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Twisted compactification of $N = 2$ 5D SCFTs to three and two dimensions from $F(4)$ gauged supergravity

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Abstract: We study supersymmetric $AdS_4 \times \Sigma_2$ and $AdS_3 \times \Sigma_3$ solutions in half-maximal gauged supergravity in six dimensions with $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ gauge group. The gauged supergravity is obtained by coupling three vector multiplets to the pure $F(4)$ gauged supergravity. The $\mathrm{SU}(2)_R$ R-symmetry together with the $\mathrm{SO}(3) \sim \mathrm{SU}(2)$ symmetry of the vector multiplets are gauged. The resulting gauged supergravity admits supersymmetric AdS_6 critical points with $\mathrm{SO}(4) \sim \mathrm{SU}(2) \times \mathrm{SU}(2)$ and $\mathrm{SO}(3) \sim \mathrm{SU}(2)_{\mathrm{diag}}$ symmetries. The former corresponds to five-dimensional $N = 2$ superconformal field theories (SCFTs) with $E_1 \sim \mathrm{SU}(2)$ symmetry. We find new classes of supersymmetric $AdS_4 \times \Sigma_2$ and $AdS_3 \times \Sigma_3$ solutions with $\Sigma_{2,3}$ being $S^{2,3}$ and $H^{2,3}$. These solutions describe SCFTs in three and two dimensions obtained from twisted compactifications of the aforementioned five-dimensional SCFTs with different numbers of unbroken supersymmetry and various types of global symmetries.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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JHEP09(2015)034

Contents

JHEP09(2015)034

1 Introduction

Field theories in six and five dimensions have been shown to posses non-trivial conformal fixed points [1, 2]. However, higher dimensional superconformal field theories (SCFTs) are not well understood as their lower dimensional analogues. The study of five-dimensional SCFTs using the AdS/CFT correspondence [3] has attracted a lot of attention both from ten and six-dimensional point of views, see for example [4–9]. And recently, the investigation of supersymmetric AdS_6 solutions has been carried out systematically in [10–12].

An approach to understand higher dimensional field theories is to make some compactification of these theories to lower dimensions. The resulting lower dimensional field theories preserving some supersymmetry are usually obtained by twisted compactifications, and the holographic study via the AdS/CFT correspondence is still applicable at least in the large N limit [13]. From string/M theory point of view, these twisted field theories can be interpreted as wrapped branes on certain curved manifolds. In many cases, there is a description in terms of lower dimensional gauged supergravities. In particular, for the present case of five-dimensional SCFTs, the effective supergravity theory is the $N = (1,1)$ $F(4)$ gauged supergravity and its matter-coupled version [14].

In this work, we will explore some aspects of twisted compactifications of five-dimensional SCFTs within the framework of half-maximal gauged supergravity in six dimensions coupled to matter multiplets [15, 16]. A similar study in the pure $F(4)$ gauged supergravity [17] have been carried out in [18] in which some $AdS_4 \times \Sigma_2$ and $AdS_3 \times \Sigma_3$

solutions have been identified along with their possible dual field theories. We will further investigate solutions of this type in the matter-coupled $F(4)$ gauged supergravity. This could presumably give rise to more general solutions than those given in [18]. The result would also provide new solutions describing IR fixed points of the RG flows from SCFTs in five dimensions to three and two-dimensional SCFTs with different numbers of supersymmetry.

As a starting point, we add three vector multiplets to the $F(4)$ gauged supergravity resulting in an $\mathrm{SU}(2)_R \times \mathrm{SU}(2) \sim \mathrm{SO}(3)_R \times \mathrm{SO}(3)$ gauge group with the first factor being the R-symmetry group. AdS_6 vacua of this theory including possible holographic RG flows between the dual SCFTs and RG flows to non-conformal field theories have already been studied in [8] and [9]. From the result in [8], there are two supersymmetric AdS_6 critial points. Both of them preserve the full sixteen supercharges, but one of them, with non-vanishing scalar fields, break the full $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ symmetry to its diagonal subgruop. These two critical points are dual to certain $N = 2$ SCFTs in five dimensions by the usual AdS/CFT correspondence.

We then proceed by looking for possible $AdS_4 \times \Sigma_2$ and $AdS_3 \times \Sigma_3$ solutions for $\Sigma_{2,3}$ being $S^{2,3}$ or $H^{2,3}$ with different residual symmetries. The resulting solutions would be dual to SCFTs in three and two dimensions obtained from twisted compactifications of the above mentioned five-dimensional SCFTs. These will give new AdS_4 and AdS_3 solutions from six-dimensional gauged supergravity and provide appropriate gravity backgrounds in the holographic study of gauge theories in five and lower dimensions.

The paper is organized as follow. We give a brief review of the $F(4)$ gauged supergravity coupled to three vector multiplets in section 2. Possible supersymmetric AdS_4 and AdS_3 solutions are given in section 3 and 4, respectively. In section 5, we give some conclusions and comments about the results. We also include an appendix describing supersymmetric AdS_6 critical points previously found in [8] as well as an analytic RG flow between them.

2 Matter coupled $N = (1, 1)$ SU(2) $\times$ SU(2) gauged supergravity in six dimensions

In this paper, we are interested in $N = (1, 1)$ gauged supergravity with $\mathrm{SU}(2) \times \mathrm{SU}(2)$ gauge group. This gauged supergravity can be obtained by coupling three vector multiplets to the pure $F(4)$ gauged supergravity constructed in [17]. The full construction by using the superspace approach can be found in [15, 16]. Apart from different metric signature $(-++++)$, we will mostly follow the notations and conventions given in [15] and [16].

The matter coupled $N = (1, 1)$ gauged supersymmetry consists of the supergravity multiplet given by

$$\left(e_\mu^a, \psi_\mu^A, A_\mu^\alpha, B_{\mu\nu}, \chi^A, \sigma\right)$$

and three vector multiplets with the field content

$$(A_\mu, \lambda_A, \phi^\alpha)^I .$$

In the above expressions, ψ_μ^A, χ^A and λ_A denote the gravitini, the spin-$\frac{1}{2}$ fields and the gauginos, respectively. All spinor fields χ^A, ψ_μ^A and λ_A as well as the supersymmetry pa-

JHEP09(2015)034

rameter ϵ^A are eight-component pseudo-Majorana spinors with indices $A, B = 1, 2$ referring to the fundamental representation of the $\mathrm{SU}(2)_R \sim USp(2)_R$ R-symmetry. Space-time and tangent space indices are denoted respectively by $\mu, \nu = 0, \ldots, 5$ and $a, b = 0, \ldots, 5$. e^a_μ and σ are the graviton and the dilaton. A^α_μ, $\alpha = 0, 1, 2, 3$, are four vector fields in the gravity multiplet. Three of these vector fields will be used to gauge the $\mathrm{SU}(2)_R$ R-symmetry. The index $I = 1, 2, 3$ labels the three vector multiplets, and finally $B_{\mu\nu}$ is the two-form field which admits a mass term.

