

Naturalness, $b \rightarrow s\gamma$, and SUSY heavy Higgses

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ABSTRACT: We explore naturalness constraints on the masses of the heavy Higgs bosons H^0 , H^\pm , and A^0 in supersymmetric theories. We show that, in any extension of MSSM which accommodates the 125 GeV Higgs at the tree level, one can derive an upper bound on the SUSY Higgs masses from naturalness considerations. As is well-known for the MSSM, these bounds become weak at large $\tan\beta$. However, we show that measurements of $b \rightarrow s\gamma$ together with naturalness arguments lead to an upper bound on $\tan\beta$, strengthening the naturalness case for heavy Higgs states near the TeV scale. The precise bound depends somewhat on the SUSY mediation scale: allowing a factor of 10 tuning in the stop sector, the measured rate of $b \rightarrow s\gamma$ implies $\tan\beta \lesssim 30$ for running down from 10 TeV but $\tan\beta \lesssim 4$ for mediation at or above 100 TeV, placing m_A near the TeV scale for natural EWSB. Because the signatures of heavy Higgs bosons at colliders are less susceptible to being “hidden” than standard superpartner signatures, there is a strong motivation to make heavy Higgs searches a key part of the LHC’s search for naturalness. In an appendix we comment on how the Goldstone boson equivalence theorem links the rates for $H \rightarrow hh$ and $H \rightarrow ZZ$ signatures.

KEYWORDS: Supersymmetry Phenomenology

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1 Introduction

The most compelling argument for the possibility of supersymmetry near the weak scale is that it allows for the possibility of *natural* electroweak symmetry breaking. This possibility, however, hinges on a number of conditions [1–4]. The tree-level electroweak symmetry breaking conditions show that the Higgs VEV should not be much larger than the higgsino mass parameter μ , so naturalness requires light higgsinos [1, 5–9]. At one loop, the top quark correction to the up-type Higgs mass parameter $m_{H_u}^2$ must be approximately canceled, requiring light stop squarks [5–8, 10–14]. Finally, the stops themselves suffer from large corrections due to a gluino loop, requiring that the gluinos must also not be too heavy [7, 8]. Although higgsinos are difficult to constrain experimentally, the search for natural SUSY has driven an extensive effort to discover stops or gluinos at the LHC. This effort has succeeded in placing stringent bounds on their possible masses and decay modes. For an up-to-date review of the implications of LHC data for SUSY, see ref. [15]. (Also see refs. [16] and [17] for recent overviews of the status of SUSY naturalness.)

When we discuss natural SUSY, we will always have in mind a scenario with relatively light stops. Loop corrections from the stops in natural scenarios are not sufficient to lift the SM-like Higgs mass to 125 GeV. As a result, we will assume that new physics *beyond* the MSSM provides a new contribution to the Higgs quartic coupling and raises the Higgs

mass at the tree level. Many options are available for this, including new F or D -term quartics [18–20].

In this paper we explore to what extent the heavy Higgs bosons of the MSSM and its extensions, H^0 , H^\pm , and A^0 , could also constitute probes of naturalness. As with the higgsino mass parameter μ , their mass terms appear in the tree-level conditions for EWSB, so naturalness will not be consistent with arbitrary values of these parameters. The reason that heavy Higgses have not joined the usual pantheon of naturalness signatures is that, in the MSSM, it is only the ratio $m_A/\tan\beta$ of their mass scale to $\tan\beta$ that is constrained, so at large $\tan\beta$ they can be out of reach of colliders without requiring any fine-tuning [6]. On the other hand, in scenarios like λ SUSY where $\tan\beta$ is order-one [19], it is known that naturalness requires the other Higgs bosons to be light [21]. The tuning cost of raising the heavy Higgs masses when $\tan\beta$ is not large was recently emphasized in ref. [22].

Our goal in this paper is to construct an argument that, in any given extension of the MSSM, even at large $\tan\beta$, there is an upper bound on the heavy Higgs masses arising from naturalness. We derive simple expressions for the fine-tuning when different possible quartic couplings are added to raise the Higgs mass to 125 GeV. The only case in which there is not an immediate bound is the MSSM-like case of an $|H_u|^4$ quartic, for which the heavy Higgses can be made heavy while simultaneously going to large $\tan\beta$. However, we will argue that measurement of $b \rightarrow s\gamma$, together with a combination of naturalness and direct constraints on other superpartner masses, allows us to cut off the large- $\tan\beta$ tail of the natural parameter space. The fact that $b \rightarrow s\gamma$ is difficult to suppress in natural SUSY due to a contribution from a loop of stops and higgsinos was emphasized in ref. [23]. Our discussion will be somewhat more general because we assume that the Higgs mass is lifted by quartic couplings beyond the MSSM, relaxing constraints on A_t assumed in that reference. Nonetheless, we will find a constraint.

Thus, there is a bound from a combination of tree-level naturalness and $b \rightarrow s\gamma$ measurements on the mass scale of heavy Higgs bosons in a natural supersymmetric theory. Unlike the tree-level constraint on higgsinos, which is difficult to exploit because they can be essentially invisible at colliders, the parameter space for natural heavy Higgses can be significantly constrained by data. Both direct searches and $\mathcal{O}(v^2/m_H^2)$ corrections to the light Higgs boson decay widths play a part in this. We close our paper with a brief look at the prospects for experimental tests of natural SUSY in these heavy Higgs search channels.

2 Tree-level fine-tuning

The most general renormalizable potential for the two Higgs doublets H_u and H_d is [24–26]:

$$\begin{aligned}
 V(H_u, H_d) = & M_u^2 |H_u|^2 + M_d^2 |H_d|^2 + (bH_u \cdot H_d + \text{h.c.}) + \frac{1}{4}\lambda_1 |H_u|^4 + \lambda_2 H_u^\dagger H_u (H_u \cdot H_d + \text{h.c.}) \\
 & + \lambda_3 |H_u|^2 |H_d|^2 + \frac{1}{2}\lambda_4 (H_u \cdot H_d + \text{h.c.})^2 + \lambda_5 |H_u \cdot H_d|^2 \\
 & + \lambda_6 H_d^\dagger H_d (H_u \cdot H_d + \text{h.c.}) + \frac{1}{4}\lambda_7 |H_d|^4.
 \end{aligned}
 \tag{2.1}$$

Here $H_u \cdot H_d$ denotes the SU(2)-invariant contraction with an antisymmetric ε symbol. One may be tempted to write another term $(H_d^\dagger H_u)(H_u^\dagger H_d)$, but this is just the linear

combination $|H_u|^2 |H_d|^2 - |H_u \cdot H_d|^2$ and can be absorbed into λ_3 and λ_5 . For simplicity we use the notation $M_u^2 \equiv |\mu|^2 + m_{H_u}^2$ and $M_d^2 \equiv |\mu|^2 + m_{H_d}^2$. In the MSSM, the nonzero tree-level quartic couplings are:

$$\lambda_1 = \lambda_7 = \frac{g^2 + g'^2}{2}; \quad \lambda_3 = \frac{g^2 - g'^2}{4}; \quad \lambda_5 = -\frac{g^2}{2}. \quad (2.2)$$

However, in the MSSM at tree level the Higgs mass is always smaller than the measured value, so we must raise it. For the most part, in this paper, we will simply assume that the Higgs mass is lifted by a new, hard SUSY-breaking contribution to one of the quartic couplings λ_i , and that beyond-MSSM physics otherwise does not affect the Higgs potential. The new term could arise from new F -terms in higher-dimension operators [20, 27] or from nondecoupling D -terms from new gauge groups [18, 28, 29].

In some cases, the detailed physics lifting the Higgs mass will also affect Higgs properties in more significant ways, e.g. when mixing with a singlet [21, 30–32] or triplet [20, 33] is important. We will not consider these models in detail, but we expect that although they may provide further experimental search channels they will not alter the basic conclusion about whether decoupling the heavy Higgs bosons is natural.

In this section we will focus on the quartic couplings $\lambda_1 |H_u|^4$ and $\lambda_5 |H_u \cdot H_d|^2$, which we view as well-motivated possibilities. We will not discuss the other cases, but a similar exercise can be carried out for all of them. The quartics λ_6 and λ_7 have effects only at small $\tan \beta$, which is disfavored because it requires a very large top Yukawa coupling. The couplings λ_3 and λ_4 have a similar effect to λ_5 , since they involve two up-type Higgs bosons and two down-type Higgs bosons. The coupling λ_2 is an interesting intermediate case, favoring moderate $\tan \beta$, but we don't know of a model in which it dominates.

