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A new look at the theory uncertainty of ϵ_K

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Abstract: The observable ϵ_K is sensitive to flavor violation at some of the highest scales. While its experimental uncertainty is at the half percent level, the theoretical one is in the ballpark of 15%. We explore the nontrivial dependence of the theory prediction and uncertainty on various conventions, like the phase of the kaon fields. In particular, we show how such a rephasing allows to make the short-distance contribution of the box diagram with two charm quarks, η_{cc}, purely real. Our results allow to slightly reduce the total theoretical uncertainty of ϵ_K, while increasing the relative impact of the imaginary part of the long distance contribution, underlining the need to compute it reliably. We also give updated bounds on the new physics operators that contribute to ϵ_K.

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Contents

1 Introduction

The study of mixing and CP violation in the K^0–$\overline{K}^0$ system was crucial for the development of the standard model (SM). The comparison of the measurement of the CP violating parameter in K^0–$\overline{K}^0$ mixing, ϵ_K, with its SM calculation provides important constraints on the CKM matrix. The observable ϵ_K also probes some of the highest new physics (NP) scales, and it gives severe constraints on explicit models of flavor. Moreover, to distinguish between possible NP interpretations of flavor anomalies, it is particularly important to know the level of consistency between the constraints on the flavor sector from K and B decay measurements.

What are the current limiting factors of the ϵ_K sensitivity to NP? How can we possibly improve them, now and in the future? The level to which we can answer these questions will have a major impact on our understanding of flavor. These limiting factors have to be looked for in the SM prediction of ϵ_K, whose uncertainty is more than an order of magnitude above the half percent precision of the experimental measurement. Part of the SM uncertainty in the ϵ_K prediction is parametric, i.e., due to the relatively poor knowledge of some of the CKM parameters, most notably A (or equivalently $|V_{cb}|$). This knowledge will be substantially improved by future measurements at Belle II and LHCb [1, 2], which will hopefully also resolve tensions between inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ [3].

Besides $|V_{cb}|$, the largest uncertainty in the SM prediction for ϵ_K originates from the calculation of η_{cc}, the QCD correction to the box diagram with two charm quarks. The NNLO calculation of this quantity [4] found a large correction and a poorly behaved perturbation series, 1, 1.38, 1.87, at leading, next-to-leading, and next-to-next-to-leading orders, respectively, and thus quoted $\eta_{cc} = 1.87 \pm 0.76$, and $|\epsilon_K| = (1.81 \pm 0.28) \times 10^{-3}$. Thus, to what extent η_{cc} is determined by short distance physics may be questioned. This resulted in different groups treating η_{cc} differently. For example CKMfitter [5, 6] uses η_{cc} quoted in ref. [4], whereas UTfit [7, 8] uses the NLO calculation of η_{cc} [9]. This contributes to the visibly different ϵ_K regions in CKMfitter and UTfit plots. Ref. [10] instead argued that $\eta_{cc} = 1.70 \pm 0.21$ was a reasonable estimate, assuming the dominance of Δm_K by the SM contribution, and using an estimate of the long-distance contribution to Δm_K. Note also that the behavior of the perturbation series, which matters for the uncertainty estimate of η_{cc}, is scheme dependent. The perturbative QCD calculations of the $\eta_{ct} = 0.496(47)$ [11] and $\eta_{tt} = 0.5765(65)$ [12] correction factors to the box diagrams with internal tt and ct quarks, respectively, appear to be better behaved.

In this paper we show that one can eliminate η_{cc} from the theoretical prediction of ϵ_K, by setting the contribution of that term to the mixing amplitude, M_{12}, purely real. While physical results are independent of such conventions, numerically some dependence remains (similar to other scheme dependences), because M_{12} and Γ_{12} are calculated using different methods. We discuss the implications of this choice on the SM uncertainty of ϵ_K and on the resulting constraints on NP, both at present and in the future.

This paper is organized as follows: in section 2 we review some definitions and formalism, making clear the approximations and phase-dependences involved. In section 3 we show how to remove the η_{cc} contribution from ϵ_K, and discuss the resulting modified predictions for ϵ_K. In section 4 we comment on implications for constraints on new physics. In section 5 summarize our findings, and conclude.

2 The state of the ϵ_K art

2.1 Definitions

The neutral kaon mass eigenstates are linear combinations of $|K^0\rangle = |d\bar{s}\rangle$ and $|\overline{K}^0\rangle = |\bar{d}s\rangle$. The time evolution of these states is described by the Schrödinger equation,

$$i \frac{d}{dt} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left(M - i \frac{\Gamma}{2} \right) \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}, \tag{2.1}$$

where the mass (M) and the decay (Γ) mixing matrices are 2×2 Hermitian matrices. The mass eigenstates are usually labeled with their lifetimes[1]

$$|K_{S,L}\rangle = p|K^0\rangle \pm q|\overline{K}^0\rangle \,, \tag{2.2}$$

[1]The sign of q is a convention, degenerate with the choice of the phase $\theta = 0$ or π in eq. (2.6). Setting the coefficients of $|K^0\rangle$ identical in $|K_L\rangle$ and $|K_S\rangle$, as done in eq. (2.2), sets another non-physical overall phase to zero.

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and they are the eigenvectors of $M - i\Gamma/2$. To write eq. (2.2) we have assumed CPT symmetry, as we do in the rest of this paper. The correspondence between the long/short lived and the heavy/light states is

$$K_L = K_{\rm heavy}\,, \qquad K_S = K_{\rm light}\,. \tag{2.3}$$

Let us define

$$\Delta m = m_L - m_S > 0\,, \tag{2.4}$$

and

$$\Delta\Gamma = \Gamma_L - \Gamma_S \simeq -\Gamma_S < 0\,. \tag{2.5}$$

Throughout this paper we keep explicitly the CP transformation phase

$$CP|K^0\rangle = e^{i\theta}|\overline{K}^0\rangle\,, \qquad CP|\overline{K}^0\rangle = e^{-i\theta}|K^0\rangle\,, \tag{2.6}$$

since both the $\theta = 0$ and $\theta = \pi$ choices are often used in the literature, and the cancellation of this is interesting to follow. The choice of the phase θ is not to be confused with the phase convention for the kaon and quark fields.

Let us define the decay amplitudes

$$A_f = \langle f|\mathcal{H}|K^0\rangle = |A_f|\, e^{i(\phi_f+\delta_f)}\,, \qquad \bar{A}_f = \langle f|\mathcal{H}|\overline{K}^0\rangle = |A_f|\, e^{i(-\phi_f+\delta_f-\theta)}\,, \tag{2.7}$$

where ϕ_f and δ_f are the weak and strong phases respectively, and the amplitude ratios[2]

$$\eta_f \equiv \frac{\langle f|\mathcal{H}|K_L\rangle}{\langle f|\mathcal{H}|K_S\rangle}\,\frac{\langle K^0|K_S\rangle}{\langle K^0|K_L\rangle} = \frac{1-(q/p)(\bar{A}_f/A_f)}{1+(q/p)(\bar{A}_f/A_f)}\,. \tag{2.8}$$

In terms of η_f for $f = \pi^+\pi^-$ and $\pi^0\pi^0$, ϵ_K and ϵ' are defined as

$$\epsilon_K = \frac{2\eta_{+-}+\eta_{00}}{3}\,, \qquad \epsilon' = \frac{\eta_{+-}-\eta_{00}}{3}\,. \tag{2.9}$$

It is η_{+-} and η_{00} which are measured (and ϵ'/ϵ is extracted from $|\eta_{00}/\eta_{+-}|^2 \simeq 1-6\,{\rm Re}(\epsilon'/\epsilon)$, valid for $|\epsilon'/\epsilon| \ll 1$).