There are 13 scalar fields parametrized by $\mathbb{R}^+ \times \mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset manifold in which the $\mathbb{R}^+ \sim \mathrm{SO}(1,1)$ part corresponds to the dilaton. Possible gauge groups are subgroups of the global symmetry group $\mathbb{R}^+ \times \mathrm{SO}(4,3)$. In the present paper, we will consider only the compact gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2) \sim \mathrm{SO}(3) \times \mathrm{SO}(3)$. The first factor is the $\mathrm{SU}(2)_R$ R-symmetry identified with the diagonal subgroup of $\mathrm{SU}(2) \times \mathrm{SU}(2) \sim \mathrm{SO}(4) \subset \mathrm{SO}(4) \times \mathrm{SO}(3)$. Following [15] and [16], we will decompose the α index into $\alpha = (0, r)$ in which $r = 1, 2, 3$. Indices r, s will become adjoint indices of the $\mathrm{SU}(2)_R$ R-symmetry.

The 12 vector multiplet scalars given by the $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset can be parametrized by the coset representative $L^\Lambda{}_\Sigma$, $\Lambda, \Sigma = 0, \ldots, 6$. We can split the index Σ, transforming by right multiplications of the local $\mathrm{SO}(4) \times \mathrm{SO}(3)$ composite symmetry, in $L^\Lambda{}_\Sigma$ to $(L^\Lambda{}_\alpha, L^\Lambda{}_I)$ and further to $(L^\Lambda{}_0, L^\Lambda{}_r, L^\Lambda{}_I)$. The vielbein of the $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset $P^I{}_\alpha$ and the $\mathrm{SO}(4) \times \mathrm{SO}(3)$ composite connections $\Omega^{\alpha\beta} = (\Omega^{rs}, \Omega^{r0})$ can be obtained from the left-invariant 1-form of $\mathrm{SO}(4,3)$

$$\Omega^\Lambda{}_\Sigma = (L^{-1})^\Lambda{}_\Pi \nabla L^\Pi{}_\Sigma, \qquad \nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma, \tag{2.1}$$

with the following identification

$$P^I{}_\alpha = (P^I{}_0, P^I{}_r) = (\Omega^I{}_0, \Omega^I{}_r). \tag{2.2}$$

The structure constants of the full $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ gauge group $f^\Lambda{}_{\Pi\Sigma}$ will be split into ϵ_{rst} and $C_{IJK} = \epsilon_{IJK}$ for the two factors $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)$, respectively. There are accordingly two coupling constants denoted by g_1 and g_2.

In order to parametrize scalar fields described by the $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset, we introduce basis elements of 7×7 matrices by

$$(e^{\Lambda\Sigma})_{\Gamma\Pi} = \delta_{\Lambda\Gamma}\delta_{\Sigma\Pi}, \qquad \Lambda, \Sigma, \Gamma, \Pi = 0, \ldots, 6\,. \tag{2.3}$$

The $\mathrm{SO}(4)$, $\mathrm{SU}(2)_R$, $\mathrm{SU}(2)$ and non-compact generators $Y_{\alpha I}$ of $\mathrm{SO}(4,3)$ are then given by

$$\begin{aligned}
\mathrm{SO}(4): \quad & J^{\alpha\beta} = e^{\beta,\alpha} - e^{\alpha,\beta}, \qquad \alpha, \beta = 0, 1, 2, 3, \\
\mathrm{SU}(2)_R: \quad & J_1^{rs} = e^{s,r} - e^{r,s}, \qquad r, s = 1, 2, 3, \\
\mathrm{SU}(2): \quad & J_2^{IJ} = e^{J+3,I+3} - e^{I+3,J+3}, \qquad I, J = 1, 2, 3, \\
& Y_{\alpha I} = e^{\alpha,I+3} + e^{I+3,\alpha}\,.
\end{aligned} \tag{2.4}$$

In this paper, we are not interested in solutions with non-zero two-form field. We therefore set $B_{\mu\nu} = 0$ from now on. The bosonic Lagrangian involving only the metric, vectors and scalar fields is given by [16]

$$\mathcal{L} = \frac{1}{4}eR - e\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}eP_{I\alpha\mu}P^{I\alpha\mu} - \frac{1}{8}ee^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu} - eV \tag{2.5}$$

JHEP09(2015)034

where $e = \sqrt{-g}$. We have written the scalar kinetic term in term of $P_\mu^{I\alpha} = P_i^{I\alpha}\partial_\mu\phi^i$, $i = 1, \ldots, 12$. The explicit form of the scalar potential is given by

$$\begin{aligned} V = & -e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^iB_i - \frac{1}{4}\left(C^I{}_tC_{It} + 4D^I{}_tD_{It}\right)\right] + m^2e^{-6\sigma}\mathcal{N}_{00} \\ & -me^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^iL_{0i}\right] \end{aligned} \tag{2.6}$$

where $\mathcal{N}_{00}$ is the 00 component of the scalar matrix $\mathcal{N}_{\Lambda\Sigma}$ defined by

$$\mathcal{N}_{\Lambda\Sigma} = L_\Lambda{}^0(L^{-1})_{0\Sigma} + L_\Lambda{}^i(L^{-1})_{i\Sigma} - L_\Lambda{}^I(L^{-1})_{I\Sigma}\,. \tag{2.7}$$

Various quantities appearing in the scalar potential and the supersymmetry transformations given below are defined as follow

$$A = \epsilon^{rst}K_{rst}, \qquad B^i = \epsilon^{ijk}K_{jk0}, \tag{2.8}$$

$$C_I{}^t = \epsilon^{trs}K_{rIs}, \qquad D_{It} = K_{0It} \tag{2.9}$$

where

$$\begin{aligned} K_{rst} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n{}_t + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_t, \\ K_{rs0} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n{}_0 + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_0, \\ K_{rIt} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^mL^n{}_t + g_2C_{IJK}L^I{}_r(L^{-1})_I{}^JL^K_t, \\ K_{0It} &= g_1\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^mL^n{}_t + g_2C_{IJK}L^I{}_0(L^{-1})_I{}^JL^K_t\,. \end{aligned} \tag{2.10}$$

Finally, we need supersymmetry transformations of χ^A, λ^I_A and ψ^A_μ to find supersymmetric bosonic solutions. These transformation rules with vanishing $B_{\mu\nu}$ field are given by

$$\begin{aligned} \delta\psi_{\mu A} = & D_\mu\epsilon_A - \frac{1}{24}\left(Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}\right)\epsilon_{AB}\gamma_\mu\epsilon^B \\ & -\frac{1}{8}\left(B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{t0}\right)\gamma^7\sigma^t_{AB}\gamma_\mu\epsilon^B \\ & +\frac{i}{16}e^{-\sigma}\left[\epsilon_{AB}(L^{-1})_{0\Lambda}\gamma_7 + \sigma^r_{AB}(L^{-1})_{r\Lambda}\right]F^\Lambda_{\nu\lambda}(\gamma_\mu{}^{\nu\lambda} - 6\delta^\nu_\mu\gamma^\lambda)\epsilon^B, \end{aligned} \tag{2.11}$$

$$\begin{aligned} \delta\chi_A = & \frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon_{AB}\epsilon^B + \frac{1}{24}\left[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB}\epsilon^B \\ & -\frac{1}{8}\left[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}\right]\gamma^7\sigma^t_{AB}\epsilon^B \\ & +\frac{i}{16}e^{-\sigma}\left[\epsilon_{AB}(L^{-1})_{0\Lambda}\gamma_7 - \sigma^r_{AB}(L^{-1})_{r\Lambda}\right]F^\Lambda_{\mu\nu}\gamma^{\mu\nu}\epsilon^B, \end{aligned} \tag{2.12}$$

