

# A new class of de Sitter vacua in type IIB large volume compactifications

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**ABSTRACT:** We construct a new class of metastable de Sitter vacua of flux compactifications of type IIB string theory. These solutions provide a natural extension of the ‘Large Volume Scenario’ anti-de Sitter vacua, and can analogously be realised at parametrically large volume and weak string coupling, using standard  $\mathcal{N} = 1$  supergravity. For these new vacua, a positive vacuum energy is achieved from the inclusion of a small amount of flux-induced supersymmetry breaking in the complex structure and axio-dilaton sector, and no additional ‘uplift’ contribution (e.g. from anti-branes) is required. We show that the approximate no-scale structure of the effective theory strongly influences the spectrum of the stabilised moduli: one complex structure modulus remains significantly lighter than the supersymmetry breaking scale, and metastability requires only modest amounts of tuning. After discussing these general results, we provide a recipe for constructing de Sitter vacua on a given compactification manifold, and give an explicit example of a de Sitter vacuum for the compactification on the Calabi-Yau orientifold realised in  $\mathbb{CP}_{11169}^4$ . Finally, we note that these solutions have intriguing implications for phenomenology, predicting no superpartners in the spectrum below  $\sim 50$  TeV, and no WIMP dark matter.

**KEYWORDS:** Flux compactifications, Supergravity Models, Superstring Vacua

ARXIV EPRINT: [1707.01095](https://arxiv.org/abs/1707.01095)

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**1 Introduction**

There are apparently many different possibilities to compactify string theory to four dimensions. Most notably, generalised electromagnetic fluxes (e.g.  $F_3$  and  $H_3$  for type IIB compactifications) can thread non-trivial cycles of the compactification geometry in a myriad of ways. For each viable compactification manifold, this results in a set of ‘flux compactifications’, each with a distinct low-energy limit (for some reviews see [1–3]). The number of effective theories in this set grows exponentially with certain topological parameters of the compactification manifold [4–7]. However, while flux compactifications produce *many* low-energy effective descriptions, it is not known whether *any* of them support solutions that are consistent with all experiments and observations. Consequently, it is not known if our

own universe is included in this set of solutions, and if so, what (if any) the distinguishing characteristics of the physically interesting solutions are.

A key challenge in determining the properties of generic flux compactifications is that most vacua arise from the topologically richest and most complicated manifolds, obscuring the connection with observation. Deformation modes of the compact geometry appear in the low-energy theory as light ‘moduli’ fields. Topologically interesting compactifications come with many moduli: typical type IIB scenarios have  $\mathcal{O}(100)$  moduli fields [8], while for F-theory scenarios that number easily reaches  $\mathcal{O}(10^5)$  [9]. Those moduli do not only appear as exotic particles in the spectrum but also control important parameters in the low energy theory such as coupling constants. This brings the need for a controlled stabilisation, with non-zero masses and fixed vacuum expectation values.

The stabilised geometry, free of tachyons, corresponds to critical points of the effective potential that satisfy a perturbative (meta-)stability condition. For a configuration with  $\partial_A V = 0$ , where  $\partial_A = \frac{\partial}{\partial \phi^A}$  for the moduli fields  $\phi^A$ , this means that the Hessian matrix,

$$\mathcal{H} = \begin{pmatrix} \partial_{A\bar{B}}^2 V & \partial_{AB}^2 V \\ \partial_{\bar{A}\bar{B}}^2 V & \partial_{\bar{A}\bar{B}}^2 V \end{pmatrix}, \tag{1.1}$$

has only positive eigenvalues. Stability is guaranteed for supersymmetric solutions. However, the accelerated expansion of the universe and the absence of observed superpartners to the Standard Model particles means that supersymmetry must be broken in solutions aiming to describe the real world. This motivates studying non-supersymmetric solutions of string compactifications. Ensuring a positive definite spectrum of the high-dimensional matrix  $\mathcal{H}$  in solutions with broken supersymmetry is a non-trivial task.

Moreover, to address the accelerated expansion of the universe, the vacuum should be a de Sitter solution with a positive cosmological constant [10, 11]. There are good reasons to believe that string compactifications allowing de Sitter solutions will be quite special and may share characteristic properties that could possibly lead to observational signatures. One piece of evidence for this comes from theories with many randomly interacting fields which can be studied by the means of random matrix theory (RMT). In such theories, metastable de Sitter vacua are exceptionally rare [12]; this conclusion remains unchanged even for typical critical points of supergravity theories with spontaneously broken supersymmetry [13]. Hence, some type of non-random structure inherited from the string compactification appears to be required to explain the accelerated expansion of the universe.<sup>1</sup>

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<sup>1</sup>Recently, several attempts have been made to model the low-energy effective theories from string theory using classes of random functions, in particular Gaussian Random Fields (GRFs). These studies have found that, at least for trivial field space geometry and a particular choice of covariance function [14], or upon ignoring the structures imposed by supergravity [15, 16], metastable vacua are more common than the random matrix theory argument suggests. In the simplest models where  $V$  is postulated to be a mean-zero Gaussian Random Field with a Gaussian covariance function, the increase in the number of vacua can be understood to arise from a rigid shift in the eigenvalue spectrum [14, 15]: for  $V = 0$ , the GRF and RMT both predict a mean-zero Wigner semi-circle spectrum, but for  $V > 0$  the spectrum of the GRF potential are shifted downwards and metastability is *more* rare in the GRF than the RMT analysis suggests. Conversely, for  $V < 0$  the spectrum is shifted to positive values and metastable vacua are more common. Hence, the GRF analysis suggests that the RMT estimate provides an *upper bound* on the frequency of metastable de Sitter vacua in random potentials.

The most studied class of metastable de Sitter solutions arising from string theory involve the ‘uplift’ of a supersymmetric anti-de Sitter solution to positive vacuum energy by the means of a set of supersymmetry breaking anti-branes placed in a warped throat of the compactification manifold. By tuning the fluxes, many moduli can be stabilised supersymmetrically at a parametrically high scale, and are rather insensitive to the uplift. The low-energy effective field theory derived from dimensional reduction of this theory can be rephrased as an interesting version of  $\mathcal{N} = 1$  supergravity [17–26]. Alternative proposals for constructing Minkowski or de Sitter vacua either deform the theory, as in the so-called Kähler uplift [27, 28], or extend its field content, by e.g. the addition of open string fields and gauge dynamics [29–46]. However, these solutions may ultimately only correspond to a particular branch of a much wider set of metastable de Sitter solutions, and it is possible that more general and simpler solutions remain to be discovered.<sup>2</sup>

The purpose of this paper is to construct a new branch of metastable de Sitter solutions of type IIB flux compactifications at large volume. These solutions are in some sense simpler than those obtained from anti-brane breaking: the de Sitter solutions presented in this paper require only the minimal set of ingredients being generated simply from spontaneous supersymmetry breaking in the moduli sector, including a comparatively small amount of supersymmetry breaking from the complex structure moduli and axio-dilaton sector. Consequently, all relevant physics should be captured by the standard  $\mathcal{N} = 1$  supergravity describing the low-energy limit of string compactifications.

The new solutions arise as a direct generalisation of those in [48] (see also [49]) by including multiple Kähler moduli and thereby allowing for a larger compactification volume that in turn is a necessary condition for our construction. Similar ideas have been explored numerically before in particular examples by allowing relevant contributions to the supersymmetry breaking from the axio-dilaton and complex structure sector, either by stabilising the Kähler sector as in the KKL<sub>T</sub> scenario [50], or by non-geometric fluxes in STU-models [51], or by modelling the spectrum by random matrix theory [52].

The de Sitter and Minkowski vacua reported here allow for an analytic treatment and extends the vacua constructed in the so-called Large Volume Scenario (LVS) [53, 54]. Notably, we will present explicit de Sitter solutions, building on LVS, for compactifications on the Calabi-Yau orientifold realised as a hypersurface in  $\mathbb{C}\mathbb{P}_{11169}^4$ .

A key benefit of these solutions is that they render almost all moduli metastable, without much tuning. This is achieved by only weakly breaking the leading order no-scale symmetry of the four-dimensional supergravity, and by utilising the lingering decoupling between the Kähler moduli sector and the complex structure moduli and the axio-dilaton.

These results deepen our understanding of the vacuum structure of the effective theories arising from flux compactifications and may have interesting applications for cosmological and particle physics models in string theory. They may also serve as a rather explicit testing ground for conjectures about non-supersymmetric vacua in string theory [55–57].

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<sup>2</sup>Certain simplifications, as the absence of quantum corrections, still seem problematic though [47].

## 2 Review of type IIB flux compactifications

In this section, we review the structure of supergravity theories in four dimensions descending from the low-energy limit of IIB string theory on a Calabi-Yau orientifold with RR and NSNS fluxes.<sup>3</sup> In particular: in section 2.1 we review the Kähler geometry of the moduli space; in section 2.2 we discuss the flux induced superpotential and possible non-perturbative corrections; and in section 2.3 we detail how physical quantities relevant for the four-dimensional effective theory are captured by the various components of the flux vector.

We consider the four-dimensional spectrum of a compactification on the orientifold  $\tilde{M}_3$  of the Calabi-Yau three-fold  $M_3$ . The low-energy degrees of freedom include the axio-dilaton  $S = C_0 + ie^{-\phi}$ , the complex structure moduli  $u^i$ , where  $i = 1, \dots, h_-^{1,2}(\tilde{M}_3)$ , and the Kähler moduli  $T^a$ , where  $a = 1, \dots, h_+^{1,1}(\tilde{M}_3)$  [58]. The corresponding low-energy theory can often be described by four-dimensional  $\mathcal{N} = 1$  supergravity in which the moduli furnish the scalar components of chiral multiplets. Our convention for indices is as follows:  $A, B$  etcetera run over all moduli fields,  $I, J$  run over all complex structure moduli and the axio-dilaton,  $i, j$  run over complex structure moduli only,  $a, b$  run over Kähler moduli only, and finally,  $s$  runs over ‘blow-up’ Kähler moduli:

$$X^A \equiv (S, u^i, T^a), \quad X^I \equiv (S, u^i), \quad T^a \equiv (T^{\text{big}}, T^s). \quad (2.1)$$

### 2.1 Moduli space geometry

The kinematics of the moduli fields are governed by a real Kähler potential,  $K$ , which at sufficiently large volume and weak string coupling can be written as the sum of three contributions,

$$K = K_{\text{dil}} + K_{\text{c.s.}} + K_K. \quad (2.2)$$

We will throughout this paper use the shorthands  $K_i = \partial_{u^i} K$ ,  $K_a = \partial_{T^a} K$ ,  $K_S = \partial_S K$ , etc, and we will also use the notation  $\tilde{K} = K_{\text{dil}} + K_{\text{c.s.}}$ . The explicit expression for (2.2) is determined by the geometry and topology of the compactification manifold, as we now briefly review.

#### 2.1.1 Axio-dilaton and complex structure deformations

The leading-order Kähler potential for the axio-dilaton  $S$  is given by,

$$K_{\text{dil}} = -\ln [-i(S - \bar{S})]. \quad (2.3)$$

Because of the  $\text{SL}(2, \mathbb{Z})$  S-duality, the axio-dilaton  $S$  can be restricted to the fundamental domain of the torus,  $\{S \in \mathbb{C}, |\text{Re } S| < \frac{1}{2}, |S| \geq 1, \text{Im}(S) > 0\}$ .