2.1 Reminder: EWSB and tuning in the tree-level MSSM

Given the potential in eq. (2.1), we can vary with respect to the VEVs of H_u^0 and H_d^0 to obtain the conditions for an electroweak symmetry breaking vacuum of VEV v . These equations, for the case of the MSSM, are:

$$M_U^2 = b \cot \beta + \frac{1}{2} m_Z^2 \cos(2\beta) \quad (2.3)$$

$$M_D^2 = b \tan \beta - \frac{1}{2} m_Z^2 \cos(2\beta). \quad (2.4)$$

The appearance of m_Z^2 here comes from assuming that only the tree-level D -term quartic couplings are present. Of course, this assumption is not consistent with the observed Higgs mass in our universe, since $m_h^2 < m_Z^2$ in the tree-level MSSM. Nonetheless, it is useful to take a quick look at tuning in this case because it is familiar and it offers a useful starting point before proceeding to theories with more general quartic terms. Adding the two EWSB equations gives

$$M_U^2 + M_D^2 = \frac{2b}{\sin(2\beta)} = m_A^2, \quad (2.5)$$

using the result one obtains by diagonalizing the pseudoscalar mass matrix. On the other hand, multiplying eq. (2.3) by $\tan^2 \beta$ and subtracting from eq. (2.4), we obtain:

$$\frac{1}{2}m_Z^2 = \frac{M_D^2 - M_U^2 \tan^2 \beta}{\tan^2 \beta - 1}. \quad (2.6)$$

In order to have a theory that is not fine-tuned, we would like the individual terms on the right-hand side to be not much larger than the terms on the left-hand side. Recalling that $M_U^2 = m_{H_u}^2 + |\mu|^2$, we can extract three conditions:

$$\begin{aligned} |\mu|^2 &\lesssim m_Z^2 \\ |m_{H_u}^2| &\lesssim m_Z^2 \\ m_{H_d}^2 &\lesssim m_Z^2 \tan^2 \beta. \end{aligned} \quad (2.7)$$

The first of these equations is the very familiar condition that higgsinos should not be much heavier than the Z boson to prevent tree-level tuning [1, 6–9, 34]. The second is unsurprising, since $\tan \beta > 1$ so that the Higgs that gets a VEV has a significant component in H_u^0 . In order to obtain a VEV at the weak scale, this Higgs should have a mass near the weak scale. The final condition receives the least attention, although it has been discussed at times in the literature (e.g. [6, 22]). It tells us that the down-type Higgs soft mass — which, at large $\tan \beta$, is approximately a measure for the mass of the states A^0, H^0 , and H^\pm — cannot be much larger than $m_Z \tan \beta$. The reason this bound typically receives less attention is that it is usually assumed that $\tan \beta$ can naturally be very large, allowing the heavy Higgs bosons to be very heavy without a large amount of fine-tuning. We think that it is timely to revisit this tree-level naturalness constraint for two reasons. First, many of the models that are frequently studied as ways of lifting the Higgs mass to 125 GeV operate best at small-to-moderate $\tan \beta$. Second, we will argue that the measurement of $b \rightarrow s\gamma$ prevents theories with very large values of $\tan \beta$ from being natural. Given such an upper bound on $\tan \beta$, a fine-tuning argument can then impose an upper bound on m_A as well. One of the main goals of this paper is to quantify this upper bound: given what we now know about $b \rightarrow s\gamma$, how heavy can the other Higgs bosons be without fine-tuning?

The fine-tuning of EWSB is typically measured in terms of the variation of either the Higgs VEV [1, 6] or the soft mass $m_{H_u}^2$ [5, 7] with respect to the input parameters. We will mostly follow the first, Barbieri-Giudice, definition to quantify the tuning of the Higgs VEV with respect to a parameter x :

$$\Delta_x \equiv \left| \frac{\partial \log v^2}{\partial \log x} \right|. \quad (2.8)$$

If $\Delta_x \gg 1$ for some parameter x , we will say that the theory is fine-tuned. This actually measures the *sensitivity* of the Higgs VEV (or, equivalently, the Z mass) to an underlying parameter. We generally think of tuning as occurring when there must be a cancelation between different contributions, requiring a delicate adjustment of different input parameters with respect to one another to achieve a result near the experimentally observed value. For recent discussions of definitions of tuning and how this computation may not always

reflect what we think of as fine-tuning, see refs. [9, 35, 36]. We expect that the Barbieri-Giudice measure is typically a fairly good, albeit imperfect, proxy for our intuitive notions of tuning.

We will now explore the tuning measure in various extensions of the MSSM. In each case, we will assume that a particular new hard-SUSY-breaking quartic coupling has been added. We will not worry much about the details of the UV completion, which we expect to have only a mild effect on the fine-tuning bounds that we infer.¹

2.2 The $\lambda_1 (H_u^\dagger H_u)^2$ extension

First we will assume that the new quartic coupling that has been added is dominantly up-type. The usual loop corrections in the MSSM obtained by integrating out stops [37–40] are of this form. It could also arise from new D -terms in conjunction with other quartics [18, 28, 29, 41–43]; its effects would dominate over those of the other quartics at large $\tan\beta$. Finally, we could consider a new source of tree-level F -terms by adding a new triplet T with appropriate hypercharge to allow an $H_u \cdot T H_u$ Yukawa coupling, and pairing T with a vector-like partner \bar{T} with a supersymmetric mass term [20, 33]. For now we will remain agnostic about the UV completion, simply assuming that such a quartic is present in the potential.

Given a new contribution $\delta\lambda_1$ to the up-type Higgs quartic (beyond the D -term contribution in eq. (2.2)), the two EWSB equations become

$$\begin{aligned} M_U^2 &= b \cot\beta + \frac{1}{2}m_Z^2 \cos(2\beta) - \frac{\delta\lambda_1}{2}v^2 \sin^2\beta, \\ M_D^2 &= b \tan\beta - \frac{1}{2}m_Z^2 \cos(2\beta). \end{aligned} \tag{2.9}$$

The new quartic term shifts the mass of the light scalar Higgs eigenstate. The full analytic formula is not very enlightening, but in the $\lambda \ll 1$, $m_A \gg m_Z$ limit we can expand it as:

$$\delta m_h^2 = \begin{cases} \delta\lambda_1 v^2 \left(1 - \frac{2}{\tan^2\beta} \left(1 + 2\frac{m_Z^2}{m_A^2}\right) + \dots\right), & \text{if } \tan\beta \gg 1. \\ \frac{1}{4}\delta\lambda_1 v^2 \left(1 + (\tan\beta - 1) \left(2 - 2\frac{m_Z^2}{m_A^2}\right) + \dots\right), & \text{if } \tan\beta \approx 1. \end{cases} \tag{2.10}$$

Alternatively, for any $\tan\beta$ we can expand the mass formula for $m_A^2 \gg m_Z^2$ as

$$m_h^2 \approx m_Z^2 \cos^2(2\beta) + \delta\lambda_1 v^2 \sin^4\beta - \frac{(2m_Z^2 \cos 2\beta - \delta\lambda_1 v^2 \sin^2\beta)^2 \sin^2(2\beta)}{4m_A^2} + \mathcal{O}(m_Z^6/m_A^4). \tag{2.11}$$

We show the contours of the lifted Higgs mass as a function of $\delta\lambda_1$ and $\tan\beta$ in figure 1.

As we expect for a term involving only the up-type Higgs, the new quartic is more efficient at raising the light Higgs boson mass to 125 GeV in the limit of large $\tan\beta$.

¹In some examples, e.g. quartic which is generated from non-decoupling D -terms, one would often need an additional fine tuning beyond the MSSM to produce the desired quartic. In the D -term scenario this is related to the fact that the non-decoupling D -term is proportional to the soft masses of heavy W'/Z' -inos. Of course we do not take this potentially model-dependent tuning into account in our analysis. However one should bear in mind that the fine tuning that we estimate is the lowest possible bound within the low energy effective theory.

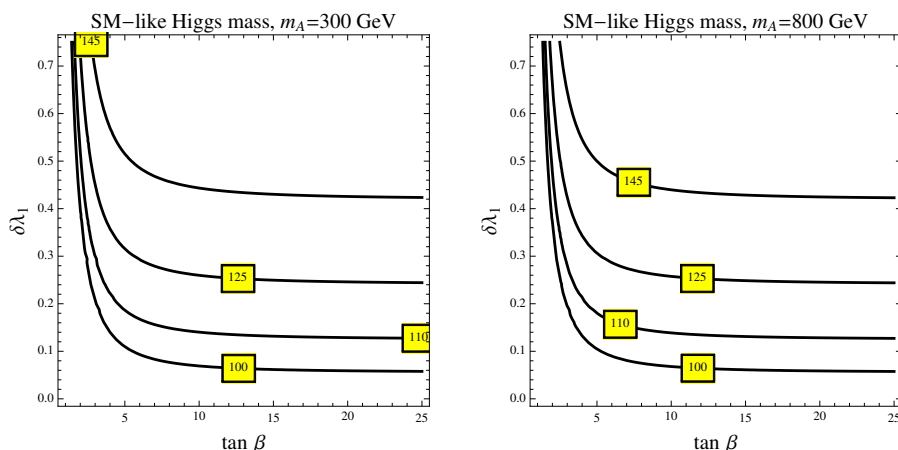


Figure 1. Contours of lifted Higgs mass when adding a new $|H_u|^4$ quartic coupling $\delta\lambda_1$, for two different choices of m_A . As expected from eq. (2.10), the dependence on m_A is small. For large $\tan\beta$ we need $\delta\lambda_1 \approx 0.24$ to lift the Higgs mass.

Furthermore, because the MSSM tree-level contribution is suppressed at small $\tan\beta$, it becomes even more difficult to obtain $\delta\lambda_1$ large enough in that case. This is illustrated by the contours of constant Higgs mass in the $(\tan\beta, \delta\lambda_1)$ plane in figure 1.