$$\begin{aligned} \delta\lambda^I_A = & -P^I_{ri}\gamma^\mu\partial_\mu\phi^i\sigma^r{}_{AB}\epsilon^B - P^I_{0i}\gamma^7\gamma^\mu\partial_\mu\phi^i\epsilon_{AB}\epsilon^B - \left(2i\gamma^7D^I{}_t + C^I{}_t\right)e^\sigma\sigma^t_{AB}\epsilon^B \\ & +2me^{-3\sigma}(L^{-1})^I{}_0\gamma^7\epsilon_{AB}\epsilon^B - \frac{i}{2}e^{-\sigma}(L^{-1})^I{}_\Lambda F^\Lambda_{\mu\nu}\gamma^{\mu\nu}\epsilon_A \end{aligned} \tag{2.13}$$

where $\sigma^{tC}{}_B$ are usual Pauli matrices, and $\epsilon_{AB} = -\epsilon_{BA}$. In our convention, the space-time gamma matrices γ^a satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \eta^{ab} = \mathrm{diag}(-1, 1, 1, 1, 1, 1), \tag{2.14}$$

JHEP09(2015)034

and $\gamma^7 = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$ with $\gamma_7^2 = \mathbf{1}$. The covariant derivative of ϵ_A is given by

$$D_\mu \epsilon_A = \partial_\mu \epsilon_A + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{i}{2}\sigma_{rAB}\left[\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right]\epsilon^B\,. \tag{2.15}$$

It should be noted that due to some difference in conventions, the above supersymmetry transformations do not coincide with those of the pure $F(4)$ gauged supergravity given in [17] when all of the fields in the vector multiplets are set to zero. However, it can be verified that the transformation rules in [17] are recovered by using the identifications

$$\gamma^\mu \to \gamma_7\gamma^\mu \qquad \text{and} \qquad \chi_A \to \gamma_7\chi_A\,. \tag{2.16}$$

The $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ gauged supergravity admits maximally supersymmetric AdS_6 critical points when $m \neq 0$. One of them is the trivial critical point at which all scalars vanish after setting $g_1 = 3m$. This critical point preserves the full $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ symmetry and should be dual to the five-dimensional SCFT with global symmetry $E_1 \sim \mathrm{SU}(2)$. Furthermore, at the vacuum, the U(1) gauge field A^0 will be eaten by the two-form field resulting in a massive $B_{\mu\nu}$ field. Another supersymmetric AdS_6 critical point preserves only the diagonal subgroup $\mathrm{SU}(2)_{\mathrm{diag}} \subset \mathrm{SU}(2)_R \times \mathrm{SU}(2)$. This critical point has been mistakenly identified as a stable non-supersymmetric AdS_6 in [8], see also the associated erratum.

Actually, the non-trivial supersymmetric critical point can also be seen from the BPS equations studied in [9], but that paper mainly considers RG flows from five-dimensional SCFTs corresponding to the trivial AdS_6 critical point to non-conformal field theories in the IR. We give the analysis of these two supersymmetric AdS_6 critical points in the appendix together with an analytic RG flow between them. This flow solution have already been studied numerically in [8]. The critical points and the flow solution are similar to the corresponding solutions in the half-maximal gauged supergravity with SO(4) gauge group in seven dimensions studied in [19].

3 AdS_4 critical points

In this section, we consider solutions of the form $AdS_4 \times S^2$ or $AdS_4 \times H^2$ with S^2 and H^2 being a two-sphere and a two-dimensional hyperbolic space, respectively. The metric takes the form of

$$ds^2 = e^{2F}dx_{1,3}^2 + e^{2G}(d\theta^2 + \sin^2\theta d\phi^2) + dr^2 \tag{3.1}$$

for the S^2 case and

$$ds^2 = e^{2F}dx_{1,3}^2 + \frac{e^{2G}}{y^2}(dx^2 + dy^2) + dr^2 \tag{3.2}$$

for the H^2 case. In both cases, the warp factors F and G are functions only of r.

The non-vanishing spin connections of the above metrics are given respectively by

$$\begin{aligned} \omega^{\hat\phi}{}_{\hat\theta} &= e^{-G}\cot\theta e^{\hat\phi}, \qquad \omega^{\hat\phi}{}_{\hat r} = G' e^{\hat\phi}, \\ \omega^{\hat\theta}{}_{\hat r} &= G' e^{\hat\theta}, \qquad \omega^{\hat\mu}{}_{\hat r} = F' e^{\hat\mu} \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} \omega^{\hat{x}}{}_{\hat{r}} &= G' e^{\hat{x}}, \qquad & \omega^{\hat{y}}{}_{\hat{r}} &= G' e^{\hat{y}}, \\ \omega^{\hat{\mu}}{}_{\hat{r}} &= F' e^{\hat{\mu}}, \qquad & \omega^{\hat{x}}{}_{\hat{y}} &= -e^{-G(r)} e^{\hat{x}} \end{aligned} \tag{3.4}$$

where $'$ denotes the r-derivative.

3.1 $N = 2$ three-dimensional SCFTs with $SO(2) \times SO(2)$ symmetry

To find supersymmetric solutions of the form $AdS_4 \times \Sigma_2$ with SO(2) × SO(2) symmetry, we turn on SO(2) × SO(2) gauge fields such that the spin connection along Σ is canceled. In the present case, there are six gauge fields (A^r, A^I) corresponding to $\mathrm{SU}(2)_R \times \mathrm{SU}(2)$ gauge group. We will turn on the following SO(2) × SO(2) gauge fields

$$A^3 = a \cos\theta d\phi \qquad \text{and} \qquad A^6 = b \cos\theta d\phi \tag{3.5}$$

for the S^2 case and

$$A^3 = \frac{a}{y} dx \qquad \text{and} \qquad A^6 = \frac{b}{y} dx \tag{3.6}$$

for the H^2 case. To avoid confusion, we have given the gauge fields using the index $\Lambda = 0, 1, \ldots, 6$.

A^3 will appear in the covariant derivative of ϵ^A since it is part of the $\mathrm{SU}(2)_R$ gauge fields. We choose this particular form of the gauge field to cancel the spin connection on Σ_2. Accordingly, the Killing spinors corresponding to unbroken supersymmetry will be constant spinors on Σ_2 provided that we impose the twist condition

$$a g_1 = 1 \tag{3.7}$$

and a set of projection conditions given below.