The complex structure moduli space is conveniently parametrised by means of projective coordinates  $z^I, I = 0, 1, \dots, h_-^{1,2}$ , which correspond to the periods of the holomorphic three-form  $\Omega$ ,

$$\vec{\Pi} = \begin{pmatrix} \int_{A^I} \Omega \\ \int_{B_I} \Omega \end{pmatrix} \equiv \begin{pmatrix} z^I \\ \mathcal{G}_I \end{pmatrix}, \quad (2.4)$$

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<sup>3</sup>We anticipate that our analysis and results extend straight-forwardly to F-theory compactifications on  $\text{CY}_4$ -manifolds.

where we chose a canonical symplectic basis  $(A^I, B_I)$  of the third homology group of  $H^3(M_3)$  and  $(\alpha_I, \beta^I)$  denote the dual cohomology group basis

$$\int_{M_3} \alpha_I \wedge \beta^J = - \int_{M_3} \beta^J \wedge \alpha_I = \delta_I^J, \quad \int_{M_3} \alpha_I \wedge \alpha_J = \int_{M_3} \beta^I \wedge \beta^J = 0. \quad (2.5)$$

The periods  $\mathcal{G}_I$  satisfy the condition  $2\mathcal{G}_I = \partial_I(z^J \mathcal{G}_J)$ , thus form the gradient of a function that is homogenous of degree two:  $\mathcal{G}_I = \partial_I \mathcal{G}$ .

The set of inhomogeneous coordinates on the complex structure moduli space is conventionally chosen as,

$$u^i = z^i/z^0, \quad i = 1, \dots, h^{1,2}. \quad (2.6)$$

Upon setting  $z^0 = 1$ , the period vector is given by.

$$\vec{\Pi} = \begin{pmatrix} 1 \\ u^i \\ 2\mathcal{F} - u^j \mathcal{F}_j \\ \mathcal{F}_i \end{pmatrix}, \quad (2.7)$$

with the prepotential  $\mathcal{F} = \mathcal{G}/(z_0)^2$ .

An  $\mathcal{N} = 1$  supergravity theory in  $d = 4$ , at low energies, is obtained by working not in  $M_3$  but on its orientifold image  $\tilde{M}_3$ , where only the involution-odd complex structure moduli are kept in the chiral spectrum. Since we will not be concerned with the details of this involution, we will simply restrict  $u^i$  to run over  $i = 1, \dots, h_-^{1,2}$ .

The complex structure-dependent contribution to the Kähler potential is given by,

$$K_{\text{c.s.}} = - \ln \left( i \int_{M_3} \Omega \wedge \bar{\Omega} \right) = - \ln \left( i \vec{\Pi}^\dagger \Sigma \vec{\Pi} \right), \quad (2.8)$$

where  $\Sigma$  denotes the symplectic metric,

$$\Sigma = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}. \quad (2.9)$$

### 2.1.2 Kähler deformations

The imaginary component of the Kähler moduli fields  $T^a$  are given by the four-cycle volumes,  $\tau^a$ , which can be defined from the Calabi-Yau manifold volume,  $\mathcal{V}$ . The volume is a homogeneous function of degree 3 in the two-cycle volumes  $t_a$ , and the four-cycle volumes are defined by  $\tau^a = \mathcal{V}_{t_a}$ . It then follows that,

$$t_a \tau^a = 3\mathcal{V}. \quad (2.10)$$

Our focus in this paper is on de Sitter vacua in the Large Volume Scenario [53, 54], for which it is convenient to consider compactifications volumes of the (strong) ‘Swiss cheese’ type,

$$\mathcal{V} = (\eta_{\text{big}} \tau^{\text{big}})^{\frac{3}{2}} - \sum_{s=1}^{N_{\text{small}}} (\eta_s \tau^s)^{\frac{3}{2}}, \quad (2.11)$$

where  $\eta_a \in \mathbb{R}$ . We have also split the index  $a = 1, \dots, h_+^{1,1}$  into one ‘big’ cycle and  $s = 1, \dots, N_{\text{small}}$  ‘small’ cycles. Here  $N_{\text{small}} = h_+^{1,1} - 1$  is the number of blow-up cycles in the compactification geometry.

The Kähler potential for the Kähler moduli is given by [59],

$$K_K = -2 \ln \left[ \mathcal{V} + \frac{\xi}{2} \left( -i \frac{S - \bar{S}}{2} \right)^{\frac{3}{2}} \right]. \quad (2.12)$$

The contribution multiplying  $\xi = -\frac{\chi(CY_3)\zeta(3)}{2(2\pi)^3}$  arises from  $(\alpha')^3$  corrections in the ten-dimensional theory. At sufficiently large volume, this term provides the dominant correction to the resulting scalar potential [60, 61]. The  $\alpha'$ -correction induces non-vanishing cross-terms between the axio-dilaton and Kähler moduli components of the metric. The relevant components of the inverse metric are given by [62],

$$K_K^{a\bar{b}} = -2 \left( \mathcal{V} + \frac{\hat{\xi}}{2} \right) \left( \frac{\partial^2 \mathcal{V}}{\partial \tau^a \partial \tau^b} \right)^{-1} + \tau^a \tau^b \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}}, \quad (2.13)$$

$$K_K^{a\bar{S}} = i \frac{3}{2} (S - \bar{S}) \tau^a \frac{\hat{\xi}}{\mathcal{V} - \hat{\xi}}, \quad (2.14)$$

$$K_K^{S\bar{S}} = -\frac{(S - \bar{S})^2}{4} \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}}, \quad (2.15)$$

where  $\hat{\xi} = \xi(-i(S - \bar{S})/2)^{3/2}$ .

## 2.2 Flux compactifications

We are interested in compactifications in which integrally quantised RR ( $F_3$ ) and NSNS ( $H_3$ ) fluxes thread some non-trivial three-cycles of  $M_3$ ,

$$\frac{1}{(2\pi)^2 \alpha'} \int_{A^I, B_I} F_3 = \vec{N}_{\text{RR}} \in \mathbb{Z}^{2(h_-^{1,2}+1)}, \quad \frac{1}{(2\pi)^2 \alpha'} \int_{A^I, B_I} H_3 = \vec{N}_{\text{NSNS}} \in \mathbb{Z}^{2(h_-^{1,2}+1)}. \quad (2.16)$$

It is conventional to introduce the complex three-form flux  $G_3 = F_3 - SH_3$ , and convenient to define the complexified flux vector as,

$$\vec{N} = - \begin{pmatrix} \int_{A^I} G_3 \\ \int_{B_I} G_3 \end{pmatrix}. \quad (2.17)$$

The fluxes contribute to the D3-charge tadpole by,

$$Q_{\text{flux}} = \frac{1}{(2\pi)^4 (\alpha')^2} \int_{M_3} H_3 \wedge F_3 = \frac{1}{(2\pi)^4 (\alpha')^2} \vec{h}^T \Sigma \vec{f} = -\frac{1}{(2\pi)^4 (\alpha')^2} K_S \vec{N}^\dagger \Sigma \vec{N}, \quad (2.18)$$

where,

$$\vec{f} = \begin{pmatrix} \int_A F_3 \\ \int_B F_3 \end{pmatrix}, \quad \text{and} \quad \vec{h} = \begin{pmatrix} \int_A H_3 \\ \int_B H_3 \end{pmatrix}. \quad (2.19)$$

The relation between  $\vec{N}$  and  $\vec{f}$  and  $\vec{h}$  is then clearly,

$$\text{Re}(\vec{N}) = -\vec{f}_3 + \text{Re}(S)\vec{h}_3, \quad \text{and} \quad \text{Im}(\vec{N}) = \text{Im}(S)\vec{h}_3. \quad (2.20)$$

Requiring that the total sum of all D3 charges vanishes in the internal space leads to a joint condition on the D3-brane content, the fluxes, and the D7-brane and O-plane configuration,

$$Q_{\text{flux}} + N_{\text{D3}} = \frac{\chi}{24}, \quad (2.21)$$

where  $N_{\text{D3}}$  denotes the net number of D3-branes, and  $\chi$  is the Euler characteristic of the Calabi-Yau fourfold that corresponds to the F-theory lift of our type IIB compactification. Fluxes that preserve supersymmetry contribute positively to this tadpole condition.

The fluxes induce a complex structure and axio-dilaton dependent energy density that in the four-dimensional theory is captured by the flux induced superpotential [63],

$$W_0 = \int_{M_3} G_3 \wedge \Omega = -\vec{N}^T \Sigma \vec{\Pi}. \quad (2.22)$$

This superpotential is exact to all orders in perturbation theory, and does not receive  $\alpha'$ -corrections. However, non-perturbative contributions from Euclidean D3-branes and gaugino condensation on stacks of D7-branes induce additional contributions:

$$W = W_0 + W_{\text{np}} = W_0 + \sum_{s=1}^{N_{\text{small}}} A_s e^{i a_s T^s}, \quad (2.23)$$

where we have specialised to compactifications of ‘Swiss-cheese’ form (2.11), and included non-perturbative corrections for all blow-up moduli  $T^s$ , but — anticipating solutions with large volume — neglected such a correction for the single ‘big cycle’  $T^{\text{big}}$ . The factor  $a_s$  is  $a_s = 2\pi$  for Euclidean D3-branes and  $a_s = 2\pi/N_{\text{D7}}$  for a stack of  $N_{\text{D7}}$  branes. The prefactors  $A_s$  depend on the axio-dilaton and the complex structure moduli. The moduli dependence can in principle be determined from the Pfaffian of one-loop corrections to the instanton action, however such a computation is very difficult. As we show below, the particular functional forms of the prefactors  $A_s$  are generically not important for our analysis and we can consistently treat them as constants.

In this paper we denote by  $D_A$  the Kähler and geometrically covariant derivative, and we use the following notation for covariant derivatives of the superpotential,

$$\begin{aligned} F_A &= D_A W = \partial_A W + K_A W, \\ Z_{AB} &= D_A F_B = \partial_A F_B + K_A F_B - \Gamma_{AB}^C F_C, \\ U_{ABC} &= D_A Z_{BC} = D_B Z_{AC} = D_C Z_{BA}. \end{aligned} \quad (2.24)$$

The supergravity F-term scalar potential is denoted by,

$$V = e^K \left( F_A \bar{F}^A - 3|W|^2 \right), \quad (2.25)$$

where indices are raised with the inverse Kähler metric  $K^{A\bar{B}}$ . The gravitino mass is given by,

$$m_{3/2} = e^{K/2} |W|. \quad (2.26)$$



### 2.3 Decomposition of flux vector

The three-form  $G_3$  is conveniently decomposed by Hodge type, and it is well known that supersymmetric fluxes are of Hodge type (2,1) and primitive [64–66]. This condition is manifest in the low-energy effective description in that, at the complex structure moduli space location of the vacuum, the flux vector  $\vec{N}$  only has non-vanishing components along the  $D_i\vec{\Pi}$  directions. The de Sitter solutions that we construct will involve non-supersymmetric flux vectors that in addition to the (2,1) components have a non-vanishing (0,3) piece (corresponding to  $W_0 \neq 0$ ), and comparatively smaller contributions along the (1,2) and (3,0) directions (corresponding to  $F_i, F_S \neq 0$ ).