To evaluate the tuning, we first take a derivative with respect to M_d^2 . We use the fact that $b = m_A^2 \sin\beta \cos\beta$ (a result that is unchanged from the MSSM case) and thus that M_d^2 can be written in terms of the physical parameters m_Z^2, m_A^2 , and $\tan\beta$ as $M_d^2 = m_A^2 \sin^2\beta - \frac{1}{2}m_Z^2 \cos(2\beta)$. The resulting expression is:

$$\left| \frac{\partial \log v^2}{\partial \log M_d^2} \right| = \frac{\cos^2\beta [2m_A^2 + m_Z^2 (1 - \cot^2\beta)] [m_A^2 \csc^2\beta + 2m_Z^2 + \delta\lambda_1 v^2]}{m_A^2 (m_Z^2 + \delta\lambda_1 v^2) + m_Z^2 \cot^2\beta [m_A^2 (\cot^2\beta - 2) + \delta\lambda_1 v^2]}. \quad (2.12)$$

This expression is not very enlightening on its own, but the main question we are interested in is: if we allow at most a given amount of fine-tuning, can we infer a bound on the physical masses of heavy particles? For this question, it is reasonable to expand the tuning measure at large m_A^2 . We will also assume that the value of $\delta\lambda_1$ is chosen to fix the Higgs mass m_h^2 as in eq. (2.11). The result is:

$$\begin{aligned} \left| \frac{\partial \log v^2}{\partial \log M_d^2} \right| &\approx \frac{1}{2} \sin^2 2\beta \frac{m_A^2 + m_h^2}{m_h^2} + \frac{1}{2} \cos^2\beta (1 - 4\cos 2\beta + \cos 4\beta) \frac{m_Z^2}{m_h^2} + \mathcal{O}(m_h^2/m_A^2) \\ &\xrightarrow{\tan\beta \rightarrow \infty} \frac{2m_A^2 + 2m_h^2 + 3m_Z^2}{m_h^2 \tan^2\beta}. \end{aligned} \quad (2.13)$$

From this we can see that if $m_A^2 \gg m_h^2$, the theory becomes very fine-tuned unless $\sin(2\beta)$ is small, which happens in the $\tan\beta \gg 1$ limit. We explicitly illustrate this point in figure 2 where we plot the contours of fine tuning as a function of m_A and $\tan\beta$. The precise value of $\delta\lambda_1$ on this plot is set by demanding $m_h = 125$ GeV.

As expected, one gets very low fine tuning for very large values of $\tan\beta$. Even now the large $\tan\beta$ region can be partially explored by the LHC, due to a robust $H^0, A^0 \rightarrow \tau^+\tau^-$

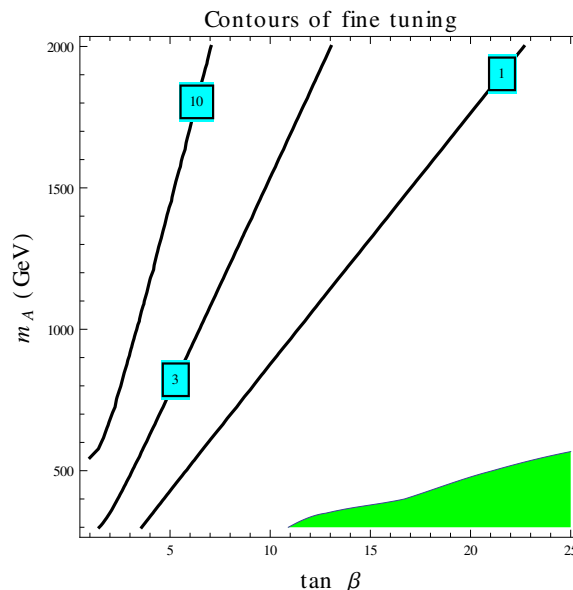


Figure 2. Contours of fine tuning of EWSB with an extra quartic $|H_u|^4$. The exact value of λ_1 is determined by demanding $m_h = 125$ GeV. The shaded green region is directly excluded by the CMS search for $H \rightarrow \tau^+\tau^-$ decay (see text for explanation).

decay mode which can be directly probed. In figure 2 we show a green region, which has been directly excluded by the CMS search [44] for $H^0 \rightarrow \tau^+\tau^-$. We anticipate that much more significant gains will be made by LHC14.

However, another important constraint on the large $\tan\beta$ region comes from the measurement of the flavor-violating decay $b \rightarrow s\gamma$, which we will explore in detail in section 3. There we will find that, for *very* low-scale SUSY breaking (mediated at $\Lambda = 10$ TeV), one can accommodate $\tan\beta \approx 30$ if one allows a factor of 10 tuning in the stop sector. (Indirect constraints from Higgs decays already force us to accept a minimum factor of about 5 tuning in the stop sector [36].) Using the formula above, we find that if we allow at most an *additional* factor of 10 tuning in EWSB, for a combined 1% tuning, we have $m_A \lesssim 8.4$ TeV. Probing such large values of m_A will require future hadron colliders, more powerful than the LHC. On the other hand, we will find in section 3.3 that with even a slightly higher mediation scale $\Lambda = 30$ TeV the bound from $b \rightarrow s\gamma$ becomes notably stronger: $\tan\beta \lesssim 10$. In this case, allowing for at most an additional factor of 10 tuning in EWSB implies $m_A \lesssim 2.8$ TeV. If we view the factor of 10 tuning in the stop sector as already deviating from naturalness, and want to ask for no *additional* tuning in EWSB, we have the stronger condition $m_A \lesssim 0.9$ TeV. Furthermore, higher mediation scales only strengthen the $\tan\beta$ upper bound from $b \rightarrow s\gamma$, so although parts of the natural parameter space may require future colliders to probe, in much of the parameter space the heavy Higgs bosons should be accessible at the LHC. Measurements of the light Higgs boson decay modes at the 14 TeV LHC with 300 fb^{-1} of data will probe the range of m_A up to about 450 GeV [45]. Heavier masses can be probed only by direct searches or higher precision measurements at the high luminosity LHC or especially future e^+e^- colliders.

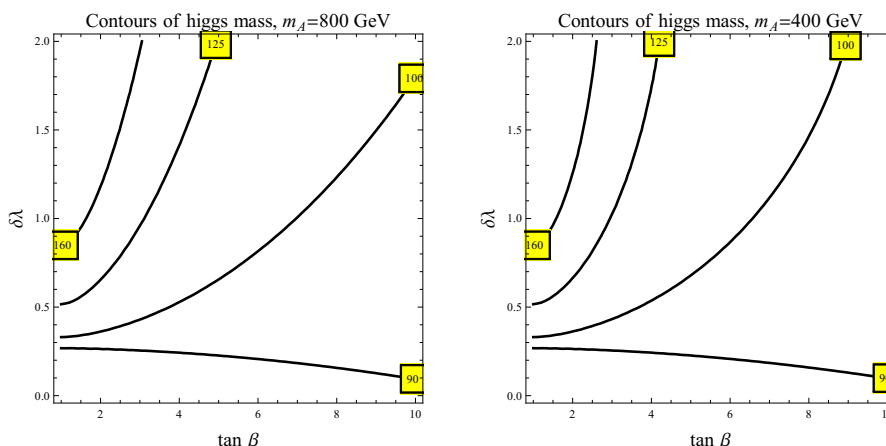


Figure 3. Contours of lifted Higgs mass when adding a new $|H_u \cdot H_d|^2$ quartic coupling $\delta\lambda_5$, for two different choices of m_A . Only moderate $\tan\beta$ values are allowed by 125 GeV Higgs.

2.3 The $\lambda_5 |H_u \cdot H_d|^2$ extension

This is the quartic extension that arises in the NMSSM or λ SUSY. It does not change the pseudoscalar mass relation $m_A^2 = 2b/\sin(2\beta)$. In this case, we find that the light Higgs mass is corrected as:

$$m_h^2 = m_Z^2 \cos^2(2\beta) + \delta\lambda_5 v^2 \sin^2(2\beta) - \frac{(m_Z^2 - \delta\lambda_5 v^2)^2 \sin^2(4\beta)}{4m_A^2} + \mathcal{O}(m_Z^6/m_A^4). \quad (2.14)$$

In this case moderate values of $\tan\beta$ are most effective for raising the Higgs mass, because the correction term involves v_d and is suppressed in the large $\tan\beta$ limit. In fact, it is impossible to get $m_h = 125$ GeV with large $\tan\beta$. On top of that, we often need large, almost non-perturbative values of $\delta\lambda_5$ in order to get the correct value of the SM-like Higgs mass.

The tuning measure in this case is:

$$\left| \frac{\partial \log v^2}{\partial \log M_d^2} \right| = \frac{M_d^2 \sin^2(2\beta) (m_A^2 \csc^2 \beta + 2m_Z^2 - 2\delta\lambda_5 v^2)}{m_A^2 m_Z^2 + (m_A^2 - \delta\lambda_5 v^2) (m_Z^2 - \delta\lambda_5 v^2) \cos(4\beta) + \delta\lambda_5 (m_A^2 + m_Z^2) v^2 - \delta\lambda_5^2 v^4}. \quad (2.15)$$

Again, this expression simplifies in the limit $m_A^2 \gg m_h^2, m_Z^2$, choosing $\delta\lambda_5$ to fix the Higgs mass m_h^2 as in eq. (2.14):

$$\left| \frac{\partial \log v^2}{\partial \log M_d^2} \right| \approx \frac{1}{2} \sin^2 2\beta \frac{m_A^2}{m_h^2} - \frac{3 - 2 \cos(2\beta) + \cos(4\beta)}{4} + \frac{m_Z^2 (1 - 4 \cos(2\beta) + \cos(4\beta))}{4m_h^2} + \mathcal{O}(m_{Z,h}^2/m_A^2). \quad (2.16)$$

This suggests that λ_5 extension is typically fine tuned, since it is not easy to find a perturbative $\delta\lambda_5$ for a light pseudo-scalar A . We show this point explicitly in figure 4. Most of the solutions for $\delta\lambda_5$ are already fine tuned, and those which are technically not fine tuned require very large values of $\delta\lambda_5$. The region with order-one values of $\delta\lambda_5$ and low fine-tuning has $m_A \lesssim 1$ TeV, so the heavy Higgs bosons may be accessible at the LHC.