For a theoretical discussion, since $K \to \pi\pi$ decays are dominated by the isospin $I = 0$ two-pion state over $I = 2$, it is convenient to define

$$\eta_I = \frac{\langle(\pi\pi)_I|\mathcal{H}|K_L\rangle}{\langle(\pi\pi)_I|\mathcal{H}|K_S\rangle}\,\frac{\langle K^0|K_S\rangle}{\langle K^0|K_L\rangle}\,, \qquad \omega \equiv \frac{\langle(\pi\pi)_{I=2}|\mathcal{H}|K_S\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}|K_S\rangle}\,. \tag{2.10}$$

The CP violating quantities ϵ_K and ϵ' can also be defined as

$$\epsilon_K = \eta_0\,, \qquad \epsilon' = \frac{\omega}{\sqrt{2}}\,(\eta_2 - \eta_0)\,. \tag{2.11}$$

The definitions in eqs. (2.9) and (2.11) are equivalent up to differences of order $|\omega\epsilon'| \sim 10^{-7}$, i.e., to a relative error of 10^{-4} for ϵ_K, and $1/22$ for ϵ' (see table 1, and use $|\omega| = |A_2/A_0|[1+\mathcal{O}(|\epsilon_K|)] \simeq 1/22$). Neglecting isospin violation, we can further write

$$\eta_{+-} = \frac{\eta_0 + \eta_2\,\omega/\sqrt{2}}{1+\omega/\sqrt{2}}\,, \qquad \eta_{00} = \frac{\eta_0 - \sqrt{2}\,\eta_2\,\omega}{1-\sqrt{2}\,\omega}\,. \tag{2.12}$$

[2] The definition $\eta_f = \langle f|\mathcal{H}|K_L\rangle/\langle f|\mathcal{H}|K_S\rangle$ is often used in the literature, and measured magnitudes and phases are quoted. However, there is an arbitrary unphysical relative phase between $|K_L\rangle$ and $|K_S\rangle$. Effectively eq. (2.8) is measured in the interference of $|K_L\rangle$ and $|K_S\rangle$ decays in regeneration experiments.

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2.2 ϵ_K, phase convention independently

We summarize here how to express ϵ_K in terms of the off-diagonal elements of the mass and width mixing matrices, M_{12} and Γ_{12} (see refs. [13, 14] for more details). We pay attention to write expressions that are independent of the phase conventions for the kaon and quark fields, and we state explicitly the approximations used.

The semileptonic CP asymmetry

$$\delta_L = \frac{\Gamma(K_L \to \pi^- \ell^+ \nu) - \Gamma(K_L \to \pi^+ \ell^- \bar{\nu})}{\Gamma(K_L \to \pi^- \ell^+ \nu) + \Gamma(K_L \to \pi^+ \ell^- \bar{\nu})} = (3.32 \pm 0.06) \times 10^{-3} \ [3] \,, \tag{2.13}$$

measures CP violation in mixing, in the limit when $A_{\pi^+\ell^-\bar{\nu}} = \bar{A}_{\pi^-\ell^+\nu} = 0$ and $|A_{\pi^-\ell^+\nu}| = |\bar{A}_{\pi^+\ell^-\bar{\nu}}|$. Note that these assumptions, valid in the SM to great accuracy, are not precisely tested yet, as the ratio $x_+ = A(\overline{K}^0 \to \pi^-\ell^+\nu)/A(K^0 \to \pi^+\ell^-\bar{\nu})$ is only constrained at the 10^{-3} level [3].[3] In this limit, the definition in eq. (2.13), and solving the eigenvalue equations imply

$$\delta_L = \frac{1 - |q/p|^2}{1 + |q/p|^2} = \frac{2\,\mathrm{Re}(\epsilon_K)}{1 + |\epsilon_K|^2} = \frac{2\,\mathrm{Im}(M_{12}^* \Gamma_{12})}{4|M_{12}|^2 + |\Gamma_{12}|^2} \,, \tag{2.14}$$

where we neglected relative higher orders in $|\omega|\epsilon'/\epsilon$. The expressions for the mass and width differences that follow from the eigenvalue equations are

$$\Delta m = 2|M_{12}| \,, \qquad \Delta\Gamma = -2|\Gamma_{12}| \,, \tag{2.15}$$

and are valid up to relative orders δ_L^2. The relative phase between M_{12} and Γ_{12} is $\pi + \mathcal{O}(\delta_L)$, since eq. (2.14) implies that its sine is small, and the eigenvalue equation $4\,\mathrm{Re}(M_{12}^*\Gamma_{12}) = \Delta m\, \Delta\Gamma < 0$ implies that its cosine is negative.

Equations (2.14) and (2.15) exhaust the information regarding kaon mixing, and $\mathrm{Im}(\epsilon_K)$ is related to CP violation in interference of decay with and without mixing. Still, ϵ_K is the observable used to constrain CP violation in K^0 mixing. The reason is that ϵ_K is measured with about 4 times smaller relative uncertainty than δ_L, and the phase of ϵ_K also depends only on mixing parameters. Indeed, the following relation for the phase ϕ_ϵ,

$$\phi_\epsilon \simeq \arctan \frac{2|M_{12}|}{|\Gamma_{12}|} \,, \tag{2.16}$$

is valid up to relative orders δ_L^2 and $|\omega^2 \epsilon'/\epsilon|$, and up to ratios of amplitudes to more than two-body final states, that do not exceed a relative contribution of 10^{-2} to ϕ_ϵ (see ref. [15] and the updated measurements in ref. [3] for details). The quantity $\arctan(-2\Delta m/\Delta\Gamma) = 43.52^\circ$ is often referred to as "superweak phase", and differs from the measured value of ϕ_ϵ by one part in 10^4, so that the error of eq. (2.16) neither exceeds that level. Using eq. (2.14) for $\mathrm{Re}(\epsilon_K)$ and eq. (2.16) for ϕ_ϵ we obtain

$$\epsilon_K = \frac{e^{i\phi_\epsilon} \sin\phi_\epsilon}{2} \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) = e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{\mathrm{Im}(-M_{12}/\Gamma_{12})}{2\,|M_{12}/\Gamma_{12}|} = e^{i\phi_\epsilon} \cos\phi_\epsilon\, \mathrm{Im}(-M_{12}/\Gamma_{12}) \,. \tag{2.17}$$

[3] This is historically called the $\Delta s = \Delta Q$ rule. In the SM it is only violated by higher orders in the weak interaction; when we discuss NP scenarios below, we neglect the impact of NP on tree-level SM processes.

Clearly, ϵ_K only depends on M_{12}/Γ_{12}, which is physical, while the phases of M_{12} and Γ_{12} separately are not. The neglected higher order terms in eq. (2.17) are also independent of phase conventions.