To see the relation between physical quantities and fluxes explicitly, we first recall that  $\{\Omega, \bar{\Omega}, D_i\Omega, \bar{D}_i\bar{\Omega}\}$  form a basis of three-forms for generic values of the moduli [67]. Correspondingly,  $\{\vec{\Pi}, \vec{\Pi}^*, D_i\vec{\Pi}, \bar{D}_i\vec{\Pi}^*\}$  form an orthogonal basis with respect to the symplectic inner product [6, 68, 69],

$$\vec{\Pi}^T \Sigma \vec{\Pi}^* = +ie^{-K_{c.s.}}, \quad (2.27)$$

$$D_i\vec{\Pi}^T \Sigma \bar{D}_j\vec{\Pi}^* = -iK_{ij}e^{-K_{c.s.}}, \quad (2.28)$$

$$\vec{\Pi}^T \Sigma D_i\vec{\Pi} = \vec{\Pi}^T \Sigma \bar{D}_i\vec{\Pi}^* = D_i\vec{\Pi}^T \Sigma D_j\vec{\Pi} = 0. \quad (2.29)$$

Hence, any vector  $\vec{V}$  in the ‘period/charge’ vector space can be expanded as follows:

$$\vec{V} = ie^{K_{c.s.}} \left( (\vec{\Pi}^\dagger \Sigma \vec{V}) \vec{\Pi} - (\vec{\Pi}^T \Sigma \vec{V}) \vec{\Pi}^* - (K^{j\bar{l}} \bar{D}_j \vec{\Pi}^\dagger \Sigma \vec{V}) D_l \vec{\Pi} + (K^{j\bar{l}} D_j \vec{\Pi}^T \Sigma \vec{V}) \bar{D}_l \vec{\Pi}^* \right),$$

for  $\vec{\Pi}$  evaluated at a generic point in the complex structure moduli space. For the flux vector  $\vec{N}$ , the expansion coefficients have direct physical interpretations,

$$\vec{N} = ie^{K_{c.s.}} \left( \frac{\bar{F}_{\bar{S}}}{K_{\bar{S}}} \vec{\Pi} - W_0 \vec{\Pi}^* - \frac{\bar{Z}^i}{K_{\bar{S}}} D_i \vec{\Pi} + F^{\bar{i}} \bar{D}_i \vec{\Pi}^* \right). \quad (2.30)$$

which follows straightforwardly from the superpotential and by using  $D_S \vec{N} = K_S \vec{N}^*$ .

We note that  $D_i D_j \Omega$  is purely (1,2) and can be expanded using only the three-forms  $\bar{D}_i \bar{\Omega}$ . Therefore the component  $Z_{ij} = D_i D_j W$  of the tensor are not free variables, but given in terms of the fluxes  $\bar{Z}_{i\bar{S}}$  as,

$$Z_{ij} = +ie^{K_{c.s.}} \kappa_{ij}^{\bar{k}} \vec{N} \Sigma \bar{D}_{\bar{k}} \vec{\Pi}^* = -\frac{i}{K_{\bar{S}}} e^{K_{c.s.}} \kappa_{ij}^{\bar{k}} \bar{Z}_{\bar{k}\bar{S}}, \quad (2.31)$$

where  $\kappa_{ijk}$  denotes the (in general field-dependent) ‘Yukawa couplings’,  $\kappa_{ijk} = \int_{M_3} \Omega \wedge \partial_{ijk}^3 \Omega$ . The flux tadpole contribution may now be expressed as,

$$(2\pi)^4 (\alpha')^2 Q_{\text{flux}} = -iK_S e^{K_{c.s.}} \left( |W_0|^2 + \frac{1}{|K_S|^2} Z_{S\bar{l}} \bar{Z}^{\bar{l}}_{\bar{S}} - F_i \bar{F}^i - \left| \frac{F_S}{K_S} \right|^2 \right). \quad (2.32)$$

We will find the decomposition of the flux vector (2.30) very useful in constructing explicit de Sitter vacua in section 5.

### 3 No-scale symmetry, slightly broken

In this section we review the implications of no-scale symmetry for the low-energy effective theory from string compactifications. Our particular focus will be on the metastable de Sitter vacua constructed in [48], for which the slightly broken no-scale symmetry bestows favourable metastability properties.

#### 3.1 No-scale symmetry

Four-dimensional  $\mathcal{N} = 1$  supergravity theories are said to be ‘no-scale’ [70–72] if a subset of the fields, here denoted  $T^a$ ,

- i)* have no Kähler potential cross-couplings with the other fields, here denoted  $X^I$ , so that  $K = K_1(X^I) + K_2(T^a)$ ,
- ii)* do not appear in the superpotential,  $W = W_0(X^I)$ ,<sup>4</sup>
- iii)* have a field-space geometry satisfying the no-scale condition,

$$K_a K^{ab} K_{\bar{b}} = 3. \tag{3.1}$$

Such theories can admit solutions to the critical point equation,  $\partial_A V = 0$ , with  $F_I = 0$  and  $F_a = K_a W$ , so that,

$$V = e^K (F_A \bar{F}^A - 3|W|^2) = e^K F_I \bar{F}^I = 0. \tag{3.2}$$

In these non-supersymmetric Minkowski solutions, the fields  $X^I$  are stabilised while the fields  $T^a$  remain unfixed. Specifically, for these solutions  $\partial_{T\bar{T}}^2 V = \partial_{T\bar{T}}^2 V = \partial_{T\bar{T}}^2 V = \partial_{T\bar{T}}^2 V = 0$ . The Hessian matrix for the fields  $X^I$  is, in the notation of equation (2.24), given by,

$$\mathcal{H} = \begin{pmatrix} \partial_{I\bar{J}}^2 V & \partial_{I\bar{J}}^2 V \\ \partial_{\bar{I}J}^2 V & \partial_{\bar{I}J}^2 V \end{pmatrix} = e^K \begin{pmatrix} (Z\bar{Z})_{I\bar{J}} + K_{I\bar{J}}|W|^2 & 2\bar{W}Z_{IJ} \\ 2W\bar{Z}_{\bar{I}\bar{J}} & (\bar{Z}Z)_{\bar{I}\bar{J}} + K_{\bar{I}\bar{J}}|W|^2 \end{pmatrix}. \tag{3.3}$$

For canonically normalised fields<sup>5</sup> it has the eigenvalues [48],

$$m_{I\pm}^2 = e^K (\lambda_I \pm |W|)^2, \tag{3.4}$$

where  $\lambda_I^2$  is an eigenvalue of  $(Z\bar{Z})_{I\bar{J}} \equiv Z_{IK}\bar{Z}_{\bar{J}}^K$ . The semi-positive-definiteness of the eigenvalues of the Hessian follows from the convexity of the potential (3.2), and it has important consequences for the metastability of de Sitter vacua constructed from theories with slightly broken no-scale symmetry.

To lowest order in the  $g_s$  and  $\alpha'$  expansions of type IIB string theory, the Kähler potential of the Kähler moduli sector is given by  $K_K = -2 \ln \mathcal{V}$ , cf. (2.12), and there are no

<sup>4</sup>A weaker form of ‘generalised no-scale condition’ takes  $W(X^I, T^a)$  but postulates that the first three derivatives of  $W$  with respect to the fields  $T^a$  vanish at the critical point [73].

<sup>5</sup>We call fields  $\tilde{\phi}^I$  canonically normalised, if they satisfy  $K_{I\bar{J}}\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}} = \delta_{I\bar{J}}\partial_\mu\tilde{\phi}^I\partial^\mu\bar{\tilde{\phi}}^{\bar{J}}$ , and we call the eigenvalues of the Hessian the masses squared.

cross-couplings between Kähler moduli and the complex structure/axio-dilaton moduli sector, i.e.  $X^I = (u^i, S)$ . The geometric condition that the volume is a homogeneous function of degree 3/2 in the four-cycle volumes, cf. equation (2.10), implies that  $K^{a\bar{b}}K_aK_{\bar{b}} = 3$ . Moreover, to this order, the superpotential is simply given by the Kähler moduli independent flux-induced contribution (2.22), and the Kähler moduli enjoy a leading order no-scale symmetry. Consequently, the complex structure moduli and the axio-dilaton have squared masses given by (3.4), once canonically normalised.

### 3.2 Approximately no-scale de Sitter vacua

In type IIB compactifications at low energies the no-scale symmetry is only approximate, broken by both string loop and sigma model corrections. For example, the leading  $(\alpha')^3$  correction captured by the Kähler potential (2.12) implies that,

$$K_a = -2\frac{\mathcal{V}_a}{\mathcal{V}} \left( 1 - \frac{\hat{\xi}}{2\mathcal{V}} \right), \tag{3.5}$$

so that the  $\alpha'$ -corrections break the no-scale condition by a small, volume suppressed, amount:

$$K^{a\bar{b}}K_aK_{\bar{b}} = 3 + \frac{3}{4}\frac{\hat{\xi}}{\mathcal{V}}. \tag{3.6}$$

This small breaking is of critical importance to the AdS vacua of the Large Volume Scenario (as we review in section 4.1), and will become equally important for our de Sitter vacua constructed in section 4. For the remainder of this section however, we take  $\hat{\xi} \rightarrow 0$  and consider a different type of no-scale breaking that arises from a postulated, small explicit *superpotential* deformation of the theory.

More precisely, following [48], we specialise to the single Kähler modulus case and take,

$$W(T, X^I) = W_0(X^I) + \delta W(T, X^I). \tag{3.7}$$

The superpotential deformation  $\delta W$  (and any derivative of it) is assumed to be small compared to  $W_0$ :  $|\delta W/W_0| \ll 1$ . The F-terms are now given by,

$$F_T = K_T W + \delta W_T, \text{ and } F_I = \epsilon W f_I, \tag{3.8}$$

where  $\epsilon \ll 1$  and  $f_I$  is a unit vector. A consistent solution of the critical point equation  $\partial_T V = 0$  takes  $\delta W$  (and any derivatives of it) to be of  $\mathcal{O}(\epsilon^2 W)$ . The critical point equations for the remaining fields,  $\partial_I V = 0$ , imply that,

$$Z_{IJ}\bar{F}^J = -\bar{W}F_I, \tag{3.9}$$

up to corrections of  $\mathcal{O}(\epsilon^2)$ . Note that  $Z_{IJ}$  is a complex symmetric tensor and hence not unitarily diagonalisable. However, equation (3.9) may be recast as an eigenvalue equation for the Hermitian matrix  $(Z\bar{Z})_{I\bar{J}}$ , the eigenvalue  $|W|^2$  and the eigenvector  $F_I$ :

$$(Z\bar{Z})_I{}^J F_J = |W|^2 F_I. \tag{3.10}$$

An immediate and important implication of equation (3.10) is that one of the eigenvalues  $\lambda_I^2$  of  $(Z\bar{Z})$  will be precisely equal to  $|W|^2$ , and hence, according to equation (3.4), there is one real degree of freedom in the complex structure and axio-dilaton sector that is massless to leading order:

$$m_{1\pm}^2 = \begin{cases} 2e^K|W|^2 + \mathcal{O}(\epsilon|W|^2) , \\ \mathcal{O}(\epsilon|W|^2) \end{cases} . \quad (3.11)$$

To order  $\mathcal{O}(\epsilon^0)$ , the massless spectrum of these solutions contains a total of three modes: the real and imaginary parts of  $T$  and  $X^{1-}$ .