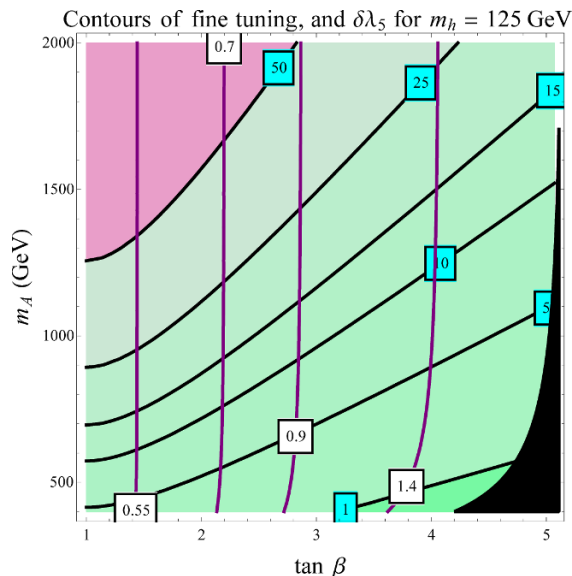


Figure 4. Contours of fine tuning of EWSB when adding a new $|H_u \cdot H_d|^2$ coupling $\delta\lambda_5$. Most of the parameter space is already fine tuned. The purple contours denote the $\delta\lambda_5$ value needed to get a 125 GeV SM-like Higgs mass. In the black region it is difficult to rely on the perturbative calculation, since it demands $\delta\lambda_5 > 2$.

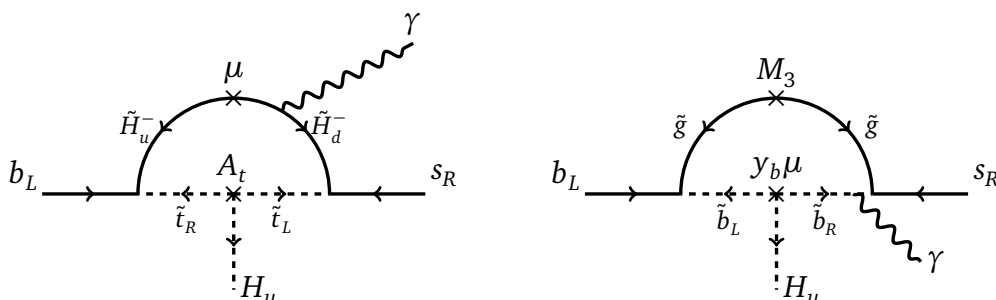


Figure 5. Diagrams contributing to the $b \rightarrow s\gamma$ process in natural SUSY theories. The higgsino has flavor violating couplings through the CKM matrix just as the W boson does, so the stop-higgsino loop at left has the same flavor factors as the SM amplitude.

3 How large can $\tan\beta$ be in natural SUSY?

The role of $b \rightarrow s\gamma$ in natural SUSY was recently emphasized in refs. [23, 46, 47]. The process receives multiple contributions in supersymmetric theories that involve an insertion of the VEV of H_u and thus are enhanced by a factor of $\tan\beta$ relative to the Standard Model amplitude [48–54]. Two of these diagrams, one with stops and higgsinos running in the loop and one with gluinos and sbottoms, are shown in figure 5. (Other diagrams involve a wino or bino running in the loop; we will ignore these terms, which are small corrections in natural parts of parameter space.)² From the loop diagram containing stops and higgsinos,

²The contributions of the charged Higgs loop is also small in most parts of the natural parameter space of m_A , if the charged Higgs is nearly degenerate with the neutral heavy Higgses. We are trying to argue

we have a correction to the matrix element scaling like:

$$\mathcal{M}_{\tilde{t},\tilde{h}}(b \rightarrow s\gamma) \sim m_t^2 \frac{A_t \mu}{m_{\tilde{t}}^4} \tan \beta. \tag{3.1}$$

The measurement of the rate for $b \rightarrow s\gamma$ puts an upper bound on this correction, which we would like to interpret as an upper bound on $\tan \beta$. Such a bound would be very weak *if* the coefficient of $\tan \beta$ could be very small. Thus, we would like to have a lower bound on the factor $\frac{A_t \mu}{m_{\tilde{t}}^4}$ in front of $\tan \beta$. Fortunately, there is an argument for each parameter that goes in the correct direction:

- $m_{\tilde{t}}$ cannot be too large because stops are needed for one-loop naturalness (canceling the top loop divergence in $m_{H_u}^2$).
- μ cannot be too small because we have a direct constraint from LEP on the possibility of light charged particles; hence $\mu \gtrsim 100$ GeV [55–58]. The LHC will potentially strengthen this constraint, although even raising the bound to 150 or 200 GeV will require a large luminosity at 14 TeV [59–62].
- Finally, A_t cannot be too small because it receives loop corrections proportional to the gluino mass M_3 . If it takes a value much smaller than these loop corrections, this would be a new source of fine-tuning. Bounds on the gluino mass are in the vicinity of 1 TeV for a variety of scenarios, both with traditional missing momentum signatures and in cases where the gluino decays to multiple jets [63–67], so it is reasonable to think that A_t should not be smaller than the radiative contribution from a 1 TeV gluino.

This tells us that naturalness, used in conjunction with the measurement of $b \rightarrow s\gamma$ and experimental bounds on the gluino mass, implies an upper bound on $\tan \beta$. We should now evaluate what this bound is, numerically. The full formula is given in a convenient form in ref. [68] (using the results of ref. [69]) and we will use it in the numerics, but first to get some intuition we will give some approximations that indicate how the correction depends on the soft parameters. We work in the limit $\mu^2 \ll m_{Q_3}^2, m_{u_3^c}^2$, introducing the notation $\overline{m}_{\tilde{t}} \equiv (m_{Q_3} m_{u_3^c})^{1/2}$ for the geometric mean of the two stop soft masses and $r = m_{Q_3}/m_{u_3^c}$ for their ratio. Then if we assume that only the stop-higgsino loop gives a significant contribution, the general formula approximately reduces to:

$$\frac{\text{Br}(B \rightarrow X_s \gamma)}{\text{Br}(B \rightarrow X_s \gamma)_{\text{SM}}} - 1 \approx 2.55 \tan \beta \frac{A_t \mu m_t^2}{\overline{m}_{\tilde{t}}^4} \left[\log \frac{\overline{m}_{\tilde{t}}}{\mu} \left(1 + 2.1 \frac{r^2 + 1}{r} \frac{\mu^2}{\overline{m}_{\tilde{t}}^2} \right) - 0.52 + \frac{1 + r^2}{2 - 2r^2} \log r - \frac{\mu^2}{\overline{m}_{\tilde{t}}^2} \left(0.76 \frac{3(r^2 + 1)}{4r} + 2.1 \frac{r^4 + 1}{2r(r^2 - 1)} \ln(r) \right) \dots \right], \tag{3.2}$$

where omitted terms are subleading in $\tan \beta$ or in $\mu^2/\overline{m}_{\tilde{t}}^2$.

that m_A cannot naturally be too large, so while the charged Higgs contribution can matter at small m_A , it is not very relevant for our argument. Therefore we will also neglect it here.

There are other loop corrections to $b \rightarrow s\gamma$, but they depend on masses that can naturally be heavy. The gluino loop shown at right in figure 5 can feel flavor violation through the squark soft mass matrices; even in an MFV scenario, these need not be universal, because — for example — $m_{\tilde{Q}}^2$ can contain a piece proportional to $V^\dagger y_u^2 V$ where y_u^2 is a diagonal matrix of up-type Yukawas [68]. However, the gluino loop involves the right-handed sbottom, which need not be light for naturalness. In fact, in some natural SUSY scenarios it must be heavy to avoid FCNCs [8, 70]. Even if we assume that the right-handed sbottom mass is near the left-handed sbottom and stop masses, we find that the gluino loop is usually subdominant to the stop-chargino loop for natural parameter values. The wino loop is suppressed by a smaller coupling as well as potentially the heaviness of the wino mass. Thus, it is reasonable for us to focus on the stop-chargino loop. In principle, other loop corrections *could* cancel it, but this is in itself a tuning.

3.1 Natural choices for A_t

The simplest estimate for the smallest natural choice of A_t , assuming running from a relatively low scale Λ , is

$$A_t^{\text{loop}} \approx -\frac{2}{3\pi^2} g_3^2 M_3 \log \frac{\Lambda}{M_3} \approx -230 \text{ GeV} \left(\frac{M_3}{1 \text{ TeV}} \right) \log_{10} \frac{\Lambda}{M_3}. \quad (3.3)$$

If we run from a higher scale, we can do a somewhat more careful estimate by resumming large logarithms.