There are two scalars which are singlet under SO(2) × SO(2) generated by J_1^{12} and J_2^{12}. The $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset representative can be written in terms of these scalars as

$$L = e^{\phi_1 Y_{03}} e^{\phi_2 Y_{33}} \,. \tag{3.8}$$

Imposing the projection conditions

$$\gamma_{\hat{r}} \epsilon_A = \epsilon_A, \qquad \gamma_7 \epsilon^A = \sigma_3^A{}_B \epsilon^B, \qquad \gamma^{\hat{\phi}\hat{\theta}} \epsilon_A = i \sigma_{3AB} \epsilon^B, \tag{3.9}$$

for the S^2 case or

$$\gamma_{\hat{r}} \epsilon_A = \epsilon_A, \qquad \gamma_7 \epsilon^A = \sigma_3^A{}_B \epsilon^B, \qquad \gamma^{\hat{x}\hat{y}} \epsilon_A = -i \sigma_{3AB} \epsilon^B, \tag{3.10}$$

JHEP09(2015)034

for the H^2 case, we find the following BPS equations

$$\phi_1' = e^{-3\sigma-\phi_1}(1-e^{2\phi_1})m, \tag{3.11}$$

$$\begin{aligned}\phi_2' = \frac{1}{4}e^{\sigma-\phi_1-\phi_2-2G}\Big[&\lambda b(1+e^{2\phi_1})(1+e^{2\phi_2})\\ &+2e^{\phi_1}(1-e^{2\phi_2})(\lambda a+2g_1e^{2\sigma+2G})\Big],\end{aligned} \tag{3.12}$$

$$\begin{aligned}\sigma' = \frac{1}{16}e^{-3\sigma-\phi_1-\phi_2-2G}\Big[&2\lambda a e^{2\sigma+\phi_1}(1+e^{2\phi_2})\\ &-\lambda b e^{2\sigma}(2e^{\phi_2}-e^{2\phi_1}+e^{2\phi_2}+e^{2\phi_1+2\phi_2}-2e^{2\phi_1+\phi_2}-1)\\ &+6me^{2G}(1-e^{2\phi_1}+2e^{\phi_2}-e^{2\phi_2}+e^{2\phi_1+2\phi_2}+2e^{2\phi_1+\phi_2})\\ &-4g_1e^{2G+4\sigma+\phi_1}(1+e^{2\phi_2})\Big],\end{aligned} \tag{3.13}$$

$$\begin{aligned}G' = \frac{1}{16}e^{-3\sigma-\phi_1-\phi_2-2G}\Big[&6\lambda a e^{2\sigma+\phi_1}(1+e^{2\phi_2})+4g_1e^{4\sigma+\phi_1+2G}(1+e^{2\phi_2})\\ &+3\lambda b e^{2\sigma}(1+e^{2\phi_1}-2e^{\phi_2}-e^{2\phi_2}-e^{2\phi_1+2\phi_2}+2e^{2\phi_1+\phi_2})\\ &-2me^{2G}(1-e^{2\phi_1}-2e^{\phi_2}-e^{2\phi_2}+e^{2\phi_1+2\phi_2}-2e^{2\phi_1+\phi_2})\Big],\end{aligned} \tag{3.14}$$

$$\begin{aligned}F' = \frac{1}{16}e^{-3\sigma-\phi_1-\phi_2-2G}\Big[&-2\lambda a e^{2\sigma+\phi_1}(1+e^{2\phi_2})+4g_1e^{4\sigma+\phi_1+2G}(1+e^{2\phi_2})\\ &-\lambda b e^{2\sigma}(1+e^{2\phi_1}-2e^{\phi_2}-e^{2\phi_2}-e^{2\phi_1+2\phi_2}+2e^{2\phi_1+\phi_2})\\ &-2me^{2G}(1-e^{2\phi_1}-2e^{\phi_2}-e^{2\phi_2}+e^{2\phi_1+2\phi_2}-2e^{2\phi_1+\phi_2})\Big]\end{aligned} \tag{3.15}$$

where $\lambda=1$ and $\lambda=-1$ for S^2 and H^2 cases, respectively.

We look for fixed point solutions satisfying $G'=\sigma'=\phi_1'=\phi_2'=0$ and $F\sim r$. From the above equations, we immediately see that these conditions require $\phi_1=0$. The $\gamma_{\hat{r}}$ projector is not necessary for constant scalars since $\gamma_{\hat{r}}$ only appears with the r-derivative. The BPS equations are automatically satisfied by the fixed point solutions without imposing the $\gamma_{\hat{r}}$ projector. Furthermore, with $\phi_1=0$, the γ_7 projection is not needed. Therefore, the AdS_4 fixed points will preserve half of the original supersymmetry corresponding to eight supercharges or $N=2$ superconformal symmetry in three dimensions.

For $\phi_1=0$, the explicit form of AdS_4 critical point is given by

$$\begin{aligned}\phi_2 &= \frac{1}{2}\ln\left[\frac{3b\pm\sqrt{a^2+8b^2}}{b-a}\right],\\ \sigma &= \frac{1}{8}\ln\left[\frac{m^2(b-a)(a+4b\mp\sqrt{a^2+8b^2})^2}{4b^2g_1^2(3b\mp\sqrt{a^2+8b^2})}\right],\\ G &= \frac{1}{8}\ln\left[\frac{b^2(b-a)^3(3b\mp\sqrt{a^2+8b^2})(a+4b\mp\sqrt{a^2+8b^2})^2}{4g_1^2m^2(a+2b\mp\sqrt{a^2+8b^2})^4}\right],\\ L_{AdS_4} &= \frac{1}{2m}\left[\frac{(b-a)m^2(a+4b\pm\sqrt{a^2+8b^2})^2}{4b^2g_1^2(3b\pm\sqrt{a^2+8b^2})}\right]^{\frac{3}{8}}.\end{aligned} \tag{3.16}$$

In the above equations, we have given a solution in the S^2 case for definiteness. A similar solution in the H^2 case can be obtained by replacing (a,b) by $(-a,-b)$ in the above solution.

JHEP09(2015)034

For $a < 0$, the solution is valid provided that $b < a$ or $b > -a$. When $a > 0$, we have a real solution for $b < a$ or $b > a$. It can be checked that there exist both $AdS_4 \times S^2$ and $AdS_4 \times H^2$ fixed points.

As an example, we give some AdS_4 solutions with a particular value of $b = 2a$ as follow:

$$AdS_4 \times S^2: \quad \phi_2 = \frac{1}{2}\ln(6+\sqrt{33}), \qquad \sigma = \frac{1}{4}\ln\left[\frac{(9+\sqrt{33})m}{4\sqrt{6+\sqrt{33}}g+1}\right],$$
$$G = \frac{1}{8}\ln\left[\frac{6a^4(213+37\sqrt{33})}{g_1^2 m^2(5+\sqrt{33})^4}\right] \tag{3.17}$$

and

$$AdS_4 \times H^2: \quad \phi_2 = \frac{1}{2}\ln(2+\sqrt{\frac{1}{3}}), \qquad \sigma = \frac{1}{4}\ln\left[\frac{(7\sqrt{3}+3\sqrt{11})m}{4\sqrt{6+\sqrt{33}}g+1}\right],$$
$$G = \frac{1}{8}\ln\left[\frac{59a^4(477+83\sqrt{33})}{g_1^2 m^2(3+\sqrt{33})^4}\right]. \tag{3.18}$$

It can also be readily verified that, by making a truncation $\phi_2 = 0$ and $b = 0$, we find only $AdS_4 \times H^2$ solution in agreement with the results of [18]. It should also be pointed out that the solutions are similar to the ones obtained in seven-dimensional gauged supergravity studied in [20] and [21, 22]. It is also possible to find a numerical RG flow solution interpolating between $\mathrm{SU}(2) \times \mathrm{SU}(2)$ AdS_6 critical point (A.8) to one of these AdS_4 critical points, but we will not give it here.