The standard model predictions for M_{12} and Γ_{12} are calculated separately, using different methods, resulting in intermediate steps that depend on phase conventions. (In contrast, in the case of B^0 and B_s^0 mixing, both M_{12} and Γ_{12} are computed by perturbative QCD methods, hence the cancellations of conventions is more apparent. In K^0 mixing, the use of chiral perturbation theory, and the separate estimation of short and long distance contributions obscure the cancellations.) The conventions that lead to the "usual" ϵ_K formula is reviewed in the rest of this section. In section 3 we use the freedom of this choice to study and minimize the uncertainties of ϵ_K.

2.3 ϵ_K in the standard phase convention

To connect the phase convention independent manifestly physical expressions in eq. (2.17) to actual calculations, we need to consider how M_{12} and Γ_{12} are computed. They are given by

$$M_{12} = \frac{1}{2m_K}\langle K^0|\mathcal{H}|\overline{K}^0\rangle\,, \qquad \Gamma_{12} = \sum_f A^*(K^0 \to f)A(\overline{K}^0 \to f)\,, \tag{2.18}$$

where f denote common final states of K^0 and $\overline{K}^0$ decay. Usually M_{12} is written as the short-distance calculation combined with the matrix element of the four-quark operator $O_1 = (\bar{d}_L\gamma_\mu s_L)^2$ in the vacuum insertion approximation, times a "bag parameter", B_K, plus corrections. The definition of B_K involves θ via [13, 16]

$$\langle K^0|(\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu s_L)|\overline{K}^0\rangle = -e^{-i\theta}\,\frac{2}{3}\,B_K(\mu)f_K^2 m_K^2\,, \tag{2.19}$$

where $B_K(\mu)$ is the usual positive real quantity. One further defines $\widehat{B}_K$, to remove the μ-dependence of $B_K(\mu)$. The width mixing, Γ_{12}, is dominated by

$$A_0^*\bar{A}_0 = e^{-i\theta}\,|A_0|^2\,e^{-2i\phi_0}\,, \tag{2.20}$$

while the subleading contributions are suppressed by $|A_2/A_0|^2 \simeq 2\times 10^{-3}$ and $\mathcal{B}(K_S \to f \neq \pi\pi) < 10^{-3}$. Equations (2.19) and (2.20) show that θ drops out of M_{12}/Γ_{12}, as it must.

In an often used CP phase convention which we also use hereafter, $\theta = \pi$ [17], and then with the usual CKM phase conventions [3], M_{12} is near the positive real axis and Γ_{12} is near the negative real axis. The weak phase, ϕ_0, of the isospin-zero amplitude, A_0, depends on hadronic matrix elements of several operators in the effective Hamiltonian. It is convenient and customary to define

$$\xi = \frac{\mathrm{Im}(A_0\,e^{-i\delta_0})}{\mathrm{Re}(A_0\,e^{-i\delta_0})}\,. \tag{2.21}$$

Without specifying phase conventions, ξ can take any values between $-\infty$ and $+\infty$, because ϕ_0 is convention dependent. In phase conventions in which $|\xi| \ll 1$ and $\mathrm{Re}(A_0\,e^{-i\delta_0}) > 0$,

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one has $\xi = \arg(A_0\, e^{-i\delta_0}) = -\frac{1}{2}\arg(-\Gamma_{12})$ up to relative orders ξ^2 (in addition to the phase-independent relative orders $\mathcal{B}(K_S \to f \neq \pi\pi)$ and $|A_2/A_0|^2$). Then

$$\arg(-M_{12}/\Gamma_{12}) = \arg(M_{12}) - \arg(-\Gamma_{12}) \simeq \frac{2\,\mathrm{Im}M_{12}}{2|M_{12}|} + 2\xi\,, \tag{2.22}$$

is valid to the required accuracy in phase conventions satisfying $\{\arg M_{12}\,, \arg\Gamma_{12}\} = \mathcal{O}(\delta_L) \ll 1 \pmod{\pi}$. Thus, starting from the manifestly convention independent eq. (2.17), choosing $\theta = \pi$ and weak phases such that $|\xi| \ll 1$, we recover the often quoted expression,

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}M_{12}}{\Delta m} + \xi\right) = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}M_{12}^{\mathrm{SD}}}{\Delta m} + \xi + \frac{\mathrm{Im}M_{12}^{\mathrm{LD}}}{\Delta m}\right) = \frac{\kappa_\epsilon\, e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\mathrm{Im}M_{12}^{\mathrm{SD}}}{\Delta m}\,. \tag{2.23}$$

We have explicitly separated the short-distance $\Delta s = 2$ contribution, M_{12}^{SD}, from ξ, and from the long-distance contribution, M_{12}^{LD}. The last term implicitly defines κ_ϵ, which is often written as [18, 19]

$$\kappa_\epsilon = \sqrt{2}\,\sin\phi_\epsilon \left(1 + \rho\,\frac{\xi}{\sqrt{2}\,|\epsilon_K|}\right). \tag{2.24}$$

2.4 Estimating ξ and ρ

Currently available estimates of ξ use either lattice QCD calculations, or the measured value of the direct CP-violating quantity, ϵ', or a combination of the two. It must be emphasized that using ϵ' as an input is only valid assuming that it is determined by the SM. (As discussed below, it is possible that ϵ' is affected by NP but ϵ_K is not, and vice versa.)

One can write ϵ' as

$$\epsilon' = \frac{i}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)}\, \sin(\phi_2 - \phi_0)\,, \tag{2.25}$$

valid up to relative orders $|A_2/A_0|$ and $|\epsilon_K|$. This expression is phase convention independent, as $\phi_2 - \phi_0$ and $\delta_2 - \delta_0$ are physical, and correctly implies $\phi_{\epsilon'} = \pi/2 + \delta_2 - \delta_0 = (42.3 \pm 1.5)^\circ$. In phase conventions in which ϕ_0 and ϕ_2 are both tiny,

$$\epsilon' = \frac{e^{i\phi_{\epsilon'}}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left[\frac{\mathrm{Im}(A_2\, e^{-i\delta_2})}{|A_2|} - \xi\right]. \tag{2.26}$$

This yields

$$\xi = \frac{\mathrm{Im}(A_2\, e^{-i\delta_2})}{|A_2|} - \sqrt{2}\,|\epsilon_K| \left|\frac{A_0}{A_2}\right| \left|\frac{\epsilon'}{\epsilon_K}\right|, \tag{2.27}$$

where the relative errors in both eqs. (2.26) and (2.27), which depend on the phase convention, are of order ξ^2. The second term in eq. (2.27) is well-known experimentally, and this expression allows using lattice calculations of A_2 instead of A_0 to estimate ξ.

Using the lattice QCD result $\mathrm{Im}(A_2\, e^{-i\delta_2}) = -6.99(0.20)(0.84) \times 10^{-13}\,\mathrm{GeV}$ [20], we obtain

$$\xi = -(1.65 \pm 0.17) \times 10^{-4} \qquad \text{(input from } \epsilon'/\epsilon \text{ measurement)}. \tag{2.28}$$

In contrast, the lattice calculation $\mathrm{Im}(A_0\, e^{-i\delta_0}) = 1.90(1.22)(1.04) \times 10^{-11}$ GeV [21], using eq. (2.21), yields

$$\xi = -(0.57 \pm 0.48) \times 10^{-4} \qquad \text{(no input from } \epsilon'/\epsilon \text{ measurement)}. \tag{2.29}$$

This difference is equivalent to the statement that the lattice QCD calculations [20, 21] show about a 2.5σ tension with ϵ', which can be further sharpened using additional inputs [22].