What we have called A_t is really a_t/y_t , where a_t is the coefficient of the three-scalar operator in the Lagrangian. Keeping only the one-loop terms involving g_3 or y_t , the RG evolution of a_t is related to that of the gluino mass by the equation (e.g. [71])

$$\frac{d}{d \log \mu} a_t = \frac{1}{16\pi^2} \left(\left(18y_t^2 - \frac{16}{3}g_3^2 \right) a_t + \frac{32}{3}y_t g_3^2 M_3 \right). \quad (3.4)$$

If we assume that $a_t \approx 0$ at some mediation scale M_{med} (as is true in a number of models, including gauge mediation), we can use this equation together with the RGEs for g_3 , y_t , and M_3 to plot the low-scale value of A_t as a function of the low-scale gluino mass parameter M_3 and the mediation scale. We show this in figure 6.

What we learn from this is that typically, the RG contribution to A_t ranges from -200 GeV to -750 GeV over a wide range of mediation scales and for gluinos near 1 TeV. Thus, a smaller trilinear coupling A_t will generally imply some tuning of a positive tree-level value at the mediation scale against a negative loop correction from gluinos. (Here we use “tree-level” loosely for the value of A_t at the mediation scale; in a given model, it may arise from loops, but we distinguish it from the contribution generated in the RGE.) We will quantify this tuning in an intuitive way. Given that A_t is a sum $A_t = A_t^{\text{tree}} + A_t^{\text{loop}}$, we can measure a tuning by the amount of cancelation:

$$\Delta_{A_t} \equiv \frac{|A_t^{\text{tree}}| + |A_t^{\text{loop}}|}{|A_t^{\text{tree}} + A_t^{\text{loop}}|}. \quad (3.5)$$

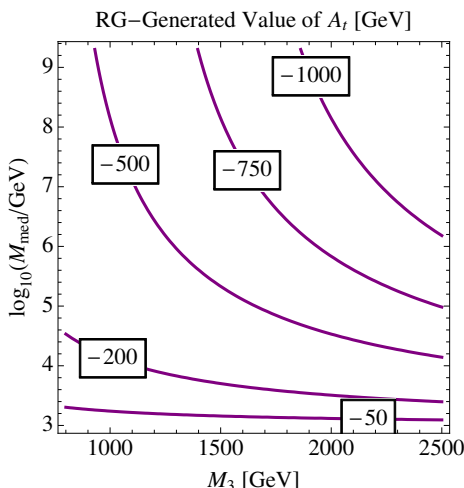


Figure 6. The low-scale value of A_t generated from solving the RGE with $A_t = 0$ at a scale M_{med} and a low-scale gluino mass M_3 .

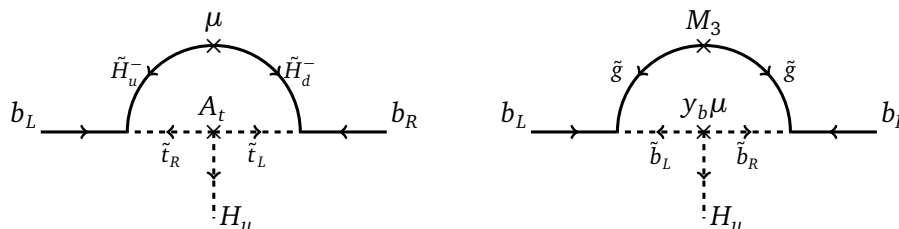


Figure 7. Diagrams contributing to the “wrong-Higgs” Yukawa coupling $H_u^\dagger Q d^c$, which can be a dominant contribution to the b -quark mass for very large $\tan \beta$.

In the regime where the two terms nearly cancel, this behaves similarly to other tuning measures like that of Barbieri and Giudice [1]. If there is no significant cancelation (e.g. if A_t at the mediation scale is much larger than the gluino-generated term), it asymptotes to 1. This is a desirable property for a tuning measure to have, because we would like to be able to compute a combined tuning in multiple variables as a product of independent tunings in each variable.

3.2 The uplifted Higgs region

As $\tan \beta$ increases, the Yukawa couplings needed to generate the b and τ masses from the VEV of H_d become large. However, a new source of masses arises from loop effects that generate the “wrong-Higgs” Yukawa couplings $H_u^\dagger Q d^c$ and $H_u^\dagger L e^c$. For sufficiently large $\tan \beta$ we can think of the b and τ masses as arising entirely for these effects, in what has been called the uplifted supersymmetric Higgs region of parameter space [72, 73]. In this part of parameter space, we must exercise some caution in our argument about the size of the $b \rightarrow s\gamma$ amplitude. The same loop diagram that generates the wrong-Higgs bottom quark Yukawa coupling also generates $b \rightarrow s\gamma$, when one external b quark is replaced by a strange quark and a photon is attached to an internal line. (Compare the loops generating

Yukawas in figure 7 to those for $b \rightarrow s\gamma$ in figure 5.) As a result, $b \rightarrow s\gamma$ is no longer enhanced by a factor of $\tan\beta$ relative to the b -quark mass, and we should be concerned that data on $b \rightarrow s\gamma$ can't actually rule out very large values of $\tan\beta$.

This concern is conceptually reasonable but proves to be numerically unfounded. The uplifted region of parameter space lies at very large values of $\tan\beta$ and also requires large values of μ , putting it outside of what we consider to be natural SUSY parameter space. In all of our computations, we will use the formulas of ref. [68], in which the corrections to the $b \rightarrow s\gamma$ amplitude are proportional to $\tan\beta/(1 + \epsilon_b \tan\beta)$, where ϵ_b in the denominator is a loop factor correcting for the wrong-Higgs contribution to the b -quark mass. The statement that the uplifted regime does not change our conclusion is that $\epsilon_b \tan\beta$ is at most $\mathcal{O}(1)$ for reasonable input parameters, whereas removing the bound at large $\tan\beta$ would require that it be $\gg 1$.

It is easy to see that naturalness is in tension with the uplifted regime by inspection of the loop corrections. The approximate result for ϵ_b in the limit $m_{Q_3}^2 = m_{u_3}^2 = m_{d_3}^2 \equiv m_q^2$ assuming $M_3^2 \gg m_q^2 \gg \mu^2$ is

$$\epsilon_b \approx \frac{1}{16\pi^2} \left\{ \frac{8g_s^2}{3} \frac{\mu}{M_3} \left[\log \frac{M_3^2}{m_q^2} \left(1 + 2 \frac{m_q^2}{M_3^2} \right) - 1 \right] + \frac{y_t^2 s_\beta^2 A_t \mu}{m_q^2} \left(1 - \frac{\mu^2}{m_q^2} \log \frac{m_q^2}{\mu^2} \right) + \dots \right\}. \tag{3.6}$$

Numerically, we expect the gluino loop contribution to ϵ_b , which is $\sim \mu/M_3$, to dominate in most of the natural SUSY parameter space. Note that for naturalness, we prefer μ as small as possible (close to 100 GeV), whereas experimentally we know that $M_3 \gtrsim 1$ TeV. Furthermore, the gluino loop drags the stop and sbottom soft masses up, so the log is rarely large. Estimating $\mu/M_3 \lesssim 0.2$ and $\log(M_3^2/m_q^2) \lesssim 3$, we see that $\epsilon_b \lesssim 10^{-2}$, so that $\epsilon_b \tan\beta$ becomes an order-one number only at $\tan\beta \sim 100$. One could try to get around this conclusion by choosing very large values of A_t to enhance the second term, but this is not very well-motivated and potentially runs into problems with vacuum stability [74]. Increasing the first term requires going to large μ and thus indicates significant tree-level tuning for electroweak symmetry breaking. In short, the uplifted regime is of little relevance for a study of natural SUSY, and will not interfere with our inference of a bound on $\tan\beta$ from $b \rightarrow s\gamma$ and naturalness.

3.3 Interpreting the experimental results on $b \rightarrow s\gamma$

For the experimental bound on $b \rightarrow s\gamma$, we will follow ref. [68] in taking the SM prediction to be [75] $\text{Br}(B \rightarrow X_s\gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$ and the experimental value to be [76, 77] $\text{Br}(B \rightarrow X_s\gamma)_{\text{exp}} = (3.43 \pm 0.22) \times 10^{-4}$. Given these values, we estimate that at 95% confidence level the ratio $R_{bs\gamma}$ of the true value of the branching ratio to its Standard Model value lies in the range

$$0.90 \leq R_{bs\gamma} \leq 1.32. \tag{3.7}$$

Because the data prefer a slightly high value relative to the Standard Model, constraints are weaker on the scenario where new physics constructively interferes with the SM amplitude. This happens when $\mu A_t > 0$. Because small A_t is easiest to achieve if the RG contribution