3.2 $N = 2$ three-dimensional SCFTs with $SO(2)$ symmetry

We now consider AdS_4 solutions that can be connected to the AdS_6 critical point with $\mathrm{SU}(2)_{\mathrm{diag}}$ symmetry (A.9) by an interpolating domain wall solution. In this case, there can be RG flows from AdS_6 critical point in (A.9) to three-dimensional SCFTs in the IR or even a flow from AdS_6 critical point (A.8) to critical point (A.9) and then to the AdS_4 points.

We look for solutions preserving $\mathrm{SO}(2)_{\mathrm{diag}}$ subgroup of $\mathrm{SO}(2) \times \mathrm{SO}(2)$ generated by $J_1^{12} + J_2^{12}$. The gauge fields for the S^2 and H^2 cases are then given respectively by

$$A^3 = a\cos\theta d\phi \qquad \text{and} \qquad A^6 = \frac{g_1}{g_2}A^3 \tag{3.19}$$

and

$$A^3 = \frac{a}{y}dx \qquad \text{and} \qquad A^6 = \frac{g_1}{g_2}A^3\,. \tag{3.20}$$

There are four $\mathrm{SO}(2)_{\mathrm{diag}}$ singlet scalars with the scalar coset representative given by

$$L = e^{\phi_1(Y_{11}+Y_{22})}e^{\phi_2 Y_{33}}e^{\phi_3 Y_{03}}e^{\phi_4(Y_{12}-Y_{21})}\,. \tag{3.21}$$

By using similar projection conditions and the relation $g_1 a = 1$ as in the previous case, we can find the corresponding BPS equations which turn out to be very complicated. To make

JHEP09(2015)034

things more manageable, we will set $\phi_3 = 0$ which can be easily checked to be a consistent truncation. Moreover, the γ_7 projector is irrelevant when $\phi_3 = 0$. Therefore, the fixed point solutions will also preserve eight supercharges. The relevant BPS equations for the remaining scalars are given by

$$\phi_4' = \frac{1}{8}e^{\sigma-2\phi_1-\phi_2-2\phi_4}(1+e^{4\phi_1})(1-e^{4\phi_4})(g_1+g_1e^{2\phi_2}+g_2-g_2e^{2\phi_2}), \tag{3.22}$$

$$\phi_1' = \frac{e^{\sigma-2\phi_1-\phi_2+2\phi_4}(1-e^{4\phi_1})}{2(1+e^{4\phi_4})}(g_1+g_1e^{2\phi_2}+g_2-g_2e^{2\phi_2}), \tag{3.23}$$

$$\begin{aligned}\phi_2' = &-\frac{1}{8g_2}e^{-\sigma-2\phi_1-\phi_2-2\phi_4-2G}\Big[-4\lambda a e^{2\phi_1+2\phi_4}(g_1+g_1e^{2\phi_2}+g_2-g_2e^{2\phi_2})\\ &-g_2e^{2\sigma+2G}\Big[g_1(1-e^{2\phi_2})(1+e^{4\phi_1}+e^{4\phi_4}+4e^{2\phi_1+2\phi_4}+e^{4\phi_1+4\phi_4})\\ &+g_2(1+e^{2\phi_2})(1+e^{4\phi_1}+e^{4\phi_4}-4e^{2\phi_1+2\phi_4}+e^{4\phi_1+4\phi_4})\Big]\Big],\end{aligned} \tag{3.24}$$

$$\begin{aligned}\sigma' = &\frac{1}{32}e^{-3\sigma-2\phi_1-\phi_2-2\phi_4-2G}\Big[48me^{2\phi_1+\phi_2+2\phi_4+2G}\\ &-(g_1-g_2)e^{4\sigma+2\phi_2+2G}(1+e^{4\phi_1}+4e^{2\phi_1+2\phi_4-2\phi_2}+e^{4\phi_4}+e^{4\phi_1+4\phi_4})\\ &-(g_1+g_2)e^{4\sigma+2G}(1+e^{4\phi_4}+e^{4\phi_4}+e^{4\phi_1+4\phi_4}+4e^{2\phi_1+2\phi_2+2\phi_4})\\ &+\frac{4\lambda a}{g_2}e^{2\sigma+2\phi_1+2\phi_4}(g_1-g_1e^{2\phi_2}+g_2+g_2e^{2\phi_2})\Big],\end{aligned} \tag{3.25}$$

$$\begin{aligned}G' = &\frac{1}{32}e^{-3\sigma-2\phi_1-\phi_2-2\phi_4-2G}\Big[16me^{2\phi_1+\phi_2+2\phi_4+2G}\\ &+(g_1-g_2)e^{4\sigma+2\phi_2+2G}(1+e^{4\phi_1}+4e^{2\phi_1+2\phi_4-2\phi_2}+e^{4\phi_4}+e^{4\phi_1+4\phi_4})\\ &+(g_1+g_2)e^{4\sigma+2G}(1+e^{4\phi_4}+e^{4\phi_4}+e^{4\phi_1+4\phi_4}+4e^{2\phi_1+2\phi_2+2\phi_4})\\ &+\frac{12\lambda a}{g_2}e^{2\sigma+2\phi_1+2\phi_4}(g_1-g_1e^{2\phi_2}+g_2+g_2e^{2\phi_2})\Big],\end{aligned} \tag{3.26}$$

$$\begin{aligned}F' = &\frac{1}{32}e^{-3\sigma-2\phi_1-\phi_2-2\phi_4-2G}\Big[16me^{2\phi_1+\phi_2+2\phi_4+2G}\\ &+(g_1-g_2)e^{4\sigma+2\phi_2+2G}(1+e^{4\phi_1}+4e^{2\phi_1+2\phi_4-2\phi_2}+e^{4\phi_4}+e^{4\phi_1+4\phi_4})\\ &+(g_1+g_2)e^{4\sigma+2G}(1+e^{4\phi_4}+e^{4\phi_4}+e^{4\phi_1+4\phi_4}+4e^{2\phi_1+2\phi_2+2\phi_4})\\ &-\frac{4\lambda a}{g_2}e^{2\sigma+2\phi_1+2\phi_4}(g_1-g_1e^{2\phi_2}+g_2+g_2e^{2\phi_2})\Big]\end{aligned} \tag{3.27}$$

where, as in the previous case, $\lambda = \pm 1$ for S^2 and H^2, respectively. When $a = 0$, $\phi_2 = \phi_1 = \phi$ and $\phi_4 = 0$, we recover the BPS equations (A.5), (A.6) and (A.7) given in the appendix.