From eqs. (2.23) and (2.24), the parameter ρ is defined as

$$\rho = 1 + \frac{1}{\xi}\, \frac{\mathrm{Im}(M_{12}^{\rm LD})}{\Delta m}\,. \tag{2.30}$$

Without a lattice computation of $M_{12}^{\rm LD}$, ρ can be estimated in the framework of chiral perturbation theory (χPT) [18] (see also [23–25]). First, one argues that all important dispersive diagrams share the same phase [18, 23], so that the phase of the absorptive and dispersive parts are related via

$$\frac{\mathrm{Im} M_{12}^{\rm LD}}{\mathrm{Re} M_{12}^{\rm LD}} \simeq \frac{\mathrm{Im}\Gamma_{12}^{\rm LD}}{\mathrm{Re}\Gamma_{12}^{\rm LD}} \simeq -2\xi(1 \pm 0.5)\,. \tag{2.31}$$

Here we keep using the 50% uncertainty quoted in ref. [18] to account for the non-aligned contributions. The dominant contribution to $\mathrm{Re} M_{12}^{\rm LD}$ comes from the $\pi\pi$ loop, which has been estimated as [18]

$$\frac{\mathrm{Re} M_{12}^{\rm LD}}{\Delta m} \simeq \frac{\mathrm{Re} M_{12}^{(\pi\pi)}}{\Delta m} \simeq 0.2 \pm 0.1\,. \tag{2.32}$$

(Preliminary lattice calculations [26] hint at a smaller role for the 2π state than the χPT estimate; refining this is important.) Equations (2.31) and (2.32) finally imply

$$\rho = 1 - 2(0.2 \pm 0.14) = 0.6 \pm 0.3\,. \tag{2.33}$$

2.5 Short distance contribution and usual evaluation of ϵ_K

Given eqs. (2.23) and (2.24) and estimates of ξ and ρ, the only remaining ingredient in making a SM prediction for ϵ_K is the expression for the short-distance contribution to M_{12} for $\theta = \pi$,

$$M_{12}^{\rm SD} = \frac{\Delta m}{\sqrt{2}}\, \widehat{C}_\epsilon \left[\lambda_t^{*2} \eta_{tt} S_0(x_t) + 2\lambda_c^* \lambda_t^* \eta_{ct} S_0(x_t, x_c) + \lambda_c^{*2} \eta_{cc} x_c\right], \tag{2.34}$$

where $\lambda_q = V_{qd} V_{qs}^*$, $x_q = [\overline{m}_q(\overline{m}_q)/m_W]^2$, the Inami-Lim functions S_0 can be found, e.g., in ref. [17], and[4]

$$\widehat{C}_\epsilon = \frac{G_F^2}{6\sqrt{2}\,\pi^2}\, \frac{m_K\, m_W^2}{\Delta m}\, f_K^2 \widehat{B}_K = (2.806 \pm 0.049) \times 10^4\,. \tag{2.35}$$

[4]The uncertainty of $\widehat{C}_\epsilon (= C_\epsilon \widehat{B}_K)$ is dominated by those of f_K^2 and $\widehat{B}_K$. Their contributions are now comparable, making the past separation of C_ϵ and $\widehat{B}_K$ less motivated.

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Taking the imaginary part of $M_{12}^{\rm SD}$, we obtain from eq. (2.23)

$$\epsilon_K = \kappa_\epsilon\, e^{i\phi_\epsilon}\, \widehat{C}_\epsilon\, |V_{cb}|^2 \lambda^2\, \bar\eta \left\{ |V_{cb}|^2 \big[(1-\bar\rho) - \lambda^2(\bar\rho - \bar\rho^2 - \bar\eta^2) \big] \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_t, x_c) - \eta_{cc} x_c \right\}, \tag{2.36}$$

where we neglected $\mathcal{O}(\lambda^{14})$ terms in the CKM expansion.[5] As is usually done, we replaced $\lambda^4 A^2$ by $|V_{cb}|^2$, which is valid in the SM, as $V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8)$ [5, 6]. The $\mathcal{O}(\lambda^2)$ correction to the leading order result, proportional to $(\bar\rho - \bar\rho^2 - \bar\eta^2)$, is severely suppressed accidentally, because $\bar\rho/(\bar\rho^2 + \bar\eta^2) = \sin^2\alpha - \frac{1}{2}\sin 2\alpha \cot\beta$ (α and β being the standard CKM angles) and α is near 90°.

Below we refer to the expression for ϵ_K in eq. (2.36) as the "usual evaluation". We discuss its central values and error budget together with that of our evaluation of ϵ_K, in section 3.2.

3 Removing η_{cc} from ϵ_K

3.1 Rephasing the evaluation of ϵ_K

With respect to the "standard" phase convention that lead to eq. (2.36), one can rephase the kaon fields as

$$|K^0\rangle \to |K^0\rangle' = e^{i\lambda_c/|\lambda_c|} |K^0\rangle, \qquad |\overline{K}^0\rangle \to |\overline{K}^0\rangle' = e^{-i\lambda_c/|\lambda_c|} |\overline{K}^0\rangle\,, \tag{3.1}$$

which has the effect of multiplying the expression for $M_{12}^{\rm SD}$ in eq. (2.34) by $\lambda_c^2/|\lambda_c|^2$, thus making the η_{cc} contribution purely real.[6] Since $|{\rm Im}(\lambda_c)/{\rm Re}(\lambda_c)| < 10^{-3}$, this rephasing has a negligible impact on the short distance contribution to Δm. However, the impact on ϵ_K is significant, which we study next.

All the results of section 2.2 are still valid, being independent of phase conventions. The results of section 2.3 and eq. (2.23) in particular are valid as well, since despite the $\mathcal{O}(1)$ changes in $\arg M_{12}$ and $\arg \Gamma_{12}$, their orders of magnitude are unchanged. In fact, in every step the phase-dependent errors never exceed a relative amount of $\mathcal{O}(\xi^2)$, and in the new phase convention $\xi' < 10^{-3}$ still holds (see below).

The consequences of the rephasing defined in eq. (3.1) are

$$\mathrm{Im} M_{12} \to \mathrm{Im} M_{12}' = \mathrm{Im} M_{12} \frac{\mathrm{Re}\lambda_c^2}{|\lambda_c^2|} + \mathrm{Re} M_{12} \frac{\mathrm{Im}\lambda_c^2}{|\lambda_c^2|} \simeq \mathrm{Im} M_{12} + 2\lambda^4 A^2 \bar\eta\, \mathrm{Re} M_{12}\,, \tag{3.2}$$

$$\xi \to \xi' = -\frac{1}{2} \frac{\mathrm{Im}(\Gamma_{12}\lambda_c^2)}{\mathrm{Re}(\Gamma_{12}\lambda_c^2)} \simeq -\frac{1}{2}\left(\frac{\mathrm{Im}\Gamma_{12}}{\mathrm{Re}\Gamma_{12}} + \frac{\mathrm{Im}\lambda_c^2}{\mathrm{Re}\lambda_c^2} \right) \simeq \xi - \lambda^4 A^2 \bar\eta\,. \tag{3.3}$$