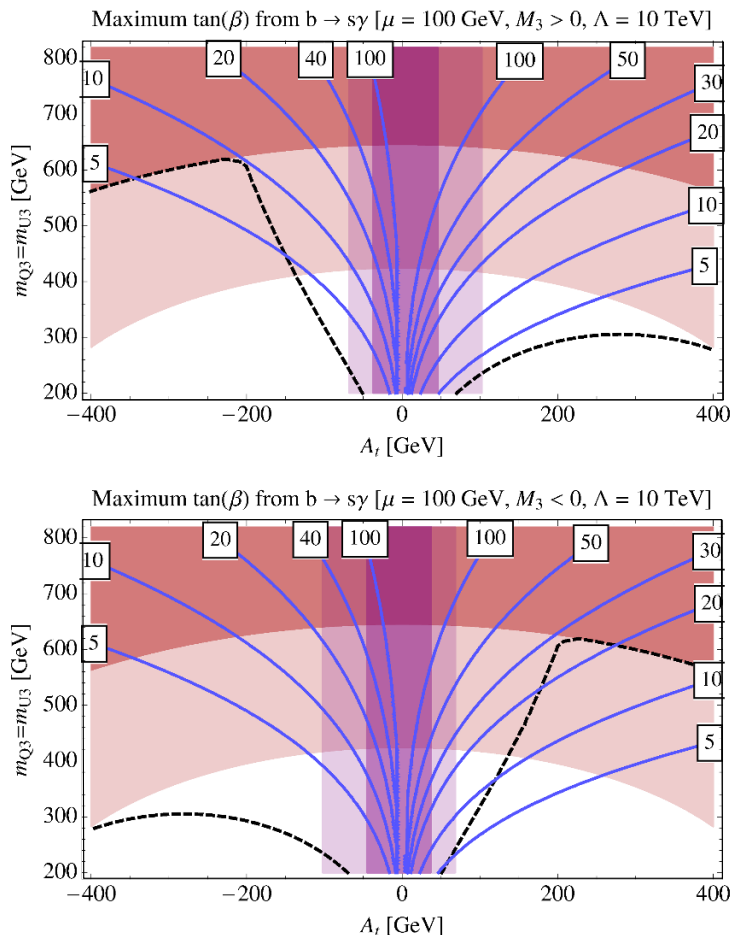


Figure 8. Constraints arising from $b \rightarrow s\gamma$. Here we have fixed $\mu = 100$ GeV (and $|M_3| = 1.3$ TeV) and plot blue solid lines for contours of the largest allowed $\tan\beta$ as a function of the stop mixing parameter A_t and the stop soft mass parameter. The shaded regions are disfavored by naturalness: the purple regions at small A_t involve tuning $\Delta_{A_t} = 5$ (lighter region) and 10 (darker region). The red shaded regions correspond to $\Delta_t = 5$ (lighter) and 10 (darker) tuning in $m_{H_u}^2$ from the stop loop contribution. The region above the black dashed lines has combined tuning $\Delta > 10$. The plots with different signs of M_3 have different tuning measures because the loop-generated A_t always has the opposite sign to M_3 .

from the gluino dominates, this corresponds to a negative sign for μM_3 . The case $\mu A_t < 0$, arising if μ and the gluino mass term have the same sign, is more strongly constrained.

We have plotted the largest allowed value of $\tan\beta$, with various naturalness constraints superimposed, in figure 8. In this figure μ is fixed to 100 GeV. We have also fixed $|M_3| = 1.3$ TeV, $m_{d_3} = 2$ TeV, $M_2 = 0.5$ TeV, and ζ (a parameter defined in ref. [68] related to the relative size of various MFV terms in the soft mass matrices) equal to 0.5. We assume that running begins at $\Lambda = 10$ TeV, a fairly extreme limit of low-scale SUSY breaking, in order to be conservative about tuning measures. Bounds on $\tan\beta$ become stronger if μ increases. The plot is relatively insensitive to the other parameters, but we have included them for concreteness. We have checked that including the stop-chargino loop alone, with

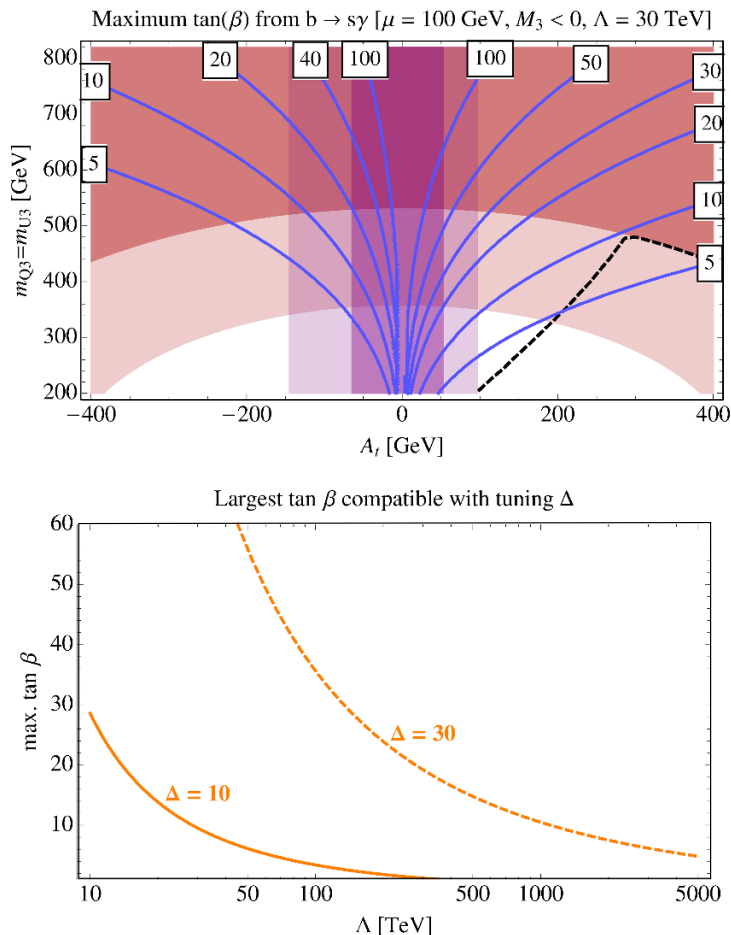


Figure 9. Constraints arising from $b \rightarrow s\gamma$. The upper plot is just like the lower panel of figure 8, except that we set the supersymmetry mediation scale Λ to 30 TeV instead of 10 TeV. The extra running means that increased tuning is required: both Δ_{A_t} and $\Delta_{\tilde{t}}$ are larger. As a result, requiring $\Delta < 10$ now imposes a stronger constraint, $\tan \beta < 9.5$. In the lower plot we show how this constraint evolves with the mediation scale, allowing for stop-sector tuning by a factor of either 10 (solid orange line) or 30 (dashed orange line). Already for a 100 TeV mediation scale the constraint is $\tan \beta < 3.4$ if we require $\Delta < 10$.

all other superpartners decoupled, makes very little difference in the result. We plot two cases with two different signs of M_3 (relative to μ). The sign of M_3 determines the sign of A_t^{loop} which enters in the tuning measure eq. (3.5).

The naturalness constraints are defined in terms of two tunings. First, large stop soft masses correspond to a tuning of the up-type Higgs soft mass parameter, which is quantified by [5, 6]

$$\Delta_{\tilde{t}} = \left| \frac{3y_t^2 m_{Q_3}^2 + m_{u_3}^2 + A_t^2}{4\pi^2 m_h^2} \log \frac{\Lambda}{m_{\tilde{t}}} \right|. \quad (3.8)$$

The second tuning arises for small values of A_t , as quantified in the expression Δ_{A_t} of eq. (3.5). In figure 8, regions of large $\Delta_{\tilde{t}}$ are shaded red and regions of large Δ_{A_t} are shaded purple. One can see that large values of $\tan \beta$ are allowed only if A_t is small or the

stop masses are large, indicating that at least one of these tuning measures is becoming large. For instance, there are two corners of parameter space where $\Delta_{A_t} = \Delta_{\tilde{t}} = 5$, one at negative A_t and one at positive A_t (where the sign is understood relative to that of μ). In the case $M_3 < 0$, at the former point, the largest 95% CL allowed value of $\tan\beta$ is less than 10; at the latter, it is about 25. Thus, as noted above, the case of positive A_t is less strongly constrained.

We have no particular reason to think that cancellations in $m_{H_u}^2$ and in A_t will happen at the same point in parameter space, although perhaps one could imagine a model in which this is true. If the tunings *are* independent, we can think of an overall tuning $\Delta = \Delta_{\tilde{t}}\Delta_{A_t}$ which is simply the product of the two individual tunings. In other words, if we have to adjust two unrelated parameters to the 10% level, this may reasonably be thought of as a 1% tuning in parameter space. With such independent tunings in mind, we have plotted dashed black contours in figure 8 that show where $\Delta = 10$. We see that the combined tuning is mildest whenever $M_3 A_t < 0$, which is driven by the fact that Δ_{A_t} prefers A_t to be either near its loop-generated value or much bigger.

The most optimistic region of parameter space has $\mu A_t > 0$ (so that the new physics contribution constructively interferes with the SM and improves agreement with data) and $A_t M_3 < 0$ (so that the trilinear can be mostly generated from the RG). From the figure, we can see that this marginally allows $\tan\beta \approx 28$ with a combined tuning $\Delta \approx 10$ coming almost entirely from the stop mass $m_{\tilde{t}} \approx 600$ GeV. The plots make it clear that allowing $\tan\beta > 30$ will require either quite heavy stops — out of the range that can be considered truly natural — or a cancelation in A_t , or both. We think that it is very conservative to conclude that *generic* natural SUSY requires $\tan\beta < 30$.