To linear order in  $\epsilon$ , the relevant part of the Hessian matrix is given by,

$$\mathcal{H} = \begin{pmatrix} \partial_{I\bar{J}}^2 V & \partial_{I\bar{J}}^2 V \\ \partial_{I\bar{J}}^2 V & \partial_{I\bar{J}}^2 V \end{pmatrix} = e^K \begin{pmatrix} (Z\bar{Z})_{I\bar{J}} + K_{I\bar{J}}|W|^2 & 2\bar{W}Z_{I\bar{J}} + U_{I\bar{J}K}\bar{F}^K \\ 2W\bar{Z}_{I\bar{J}} + \bar{U}_{I\bar{J}K}F^{\bar{K}} & (\bar{Z}\bar{Z})_{I\bar{J}} + K_{I\bar{J}}|W|^2 \end{pmatrix} . \quad (3.12)$$

The field  $X^{1-}$  is then lifted at  $\mathcal{O}(\epsilon)$  with the squared mass,

$$m_{1-}^2 = e^K \text{Re} (U_{I\bar{J}K} \bar{f}^I \bar{f}^J \bar{f}^K \bar{W}) \epsilon , \quad (3.13)$$

in the generic case of  $U_{I\bar{J}K} \bar{f}^I \bar{f}^J \bar{f}^K \sim \mathcal{O}(1)$ . This squared mass can be made positive by a moderate tuning of the phase of the superpotential.

In sum, the full spectrum of these solutions then consists of three sets of fields:

- All  $X^I$ , except for  $X^{1-}$ , receive positive squared masses at  $\mathcal{O}(\max(e^K \lambda_I^2, e^K |W|^2))$ , and are not destabilised by the inclusion of small amounts of supersymmetry breaking in the complex structure and axio-dilaton sector.
- The field  $X^{1-}$  is lighter than the other fields in this sector, but can be stabilised at  $\mathcal{O}(F_I/W) \sim \epsilon$  by a small amount of tuning.
- The real and imaginary parts of  $T$  are lifted at  $\mathcal{O}(\epsilon^2)$  and the corresponding eigenvalues of the Hessian matrix are given by,

$$m_{T\pm}^2 = \left( -\frac{4K_T}{3} \text{Re}(\bar{W}\delta W_T) \pm |K^T \bar{W} \delta W_{TTT} - \frac{4K_T}{3} \text{Re}(\bar{W}\delta W_T)| \right) e^K . \quad (3.14)$$

As shown in [48], both these eigenvalues can be rendered positive. However, in the case of a single Kähler modulus, the tuning is somewhat restrictive. For example, in the case of  $\delta W$  arising from a single non-perturbative effect,  $\delta W = A(X^i) \exp(iaT)$ , positivity of  $m_{T-}^2$  requires  $a\tau < \sqrt{2}$ , while de Sitter minima have  $a\tau > 1$ . These conditions restrict the values of  $\epsilon$  that are realisable, given the vacuum expectation values of  $A$  and  $W_0$ :  $1.47|A|/|W_0| < \epsilon^2 < 1.57|A|/|W_0|$ . Hence, in this case  $\epsilon \ll 1$  requires  $|A| \ll W_0$ .

In this paper, we will go beyond the discussion of [48] by including no-scale breaking effects in both the superpotential and the Kähler potential, and we will consider compactifications with an arbitrary number of moduli. In this arguably more interesting case, we will see that the conditions for metastable de Sitter vacua can be greatly relaxed.

## 4 De Sitter vacua at large volume

In this section, we describe how the prescription of reference [48] can be generalised to produce metastable de Sitter vacua at exponentially large volume, thereby extending the non-supersymmetric AdS vacua of [53].

### 4.1 Anti-de Sitter vacua in the Large Volume Scenario

We briefly recall the prescription of the Large Volume Scenario (LVS) for constructing non-supersymmetric AdS vacua [53]. Starting from the  $\alpha'$ -corrected Kähler potential (2.2) and the flux-induced superpotential (2.23), the complex structure and axio-dilaton are stabilised by requiring that the three-form flux background is supersymmetric, i.e.  $F_I = 0$ . To zeroth order in  $\alpha'$ , the Kähler moduli sector is no-scale, and the squared masses of the complex structure moduli and the axio-dilaton are given by equation (3.4). Under these circumstances (3.10) does not imply  $\lambda_I = |W|$ , and generically, the complex structure moduli and the axio-dilaton are stabilised at the scale  $\mathcal{O}(\max(e^K \lambda_I^2, e^K |W|^2))$  for each  $\lambda_I$ . These moduli are then generically decoupled from the details of the Kähler moduli stabilisation [74–76].

Secondly, the axions of the ‘small’ cycles,  $\text{Re}(T^s)$ , are lifted by contributions to the potential of the form  $e^K (W_{np} \bar{W} + \text{c.c.})$ . Minimising the resulting cosine potentials results in a scalar potential only involving the overall volume  $\mathcal{V}$  and the ‘small’ Kähler moduli,  $\tau^s$ . The leading contributions to the scalar potential in an expansion in powers of the inverse volume are,

$$\begin{aligned}
 V_{\text{LVS}} &= e^{\tilde{K}} \left[ \frac{3\xi(\text{Im}(S))^{3/2}}{4\mathcal{V}^3} |W_0|^2 - \sum_s^{N_{\text{small}}} \frac{4a_s \tau^s |A_s W_0|}{\mathcal{V}^2} e^{-a_s \tau^s} + \sum_s^{N_{\text{small}}} \frac{8\sqrt{\tau^s}}{3\eta_s^{3/2} \mathcal{V}} a_s^2 |A_s|^2 e^{-2a_s \tau^s} \right], \\
 &\equiv \frac{a}{\mathcal{V}^3} - \frac{\sum_{s=1}^{N_{\text{small}}} b_s x_s e^{-x_s}}{\mathcal{V}^2} + \frac{\sum_{s=1}^{N_{\text{small}}} c_s \sqrt{x_s} e^{-2x_s}}{\mathcal{V}}, \tag{4.1}
 \end{aligned}$$

where we have introduced the notation  $x_s = a_s \tau^s$ . To consistently neglect higher order instanton corrections, we only consider solutions with  $x_s > 1$  for all  $s$ .

The critical point equations  $\partial_{x_s} V = 0$  are solved by,

$$\mathcal{V}_{\text{min}} = 2 \frac{b_s}{c_s} e^{x_s} \sqrt{x_s} \left( \frac{x_s - 1}{4x_s - 1} \right) \approx \frac{b_s}{2c_s} e^{x_s} \sqrt{x_s}, \tag{4.2}$$

for each  $x_s$ . In the last step we approximate  $x_s \gg 1$ . Using the above in  $\partial_{\mathcal{V}} V = 0$ , implies that,

$$a = 4 \sum_{s=1}^{N_{\text{small}}} \frac{b_s^2}{c_s} x_s^{5/2} \frac{(x_s - 1)}{(4x_s - 1)^2} \approx \frac{1}{4} \sum_{s=1}^{N_{\text{small}}} \frac{b_s^2}{c_s} x_s^{3/2}. \tag{4.3}$$

This critical point is the minimum of the potential for  $\mathcal{V}$ . Away from the critical point the potential approaches zero from below in the limit  $x_s \rightarrow \ln \mathcal{V}$  and  $\mathcal{V} \rightarrow \infty$ :

$$V_{\text{LVS}} \sim -\frac{\ln \mathcal{V}}{\mathcal{V}^3} \rightarrow 0. \tag{4.4}$$

Consequently, the minimum is a non-supersymmetric AdS vacuum. The leading-order vacuum energy is given by,

$$V_{\text{LVS}}|_{\text{min}} = -\frac{2}{\mathcal{V}_{\text{min}}^3} \left( \sum_s \frac{b_s^2(x_s - 1)x_s^{3/2}}{c_s(4x_s - 1)^2} \right) \approx -\frac{1}{8\mathcal{V}_{\text{min}}^3} \sum_s \frac{b_s^2}{c_s} \sqrt{x_s}. \quad (4.5)$$

## 4.2 Large volume extrema

We now generalise the prescription of the Large Volume Scenario to construct de Sitter vacua with an exponentially large volume. The key idea is, just as discussed in section 3.2, to consistently include a small amount of flux-induced supersymmetry breaking in the complex structure and axio-dilaton sector, as parametrised by the F-terms,

$$F_I = \epsilon W f_I, \quad (4.6)$$

with  $\epsilon \ll 1$  a small parameter and  $f_I$  a unit vector. Our construction generalises that of [48] by including multiple Kähler moduli and additional no-scale breaking from  $\alpha'$ -corrections to the Kähler potential.

An immediate concern is that non-vanishing F-terms for the complex structure and the axio-dilaton potentially destabilise the potential and cause decompactification. The additional F-terms,  $F_I \bar{F}^I = F_i \bar{F}^i + F_S \bar{F}^S$ , contribute to the potential like,

$$V = V_{\text{LVS}} + \frac{e^{\tilde{K}} F_I \bar{F}^I}{\mathcal{V}^2}. \quad (4.7)$$

Since  $V_{\text{LVS}} \sim \mathcal{V}^{-3}$ , we see that generic amounts of supersymmetry breaking,  $F_I \bar{F}^I \sim |W|^2$ , source a dominant run-away potential for the volume.<sup>6</sup> If the supersymmetry breaking in the complex structure and axio-dilaton sector is suitably small, however,  $F_I \bar{F}^I$  can provide the positive energy contribution necessary to lift the LVS AdS vacua to positive vacuum energy, without causing destabilisation.

For  $F_I = \epsilon W f_I$  as in (4.6), we have  $F_I \bar{F}^I = \epsilon^2 |W|^2$  and the condition that at the minimum the extra contribution to the potential is of the same order as the LVS contribution requires that:

$$\epsilon = \mathcal{O}(\mathcal{V}_{\text{min}}^{-1/2}). \quad (4.8)$$

Such a small contribution can be achieved by the tuning of fluxes: for continuous fluxes, such tuning is always possible; for quantised fluxes, we expect this be possible in compactifications with many cycles and a large flux tadpole. We emphasise, however, that these new F-terms are the consequence of non-supersymmetric three-form fluxes, and not the result of an ubiquitous small backreaction from the non-SUSY solution in the Kähler sector,

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<sup>6</sup>The existence of run-away directions in the moduli space has since long been identified as a critical aspect of string compactifications [77]. A detailed analysis of the  $F_I \bar{F}^I \sim |W|^2$  case is intricate: for comparatively small volumes and large supersymmetry breaking the various contributions to the potential become of similar magnitude and couplings between the axio-dilaton, complex structure and the Kähler moduli are all important [74, 75]. Moreover, the applicability of the four-dimensional  $\mathcal{N} = 1$  supergravity as an effective description of the low-energy theory becomes questionable as the hierarchy between the compactification scale and the supersymmetry breaking scale decreases. We will not be concerned about this regime in this paper.

which induces even smaller F-terms of the order of  $F_I \bar{F}^I \sim \mathcal{O}(|W|^2/\mathcal{V}_{\min}^2) \sim \mathcal{O}(\epsilon^4|W|^2)$  in the standard Large Volume Scenario.

