G}_{\Delta}(z', \bar{z}') \right\rangle &= \left(\prod_{n=1}^N \int d\omega_n \omega_n^{\Delta_n - 1} \right) f(\Delta) \int d\omega' \omega'^{\Delta - 1} \delta^{(4)} \left(\sum_{n=1}^N \omega_n q_n + \omega' q' \right) \\ &\times \mathcal{M}_{\ell_1 \dots \ell_N, \ell = -2}(\omega_n, z_n, \bar{z}_n; \omega', z', \bar{z}') \end{aligned} \tag{3.3}$$

where $\mathcal{M}_{\ell_1 \dots \ell_N, \ell = -2}$ is the Feynman's matrix element for the scattering of N gauge bosons with helicities ℓ_n and one helicity $\ell = -2$ graviton. We omit gauge indices and gauge

³Actually, $\Delta = 0$ modes are pure diffeomorphisms, so it would be more appropriate to call them ‘‘Goldstone’’ modes instead of gravitons.

operator normalization factors because they are not relevant to our discussion, and stick to one partial amplitude corresponding say to the canonical ordering inside the Chan-Paton trace factor. Recall that the dimensions of gauge boson operators are fixed to $\Delta_n = 1 + i\lambda_n$.

Next, we take the shadow transform. Since we are interested in the $\Delta = 0$ limit and $f(\Delta)$ is already of order Δ , we can evaluate this transform as if Δ were vanishing:⁴

$$\left\langle \prod_{n=1}^N \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \tilde{\mathcal{G}}_{\Delta}(z, \bar{z}) \right\rangle = \frac{3!}{2\pi} \int d^2 z' \frac{1}{(z - z')^4} \left\langle \prod_{n=1}^N \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{G}_{\Delta}(z', \bar{z}') \right\rangle \quad (3.4)$$

Since our goal is to extract the OPE, we are interested in the pole behavior of the above expression in the limit when z approaches one of other points, say $z \rightarrow z_N$. A short reflection reveals that poles will appear in the integration region where $z' \rightarrow z_N$. Looking back at eq. (3.3), we see that this is the collinear limit of $\mathcal{M}_{\ell_1 \dots \ell_N, \ell=-2}(p_n, p')$ when the graviton's four-momentum $p' = \omega' q'$ becomes parallel to the momentum $p_N = \omega_N q_N$ of the N th gauge boson. The matrix element \mathcal{M} is singular in this limit in a rather peculiar way. The pole terms $(\bar{z}' - \bar{z}_N)^{-1}$ are always accompanied by factors $(z' - z_N)$ so they are singular only if z and \bar{z} are considered as independent variables, otherwise it is just a phase ambiguity reflecting an azimuthal asymmetry [25]. This singularity is important for the shadow transform (3.4). It can be extracted [26] from the Feynman diagrams in which the graviton is radiated away by the N th gauge boson. One finds

$$\mathcal{M}(1, 2, \dots, N^-, p'^{--}) = \frac{z' - z_N}{\bar{z}' - \bar{z}_N} \frac{\omega_P}{\omega'} \mathcal{M}(1, 2, \dots, P^-) + \dots, \quad (3.5)$$

$$\mathcal{M}(1, 2, \dots, N^+, p'^{--}) = \frac{z' - z_N}{\bar{z}' - \bar{z}_N} \frac{\omega_N^2}{\omega' \omega_P} \mathcal{M}(1, 2, \dots, P^+) + \dots, \quad (3.6)$$

where we omitted non-singular terms. We use the notation of ref. [11], with \pm superscripts referring to particle helicities. Here, P is the combined momentum of the collinear pair:

$$P = p_N + p' = \omega_N q_N + \omega' q' = \omega_P q_P, \quad (3.7)$$

with

$$\omega_P = \omega_N + \omega' \quad q_P = q_N = q' \quad (z_P = z_N = z', \bar{z}_P = \bar{z}_N = \bar{z}'). \quad (3.8)$$

We begin with the case of $\ell_N = -1$, with the collinear limit given in (3.5). After inserting it into eqs. (3.3), (3.4), we obtain

$$\begin{aligned} \left\langle \prod_{n=1}^{N-1} \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{O}_{-1}(z_N, \bar{z}_N) \tilde{\mathcal{G}}_{\Delta}(z, \bar{z}) \right\rangle &= \left(\prod_{n=1}^{N-1} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \times \\ &\times \frac{f(\Delta)}{2\pi} \int d\omega_N \omega_N^{\Delta_N - 1} \int d\omega' \omega'^{\Delta - 2} \omega_P \int d^2 z' \frac{3!}{(z - z')^4} \frac{z'_N}{\bar{z}'_N} \times \\ &\times \delta^{(4)} \left(\sum_{n=1}^{N-1} \omega_n q_n + \omega_P q_P \right) \mathcal{M}(1, 2, \dots, P^-), \end{aligned} \quad (3.9)$$