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[5] We use the expansion of the CKM matrix valid to all orders [5, 6], which implies

$$\lambda_c = -\lambda \left[1 - \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) \right] - i\bar\eta A^2 \lambda^5 \left[1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) \right],$$
$$\lambda_t = -A^2\lambda^5 \left[1 - \bar\rho + \frac{\lambda^2}{2}(1 - 3\bar\rho + 2\bar\rho^2 + 2\bar\eta^2) + \mathcal{O}(\lambda^4) \right] + i\bar\eta A^2 \lambda^5 \left[1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) \right], \tag{2.37}$$

[6] The definition of kaons in terms of quarks introduces two further non-physical arbitrary phases α and $\tilde\alpha$ ($|K^0\rangle = e^{i\alpha}|d\bar s\rangle$, $|\overline{K}^0\rangle = e^{i\tilde\alpha}|\bar d s\rangle$). If they are set to zero, then eq. (3.1) can also be obtained by choosing a CKM matrix convention where $V_{cd}V_{cs}^*$ is real, e.g., $V_{\rm CKM}' = V_{\rm CKM} \times \mathrm{diag}\big(1,\, \lambda_c/|\lambda_c|,\, 1\big)$.

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Both in $\mathrm{Im}M'_{12}$ and in ξ', the uncertainties due to neglected terms are negligible. Thus, the short-distance contribution to M_{12} becomes

$$M_{12}^{\mathrm{SD}\,\prime} = \frac{\Delta m}{\sqrt{2}}\,\widehat{C}_\epsilon\left[\frac{\lambda_t^{*2}\lambda_c^2}{|\lambda_c|^2}\,\eta_{tt}S_0(x_t) + 2\lambda_c\lambda_t^*\,\eta_{ct}S_0(x_t,x_c) + |\lambda_c|^2\,\eta_{cc}x_c\right], \tag{3.4}$$

and the η_{cc} term does not contribute to the imaginary part.

For the long-distance contribution to M_{12}, we can use the same estimate as in ref. [18] to obtain $\rho = 0.6 \pm 0.3$, as reviewed in section 2.4. We then obtain

$$\mathrm{Im}M_{12}^{\mathrm{LD}\,\prime} = -2\big[\xi(1\pm 0.5) - \lambda^4A^2\bar\eta\big]\,\mathrm{Re}M_{12}^{\mathrm{LD}} = -2(\xi' \pm 0.5\,\xi)\,\mathrm{Re}M_{12}^{\mathrm{LD}}\,, \tag{3.5}$$

where in the first equality we used eqs. (2.31) and (3.2), and in the second equality eq. (3.3). For simplicity, we define

$$\kappa'_\epsilon = \sqrt{2}\sin\phi_\epsilon \times \left(1 + \rho'\frac{\xi'}{\sqrt{2}|\epsilon_K|}\right), \tag{3.6}$$

with

$$\rho' = 1 + \frac{1}{\xi'}\frac{\mathrm{Im}(M_{12}^{\mathrm{LD}\prime})}{\Delta m} = 1 - 2\left(1 \pm 0.5\,\frac{\xi}{\xi'}\right)(0.2\pm 0.1)\,, \tag{3.7}$$

where in the second equality we used eqs. (3.5) and (2.32). Numerically, we find

$$\rho' = 0.6 \pm 0.2\,, \tag{3.8}$$

where the uncertainty of ρ' coming from the CKM inputs (contained in ξ') is negligible.

Thus, we finally obtain

$$\epsilon_K = \kappa'_\epsilon\,e^{i\phi_\epsilon}\,\widehat{C}_\epsilon\,|V_{cb}|^2\lambda^2\,\bar\eta\left\{|V_{cb}|^2\big[(1-\bar\rho) - \lambda^2(\bar\rho - \bar\rho^2 - \bar\eta^2)\big]\eta_{tt}S_0(x_t) + \eta_{ct}S_0(x_t,x_c)\right\}, \tag{3.9}$$

to which we refer below as "our evaluation". For convenience, we report our evaluation in a ready-to-use form in eqs. (5.2)–(5.4) in section 5.

3.2 Numerical results and discussion

We collect in table 1 the inputs used from experimental measurements, as well as from perturbative and lattice computations. Concerning CKM parameters, the SM prediction of ϵ_K is obtained using the parameters that result from the full CKM fit. In fact, their best-fit values are practically unaffected by the exclusion of ϵ_K from the fit inputs [30]. If one wants instead to account for possible NP contributions in the CKM fit, and obtain a prediction for ϵ_K that is as independent as possible of such NP, then one should use the values of the CKM parameters that come from a fit to tree-level observables only. In this second approach, the only assumption about NP is that it affects negligibly observables that are dominated by tree-level processes in the SM. We show the values of the CKM parameters in these two cases in table 2.[7] The increased uncertainty in $|V_{cb}|$ and $\bar\eta$, when not determined from the CKM fit, reflects the tension between exclusive and inclusive determinations of $|V_{cb}|$ and $|V_{ub}|$.

[7]CKMfitter [6] performs several fits, using only tree-level observables to determine $\bar\eta$ and $\bar\rho$. Conservatively, we use the one where the only angle measurement included is $\gamma(DK)$, and that combines the measured values of $|V_{ub}|$, for consistency with our treatment of $|V_{cb}|$. CKMfitter plots the fit results, without quoting numerical results. The values in table 2 are read off from the plot, which is sufficient for our purposes, given the large uncertainties.

Parameter	value	source
Δm	$3.484(6) \times 10^{-12}$ MeV	[3]
m_{K^0}	$497.614(24)$ MeV	[3]
$\Delta\Gamma$	$7.3382(33) \times 10^{-12}$ MeV	[3]
$\lvert\epsilon_K\rvert$	$(2.228 \pm 0.011) \times 10^{-3}$	[3]
ϕ_ϵ	$(43.52 \pm 0.05)^\circ$	[3]
$\lvert\epsilon'/\epsilon\rvert$	$(1.66 \pm 0.23) \times 10^{-3}$	[3]
$\lvert A_0/A_2\rvert$	$22.45(6)$	[3, 27]
$\lvert A_0\rvert$	$3.32(2) \times 10^{-7}$ GeV	[3, 27]
η_{cc}	$1.87(76)$	[4]
η_{ct}	$0.496(47)$	[11]
η_{tt}	$0.5765(65)$	[12]
$\overline{m}_t(\overline{m}_t)$	$162.3(2.3)$ GeV	[28]
$\overline{m}_c(\overline{m}_c)$	$1.275(25)$ GeV	[3]
$\widehat{B}_K$	$0.7661(99)$	[29]
f_K	$156.3(0.9)$ MeV	[29]
$\mathrm{Im}\,(A_2 e^{-i\delta_2})$	$-6.99(0.20)(0.84) \times 10^{-13}$ GeV	[20]
$\mathrm{Im}\,(A_0 e^{-i\delta_0})$	$-1.90(1.22)(1.04) \times 10^{-11}$ GeV	[21]

Table 1. Inputs used for the calculation of ϵ_K.

CKM parameters	SM CKM fit [6]	tree-level only	
λ	0.22543 ± 0.00037	0.2253 ± 0.0008	[3]
$\lvert V_{cb}\rvert (= A\lambda^2)$	$(41.80 \pm 0.51) \times 10^{-3}$	$(41.1 \pm 1.3) \times 10^{-3}$	[3]
$\bar\eta$	0.3540 ± 0.0073	0.38 ± 0.04	[6]
$\bar\rho$	0.1504 ± 0.0091	0.115 ± 0.065	[6]

Table 2. The CKM parameters used as inputs. Using the SM CKM fit results assumes that the SM determines all observables. The tree-level inputs are applicable even if TeV-scale new physics affects the loop-mediated processes.