We begin with a simple solution for $\phi_4 = 0$. There are two possibilities for the critical points to occur depending on the values of the coupling constants g_1 and g_2. The critical point that can be connected to the AdS_6 critical point (A.9) for which $g_2 < -g_1$ and

JHEP09(2015)034

$g_1 = 3m > 0$ is given by

$$\begin{aligned}
\phi_2 &= \frac{1}{2}\ln\left[\frac{g_2+g_1}{g_2-g_1}\right], \qquad \phi_1 = \pm\frac{1}{2}\ln\left[\frac{g_2+g_1}{g_2-g_1}\right], \\
\sigma &= \frac{1}{4}\ln\left[-\frac{2m\sqrt{g_2^2-g_1^2}}{g_1 g_2}\right], \qquad G = \frac{1}{2}\ln\left[-\frac{a}{g_2 m}\sqrt{-\frac{m(g_2^2-g_1^2)^{\frac{3}{2}}}{2g_1 g_2}}\right], \\
L_{AdS_4} &= \frac{1}{2m}\left[-\frac{2m\sqrt{g_2^2-g_1^2}}{g_1 g_2}\right]^{\frac{3}{4}}.
\end{aligned} \tag{3.28}$$

It can be verified that, in this case, only $AdS_4 \times H^2$ solutions are possible. The other possibility with positive g_2 however does not give any real solutions.

For a particular value of $g_2 = g_1$, there is an $AdS_4 \times H^2$ solution given by

$$\begin{aligned}
\phi_2 &= -\frac{1}{2}\ln 3, \qquad \phi_1 = 0, \\
\sigma &= \frac{1}{8}\ln 3 + \frac{1}{4}\ln\left[\frac{m}{g_1}\right], \qquad G = -\frac{1}{2}\ln\left[\frac{2g_1}{3^{\frac{3}{4}}a}\sqrt{\frac{m}{g_1}}\right].
\end{aligned} \tag{3.29}$$

For non-zero ϕ_4, there is a class of solutions parametrized by ϕ_4. The explicit form of these solutions for $g_2 < -g_1$ and $g_1 = 3m > 0$ is given by

$$\begin{aligned}
\phi_2 &= \frac{1}{2}\ln\left[\frac{g_2+g_1}{g_2-g_1}\right], \qquad \sigma = \frac{1}{4}\ln\left[-\frac{2m\sqrt{g_2^2-g_1^2}}{g_1 g_2}\right], \\
G &= \frac{1}{2}\ln\left[-\frac{a}{g_2 m}\sqrt{-\frac{m(g_2^2-g_1^2)^{\frac{3}{2}}}{2g_1 g_2}}\right], \\
\phi_1 &= \frac{1}{2}\ln\left[\frac{2e^{2\phi_4}(g_1^2+g_2^2)+\sqrt{4e^{4\phi_4}(g_1^2+g_2^2)^2-(1+e^{4\phi_4})^2(g_2^2-g_1^2)^2}}{(g_2^2-g_1^2)(1+e^{4\phi_4})}\right].
\end{aligned} \tag{3.30}$$

This solution is also $AdS_4 \times H^2$ as in the previous case and valid for

$$\frac{(g_2+g_1)^2}{(g_2-g_1)^2} \leq e^{4\phi_4} \leq \frac{(g_2-g_1)^2}{(g_2+g_1)^2}. \tag{3.31}$$

In this case, the scalar ϕ_4 is not determined by the BPS equations. Consider this critical point to be an IR fixed point of the five-dimensional SCFTs corresponding to the AdS_6 critical point (A.9), we can see that ϕ_4 corresponds to a marginal deformation since ϕ_4 is massless as can be seen from the scalar masses given in [8]. It should also be noted that when the symmetry is reduced to SO(2), only solutions with a hyperbolic space are possible. This is similar to the seven-dimensional results studied in [20–22].

4 AdS_3 critical points

We now look for a gravity dual of five-dimensional SCFTs compactified on a three-manifold Σ_3 which can be S^3 or H^3. The IR effective theories would be two-dimensional field theories.

JHEP09(2015)034

We particularly look for the gravity solutions corresponding to conformal field theories in the IR, so the gravity solutions will take the form of $AdS_3 \times \Sigma_3$.

The metrics and the associated spin connections for each case are given by

$$ds_7^2 = e^{2F} dx_{1,2}^2 + dr^2 + e^{2G} \left[d\psi^2 + \sin^2\psi (d\theta^2 + \sin^2\theta d\phi^2) \right] \tag{4.1}$$

for $\Sigma_3 = S^3$ with the spin connections

$$\begin{aligned}
&\omega^{\hat{\mu}}{}_{\hat{r}} = F' e^{\hat{\mu}}, \qquad \omega^{\hat{\psi}}{}_{\hat{r}} = G' e^{\hat{\psi}}, \qquad \omega^{\hat{\theta}}{}_{\hat{r}} = G' e^{\hat{\theta}}, \\
&\omega^{\hat{\phi}}{}_{\hat{r}} = G' e^{\hat{\phi}}, \qquad \omega^{\hat{\phi}}{}_{\hat{\theta}} = e^{-G} \frac{\cot\theta}{\sin\psi} e^{\hat{\phi}}, \\
&\omega^{\hat{\phi}}{}_{\hat{\psi}} = e^{-G} \cot\psi e^{\hat{\phi}}, \qquad \omega^{\hat{\theta}}{}_{\hat{\psi}} = e^{-G} \cot\psi e^{\hat{\theta}}
\end{aligned} \tag{4.2}$$

and

$$ds_7^2 = e^{2F} dx_{1,2}^2 + dr^2 + \frac{e^{2G}}{y^2}(dx^2 + dy^2 + dz^2) \tag{4.3}$$

for $\Sigma_3 = H^3$ with the spin connections given by

$$\begin{aligned}
&\omega^{\hat{z}}{}_{\hat{r}} = G' e^{\hat{z}}, \qquad \omega^{\hat{y}}{}_{\hat{r}} = G' e^{\hat{y}}, \qquad \omega^{\hat{x}}{}_{\hat{r}} = G' e^{\hat{x}}, \\
&\omega^{\hat{x}}{}_{\hat{y}} = -e^{-G} e^{\hat{x}}, \qquad \omega^{\hat{z}}{}_{\hat{y}} = -e^{-G} e^{\hat{z}}, \qquad \omega^{\hat{\mu}}{}_{\hat{r}} = F' e^{\hat{\mu}} .
\end{aligned} \tag{4.4}$$

4.1 $N = (1,1)$ two-dimensional SCFTs with $SO(3)$ symmetry

We first look at solutions preserving $\mathrm{SO}(3)_{\mathrm{diag}} \subset \mathrm{SO}(3)_R \times \mathrm{SO}(3)$ symmetry. The $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset representative is given in (A.2). We then proceed by turning on $\mathrm{SO}(3)_{\mathrm{diag}}$ gauge fields to cancel the spin connections on Σ_3 as in the previous section.

For the S^3 case, we choose the $\mathrm{SU}(2)_R$ gauge fields to be

$$A^1 = a \cos\psi \sin\theta d\phi, \qquad A^2 = b \cos\theta d\phi, \qquad A^3 = c \cos\psi d\theta \tag{4.5}$$

while, for the H^3 case, they are given by

$$A^1 = \frac{a}{y} dz, \qquad A^2 = 0, \qquad A^3 = \frac{b}{y} dx . \tag{4.6}$$

In both cases, the SO(3) gauge fields are related to the $\mathrm{SU}(2)_R$ gauge fields by

$$A^I = \frac{g_1}{g_2} A^r . \tag{4.7}$$

The two sets of gauge fields implement the $\mathrm{SO}(3)_{\mathrm{diag}}$ gauge fields. Furthermore, the twist condition implies $a = b = c$ and $g_1 a = 1$.