To construct perturbatively metastable vacua, we need to solve the critical point equations and ensure that the Hessian matrix has no negative eigenvalues. With non-vanishing supersymmetry breaking in the complex structure and axio-dilaton sector, the critical point equation for these moduli fields becomes,

$$\partial_I V = \partial_I V_{\text{LVS}} + e^{\tilde{K}} \frac{(\bar{F}^J Z_{IJ} + F_I \bar{W})}{\mathcal{V}^2}, \tag{4.9}$$

where we have used that  $\partial_I (e^{\tilde{K}} F_J \bar{F}^J) = e^{\tilde{K}} (\bar{F}^J Z_{IJ} + F_I \bar{W})$ . Since  $(\bar{F}^J Z_{IJ} + F_I \bar{W}) \sim \mathcal{V}_{\min}^{-1/2}$  and  $\partial_I V_{\text{LVS}}$  scales like  $\mathcal{V}_{\min}^{-3}$ , the second term provides the leading order contribution to the critical point equation in an expansion in the inverse volume. Thus, up to corrections of  $\mathcal{O}(\mathcal{V}_{\min}^{-1})$ , the equation  $\partial_I V = 0$  implies that,<sup>7</sup>

$$\bar{F}^J Z_{IJ} = -\bar{W} F_I. \tag{4.10}$$

This is precisely equation (3.9). The implications of this equation are again very important: the three-form fluxes can stabilise most of the fields in the complex structure and axio-dilaton sector with squared masses of  $\sim 1/\mathcal{V}_{\min}^2$ , but a consistent inclusion of the additional supersymmetry breaking effects requires one eigenvalue of  $(Z\bar{Z})$  to be equal to  $|W|^2$ . So, again, one real component of these fields is unstabilised at leading order. We will again denote this component by  $X^{1-}$  and we will see in section 4.3 that this field is lifted at subleading order and can be stabilised by very modest amounts of tuning.

Granted a solution of equation (4.10) (we give an explicit prescription for solving it in section 5), we now turn to the critical point equation for the Kähler moduli. The scalar potential is now given by,

$$V = \frac{\tilde{a}}{\mathcal{V}^3} - \frac{\sum_s \tilde{b}_s x_s e^{-x_s} - \epsilon^2 \tilde{f}^2}{\mathcal{V}^2} + \frac{\sum_s \tilde{c}_s \sqrt{x_s} e^{-2x_s}}{\mathcal{V}}, \tag{4.11}$$

where,  $\tilde{a}$ ,  $\tilde{b}$ ,  $\tilde{c}$ ,  $\tilde{f}$  are complex structure and axio-dilaton dependent (but Kähler moduli independent) parameters:

$$\tilde{a} = \frac{3}{4} e^{\tilde{K}} \xi (\text{Im}(S))^{3/2} |W_0|^2, \quad \tilde{b}_s = 4e^{\tilde{K}} |A_s W_0|, \tag{4.12}$$

$$\tilde{c}_s = \frac{8}{3} e^{\tilde{K}} \frac{a_s^{\frac{3}{2}} |A_s|^2}{\eta_s^{3/2}}, \quad \epsilon^2 \tilde{f}^2 = e^{\tilde{K}} F_I \bar{F}^I. \tag{4.13}$$

Clearly, while the new  $\tilde{f}^2 = \exp(\tilde{K})|W_0|^2$  contribution affects the equation  $\partial_{\mathcal{V}} V = 0$ , it does not enter into the critical point equations for the ‘small’ Kähler moduli,  $\partial_{x_s} V = 0$ . Hence, we again find,

$$\mathcal{V}_{\min} = 2 \frac{\tilde{b}_s}{\tilde{c}_s} e^{x_s} \sqrt{x_s} \left( \frac{x_s - 1}{4x_s - 1} \right) \approx \frac{\tilde{b}_s}{2\tilde{c}_s} e^{x_s} \sqrt{x_s}, \tag{4.14}$$

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<sup>7</sup>Since  $Z_{SS} \equiv 0$  in general,  $F_I$  cannot be exactly aligned with the axio-dilaton direction, but needs to have some non-vanishing component in the complex structure sector.

for each  $x_s$ , as in equation (4.2). Equation (4.3) instead is generalised to,

$$\tilde{a} + \frac{2}{3}\epsilon^2 \tilde{f}^2 \mathcal{V}_{\min} = 4 \sum_{s=1}^{N_{\text{small}}} \frac{b_s^2}{c_s} x_s^{5/2} \frac{(x_s - 1)}{(4x_s - 1)^2} \approx \frac{1}{4} \sum_{s=1}^{N_{\text{small}}} \frac{b_s^2}{c_s} x_s^{3/2}. \quad (4.15)$$

The implicit solutions of the critical point equations (cf. (4.10), (4.14) and (4.15)) do not guarantee that the solution is a local minimum of the potential. To investigate the metastability of these we now turn to the Hessian matrix.

### 4.3 Metastable vacua

Perturbative metastability requires that all the eigenvalues of the Hessian matrix are positive. A key concern is the magnitude of the off-diagonal terms in the mass matrix, which lead to mass splittings that can cause destabilisation. To estimate the importance of these effects, it often suffices to recall that to second order in matrix perturbation theory for the matrix  $\mathcal{H}_{AB} = \mathcal{H}_{AB}^{(0)} + \delta\mathcal{H}_{AB}$ , the perturbed eigenvalues are given by,

$$m_A^2 = m_{(0),A}^2 + \delta\mathcal{H}_{AA} + \sum_{B \neq A} \frac{|\delta\mathcal{H}_{AB}|^2}{m_{(0),A}^2 - m_{(0),B}^2}, \quad (4.16)$$

where  $m_{(0),A}^2$  denotes the eigenvalues of the unperturbed matrix  $\mathcal{H}_{AB}^{(0)}$ . We will now show that the volume scaling of the various terms in the Hessian matrix makes ensuring perturbative stability a rather simple task.

Most of the fields in the complex structure moduli and the axio-dilaton sector are stabilised at a high scale. The Hessian matrix to zeroth order in  $\epsilon$  is again given by (3.12), and these fields have canonically normalised squared masses,<sup>8</sup>

$$m_{I\pm}^2 = \frac{e^{\tilde{K}} (\lambda_I \pm |W_0|)^2}{\mathcal{V}^2} + \mathcal{O}(1/\mathcal{V}^{5/2}). \quad (4.17)$$

However, since the critical point equation (4.10) requires that  $(Z\bar{Z})$  has one eigenvalue equal to  $|W|^2$ , one real degree of freedom in this sector,  $X^{1-}$ , is lifted at  $\mathcal{O}(1/\mathcal{V}^{5/2})$  precisely as discussed in section 3.2. The leading order squared mass is again given by,

$$m_{1-}^2 = \frac{e^{\tilde{K}} \text{Re}(U_{IJK} \bar{f}^I \bar{f}^J \bar{f}^K \bar{W})}{\mathcal{V}^2} \epsilon. \quad (4.18)$$

In general we expect the contribution  $U_{IJK} \bar{f}^I \bar{f}^J \bar{f}^K$  to be order one. If this contribution is sub-leading, however, this mode will be lifted at  $\mathcal{O}(1/\mathcal{V}_{\min}^3)$  and a more complicated analysis is required. This could occur if  $D_k(\kappa_{ijl} e^{K_{c.s.}}) = 0$ , which holds if the moduli space is a symmetric space with a covariantly constant Riemann tensor. This is the case for the *STU*-model considered in [48], but for the moduli space of a more general Calabi-Yau compactification,  $D_k(\kappa_{ijl} e^{K_{c.s.}}) \neq 0$ .<sup>9</sup>

<sup>8</sup>The volume scaling of the masses in this sector is unaffected by the canonical normalisation.

<sup>9</sup>We note that, contrary to an aside assertion of [7], generically  $D_k(\kappa_{ijl} e^{K_{c.s.}}) \neq 0$  in the large complex structure limit of the compactification manifold.



The stability of the Kähler moduli sector can be understood from the volume scaling of the elements of the full Hessian matrix. We here, for simplicity, consider non-canonically normalised fields; canonical normalisation does not change the metastability of a critical point. Cross-terms between the ‘small’ Kähler moduli on the one hand and the axio-dilaton and complex structure moduli on the other arise only from  $V_{\text{LVS}}$  in equation (4.7), and hence  $\partial_{I\tau^s}^2 V \sim \partial_{\bar{I}\tau^s} V \sim 1/\mathcal{V}^3$ . Cross-terms with the overall volume modulus scale like  $\partial_{IV}^2 V \sim \partial_{\bar{I}V}^2 V \sim 1/\mathcal{V}^4$ .

The Hessian matrix is then schematically given by,

$$\mathcal{H} = \left( \begin{array}{cc|cc} m_{I\pm}^2 & \vec{0} & \partial_{I\tau^s}^2 V & \partial_{IV}^2 V \\ \vec{0}^T & m_{1-}^2 & \partial_{I\tau^s}^2 V & \partial_{IV}^2 V \\ \hline \partial_{I\tau^s}^2 V & \partial_{I\tau^s}^2 V & \partial_{\tau^s\tau^s}^2 V & \partial_{\tau^s\mathcal{V}}^2 V \\ \partial_{IV}^2 V & \partial_{IV}^2 V & \partial_{\tau^s\mathcal{V}}^2 V & \partial_{\mathcal{V}\mathcal{V}}^2 V \end{array} \right) = \mathcal{O}(\mathcal{V}^{-2}) \left( \begin{array}{cc|cc} 1 & \vec{0}^T & \mathcal{O}(\mathcal{V}^{-1}) & \mathcal{O}(\mathcal{V}^{-2}) \\ \vec{0} & \mathcal{O}(\mathcal{V}^{-\frac{1}{2}}) & \mathcal{O}(\mathcal{V}^{-1}) & \mathcal{O}(\mathcal{V}^{-2}) \\ \hline \mathcal{O}(\mathcal{V}^{-1}) & \mathcal{O}(\mathcal{V}^{-1}) & \mathcal{O}(\mathcal{V}^{-1}) & \mathcal{O}(\mathcal{V}^{-2}) \\ \mathcal{O}(\mathcal{V}^{-2}) & \mathcal{O}(\mathcal{V}^{-2}) & \mathcal{O}(\mathcal{V}^{-2}) & \mathcal{O}(\mathcal{V}^{-3}) \end{array} \right). \tag{4.19}$$

Using equation (4.16), we see that cross-terms between the complex structure and axio-dilaton sector and the Kähler moduli contribute only with sub-leading corrections. The ‘small’ Kähler moduli squared masses are corrected at order  $\mathcal{O}(\mathcal{V}^{-7/2})$ , but the diagonal contributions are of  $\mathcal{O}(\mathcal{V}^{-3})$ . The overall volume squared mass scales is corrected at  $\mathcal{O}(\mathcal{V}^{-11/2})$ , but the leading order contributions from the Kähler sector enter at  $\mathcal{O}(\mathcal{V}^{-5})$ . Thus, we can consistently neglect cross-couplings between the axio-dilaton and complex structure moduli on the one hand, and the Kähler moduli on the other. We also note that the volume scalings of equation (4.19) are still satisfied for moduli dependent prefactors  $A_s(S, u^i)$  of equation (2.23), and we expect the decoupling to apply also in this more general case.

Focussing on the Kähler moduli sector, equations (4.16) and (4.19) imply that the small Kähler moduli are stabilised at  $\mathcal{O}(1/\mathcal{V}^3)$ , and that cross-couplings with the overall volume modulus only lead to very small corrections. Since the potential (4.1) is sum-separable in the fields  $\tau^s$ , the only non-vanishing elements of the Hessian matrix are the diagonal values which (again for non-canonically normalised fields) are given by,

$$\partial_{\tau^s\tau^s}^2 V|_{\min} = \frac{a_s^2 \tilde{b}_s^2 (x_s - 1) (x_s (8x_s^2 - 6x_s + 3) + 1)}{\tilde{c}_s \mathcal{V}_{\min}^3 (4x_s - 1)^2 \sqrt{x_s}} \approx \frac{a_s^2 \tilde{b}_s^2 x_s^{\frac{3}{2}}}{2\tilde{c}_s \mathcal{V}_{\min}^3}, \tag{4.20}$$

with no sum on  $s$ . Here we have used equations (4.14) and in the last step we took  $x_s \gg 1$ . Equation (4.20) clearly implies positive squared masses for the small Kähler moduli, just as in the standard Large Volume Scenario.