Thus, the usual evaluation eq. (2.36) and our evaluation eq. (3.9) lead to the SM predictions for ϵ_K shown in table 3. When interested in the SM prediction for ϵ_K, we use the more precise value of ξ, determined using the measured value of ϵ'/ϵ as an input (in line with the assumption that the SM accounts for all flavor measurements). In the determination where we allow for NP, instead, we use the lattice value of $\mathrm{Im}(A_0)$ to determine ξ, instead of the measured ϵ'/ϵ. For convenience, we also report in table 3 the values of ξ, κ_ϵ and ξ', κ'_ϵ in our evaluation that correspond to these choices. Finally, the various sources of uncertainties in ϵ_K and their relative impacts are shown in table 4. The total error of ϵ_K is obtained by adding all contributions in quadrature.

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	CKM inputs	$\lvert\epsilon_K\rvert \times 10^3$	$\kappa_\epsilon^{(\prime)}$	$\xi^{(\prime)} \times 10^4$
Usual evaluation	tree-level	2.30 ± 0.42	0.963 ± 0.010	-0.57 ± 0.48
	SM CKM fit	2.16 ± 0.22	0.943 ± 0.016	-1.65 ± 0.17
Our evaluation	tree-level	2.38 ± 0.37	0.844 ± 0.044	-6.99 ± 0.92
	SM CKM fit	2.24 ± 0.19	0.829 ± 0.049	-7.83 ± 0.26

Table 3. Present value of ϵ_K in the usual evaluation (upper part) and in our evaluation (lower part). For convenience, we also show the values of the quantities κ_ϵ and ξ defined in eqs. (2.24) and (2.21) in the upper part, and κ'_ϵ and ξ' defined in eqs. (3.6) and (3.3) in the lower part.

	CKM inputs	η_{cc}	η_{ct}	$\kappa_\epsilon^{(\prime)}$	m_t	m_c	$f_K^2 \widehat{B}_K$	$\lvert V_{cb}\rvert$	$\bar\eta$	$\bar\rho$	$\lvert\Delta\epsilon_K/\epsilon_K\rvert_{\text{tot.}}$
Usual evaluation	tree-level	7.3%	4.0%	1.1%	1.7%	0.8%	1.7%	11.1%	10.4%	5.4%	18.4%
	SM CKM fit	7.4%	4.0%	1.7%	1.7%	0.8%	1.7%	4.2%	2.0%	0.8%	10.2%
Our evaluation	tree-level	—	3.4%	5.2%	1.5%	1.2%	1.7%	9.5%	8.9%	4.5%	15.6%
	SM CKM fit	—	3.4%	5.9%	1.5%	1.3%	1.7%	3.6%	1.7%	0.7%	8.4%

Table 4. The present error budget of ϵ_K in the usual evaluation (upper part) and using our evaluation (lower part). The parameters with a corresponding uncertainty above 1% are shown.

As expected, the central values of ϵ_K in table 3 vary according to the strategy used to compute ϵ_K (our vs. usual evaluation, and SM CKM fit vs. tree-level inputs). The central values are actually all within 1σ of each other, and of the experimental central value $\lvert\epsilon_K\rvert^{(\text{exp})} = 2.228 \times 10^{-3}$. Note that the latest determination of V_{cb} from $B \to D\ell\bar\nu$, $\lvert V_{cb}\rvert = 40.8(1.0) \times 10^{-3}$ [31], reduces the tension with its inclusive determination (however, that from $B \to D^*\ell\bar\nu$ remains lower; see, e.g., ref. [32] for more discussions). Table 3 also shows that in our evaluation the uncertainty in the long distance contribution to ϵ_K (i.e., $\kappa_\epsilon^{(\prime)} \neq 1$) is relatively more important than in the usual evaluation. In the latter case, the η_{cc} term contributes to ϵ_K with a negative sign, and its removal in our evaluation is compensated by an increase in the imaginary part of the long-distance contribution. Table 4 makes the usefulness of our evaluation of ϵ_K manifest:

- ◇ Given state-of-the-art inputs, our evaluation eq. (3.9) slightly reduces the relative uncertainties of ϵ_K with respect to the usual one in eq. (2.36);
- ◇ The gain in relative uncertainty from the removal of η_{cc} is partially compensated by an increase in the uncertainty from κ_ϵ, which is dominated by the uncertainty of the long-distance contribution $\text{Im}(M_{12}^{\text{LD}})$. (See sections 2.4 and 3.1 for its estimate, in the usual and in our evaluation respectively.)

These observations highlight the importance of achieving a better theoretical control of the long-distance contribution to M_{12}. While some progress could already be attained with tools like χPT, a significant step forward probably requires an effort from lattice QCD (recent attempts in this direction have appeared in refs. [26, 33, 34]). The importance of such an effort is even greater considering future prospects for the ϵ_K uncertainty, which, with the removal of η_{cc}, is dominated by the CKM parameters. Within the next decade it

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should be possible to measure $|V_{cb}|$ with an uncertainty of about 0.3×10^{-3} [1, 35], to be compared with 1.3×10^{-3} in table 2. This would correspond to a reduced contribution to the ϵ_K error budget,

$$\left|\frac{\Delta\epsilon_K}{\epsilon_K}\right|_{\Delta|V_{cb}|=0.3\times10^{-3}} = 2.2\%\,, \tag{3.10}$$

in our evaluation of ϵ_K (2.6% in the usual one). Similarly, tree-dominated measurements will determine γ and $|V_{ub}|$ with much better precision [1, 2, 35], which will translate to an uncertainty of ϵ_K due to CKM elements comparable to the current SM CKM fit in table 4. Finally, different lattice QCD calculations of $\widehat{B}_K$ obtain different results for its uncertainty [36–38], which, however, do not exceed the 2–3% percent level and are thus subdominant in the error budget of ϵ_K. (A more acute tension is present for the bag parameters of non-SM operators, see section 4.)

3.3 Further comments on the rephasing

We collect here some remarks that are not strictly necessary to the previous discussion, but that might help to make it clearer.

- Looking at table 3, it may appear counterintuitive that larger ξ uncertainties correspond to more precise values of κ_ϵ. That is the case because, when the ξ uncertainty is larger, the ξ central value is accidentally smaller. The larger impact on the κ_ϵ uncertainty comes from ρ, which multiplies ξ, and so its central value also impacts the error budget.

- The rephasing of kaon and quark fields is independent of the freedom to remove the charm or up (or top) contribution, via unitarity of the CKM matrix. The standard choice is to eliminate the u-quark contribution, $\lambda_u = -\lambda_t - \lambda_c$, which we also followed. The possibility to use CKM unitarity to remove λ_c, instead of λ_u, has been emphasized in ref. [33] (see appendix A of that paper). With that choice, $M_{12}^{\rm SD}$ contains terms proportional to λ_t^{*2}, λ_u^{*2} and $\lambda_t^*\lambda_u^*$, and the second one will not contribute to ϵ_K, since λ_u is real in the standard phase convention.