Using the following projectors

$$\begin{aligned}
H^3 : \quad & \gamma_{\hat{r}} \epsilon_A = \epsilon_A, \qquad \gamma_{\hat{x}\hat{y}} \epsilon_A = -i\sigma^3_{AB} \epsilon^B, \\
& \gamma_{\hat{z}\hat{y}} \epsilon_A = -i\sigma^1_{AB} \epsilon^B, \qquad \gamma_{\hat{x}\hat{z}} \epsilon_A = -i\sigma^2_{AB} \epsilon^B,
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
S^3 : \quad & \gamma_{\hat{r}} \epsilon_A = \epsilon_A, \qquad \gamma_{\hat{\theta}\hat{\psi}} \epsilon_A = i\sigma^3_{AB} \epsilon^B, \\
& \gamma_{\hat{\phi}\hat{\psi}} \epsilon_A = i\sigma^1_{AB} \epsilon^B, \qquad \gamma_{\hat{\phi}\hat{\theta}} \epsilon_A = i\sigma^2_{AB} \epsilon^B,
\end{aligned} \tag{4.9}$$

JHEP09(2015)034

we obtain the BPS equations

$$\phi' = \frac{1}{4g_2}e^{-\sigma-3\phi-2G}[(1+e^{2\phi})g_1+(1-e^{2\phi})g_2][2\lambda a e^{2\phi}+g_2 e^{2\phi+2G}(1-e^{4\phi})], \tag{4.10}$$

$$\sigma' = \frac{3}{8g_2}\lambda a e^{-\sigma-\phi-2G}[(1-e^{2\phi})g_1+(1+e^{2\phi})g_2]-\frac{3}{2}me^{-3\sigma} -\frac{1}{16}[(1+e^{2\phi})^3 g_1+(1-e^{2\phi})^3 g_2], \tag{4.11}$$

$$G' = \frac{1}{16}e^{\sigma-3\phi}[(1+e^{2\phi})^3 g_1+(1-e^{2\phi})^3 g_2]+\frac{1}{2}me^{-3\sigma} +\frac{5}{8g_2}\lambda a e^{-\sigma-\phi-2G}[(1-e^{2\phi})g_1+(1+e^{2\phi})g_2], \tag{4.12}$$

$$F' = \frac{1}{16}e^{\sigma-3\phi}[(1+e^{2\phi})^3 g_1+(1-e^{2\phi})^3 g_2]+\frac{1}{2}me^{-3\sigma} -\frac{3}{8g_2}\lambda a e^{-\sigma-\phi-2G}[(1-e^{2\phi})g_1+(1+e^{2\phi})g_2] \tag{4.13}$$

where $\lambda = \pm 1$ for S^3 and H^3 as in the previous cases. Note also that, in both cases, the last projector is not independent from the second and the third ones. Therefore, fixed point solutions will preserve four supercharges or equivalently $N = (1,1)$ superconformal symmetry in two dimensions. The analysis of unbroken supersymmetry can be done in a similar manner to that given in [18].

We begin with a simple fixed point solution with $g_2 = g_1$. In this case, only $AdS_3 \times H^3$ solution exists and is given by

$$\phi = -\frac{1}{4}\ln 3, \qquad \sigma = \frac{1}{16}\ln 3 - \frac{1}{4}\ln\left[\frac{g_1}{m}\right], \\ G = \frac{3}{16}\ln 3 - \frac{1}{2}\ln\left[\frac{g_1}{a}\sqrt{\frac{m}{g_1}}\right]. \tag{4.14}$$

For $g_2 \neq g_1$, we find two classes of solutions. In the first class, only $AdS_3 \times H^3$ is possible and given by

$$\phi = \frac{1}{2}\ln\left[\frac{g_1+g_2}{g_2-g_1}\right], \qquad \sigma = \frac{1}{2}\ln\left[\frac{-3m\sqrt{g_2^2-g_1^2}}{2g_1g_2}\right], \\ G = \frac{1}{2}\ln\left[-\frac{a(g_2^2-g_1^2)^{\frac{3}{4}}}{g_2}\sqrt{-\frac{3}{2g_1g_2m}}\right] \tag{4.15}$$

where we have chosen $g_1 > 0$ and $g_2 < -g_1$. For positive g_2 with $g_2 > g_1$ and $g_1 > 0$, we find another $AdS_3 \times H^3$ solution

$$\phi = \frac{1}{2}\ln\left[\frac{g_1+g_2}{g_2-g_1}\right], \qquad \sigma = \frac{1}{2}\ln\left[\frac{3m\sqrt{g_2^2-g_1^2}}{2g_1g_2}\right], \\ G = \frac{1}{2}\ln\left[\frac{a(g_2^2-g_1^2)^{\frac{3}{4}}}{g_2}\sqrt{-\frac{3}{2g_1g_2m}}\right]. \tag{4.16}$$

JHEP09(2015)034

The second class of solutions is given by

$$\sigma = \frac{3}{4}\phi - \frac{1}{4}\ln\left[\frac{(1+e^{6\phi})g_1 + (1-e^{6\phi})g_2}{6m}\right], \tag{4.17}$$

$$G = \phi - \frac{1}{2}\ln\left[\frac{g_2 e^{\frac{3\sigma}{2}}(e^{4\phi}-1)}{\lambda a}\sqrt{\frac{3m}{2[(1+e^{6\phi})g_1 + (1-e^{6\phi})g_2]}}\right] \tag{4.18}$$

where ϕ is a solution to the following equation

$$(1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi})g_1 + (1 + 3e^{2\phi} - 3e^{4\phi} - e^{6\phi})g_2 = 0\,. \tag{4.19}$$

The explicit form of ϕ can be written, but we refrain from giving such a complicated expression. We will however give some examples of the solutions. Using the relation $g_1 = 3m$ and choosing $g_2 = \frac{1}{2}g_1$, we find an $AdS_3 \times S^3$ solution characterized by

$$\phi = 0.9645, \qquad \sigma = -0.528, \qquad G = -0.4309 - \frac{1}{2}\ln\left[\frac{m}{a}\right]. \tag{4.20}$$

It is also possible to obtain an $AdS_3 \times H^3$ solution for $g_2 = \frac{1}{2}g_1$. This solution is given by

$$\phi = -0.5732, \qquad \sigma = -0.2723, \qquad G = -0.5732 - \frac{1}{2}\ln\left[\frac{0.3278m}{a}\right]. \tag{4.21}$$

4.2 $N = (1,0)$ two-dimensional SCFTs with $SO(3) \times SO(2)$ symmetry

We now look for a larger residual symmetry. Although the dilaton σ is a singlet of $\mathrm{SO}(3)_R \times \mathrm{SO}(3)$, the second SO(3) gauge fields cannot be turned on without turning on some vector multiplet scalars as can be seen from the supersymmetry transformation of λ^I_A. On the other hand, no vector multiplet scalar is a singlet of the SO(3) symmetry. We can at most have $\mathrm{SO}(3)_R \times \mathrm{SO}(2)$ symmetry with SO(2) being a subgroup of SO(3). Among the 12 scalars in $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$, there is only one $\mathrm{SO}(3)_R \times \mathrm{SO}(2)$ singlet. This corresponds to the non-compact generator Y_{03}. We then parametrize the coset representative as

$$L = e^{\Phi Y_{03}}\,. \tag{4.22}$$

The $\mathrm{SO}(3)_R$ gauge fields are the same as in the previous case while the SO(2) gauge field will be chosen to be

$$A^6 = b\cos\psi \qquad \text{or} \qquad A^6 = \frac{b}{y}dx \tag{4.23}$$

for $\Sigma_3 = S^3$ and $\Sigma_3 = H^3$, respectively.