We now turn to the metastability of the overall volume, and the possibility to get Minkowski or de Sitter vacua from this construction.

#### 4.4 Metastability condition and de Sitter vacua

The leading order vacuum energy in the presence of  $\tilde{f} \neq 0$  is given by,

$$V|_{\min} = \frac{1}{\mathcal{V}_{\min}^3} \left( \frac{1}{3} \epsilon^2 \tilde{f}^2 \mathcal{V}_{\min} - 2 \sum_s \frac{\tilde{b}_s^2 (x_s - 1) x_s^{3/2}}{\tilde{c}_s (4x_s - 1)^2} \right). \tag{4.21}$$

Clearly, a non-negative vacuum energy then requires,

$$6 \sum_s \frac{\tilde{b}_s^2 (x_s - 1) x_s^{3/2}}{\tilde{c}_s (4x_s - 1)^2} \leq \epsilon^2 \tilde{f}^2 \mathcal{V}_{\min}. \quad (4.22)$$

However,  $\tilde{f}$  is also bounded from above: the lightest mode, which coincides at leading order with the overall volume modulus, is destabilised by too large flux-induced F-terms. More precisely, the second derivative of the potential with respect to the volume is given by,

$$\partial_{\mathcal{V}}^2 V|_{\min} = \frac{2}{\mathcal{V}_{\min}^5} \left( 2 \sum_s \frac{\tilde{b}_s^2 x_s^{3/2} (2x_s^2 - x_s - 1)}{\tilde{c}_s (4x_s - 1)^2} - \epsilon^2 \tilde{f}^2 \mathcal{V}_{\min} \right). \quad (4.23)$$

The corresponding eigenvalue of the Hessian matrix gives the squared mass. For a single small blow-up modulus and non-canonically normalised fields it is given by,

$$m_{\mathcal{V}}^2 = \frac{2}{\mathcal{V}_{\min}^5} \left( \frac{6\tilde{b}_1^2 x_1^{5/2} (12x_1^3 - 23x_1^2 + 16x_1 - 5)}{c_1 (4x_1 - 1)^2 (8x_1^3 - 6x_1^2 + 3x_1 + 1)} - \epsilon^2 \tilde{f}^2 \mathcal{V}_{\min} \right). \quad (4.24)$$

Thus, metastability requires,

$$\epsilon^2 \tilde{f}^2 \mathcal{V}_{\min} < \frac{6\tilde{b}_1^2 x_1^{5/2} (12x_1^3 - 23x_1^2 + 16x_1 - 5)}{c_1 (4x_1 - 1)^2 (8x_1^3 - 6x_1^2 + 3x_1 + 1)}. \quad (4.25)$$

Defining,

$$h_s(x_s) = \frac{\tilde{b}_s^2 (x_s - 1) x_s^{3/2}}{\tilde{c}_s (4x_s - 1)^2} \geq 0, \quad (4.26)$$

the constraints (4.22) and (4.25) can be written as,

$$1 \leq \frac{\epsilon^2 \tilde{f}^2 \mathcal{V}}{6h_1(x_1)} < \frac{x_1(5 + x_1(12x_1 - 11))}{1 + x_1(8x_1^2 - 6x_1 + 3)}. \quad (4.27)$$

The right-hand-side is a monotonic function of  $x_1 \geq 1$  that is equal to 1 for  $x_1 = 1$  and asymptotes to  $3/2$  for  $x_1 \rightarrow \infty$ . Thus for any  $x_1 = a_1 \tau^1 > 1$  there always exists a range of values for  $\epsilon^2$  for which we have stable dS vacua. This result is easily generalised to the several blow-up moduli situation in the case we are interested in, namely when these are supported by individual non-perturbative effects. In this case, the effects of the blow-up moduli on the volume modulus mass simply ‘add up’, and the generalisation of equation (4.27) is given by,

$$\sum_s h_s(x_s) \leq \frac{\epsilon^2 \tilde{f}^2 \mathcal{V}}{6} < \sum_s \left( \frac{x_s(5 + x_s(12x_s - 11))}{1 + x_s(8x_s^2 - 6x_s + 3)} h_s(x_s) \right). \quad (4.28)$$

By the same reasoning as in the single blow-up modulus case, there is always a permitted range of values for  $\epsilon$  that gives rise to metastable de Sitter vacua. From equation (4.28), we see that this range grows with the inclusion of additional Kähler moduli.

## 5 An explicit example for $\mathbb{CP}_{11169}^4$

In this section, we give a general method for constructing examples of the new class of metastable de Sitter vacua presented in this paper, and we illustrate this method by an explicit example.

### 5.1 Method

The class of vacua constructed in this paper can be obtained explicitly in a straightforward, step-by-step procedure, at least for continuous fluxes. Key to this is the decomposition of the flux vector (2.30), which allows us to engineer a consistent supersymmetry breaking flux background. We here present the detailed steps of this prescription.

#### Method:

1. Pick a point  $p$  in the complex structure and axio-dilaton moduli space at which the vacuum should be realised; evaluate the basis vectors  $\{\vec{\Pi}, \vec{\Pi}^*, D_i \vec{\Pi}, \bar{D}_i \vec{\Pi}^*\}_p$  at this point.
2. Turn on fluxes along the  $D_i \vec{\Pi}|_p$  directions only. According to (2.30), this corresponds to freely choosing the values of  $Z_{iS}|_p$ . These fluxes are supersymmetric with  $W_0|_p = 0$ , and the flux vector is now given by,

$$\vec{N} = -ie^{K_{c.s.}} \left( \frac{\bar{Z}^i \bar{S}}{K \bar{S}} D_i \vec{\Pi} \right) \Big|_p. \tag{5.1}$$

Given (2.31), this also specifies  $Z_{ij}|_p$ . In fact, since  $Z_{SS} = 0$  this determines the entire tensor  $Z_{IJ}|_p$  in the complex structure and axio-dilaton sector.

3. Solve the equation,

$$v^{*I} Z_{IJ}|_p = -\lambda^* v_J, \tag{5.2}$$

for  $v_I$  and  $\lambda$ , with  $v_I (K^{I\bar{J}}|_p) v_{\bar{J}}^* = 1$ . The vector  $v_J$  is then an eigenvector of  $(Z\bar{Z})_I^J|_p$  with the eigenvalue  $|\lambda|^2$ . We now take  $W_0|_p = \lambda$  and add to the flux vector the corresponding contribution along the  $\vec{\Pi}^*|_p$  direction,

$$\vec{N} = -ie^{K_{c.s.}} \left( W_0 \vec{\Pi}^* + \frac{\bar{Z}^i \bar{S}}{K \bar{S}} D_i \vec{\Pi} \right) \Big|_p. \tag{5.3}$$

The flux now has (2,1) and (0,3) components, and will generically induce non-vanishing F-terms for the Kähler moduli, but not for the complex structure moduli or the axio-dilaton.

4. Fix the flux induced contribution to the D3-tadpole, cf. (2.32), to any desired value by re-scaling  $Z_{Si}|_p$  and  $W_0|_p$ . We will further slightly deform this flux background, but this will only change the tadpole by a small amount of  $\mathcal{O}(\epsilon^2)$  that we will not be concerned with.

5. For the choice of branes and instantons supporting the non-perturbative effects in (2.23) and the flux-induced superpotential following from (2.22) and (5.3), find the non-supersymmetric AdS vacuum following the Large Volume Scenario prescription reviewed in section 4.1. Compute the magnitude of the uplift required to achieve a semi-positive definite vacuum energy.
6. Take  $F_I = \epsilon W v_I|_p$  and choose  $\epsilon \ll 1$  so that these F-terms can provide the right level of uplift. Add fluxes along the  $\vec{\Pi}|_p$  and  $\vec{D}_i \vec{\Pi}^*|_p$  directions so that the full flux vector is given by (2.30). These fluxes are now constructed to satisfy the non-supersymmetric critical point equation (3.9).
7. Minimise the full potential, dependent on both Kähler moduli and complex structure moduli, to find the perturbatively stable de Sitter vacuum.<sup>10</sup>

## 5.2 Example

We here illustrate our general prescription to construct de Sitter vacua by presenting an explicit example using the Calabi-Yau threefold obtained as a hypersurface in the projective space  $\mathbb{CP}_{11169}^4$ . This Calabi-Yau has  $h^{1,1} = 2$  and  $h^{2,1} = 272$ , but to render the model tractable, we follow the discussion of this example in [53] and consistently set all but two of the complex structure moduli to zero.

### 5.2.1 $\mathbb{CP}_{11169}^4$

The geometry of the axio-dilaton moduli space of this compactification is determined by the pre-potential which in the ‘large complex structure expansion’ is given by,

$$\mathcal{F} = -\frac{3}{2}(u^1)^3 - \frac{3}{2}(u^1)^2 u^2 - \frac{1}{2}u^1(u^2)^2 + \frac{9}{4}(u^1)^2 + \frac{3}{2}u^1 u^2 + \frac{17}{4}u^1 + \frac{3}{2}u^2 - i\zeta(3)\frac{135}{4\pi^3}. \quad (5.4)$$

In addition, the prepotential receives instanton corrections that are exponentially suppressed for sufficiently large  $\text{Im}(u^i)$ , and will not be important for our example. The Kähler potential for the complex structure moduli is then explicitly given by,

$$K_{\text{c.s.}} = -\log \left[ \frac{i}{2}(u^1 - \bar{u}^1)(3(u^1 - \bar{u}^1)(u^2 - \bar{u}^2) + 3(u^1 - \bar{u}^1)^2 + (u^2 - \bar{u}^2)^2) + \frac{135\zeta(3)}{\pi^3} \right].$$

The compactification volume is expressed in terms of four-cycle moduli as,

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left( \left( -i \frac{T^{\text{big}} - \bar{T}^{\text{big}}}{2} \right)^{3/2} - \left( -i \frac{T^s - \bar{T}^s}{2} \right)^{3/2} \right), \quad (5.5)$$

and the Kähler potential is given by equation (2.12) with  $\xi = -\frac{\chi(\text{CY}_3)\zeta(3)}{2(2\pi)^3} \approx 1.31$ .

---

<sup>10</sup>Due to the coupling between the Kähler sector and the axio-dilaton and complex structure sectors, the axio-dilaton and complex structure vevs at the de Sitter vacuum are slightly shifted from their previously chosen values at the point  $p$ .

The flux-induced superpotential is given by,

$$\begin{aligned}
 W_0 = & \frac{1}{4}(h_0 S - f_0) \left( 6(u^1)^3 + 6(u^1)^2 u^2 + u^1 (2(u^2)^2 + 17) + 6 \left( u^2 - \frac{45i\zeta(3)}{\pi^3} \right) \right) \\
 & + \frac{1}{4}(f_1 - h_1 S) (18(u^1)^2 + 6u^1(2u^2 - 3) + 2(u^2)^2 - 6u^2 - 17) \\
 & + \frac{1}{2}(f_2 - h_2 S) (3(u^1)^2 + u^1(2u^2 - 3) - 3) + f_3 - h_3 S + (f_4 - h_4 S)u^1 \\
 & + (f_5 - h_5 S)u^2, \tag{5.6}
 \end{aligned}$$

and non-perturbative effects are assumed to be supported by a stack of ten D7-branes wrapping the blow-up cycle; we take,

$$a_s = \frac{2\pi}{10}, \quad A_s = 1. \tag{5.7}$$

### 5.2.2 Step-by-step procedure

We here give a very detailed description of the construction of a single de Sitter vacuum in  $\mathbb{CP}_{11169}^4$ . Throughout this example, we work with continuous fluxes and present our results with three significant digits, however, we keep working at each step with numbers with higher precision.