  However, the expression for ϵ_K obtained using $\lambda_c = -\lambda_t - \lambda_u$ cannot yet be used to make precise predictions, since the coefficients analogous to η_{tt} and η_{ct} have not been computed. Ref. [33] argued that they would not have large uncertainties, and that the related lattice calculations would become more accurate, due to the suppression of the perturbative contribution for momenta smaller than m_c. While this could justify pursuing that path, using $\lambda_c = -\lambda_t - \lambda_u$ renders the top contribution sensitive to the m_c scale, which is generically associated with larger uncertainties. Our evaluation relies instead on well established results, and allows immediate quantitative predictions.

- One may wonder if a rephasing other than that in eq. (3.1) could reduce the ϵ_K uncertainty even further. Instead of eq. (3.1), an optimal choice might reduce but not eliminate the η_{cc} contribution to ${\rm Im}M_{12}^{\rm SD}$, and the combined uncertainty due to

η_{cc} and κ_ϵ may decrease. To explore this, let us define the general rephasing

$$|K^0\rangle \to |K^0\rangle' = e^{i\,a\,\lambda_c/|\lambda_c|}|K^0\rangle, \qquad |\overline{K}^0\rangle \to |\overline{K}^0\rangle' = e^{-i\,a\,\lambda_c/|\lambda_c|}|\overline{K}^0\rangle\,, \tag{3.11}$$

where the usual evaluation corresponds to $a = 0$, and our evaluation to $a = 1$. We can choose a to minimize the total uncertainty of ϵ_K. We find that the optimal values are $a \approx 1.0$ and $a \approx 0.7$ for the cases of tree-level and SM CKM fit inputs, respectively. The resulting total uncertainties for the latter case is $|\Delta\epsilon_K/\epsilon_K|_{\text{total}} = 7.9\%$, to be compared with 8.4% of the case $a = 1$ in table 4. The corresponding central ϵ_K value is 2.23×10^{-3}.

4 Constraints on new physics

If a pattern of deviations from the SM is given, like in a specific model of flavor, then the correct strategy to study flavor and CP constraints would be to perform a fit to the SM + NP parameters (see, e.g., ref. [35]). Here we would like to derive consequences for NP that are of a more general validity, and do not need the specification of a model. Therefore, we take an effective field theory (EFT) approach, and comment on explicit NP models at the end of this section. We parametrize the NP contribution to K^0 mixing in terms of dimension-six operators, suppressed by a mass scale squared, Λ^2. The operator basis we consider consists of O_1, defined before eq. (2.19), and

$$O_2 = (\bar d_R s_L)^2, \quad O_3 = (\bar d_R^\alpha s_L^\beta)(\bar d_R^\beta s_L^\alpha)\,, \quad O_4 = (\bar d_R s_L)(\bar d_L s_R)\,, \quad O_5 = (\bar d_R^\alpha s_L^\beta)(\bar d_L^\beta s_R^\alpha)\,, \tag{4.1}$$

where α, β are color indices, that are implicit when their contraction is between Lorentz-contracted fields. The observable most sensitive to $O_{1,\ldots,5}$ is ϵ_K, so our procedure is consistent (Δm, also sensitive to NP in K^0 mixing, suffers from larger long-distance and η_{cc} uncertainties).

To derive bounds on the operators in eq. (4.1), we need both their matrix elements between two kaon states at a certain low scale μ, and the running of their Wilson coefficients from Λ down to that scale. The matrix elements are defined in terms of the bag parameters, with $B_1 = B_K$ of eq. (2.19), and

$$\langle K^0|O_i(\mu)|\overline{K}^0\rangle = \frac{a_i}{4}\, B_i(\mu)\, \frac{m_K^4 f_K^2}{[m_s(\mu) + m_d(\mu)]^2}\,, \qquad i = 2,\ldots,5\,, \tag{4.2}$$

with $a_i = \{-5/3,\ 1/3,\ 2,\ 2/3\}$. Recent calculations obtained partly consistent results [37, 39–41], while a 30–40% tension between calculations of B_4 and B_5 remains (as it was already the case nearly a decade ago [42, 43]). For definiteness, we use here the values obtained in ref. [37] (in the $\overline{\text{MS}}$ scheme), shown in table 5, together with the quark masses used.

We assume that only one operator deviates from the SM at the high scale Λ, with a purely imaginary coefficient. We run it down to the scale $\mu = 3\,\text{GeV}$, at which the matrix elements are given. Because of the large uncertainties of the bag parameters B_i, we use the LO running and mixing of the Wilson coefficients of $O_{1,\ldots,5}$ [44, 45] (see refs. [46, 47] for a consistent treatment of the Wilson coefficients together with the bag parameters at NLO).

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Quark masses (at 3 GeV)		Bag parameters (at 3 GeV)				
$\overline{m}_s$	$\overline{m}_d$	B_1	B_2	B_3	B_4	B_5
86.5 MeV	4.4 MeV	0.506	0.46	0.79	0.78	0.49

Table 5. Inputs used for setting bounds on NP from ϵ_K. Both the bag parameters [37] and the quark masses are in the $\overline{\text{MS}}$ scheme; the latter are obtained by NLO running from the values at 2 GeV given in ref. [3].

$\text{Im}(C_i)\,\frac{(3\ \text{TeV})^2}{\Lambda^2} < X$	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$
tree-level $X =$	2.4×10^{-8}	3.3×10^{-10}	1.2×10^{-9}	7.5×10^{-11}	2.4×10^{-10}
SM CKM fit $X =$	1.2×10^{-8}	1.7×10^{-10}	6.2×10^{-10}	3.9×10^{-11}	1.2×10^{-10}

Table 6. Upper bounds from ϵ_K on the imaginary parts of the Wilson coefficients of the operators $\mathcal{O}_{1,\dots,5}$, run down to 3 GeV from a scale of 3 TeV. For each operator we give the bound both from the tree-level CKM inputs and from the SM CKM inputs.

We then express the constraints from ϵ_K as lower bounds on Λ, requiring the NP contribution to the experimental measured value of ϵ_K to be less than twice the theoretical uncertainties in table 4, i.e., 31% for tree-level inputs and 16% for SM CKM fit inputs (keeping in mind the last point of section 3.3). We ignore the differences between the experimental central value of ϵ_K and the theoretical predictions, because it is small and depends anyway on the CKM parameters resulting from a specific fit, and because this way the constraint on NP is independent of its sign.

The results are shown in figure 1, both for the SM CKM fit and for tree-level inputs, as darker (right) and lighter (left) histograms, respectively. From the point of view of NP, the former case assumes ϵ_K to be the most sensitive observable to flavor violation, and the second one is more conservative and only requires NP not to substantially affect processes that are tree-level in the SM. The operator most constrained by ϵ_K is O_4, which probes scales near 10^6 TeV.

In addition we show, in table 6, the resulting bounds on the imaginary part of the Wilson coefficients C_i of the operators $\mathcal{O}_i$, for a fixed scale $\Lambda = 3$ TeV. That is useful for the reader interested in models with new degrees of freedom not too far from the TeV scale. In fact, the running from the scales shown in figure 1 down to 3 TeV is a sizable effect, which yields differences of order 50% or larger in the constraints on the Wilson coefficients. The same differences are, instead, below the 10% level if the running is performed from 3 TeV to, say, 1 or 10 TeV.