⁴The shadow formula valid for arbitrary (h, \bar{h}) can be found in ref. [23].

where $z'_N \equiv z' - z_N$ and $\bar{z}'_N \equiv \bar{z}' - \bar{z}_N$. We integrate by parts over z' two times:

$$\int d^2 z' \frac{3!}{(z - z')^4} \frac{z'_N}{\bar{z}'_N} \longrightarrow \int d^2 z' \frac{1}{(z - z')^2} \left[\partial_{z'}^2 \left(\frac{z'_N}{\bar{z}'_N} \right) + 2 \partial_{z'} \left(\frac{z'_N}{\bar{z}'_N} \right) \partial_{z'} + \left(\frac{z'_N}{\bar{z}'_N} \right) \partial_{z'}^2 \right]. \quad (3.10)$$

The first term yields

$$\partial_{z'}^2 \left(\frac{z'_N}{\bar{z}'_N} \right) = 2\pi \delta^{(2)}(z' - z_N) \quad (3.11)$$

and the integral becomes localized at $z' = z$, giving rise to a double pole of the correlator (3.9) at $z = z_N$. In the second term, with

$$\partial_{z'} \left(\frac{z'_N}{\bar{z}'_N} \right) = \frac{1}{\bar{z}'_N} \quad (3.12)$$

we integrate by parts one more time:

$$\int \frac{d^2 z'}{(z - z')^2} \frac{2}{\bar{z}'_N} \partial_{z'} \longrightarrow - \int d^2 z' \left[\frac{4\pi}{z - z'} \delta^{(2)}(z' - z_N) \partial_{z'} + \frac{2}{\bar{z}'_N (z - z')} \partial_{z'}^2 \right]. \quad (3.13)$$

Here, the first term gives rise to a single pole, while the second term, although it is logarithmically divergent at $z = z_N$, does not contribute any poles. Actually, the third term in (3.10) contains a similar logarithmic divergence. These two logarithmic terms disappear, however, in the $\Delta = 0$ limit because they contain $\partial_{z'}^2$, acting on the remaining part of the integrand. As explained below, each $\partial_{z'}$ derivative brings one power of ω' , therefore the corresponding ω' integrals are finite. In the $\Delta = 0$ limit, they are suppressed by $f(\Delta)$ normalization factor (2.13) which is of order $\mathcal{O}(\Delta)$.

In eq. (3.13), the derivative $\partial_{z'}$ on the right hand side acts on the remaining part of the integrand, that is the momentum-conserving delta function and the matrix element \mathcal{M} , cf. eq. (3.9). Both these functions depend on $p' = \omega' q'(z')$ through the combination $P = p' + p_N$, see eqs. (3.7), (3.8). For any such function

$$\partial_{z'} = \frac{\omega'}{\omega_P} \partial_{z_P} \quad (3.14)$$

when, as in our case, the derivative is evaluated at $q' = q_P$.⁵ After putting this altogether, we find that after the shadow transform, eq. (3.9) yields

$$\begin{aligned} & \left\langle \prod_{n=1}^{N-1} \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{O}_{-1}(z_N, \bar{z}_N) \tilde{\mathcal{G}}_{\Delta}(z, \bar{z}) \right\rangle = \left(\prod_{n=1}^{N-1} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \quad (3.15) \\ & \times f(\Delta) \left[\frac{1}{(z - z_N)^2} \int d\omega_N \omega_N^{\Delta_N - 1} \int d\omega' \omega'^{\Delta - 2} \omega_P \delta^{(4)} \left(\sum_{n=1}^{N-1} \omega_n q_n + \omega_P q_P \right) \mathcal{M}(1, 2, \dots, P^-) \right. \\ & \left. - \frac{2}{z - z_N} \int d\omega_N \omega_N^{\Delta_N - 1} \int d\omega' \omega'^{\Delta - 1} \partial_{z_P} \left\{ \delta^{(4)} \left(\sum_{n=1}^{N-1} \omega_n q_n + \omega_P q_P \right) \mathcal{M}(1, 2, \dots, P^-) \right\} \right] \end{aligned}$$

⁵Actually, \mathcal{M} depends on the momentum spinor λ_P . Nevertheless, since this amplitude is extracted by factorizing on the gauge boson pole in the (p_N, p') channel, cf. eqs. (3.5), (3.6), the dependence on λ_P is reduced to a simple polarization-dependent factor, while the other factor depends on vector P . After checking these factors, one finds that eq. (3.14) remains valid.