In fact, we are usually understating the required cancelation in A_t , because most reasonable models will run from a higher UV scale and generate values of A_t a factor of 2 or more larger than we have considered. Even a slightly larger amount of running produces a significantly stronger conclusion, as we illustrate in figure 9. Beginning the RGE at 30 TeV instead of 10 TeV produces a larger value of A_t^{loop} and also increases the stop-generated contribution to $m_{H_u}^2$. In this case, the conclusion is already that $\tan\beta < 10$. We show how the bound on $\tan\beta$ changes with the mediation scale in the lower panel of figure 9. Running from 100 TeV already requires $\tan\beta < 3.4$ for consistency with an overall tuning $\Delta < 10$. (In fact, the bound already hits $\tan\beta = 1$ when the mediation scale $\Lambda \approx 350$ TeV, indicating that models of high-scale SUSY breaking will require significant fine-tuning for compatibility with the $b \rightarrow s\gamma$ measurement.) Although the choice of a tuning measure is to some extent a matter of taste, it is clear that accommodating $\tan\beta \gtrsim 10$ requires both *very* low-scale mediation and a mild tuning. We also show, with the dashed orange line in the lower panel of figure 9, that allowing for more tuning significantly increases the range of allowed $\tan\beta$. If we allow $\Delta = 30$ rather than 10, we can accommodate $\tan\beta = 30$ even with running from 100 TeV, and $\tan\beta = 10$ even with running from 1000 TeV. Still, high-scale SUSY breaking is highly constrained even allowing for this larger amount of tuning.

As a simpler estimate, we can use the one loop RG approximation eq. (3.3) for A_t^{loop} , write the average stop mass in terms of the tuning $\Delta_{\tilde{t}}$ from eq. (3.8), and use the leading

term in equation (3.2) to estimate that

$$2.55 \tan \beta \frac{A_t \mu m_t^2}{\overline{m}_t^4} \log \frac{\overline{m}_t}{\mu} \lesssim 0.32, \quad (3.9)$$

implying

$$\tan \beta \lesssim 28 \left(\frac{\Delta_{\tilde{t}}}{10} \right)^2 \left(\frac{100 \text{ GeV}}{\mu} \right) \left(\frac{1.3 \text{ TeV}}{|M_3|} \right) \frac{2}{\log \frac{\overline{m}_t}{\mu}} \frac{2}{\log \frac{\Lambda}{|M_3|}} \left(\frac{2}{\log \frac{\Lambda}{\overline{m}_t}} \right)^2. \quad (3.10)$$

This is a useful check that the more detailed numerical results are reasonable. The $(\log \Lambda)^{-3}$ behavior explains the rapid improvement of the bound as we increase Λ above 10 TeV that we saw in figure 9.

3.4 Comment on $B_s \rightarrow \mu^+ \mu^-$

This very rare process is often quoted as the best possible constraint on SUSY with large $\tan \beta$. Indeed the most important SUSY contribution to $B_s \rightarrow \mu^+ \mu^-$ is proportional to $\tan^3 \beta$ [78, 79], and therefore is naively expected to be very sensitive to natural SUSY. However we find that all the constraints that we get from this process are subdominant to $b \rightarrow s\gamma$ constraints. There is a simple explanation for why this happens. Although the matrix element is enhanced by $\tan^3 \beta$, it is also suppressed by m_A^2 . As we have learned in section 2, in the large $\tan \beta$ limit the fine tuning of EWSB stays approximately constant along the contours of $m_A \tan \beta = \text{const}$. Therefore, effectively the matrix element is enhanced only by a single power of $\tan \beta$, precisely as is $b \rightarrow s\gamma$.

On the other hand, the rate of $B_s \rightarrow \mu^+ \mu^-$ is measured to much worse precision than $b \rightarrow s\gamma$. While the process $b \rightarrow s\gamma$ is measured to the precision of better than 10%, ref. [80] gives the following 95% CL bound on $B_s \rightarrow \mu^+ \mu^-$:

$$1.1 \times 10^{-9} < BR(B_s \rightarrow \mu^+ \mu^-)_{exp} < 6.4 \times 10^{-9} \quad (3.11)$$

Based on the SM prediction [81]

$$BR(B_s \rightarrow \mu^+ \mu^-)_{SM} = (3.32 \pm 0.17) \times 10^{-9}, \quad (3.12)$$

from these equations we estimate that at 95% confidence level

$$0.31 < R_{B_s \rightarrow \mu^+ \mu^-} < 1.95. \quad (3.13)$$

The lower bound is meaningless in large $\tan \beta$ regime: in SUSY one cannot get $R_{B_s \rightarrow \mu^+ \mu^-}$ smaller than 0.5, unless the SUSY contribution is dominated by the Z -penguin.³ On the

³The reason for this is that there is a contribution with H^0 exchange that interferes destructively with the Standard Model, and a contribution with A^0 exchange that does not interfere and is equal to the H^0 amplitude to the extent that $m_A \approx m_{H^0}$. Thus the squared matrix element goes as $|A_{SM} - A_{NP}|^2 + |A_{NP}|^2 \geq \frac{1}{2} |A_{SM}|^2$. If the Z penguin contribution matters, this argument is no longer strictly true. But the Z penguin goes only as $\tan^2 \beta$ and is suppressed by the mass insertion δ_{LR} in the up sector (see [82] for relevant expressions), so is generally expected to be less important.

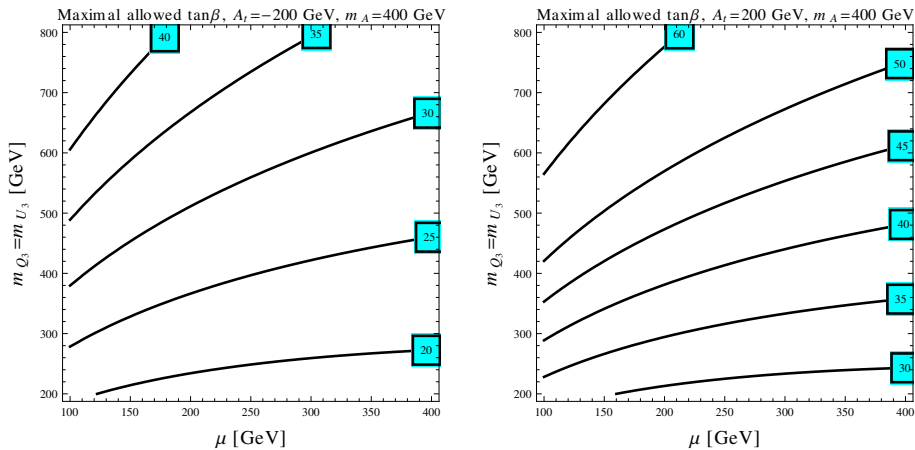


Figure 10. Constraints arising from $B_s \rightarrow \mu^+ \mu^-$ for $m_A = 400$ GeV.

other hand the upper bound is weak, allowing $\mathcal{O}(1)$ deviations from the SM-predicted values. Therefore the bounds on $\tan \beta$ one gets from this process are much weaker than those one gets from $b \rightarrow s\gamma$. To illustrate these points we show these bounds, considering (as in the $b \rightarrow s\gamma$ example) only the higgsino loop contribution, in figure 10, showing the maximal allowed values of $\tan \beta$ for $m_A = 400$ GeV. We see that these constraints are very clearly subdominant to the $b \rightarrow s\gamma$ constraints in figure 8. At higher m_A these constraints quickly decouple.

3.5 The Dirac loophole

If the gluino has a *Dirac mass* rather than the standard Majorana mass, our argument breaks down because A_t can naturally be smaller, protected by R -symmetry. Supersymmetry with Dirac gauginos has received a great deal of recent attention: see (for example) refs. [8, 83–94]. An R -symmetry forbids a μ -term as the higgsino mass, so these models typically involve new doublets that pair with the usual Higgs doublets to form massive higgsinos. The A -term is also forbidden. Depending on how and to what extent the R -symmetry is broken, a remnant of our argument may survive in these models. For the most part, however, we expect that these models evade our argument and that a detailed look at the EWSB conditions and naturalness in these theories will require a completely different perspective. Some aspects of naturalness in such theories have been addressed in refs. [91, 92, 94]. Although this loophole exists, models with Dirac gauginos are necessarily more baroque than traditional SUSY models, and we do not feel that they undermine the motivation for viewing heavy Higgs bosons as key channels in which to search for naturalness.

4 Outlook

The traditional harbingers of SUSY naturalness are higgsinos at tree level, stops at one loop, and gluinos at two loops. Higgsinos, being produced only through the electroweak interactions, are difficult to constrain at hadron colliders. Stops and, especially, gluinos

are easier to search for directly due to their large QCD cross sections. But if R -parity is violated, the spectrum is compressed, or decays go through a hidden sector, traditional missing momentum searches for stops and gluinos can be dramatically weakened. Optimized searches for these “hidden SUSY” cases are receiving increased attention. One of our goals in this paper is to argue that searches for heavy Higgs bosons provide another way to address such scenarios.

Heavy Higgs bosons, unlike superpartners, have predictable decays to pairs of Standard Model particles. The neutral boson H^0 will decay to τ 's and b 's at large $\tan\beta$, and to tops, light Higgs bosons, Z bosons and W bosons at smaller $\tan\beta$. The ZZ “golden channel” is one interesting search mode, and its rate is linked to the hh channel by the Goldstone equivalence theorem, as explained in appendix A. Although extensions of the MSSM might open new decay modes of the heavy Higgses, it seems unlikely that these decays dominate, especially given the SM-like nature of the light Higgs as measured so far. Thus, heavy Higgs searches offer a window on naturalness that is less easily dodged by clever model-building than other SUSY searches.