We end this section with comments concerning the sensitivity of ϵ_K to explicit and widely studied NP flavor models:

- ◇ Composite Higgs models with partial compositeness (see, e.g., [48]) constitute a case where ϵ_K is the most sensitive observable to flavor and CP violation [49, 50], unless a flavor symmetry is imposed on the strong sector [50–53]. Then it is reasonable to derive bounds from ϵ_K using inputs from a CKM fit that assumes the SM, and corre-

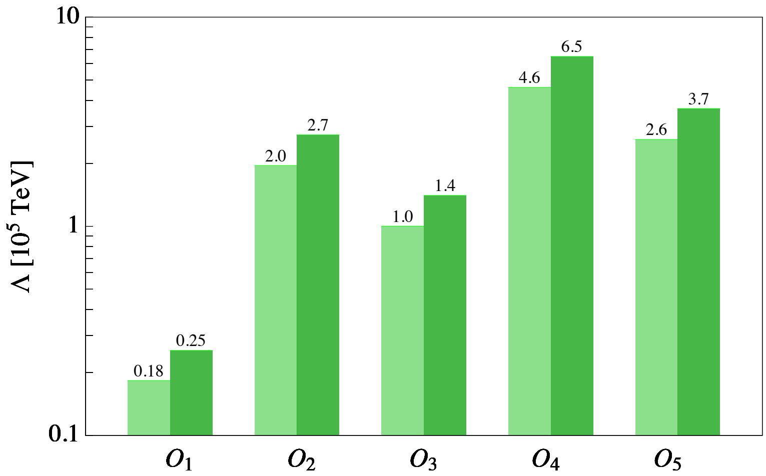

Figure 1. Lower bounds from ϵ_K on the new physics scale Λ suppressing each of the operators $O_{1,\ldots,5}$, in units of 10^5 TeV. For each operator we give on the left (lighter green) the bound from tree-level CKM inputs, on the right (darker green) the bound from SM CKM inputs.

sponds to the $\sim 8\%$ theory error in table 4. This procedure implies for example that, in the language of ref. [50] and with an anarchic flavor structure in the strong sector, ϵ_K constrains composite fermion resonances to have masses larger than $\sim 30\,\text{TeV}$.

◇ Other motivated cases are models realizing a "CKM-like" pattern of flavor and CP violation, with SM-like suppressions for the operators present in the SM, and vanishing $O_{2,\ldots,5}$. As argued in ref. [54], they consist either in $\mathrm{U}(3)^3$ [55–57], or in $\mathrm{U}(2)^3$ [52, 58] models, all the other symmetries being equivalent to them. In these models there is not a clear hierarchy between observables in sensitivity to NP. The correct procedure to analyze the impact of flavor and CP violation is, therefore, to perform a fit to the SM+NP flavor parameters, using the theoretical prediction of ϵ_K (see eqs. (5.2)–(5.4) for ready-to-use expressions).

◇ More specifically, while in general the scales probed by ϵ_K are higher than those probed by ϵ'/ϵ, in CKM-like models an EFT analysis shows [59] that ϵ'/ϵ is more sensitive to NP than ϵ_K. However, in concrete realizations it is not difficult to reverse this conclusion, for example in supersymmetry with the first two generations heavier than the third one [59].

5 Conclusions and outlook

Without any clear deviation from the CKM picture of flavor and CP violation, it is hard, if not impossible, to shed light on a more fundamental theory of flavor. Among all observables, ϵ_K probes some of the highest energies, and puts some of the most severe constraints on explicit flavor models. It is therefore important to improve its SM prediction, which has a much larger uncertainty than its experimental determination.

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The theory uncertainty of ϵ_K depends on the uncertainty of CKM parameters, most notably on that of A (or equivalently $|V_{cb}|$). The largest non-parametric uncertainty until now has been due to the perturbative QCD correction to the box diagram with two charm quarks, η_{cc}. We showed that the dependence of ϵ_K on η_{cc} can be removed via a rephasing of the kaon fields, which makes this contribution to M_{12} purely real. In other words, in our phase convention, the contribution to ϵ_K from dimension-six operators always contains the top mass scale. The resulting uncertainty of the SM prediction of ϵ_K is slightly reduced and, perhaps more importantly, the largest source of non-parametric error now comes from the long distance contribution to M_{12}. Thus, our formulation highlights the importance to achieve a better theoretical control of the latter, possibly using lattice QCD. The case is further strengthened by the precision with which the CKM inputs are expected to be measured at Belle II and LHCb.

In section 2, we reviewed the derivation of the SM prediction for ϵ_K, explicitly exhibiting the phase convention dependences and the approximations used. Our evaluation is presented in section 3, together with its numerical consequences for the central values and uncertainties of ϵ_K summarized in table 3. The detailed error budget of ϵ_K, in our evaluation, is compared with the conventional one in table 4.

Finally, we provided updated constraints on new physics contributions to ϵ_K in section 4, taking full advantage of the rephasing freedom. We also discussed how they apply to CKM-like models, and to composite Higgs models with an anarchic flavor structure. The constraints in figure 1 and table 6 provide a well-defined quantification of the ϵ_K sensitivity to NP, and are obtained from imposing

$$|\epsilon_K|^{(\mathrm{NP})} < \begin{cases} 0.31\,|\epsilon_K|^{(\mathrm{exp})} & \text{(tree-level inputs)}\,, \\ 0.16\,|\epsilon_K|^{(\mathrm{exp})} & \text{(SM CKM fit inputs)}\,, \end{cases} \tag{5.1}$$

as discussed in section 4.

Such an analysis ignores the pattern and correlations typical of specific NP realizations. For the convenience of the reader interested in such an analysis, that needs the CKM parameters coming from its own SM + NP fit, we report here our ready-to-use expression for ϵ_K without η_{cc},

$$\epsilon_K = \kappa'_\epsilon\, e^{i\phi_\epsilon}\, \widehat{C}_\epsilon\, |V_{cb}|^2 \lambda^2\, \bar\eta\, \Big\{ |V_{cb}|^2 \big[(1-\bar\rho) - \lambda^2(\bar\rho - \bar\rho^2 - \bar\eta^2)\big]\eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_t, x_c) \Big\}, \tag{5.2}$$

where κ'_ϵ is given using either the measured ϵ'/ϵ value as an input or using only SM lattice inputs by

$$\kappa'_\epsilon = \begin{cases} 0.834 - 0.11\Delta \pm (0.047 + 0.036\Delta)\,, & (\epsilon'/\epsilon \text{ and lattice } \mathrm{Im}(A_2) \text{ input})\,, \\ 0.854 - 0.11\Delta \pm (0.041 + 0.035\Delta)\,, & (\text{lattice } \mathrm{Im}(A_0) \text{ input})\,, \end{cases} \tag{5.3}$$

and

$$\Delta = \frac{\bar\eta}{0.35} \left(\frac{|V_{cb}|}{41 \times 10^{-3}} \right)^2 - 1\,. \tag{5.4}$$

Equations (5.2) and (5.3), and the inputs in table 1 (which imply $\widehat{C}_\epsilon = (2.806 \pm 0.049) \times 10^4$), allow making predictions for ϵ_K for the preferred values of CKM parameters λ, $|V_{cb}| = A\lambda^2$, $\bar\eta$, and $\bar\rho$.

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