1. We choose to construct a vacuum at  $p = \{S = u^1 = u^2 = 2i\}$ , i.e. we set the axion vevs in this sector to zero. The relevant period vectors are,

$$\vec{\Pi} \approx \begin{pmatrix} 1 \\ 2i \\ 2i \\ -19.1i \\ 36.3 + 12i \\ 11.5 + 3i \end{pmatrix}, \quad D_1 \vec{\Pi} \approx \begin{pmatrix} 0.558i \\ -0.117 \\ -1.12 \\ -17.1 \\ -2.20 - 3.76i \\ -0.175 - 1.58i \end{pmatrix}, \quad D_2 \vec{\Pi} \approx \begin{pmatrix} 0.174i \\ -0.349 \\ 0.651 \\ -5.16 \\ -0.594 - 1.67i \\ -0.523 + 0.00669i \end{pmatrix}.$$

2. We choose to turn on fluxes so that  $Z_{S1}|_p = 2.4$  and  $Z_{S2}|_p = -5.43$ . The full tensor  $Z_{IJ}|_p$  is then given by,

$$Z_{IJ}|_p \approx \begin{pmatrix} 0 & 2.4 & -5.43 \\ 2.4 & -7.24 & -2.41 \\ -5.43 & -2.41 & 5.57 \end{pmatrix}. \tag{5.8}$$

The flux vector is now given by equation (5.1). The RR and NSNS fluxes expressed in the original basis are given by,

$$\vec{f} \approx \begin{pmatrix} 0 \\ 6.03 \\ -18.7 \\ 3.65 \\ -0.9 \\ 9.04 \end{pmatrix}, \quad \vec{h} \approx \begin{pmatrix} 0.113 \\ 0 \\ 0 \\ 0 \\ -5.55 \\ 4.46 \end{pmatrix}. \tag{5.9}$$

By construction, these fluxes give  $W|_p = F_I|_p = 0$ .

3. The equation  $v^{*I}Z_{IJ}|_p = -\lambda^*v_J$  has several solutions, and  $\lambda$  can be chosen as the positive or negative square-root for either of the three eigenvalues of  $(Z\bar{Z})_I^J|_p$ . We take  $\lambda \equiv \lambda_1 \approx -104$ , corresponding to the middle eigenvalue, the others being  $\lambda_2 \approx -53.6$  and  $\lambda_3 \approx -1230$ . The vector  $v_I$  is then given by,

$$v_I \approx \begin{pmatrix} -0.196i \\ +0.189i \\ +0.0365i \end{pmatrix}, \tag{5.10}$$

which is unit normalised with respect to the field space metric. We identify our chosen value of  $\lambda$  with the desired vacuum expectation value of the flux superpotential, and turn on (0,3) fluxes according to (5.3). The RR and NSNS fluxes are now given by,

$$\vec{f} \approx \begin{pmatrix} 0 \\ 6.93 \\ -17.8 \\ -5.03 \\ 4.55 \\ 10.4 \end{pmatrix}, \quad \vec{h} \approx \begin{pmatrix} -0.114 \\ 0 \\ 0 \\ 0 \\ -13.8 \\ 1.85 \end{pmatrix}. \tag{5.11}$$

4. In this example, the flux induced tadpole becomes,

$$(2\pi)^4(\alpha')^2 Q_{\text{flux}} = \vec{h}^T \Sigma \vec{f} \approx 129. \tag{5.12}$$

This is (by our ostensibly prescient choice of  $Z_{S_i}$ ) already consistent with the tadpole constraint from a known embedding of  $\mathbb{CP}_{11169}^4$  into F-theory [78]. However, due to the linearity of the problem, other tadpole constraints corresponding to other 7-brane configurations can be satisfied by simply rescaling  $W_0|_p$  and  $Z_{S_i}|_p$ .

5. To estimate the magnitude of the required uplift, we minimise the LVS scalar potential (4.1) for the Kähler moduli sector alone, given  $W_0|_p = -104$ ,<sup>11</sup> and find a non-supersymmetric AdS solution.<sup>12</sup> In this example, the volume of the AdS vacuum is given by,  $\mathcal{V}|_p \approx 6360$  and the leading order ‘cosmological constant’ is given by,

$$V|_p \approx -1.15 \times 10^{-11}. \tag{5.13}$$

In point 6 below, the inclusion of non-vanishing F-terms for the complex structure moduli and the axio-dilaton will shift the vev of the volume. Nevertheless, the volume of the AdS vacuum indicates the rough size of the desired uplift:  $\epsilon \sim \mathcal{V}^{-1/2}|_p \approx 1/80$ .

<sup>11</sup>In the literature (for example [79]), a commonly used definition of  $W_0$  used in studies of Kähler moduli stabilisation includes a normalisation factor from the complex structure and axio-dilaton Kähler potential. In our example,  $(e^{\tilde{K}/2}W_0)|_p = -3.44$ .

<sup>12</sup>By construction, this solution has a leading-order flat direction in the axio-dilaton and complex structure sector.

$m_{3+}^2$	$m_{3-}^2$	$m_{1+}^2$	$m_{2+}^2$	$m_{2-}^2$	$m_{1-}^2$
$1.61 \times 10^{-5}$	$1.15 \times 10^{-5}$	$3.56 \times 10^{-7}$	$2.12 \times 10^{-7}$	$2.20 \times 10^{-8}$	$2.23 \times 10^{-9}$

**Table 1.** Canonically normalised squared masses for the axio-dilaton and complex structure moduli in natural units. We labeled the masses squared as in section 3.

6. We now turn on fluxes that are, at  $p$ , not supersymmetric in the complex structure and axio-dilaton sector. To satisfy the leading order critical point equation, cf. equation (3.9), these have to be proportional to  $v_I$ . We choose  $\epsilon^2 = F_I \bar{F}^I / |W|^2 = (1/120)^2$ , and find the slightly corrected fluxes,

$$\vec{f} \approx \begin{pmatrix} 0 \\ 6.94 \\ -17.7 \\ -4.98 \\ 4.60 \\ 10.4 \end{pmatrix}, \quad \vec{h} \approx \begin{pmatrix} -0.114 \\ 0 \\ 0 \\ 0 \\ -13.7 \\ 1.88 \end{pmatrix}. \quad (5.14)$$

7. Finally, we directly minimise the full scalar potential given the fluxes (5.14). We consistently find a critical point near, but not exactly at  $p$ , with relative deviations of  $\mathcal{O}(\mathcal{V}^{-1/2})$  due to the mixing with the Kähler moduli. The moduli vevs at the critical point are given by,

$$S \approx 1.96i, \quad u^1 \approx 2.03i, \quad u^2 \approx 1.97i, \quad (5.15)$$

$$\tau^s \approx 9.64, \quad \mathcal{V} \approx 11300. \quad (5.16)$$

The ‘small’ Kähler modulus axion is stabilised at zero (since  $W_0 < 0$  and  $A_s > 1$ ), and the axion of the ‘large’ cycle remains unfixed. The vacuum expectation value of the potential at the critical point is positive,

$$V \approx 9.03 \times 10^{-14}. \quad (5.17)$$

The eigenvalues for the Hessian matrix for canonically normalised fields are given in tables 1 and 2. The spectrum is positive semi-definite, with only the axion of the large Kähler modulus still massless.

We close this section by discussing the consistency of our analytical discussion in section 4 with the spectra of tables 1 and 2.

The canonically normalised masses for the axio-dilaton and complex structure moduli are well described by the analytical expressions reported in [48] and reviewed in sections 3 and 4, in particular equations (4.17) and (4.18) reproduce the masses squared in table 1 at the percent level.

$m_{\text{Re}(T^s)}^2$	$m_{\text{Im}(T^s)}^2$	$m_{\text{Im}(T^{\text{big}})}^2$	$m_{\text{Re}(T^{\text{big}})}^2$
$1.18 \times 10^{-5}$	$1.10 \times 10^{-5}$	$7.06 \times 10^{-12}$	0

**Table 2.** Canonically normalised squared masses for the Kähler moduli. As in standard LVS, the masses of  $T^s$  are of the same order as the masses in the axio-dilaton and complex structure sector. As previously discussed in subsection 4.4, there is a leading order mass mixing between the overall volume and the small cycle, so that the eigenvalue denoted by  $m_{\text{Im}(T^{\text{big}})}^2$  corresponds to an eigenvector with leading entries along the  $\text{Im}(T^{\text{big}})$  and  $\text{Im}(T^s)$  directions.

Similarly, we can compare the squared masses for the small Kähler modulus with the expression given in (4.20). For a single small Kähler modulus and without the assumption  $x_s \gg 1$ , we find the canonically normalised mass squared

$$\frac{1}{2} K^{T^s \bar{T}^s} \partial_{\tau^s \bar{\tau}^s}^2 V \approx 1.10 \times 10^{-5}, \quad (5.18)$$

which is in perfect agreement with the result above.

As a final check of our solution, we can calculate the canonically normalised mass for the overall volume from the simplified scalar potential given in equation (4.11). This requires us to determine  $\epsilon^2 \tilde{f}^2 = e^{\tilde{K}} F_I \bar{F}^I$ . We find for our example,

$$e^{\tilde{K}} F_I \bar{F}^I = 0.00109 \left( 0.793 + \frac{828}{\mathcal{V}} + \frac{314000}{\mathcal{V}^2} \right), \quad (5.19)$$

where the terms that are subleading in powers of the volume arise from the inverse Kähler metric in  $F_I \bar{F}^I = F_I K^{I\bar{J}} \bar{F}_{\bar{J}}$ . However, the potential of equation (4.11) is only valid to leading order in the volume, and keeping only the leading order contribution to  $e^{\tilde{K}} F_I \bar{F}^I$  we have  $\epsilon^2 \tilde{f}^2 = e^{\tilde{K}} F_I \bar{F}^I = 0.000866$ . We then find for the volume modulus the canonically normalised squared mass<sup>13</sup>  $m_{\text{Im}(T^{\text{big}})}^2 \approx 1.50 \times 10^{-12}$ . This value is still roughly consistent with our full result above, but shows the largest deviation from the true value than our analytical estimates of the other moduli masses. This is explained by the only moderately large volume in our explicit example, and the fact that the volume modulus is the lightest modulus and hence the most sensitive one to small corrections that arise due to the large volume approximation in section 4.

Comparing with the metastability analysis in subsection 4.4, we find from equation (4.27) for the final values the range,

$$9.38 < \epsilon^2 \tilde{f}^2 \mathcal{V} < 13.6, \quad (5.20)$$

where for our solution we have  $\epsilon^2 \tilde{f}^2 \mathcal{V} \approx 10.7$ .

As discussed above, we did not try to get the fluxes to be simultaneously integer quantised and consistent with a given, moderately large flux tadpole. We expect that the flux tuning required to make  $\epsilon \ll 1$  can be realised using quantised fluxes in compactifications with more moduli (or substantially larger flux tadpoles) than the example we have considered.

<sup>13</sup>As discussed in subsection 4.4, there is a non-negligible mass mixing with the small cycle so we have to diagonalise the Hessian. Note, that the eigenvalue given in (4.24) corresponds to non-canonically normalised fields so it does not give the value we quote here.