One recent survey of the reach of LHC Run II for heavy Higgs bosons is ref. [95], which shows that the $H \rightarrow \tau^+\tau^-$ channel can reach above 1 TeV for large $\tan\beta$ while $H \rightarrow t\bar{t}$ can reach above 1 TeV at small $\tan\beta$. The intermediate $\tan\beta$ regime is more difficult to probe and could deserve increased effort, given the added motivation that arises when viewing these searches as an additional probe of naturalness. Other recent theoretical work on signals of heavy Higgs bosons includes refs. [96–101].

It is interesting to ask to what extent our naturalness bounds on heavy Higgs masses can be improved in the future. We do not expect significant theoretical improvements in the Standard Model prediction of $b \rightarrow s\gamma$ in the future, due to irreducible uncertainties [102, 103]. The currently less constraining measurement of $B_s \rightarrow \mu^+\mu^-$ might play a more interesting role in the future. The LHCb result [80] is dominated by statistical uncertainties. Future data is expected to improve the error bar to 10% precision [104]. With such an improved measurement, the constraints from $B_s \rightarrow \mu^+\mu^-$ would become an important supplement to the $b \rightarrow s\gamma$ bound in naturalness arguments regarding the large $\tan\beta$ region.

Another way that our arguments could become somewhat stronger in the future is through an improved lower bound on the higgsino mass parameter μ , which will in turn require smaller values of $\tan\beta$ to accommodate the same constraint on $b \rightarrow s\gamma$. However, higgsino searches are difficult. A more promising route to a stronger bound is through improved bounds on gluino masses, since these feed into A_t at loop level. Although gluino signals are susceptible to being “hidden” in various ways, they are less so than stops, and in fact bounds on gluinos exist even with complicated decay chains lacking missing energy [67]. These bounds should improve early in Run 2 of the LHC, which will allow a stronger statement to be made about the heavy Higgs masses expected from naturalness arguments.

We emphasize that heavy Higgs boson searches provide a robust way to constrain natural models of supersymmetry. Although they are already a part of the LHC's suite of new physics searches, we believe that they should be viewed as part of the naturalness program and accorded a correspondingly intense focus.

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A $H \rightarrow hh$, $H \rightarrow ZZ$, and Goldstone equivalence: a comment on branching ratios

One interesting search channel for a heavy Higgs is $H \rightarrow hh$, which is particularly appealing since the Standard Model rate for events with two Higgs bosons is very small [98, 105–112]. On the other hand, the dominant Higgs decay is to $b\bar{b}$, a challenging signal to pull out of background. Given that the very clean $h \rightarrow ZZ^* \rightarrow 4\ell$ channel played a key role in the discovery of the 125 GeV Higgs boson, it is interesting to ask when, and to what extent, the $H \rightarrow hh$ decay mode dominates over $H \rightarrow ZZ$. Answers to this question may be extracted from the literature, but are often expressed in rather technical forms. For example, in ref. [98], we learn that the coupling g_{Hhh} is proportional to $(3m_A^2 - 2m_h^2 - m_H^2)(\cos(2\beta - 2\alpha) - \cot(2\beta)\sin(2\beta - 2\alpha)) - m_A^2$. Even an MSSM aficionado might have to resort to numerical estimates to have much intuition for what such an expression means. On the other hand, numerically, one can see from plots (e.g. in refs. [95] or [98]) that $\Gamma(H \rightarrow hh)$ is typically about an order of magnitude larger than $\Gamma(H \rightarrow ZZ)$.

In fact, in most models it will be true that $\Gamma(H \rightarrow hh) \approx 9 \Gamma(H \rightarrow ZZ)$, which follows straightforwardly from the Goldstone boson equivalence theorem. Corrections are expected to be of order m_h^2/m_H^2 . This result is likely known to experts but we have not seen it in the literature, so we will explain it here. It offers a useful rule-of-thumb for experimentalists considering whether to undertake a search for Higgs pair production. Assuming this factor of 9 between the heavy Higgs branching ratios, one can ask whether a planned search for Higgs pair production can beat the cleaner, but rarer, $ZZ \rightarrow 4\ell$ signal.

The factor of 9 in the rate comes from a combinatoric factor of 3 in the amplitude that we can explain using a strategy that has appeared in ref. [45], namely working in the basis of VEV eigenstates. We will denote by h the linear combination of fields that has a VEV, and H the orthogonal combination:

$$h = \sin \beta H_u + \cos \beta H_d^\dagger = \begin{pmatrix} iG^+ \\ (v + h^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad (\text{A.1})$$

$$H = -\cos \beta H_u + \sin \beta H_d^\dagger = \begin{pmatrix} iH^+ \\ (H^0 + iA^0)/\sqrt{2} \end{pmatrix}. \quad (\text{A.2})$$

Notice that we are working not just with the real components of the Higgs fields but with *entire* $SU(2)_L$ doublets. Furthermore, the real scalar Higgs modes h^0 and H^0 contained in

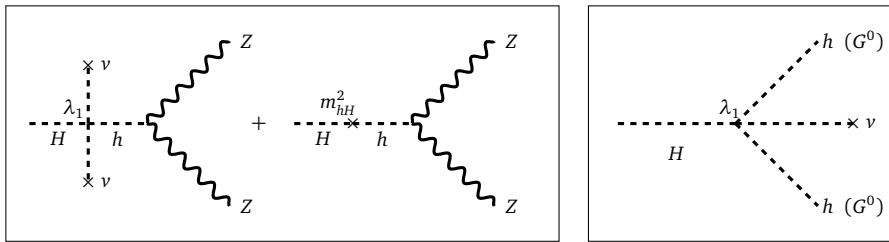


Figure 11. Left: the decay $H \rightarrow ZZ$ in unitary gauge, for which the VEVless eigenstate H first mixes into the eigenstate h and then couples through its VEV to $Z_\mu Z^\mu$. Right: the decay $H \rightarrow hh$ and the related decay to two Goldstone modes. A relative factor of 3 arises from the combinatoric choice of which h leg to replace by a vev in $H \rightarrow hh$.

h and H will not be mass eigenstates, in general. On the other hand, the three Goldstone degrees of freedom G^0, G^\pm for electroweak symmetry breaking are entirely contained in h , and only the real scalar mode h^0 in h has couplings to W^\pm and Z bosons of the form $h^0 V_\mu V^\mu$. Given LHC data, we know that the VEV eigenstates are approximately the same as the mass eigenstates; in other words, we are in the “alignment limit” $\cos(\beta - \alpha) \ll 1$, because the light Higgs is observed to couple to particles proportional to their masses as in the SM [98]. As a result we can think of the heavy Higgs boson as living mostly in H . The decays $H^0 \rightarrow h^0 h^0$ and $H^0 \rightarrow Z_L Z_L$, where we use the Goldstone equivalence theorem to relate the decay rate to longitudinal Z bosons to decays to the Goldstone mode G^0 inside h , both arise from a quartic term in the potential containing one copy of H and three of h :

$$V \supset \tilde{\lambda}_1 \left(H^\dagger h + h^\dagger H \right) h^\dagger h \supset \tilde{\lambda}_1 \left(v H^0 G^+ G^- + \frac{v}{2} H^0 G^0 G^0 + \frac{3v}{2} H^0 h^0 h^0 + H^0 h^0 G^+ G^- + \dots \right). \quad (\text{A.3})$$

Thus, there is a relative factor of 3 in the Feynman rule for H^0 to two Higgs bosons relative to H^0 to two Goldstones. In the first case we have three h factors in the potential, one of which must be replaced by a VEV and two with a physical Higgs boson. The combinatoric factor of 3 comes from the fact that we can replace any of the three h ’s with a VEV. In the second case we again replace one with a VEV, but the other two with Goldstones. The difference is that H^0 lives in the real part of H and so must be paired with either an h or a v in the hermitian $H^\dagger h + h^\dagger H$ factor; the two Goldstones must go in the $h^\dagger h$ factor, and so we have no combinatoric freedom in this case. (Let us also mention in passing that eq. (A.3) leads to several three-body decays of the heavy Higgs, suppressed by phase space but not by couplings; the phenomenology of such decays could be interesting, and is as far as we know unexplored.)

The relative decay rate is also easy to understand in unitary gauge, as shown in figure 11. In this case another contribution arises from the mass mixing of H and h , but this is related to the coupling $\tilde{\lambda}_1$ by a tadpole cancelation condition. In other words, our choice of H as the eigenstate with zero VEV relates the terms $m_{Hh}^2 (H^\dagger h + h^\dagger H)$ and $\tilde{\lambda}_1 (H^\dagger h + h^\dagger H) (h^\dagger h)$ in the potential. In particular, the coupling for $H \rightarrow hh$ vanishes in the limit $m_{Hh}^2 \rightarrow 0$, which is the exact alignment limit where VEV eigenstates are mass eigenstates; this is reflected in the factors of $\cos(\beta - \alpha)$ in the g_{Hhh} coupling in e.g.

ref. [98]. A little algebra shows that the unitary gauge calculation matches the Goldstone equivalence estimate up to terms of order $m_{Z,h}^2/m_H^2$, as expected on general grounds.

The case of a singlet scalar decaying to hh and ZZ is similar, but the combinatoric factor of 3 no longer exists, so we expect the branching ratios to be approximately equal.

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