Now we can change the energy integration variables from (ω_N, ω') to $(\omega_P = \omega_N + \omega', \omega')$:

$$\int_0^\infty d\omega_N \int_0^\infty d\omega' \cdots \longrightarrow \int_0^\infty d\omega_P \int_0^{\omega_P} d\omega' \cdots \quad (3.16)$$

and integrate over graviton's energy ω' , with the result

$$\begin{aligned} \left\langle \prod_{n=1}^{N-1} \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{O}_{-1}(z_N, \bar{z}_N) \tilde{\mathcal{G}}_\Delta(z, \bar{z}) \right\rangle &= \left(\prod_{n=1}^{N-1} \int d\omega_n \omega_n^{\Delta_n - 1} \right) \int d\omega_P \omega_P^{\Delta_N + \Delta - 1} \times \\ &\times f(\Delta) \left[\frac{B(\Delta_N, \Delta - 1)}{(z - z_N)^2} \delta^{(4)} \left(\sum_{n=1}^{N-1} \omega_n q_n + \omega_P q_P \right) \mathcal{M}(1, 2, \dots, P^-) \right. \\ &\left. - 2 \frac{B(\Delta_N, \Delta)}{z - z_N} \partial_{z_P} \left\{ \delta^{(4)} \left(\sum_{n=1}^{N-1} \omega_n q_n + \omega_P q_P \right) \mathcal{M}(1, 2, \dots, P^-) \right\} \right], \end{aligned} \quad (3.17)$$

where B denotes Euler's beta function. At this point, in order to turn the shadow operator $\tilde{\mathcal{G}}_\Delta(z, \bar{z})$ into the energy-momentum tensor $T(z)$, we take the $\Delta = 0$ limit. With the factor $f(\Delta)$ given in eq. (2.13),

$$f(\Delta) B(\Delta_N, \Delta - 1) \rightarrow \frac{\Delta_N - 1}{2}, \quad f(\Delta) B(\Delta_N, \Delta) \rightarrow -\frac{1}{2}. \quad (3.18)$$

Note that on the r.h.s. of eq. (3.17), the energy of the N th gauge boson is ω_P and its momentum is $P = \omega_P q_P$, with $q_P = q_N$. Furthermore, the amplitude and its derivative are evaluated at $z_P = z_N$. As a result,

$$\begin{aligned} \left\langle \prod_{n=1}^{N-1} \mathcal{O}_{\ell_n}(z_n, \bar{z}_n) \mathcal{O}_{-1}(z_N, \bar{z}_N) T(z) \right\rangle &= \left(\prod_{n=1}^N \int d\omega_n \omega_n^{\Delta_n - 1} \right) \\ &\times \left[\frac{h_N}{(z - z_N)^2} \delta^{(4)} \left(\sum_{n=1}^N \omega_n q_n \right) \mathcal{M}(1, 2, \dots, N^-) \right. \\ &\left. + \frac{1}{z - z_N} \partial_{z_N} \left\{ \delta^{(4)} \left(\sum_{n=1}^N \omega_n q_n \right) \mathcal{M}(1, 2, \dots, N^-) \right\} \right], \end{aligned} \quad (3.19)$$

where we used the fact that for $\ell_N = -1$, $h_N = (\Delta_N - 1)/2$.

The OPE (3.2) follows from the $z \rightarrow z_N$ singularities of the energy-momentum tensor correlator (3.19). A similar proof can be carried out in the case of $\ell_N = +1$ [$h_N = (\Delta_N + 1)/2$], by using the collinear limit (3.6).

4 Conclusions

The change of basis from plane waves to conformal wave packets, accomplished by using Mellin transformations, converts Yang-Mills amplitudes into celestial amplitudes. The basic idea of CCFT holography is to regard these amplitudes as the correlation functions of the operators associated to external particles. By construction, these operators must transform as quasi-primaries under the (global) $SL(2, \mathbb{C})$ inherited from four-dimensional

Lorentz symmetry. It is by no means obvious that these operators transform as Virasoro primaries because \mathcal{CS}^2 diffeomorphisms are generated by *a priori* unknown CCFT energy-momentum tensor $T(z)$. In this work, guided by refs. [21, 22], we assumed that $T(z)$ is given by the shadow of the $\Delta = 0$ graviton operator representing pure (large) diffeomorphisms. We showed that under this assumption, the operators representing gauge bosons do indeed transform as Virasoro primaries.

As mentioned before, the assumed form of $T(z)$ was mainly based on the arguments involving soft theorems, in the framework of quantum field theory formulated in curvilinear coordinates in which \mathcal{CS}^2 diffeomorphisms appear as a part of larger BMS symmetry. It would be very enlightening to discuss this symmetry in a purely CCFT setup, by studying the OPE of TT products. This OPE has already been discussed in the context of double-soft limits in ref. [27]. In our approach, it should emerge from the collinear limit of two shadow operators. Unfortunately, unlike in the case of a single shadow, understanding the collinear behaviour of Einstein-Yang-Mills amplitudes is not sufficient for extracting the collinear singularities of shadow products. We hope to address this issue in the future.

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