## 6 Visible sector soft terms

In this paper we have focussed on the generation of metastable de Sitter vacua with stabilised moduli, but compactifications of string theory hoping to describe the real world also need to include matter and gauge fields compatible with the Standard Model of particle physics. Such a ‘visible sector’ may for example be generated by intersecting branes or by localising branes at singularities in the compactification geometry. Our mechanism for generating metastable de Sitter vacua has important consequences for the properties of the soft supersymmetry breaking terms in the visible sector, as we briefly outline in this section.

For concreteness, we focus on the well-studied case of visible sectors generated by branes at singularities, cf. [62, 80–84] for earlier work on such models in the Large Volume Scenario. We denote the Kähler modulus that resolves the cycle by  $T^{\text{SM}}$ , with the four-cycle volume controlled by  $\text{Im}(T^{\text{SM}}) = \tau^{\text{SM}}$ . Due to the issues of large-scale breaking of the Standard Model symmetries raised in [85], the new modulus  $T^{\text{SM}}$  is taken to be distinct from the moduli  $T_s$  that are stabilised by non-perturbative effects. The modulus  $T^{\text{SM}}$  is instead assumed to be stabilised supersymmetrically using D-terms, giving  $F_{T^{\text{SM}}} = 0$ . In this case, the soft supersymmetry breaking parameters determining the low-energy phenomenology of the model are generated by couplings to the supersymmetry breaking bulk moduli,  $T^{\text{big}}$ ,  $u^i$  and  $S$ , which can be computed using the general formulae of [86–88].

We begin by computing the induced gaugino masses in our scenario. For branes localised at a collapsed singularity, the holomorphic gauge kinetic function is given by,

$$f = c_1 S + c_2 T^{\text{SM}}. \tag{6.1}$$

The gaugino masses are then given by,

$$M_{1/2} = \frac{1}{2\text{Im}(f)} e^{K/2} \bar{F}^A \partial_A f = \frac{1}{2\text{Im}(f)} e^{K/2} \bar{F}^S \partial_S f \sim \frac{m_{3/2}}{\mathcal{V}_{\text{min}}^{1/2}}. \tag{6.2}$$

While suppressed with respect to  $m_{3/2}$ , these fields are heavier by a factor of  $\sqrt{\mathcal{V}_{\text{min}}}$  than usually assumed in the Large Volume Scenario.

For chiral visible sector fields  $C^a$  with a diagonal matter metric,

$$K = \sum_{\alpha} \tilde{K}_{\alpha} C^{\alpha} \bar{C}^{\bar{\alpha}} + \dots, \tag{6.3}$$

the soft masses for the non-canonically normalised fields in an approximately Minkowski vacuum are given by [87],

$$m_{\alpha}^2 = m_{3/2}^2 - e^K \bar{F}^A F^{\bar{B}} \partial_{A\bar{B}}^2 \log(\tilde{K}_{\alpha}). \tag{6.4}$$

Thus, determining the soft masses requires knowing the moduli dependence of the matter Kähler metric.

Arguments based on holomorphicity and locality suggest that for visible sectors realised at branes at singularities [89],

$$\tilde{K}_{\alpha} = h_{\alpha}(X^I, \bar{X}^{\bar{I}}) e^{K/3} \kappa_{\alpha}, \tag{6.5}$$

for  $X^I = (S, u^i)$ . Here  $\kappa_\alpha$  is assumed to not depend on bulk moduli. For  $h_\alpha$  a moduli-independent constant, this Kähler metric is of the ‘sort-of-sequestered’ form [90], implying vanishing soft masses, just as in the scenario of sequestered supersymmetry breaking of reference [91]. However, sequestering in string compactifications is expected to be at best approximate, and non-vanishing soft terms can be induced by e.g.  $\alpha'$  corrections [62, 92], superpotential de-sequestering [90, 93], or, in our case, a non-constant function  $h_\alpha(X^I, \bar{X}^{\bar{I}})$ . These de-sequestering effects can dominate over anomaly mediated contributions to the soft masses. Explicit computations of the matter metric in certain toroidal orbifolds indicate that, at least in these cases,  $h_\alpha$  is a non-constant function [94–96].

For  $h_\alpha$  non-constant, the soft terms are generated as, schematically,

$$m_\alpha^2 \sim \frac{m_{3/2}^2}{\mathcal{V}_{\min}} \left( \frac{\partial \bar{\partial} h_\alpha}{h_\alpha} - \frac{\partial h_\alpha \bar{\partial} h_\alpha}{h_\alpha^2} \right) \sim M_{1/2}^2. \tag{6.6}$$

In this case, we also expect that,<sup>14</sup>

$$(A_{\alpha\beta\gamma})^2 \sim B\mu \sim \frac{m_{3/2}^2}{\mathcal{V}_{\min}} \sim M_{1/2}^2. \tag{6.7}$$

This scenario has far-reaching phenomenological implications.<sup>15</sup> By virtue of coupling with gravitational strength interactions to all sectors of the theory, moduli tend to be displaced from their vacuum expectation values during inflation, and subsequently come to oscillate around the vacuum, red-shifting like matter before eventually decaying. This way, long-lived moduli can come to dominate the energy density of the universe, but — if too long-lived — may fail to reheat the Standard Model at temperatures sufficient for Big Bang Nucleosynthesis. This is the ‘cosmological moduli problem’ [97–99]. For moduli with generic, gravitational strength interactions, the corresponding bound on the decay rate implies that  $m \gtrsim 50$  TeV. The cosmological moduli problem applies in particular to the overall volume modulus, so that,

$$m_\mathcal{V} \sim \frac{m_{3/2}}{\sqrt{\mathcal{V}_{\min}}} \gtrsim 50 \text{ TeV}. \tag{6.8}$$

However, according to equations (6.6) and (6.7), the soft terms are all at the same scale as the volume modulus,  $m_\mathcal{V} \sim m_{\text{soft}} \sim A \sim \sqrt{B\mu}$ , implying that,

$$m_{\text{soft}} \gtrsim 50 \text{ TeV}. \tag{6.9}$$

This lower bound on the soft masses has obvious implications for laboratory searches for supersymmetry, but also constrains the particle nature of dark matter. The lightest superparticle (LSP) in supersymmetric extensions of the Standard Model is the prime WIMP candidate, but in our scenario equation (6.9) implies that it would hardly lie in the WIMP window. Hence, dark matter cannot be the standard WIMP, but may well be realised by e.g. axions.

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<sup>14</sup>For  $h_\alpha$  constant and vanishing superpotential cross-couplings between the supersymmetry breaking moduli and the matter fields, equation (6.5) also implies the vanishing of the leading order contribution to the soft A-terms and, in the absence of a  $\mu$ -term, also a vanishing soft  $B\mu$  term.

<sup>15</sup>We are very grateful to Michele Cicoli for discussions on this point.

## 7 Conclusions

In this paper we have proposed an extension of the standard Large Volume Scenario in which we relaxed the assumption on exactly vanishing F-terms in the axio-dilaton and complex structure sector. The additional amount of supersymmetry breaking in this sector has to be small,  $F^2 \sim \mathcal{O}(1/\mathcal{V})$ , in order to not destabilise the Kähler sector. We showed that the non-supersymmetric critical point equation for the axio-dilaton and the complex structure moduli has direct implications for the moduli spectrum of the compactification, and forces one real field in this sector to be lighter than in the standard Large Volume Scenario. Just as in [48] however, this field can be stabilised by a moderate tuning of a phase. This ensures that the decoupling of the axio-dilaton and complex structure sector from the Kähler sector is still essentially the same as in the standard Large Volume Scenario.

The inclusion of additional sources of spontaneous supersymmetry breaking leads to the following schematic potential for the Kähler moduli:

$$V \sim \frac{F^2}{\mathcal{V}^2} + \frac{\tilde{a}}{\mathcal{V}^3} - \frac{\tilde{b}\tau e^{-a\tau}}{\mathcal{V}^2} + \frac{\tilde{c}\sqrt{\tau}e^{-2a\tau}}{\mathcal{V}}. \tag{7.1}$$

We find that the new F-term breaking can lead, for a finite range of  $F^2$ , to metastable de Sitter vacua that do not require any additional uplift (like for example an anti-D3-brane). Thus our construction constitutes a new class of de Sitter vacua in string theory.

In section 5, we presented a general method for explicitly constructing examples of this class of vacua, and we illustrated this method for the particular case of compactifications on the Calabi-Yau constructed as a hypersurface in  $\mathbb{CP}_{11169}^4$ . This method relies on the continuous flux approximation, but we expect that examples with quantised fluxes can be constructed in compactifications with a larger number of three-cycles and a large flux tadpole. It would be very interesting to explicitly construct such examples.

We also discussed potential implication for supersymmetry breaking soft terms for visible sectors realised through branes at local singularities [83, 92]. Our solutions invoke larger supersymmetry breaking contributions from the axio-dilaton and complex structure fields compared to the usual LVS construction. This leads to larger gaugino masses than usually assumed in the Large Volume Scenario,

$$M_{1/2} \sim \frac{m_{3/2}}{\sqrt{\mathcal{V}_{\min}}}, \tag{7.2}$$

and soft terms of the same order,

$$m_{\text{soft}}^2 \sim M_{1/2}^2 \sim A^2 \sim B\mu. \tag{7.3}$$

Since also  $m_{\mathcal{V}} \sim m_{\text{soft}}$ , the resolution of the cosmological moduli problem suggest that these soft terms are  $\gtrsim 50$  TeV, which predicts null results at present laboratory searches for supersymmetry, and a non-WIMP origin of dark matter. It would be interesting to study the phenomenology and cosmology of this scenario in more detail.

Our present analysis applies to Calabi-Yau manifolds of strong Swiss-cheese type with an arbitrary number of small cycles. It would be interesting to check whether it also extends to more general compactification topologies.

We consider de Sitter solutions in four-dimensional effective theories derived from string compactifications. It would be interesting to understand the full ten-dimensional solution. Such a solution requires a ten-dimensional description of the non-perturbative gaugino condensate arising from strong gauge dynamics on stacks of D7-branes. Recent discussions in ten dimensions show possible subtle issues in particular constructions, over which one desires full control in ten dimensions to justify our small corrections [100–102]. It remains an open question whether it is consistent to add those non-perturbative quantum corrections on no-scale backgrounds after  $\alpha'$  corrections, as pointed out recently in [103].

## Acknowledgments

We would like to thank M. Dias, J. Frazer, R. Kallosh, L. McAllister, F. Quevedo, K. Sousa and A. Westphal for illuminating discussions. We are grateful to C. Brodie and M. Cicoli for comments on the draft and for discussions. DG thanks the Institute for Theoretical Physics, TU Wien, for hospitality and The Abdus Salam International Centre for Theoretical Physics for hospitality and support. BV and TW thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support. DM and TW are grateful for the hospitality of the Aspen Center for Physics, which is supported by National Science Foundation grant PHY-1607611. BV, DM and TW are grateful to the Lorentz center for hospitality during the completion of this work. DG was partially supported by UPTC-DIN grant no. SGI-2145. DM is supported by a Stephen Hawking Advanced Fellowship at the Centre for Theoretical Cosmology at the University of Cambridge. BV was supported during this work by: the European Commission through the Marie Curie Intra-European fellowship 328652-QM-sing and Starting Grant of the European Research Council (ERC-2011-SrG 279617 TOI), in part by the Interuniversity Attraction Poles Programme initiated by the Belgian Science Policy (P7/37) and in part by the European Research Council grant no. ERC-2013-CoG 616732 HoloQosmos, and the KU Leuven C1 grant Horizons in High-Energy Physics. TW is supported by the Austrian Science Fund (FWF): P 28552.

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