Apart from the projectors in (4.8) and (4.9), in this case, we need to impose an additional projector involving γ_7 namely

$$\gamma_7 \epsilon^A = \sigma_3{}^A{}_B \epsilon^B\,. \tag{4.24}$$

The critical points would then preserve only half of the supersymmetry in the previous case. This corresponds to $N = (1,0)$ superconformal symmetry in two dimensions.

JHEP09(2015)034

With all these and $\lambda = \pm 1$ for S^3 and H^3, respectively, we find the following BPS equations

$$\Phi' = \frac{1}{2}e^{-3\sigma-\Phi-2G}\left[\lambda b e^{2\sigma}(1+e^{2\Phi}) + 2me^{2G}(1-e^{2\Phi})\right], \tag{4.25}$$

$$\begin{aligned}\sigma' = \frac{1}{8}e^{-3\sigma-\Phi-2G}&\left[6\lambda e^{2\sigma+\Phi} + \lambda b e^{2\sigma}(e^{2\Phi}-1)\right.\\ &\left.+2e^{2G}(3m - 2g_1 e^{4\sigma+\Phi} + 3me^{2\Phi})\right],\end{aligned} \tag{4.26}$$

$$\begin{aligned}G' = \frac{1}{8}e^{-3\sigma-\Phi-2G}&\left[3\lambda b e^{2\sigma}(e^{2\Phi}-1) + 10\lambda a e^{2\sigma+\Phi}\right.\\ &\left.+2e^{2G}(2g_1 e^{4\sigma+\Phi} + m + me^{2\Phi})\right],\end{aligned} \tag{4.27}$$

$$\begin{aligned}F' = \frac{1}{8}e^{-3\sigma-\Phi-2G}&\left[-\lambda b e^{2\sigma}(e^{2\Phi}-1) - 6\lambda a e^{2\sigma+\Phi}\right.\\ &\left.+2e^{2G}(2g_1 e^{4\sigma+\Phi} + m + me^{2\Phi})\right].\end{aligned} \tag{4.28}$$

When $\Phi = 0$ and $b = 0$, the above equations reduce to the pure $F(4)$ gauged supergravity which admits only $AdS_3 \times H^3$ solutions in agreement with [18].

We find an $AdS_3 \times \Sigma_3$ fixed point given by

$$\sigma = \frac{1}{4}\ln\left[\frac{e^{-\Phi}[3ae^{\Phi}(e^{2\Phi}-1) + 2bm(1+e^{2\Phi}+e^{4\Phi})]}{g_1 b(1+e^{2\Phi})}\right], \tag{4.29}$$

$$G = \frac{1}{4}\ln\left[\frac{be^{-\Phi}(1+e^{2\Phi})[3ae^{\Phi}(e^{2\Phi}-1) + 2b(1+e^{2\Phi}+e^{4\Phi})]}{4g_1 m(e^{2\Phi}-1)^2}\right], \tag{4.30}$$

$$\Phi = \ln\left[\frac{\sqrt{2a^2+2b^2+2a\sqrt{a^2-2b^2}} - a \pm \sqrt{a^2-2b^2}}{2b}\right]. \tag{4.31}$$

The above solution is written for the S^3 case. To find the solution in the H^3 case, (a, b) should be replaced by $(-a, -b)$ in all of the above expressions. The solution is valid for non-vanishing a and b with $-\sqrt{\frac{a^2}{2}} \leq b \leq \sqrt{\frac{a^2}{2}}$.

From the analysis, it turns out that only $AdS_3 \times H^3$ solutions are possible. As an example of explicit solutions, we take $a > 0$ and choose $b = \pm\frac{a}{\sqrt{2}}$. The solutions are given by

$$\begin{aligned}\Phi &= \ln\left[\frac{\sqrt{3}-1}{\sqrt{2}}\right], \qquad \sigma = \frac{1}{4}\ln\left[\frac{2\sqrt{2}m(2-\sqrt{3})}{(2\sqrt{3}-3)g_1}\right],\\ G &= \ln\left[\left(\frac{3}{8}\right)^{\frac{7}{32}}\sqrt{\frac{a}{m}}\left(\frac{m}{g_1}\right)^{\frac{1}{16}}\right]\end{aligned} \tag{4.32}$$

for $b = \frac{a}{\sqrt{2}}$ and

$$\begin{aligned}\Phi &= \ln\left[\frac{\sqrt{3}+1}{\sqrt{2}}\right], \qquad \sigma = \frac{1}{4}\ln\left[\frac{2\sqrt{2}m}{\sqrt{3}g_1}\right],\\ G &= \ln\left[\left(\frac{3}{8}\right)^{\frac{1}{8}}\sqrt{\frac{a}{m}}\left(\frac{m}{g_1}\right)^{\frac{1}{4}}\right]\end{aligned} \tag{4.33}$$

for $b = -\frac{a}{\sqrt{2}}$. The solution in this case does not have an analogue in seven dimensions since there is no $\mathrm{SO}(3)_R$ scalar in that case.

JHEP09(2015)034

5 Conclusions

We have classified supersymmetric AdS_4 and AdS_3 solutions of $N = (1, 1)$ six-dimensional gauged supergravity coupled to three vector multiplets with $\mathrm{SU}(2) \times \mathrm{SU}(2)$ gauge group. Depending on the values of the two gauge couplings, there are both $AdS_4 \times S^2$ and $AdS_4 \times H^2$ solutions with $\mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry and $AdS_4 \times H^2$ solutions with $\mathrm{SO}(2)$ symmetry. All solutions preserve eight of the original sixteen supercharges and are dual to $N = 2$ SCFTs in three dimensions. For $AdS_3 \times \Sigma_3$ solutions, we have found new $AdS_3 \times S^3$ and $AdS_3 \times H^3$ solutions preserving four supercharges and $\mathrm{SO}(3)$ symmetry. These solutions correspond to $N = (1, 1)$ SCFTs in two dimensions. For $\mathrm{SO}(3) \times \mathrm{SO}(2)$ symmetry, only $AdS_3 \times H^3$ solutions exist with $\frac{1}{8}$ supersymmetry unbroken. These solutions provide gravity duals of $N = (1, 0)$ SCFTs in two dimensions. Apart from the $\mathrm{SO}(3) \times \mathrm{SO}(2)$ AdS_3 fixed point, the solutions are very similar to those of $N = 2$ $\mathrm{SO}(4)$ gauged supergravity in seven dimensions [20].

All of these solutions correspond to IR fixed points of five-dimensional SCFTs with global symmetry $\mathrm{SU}(2)$ in lower dimensional space-time. There should be RG flows describing twisted compactifications of these SCFTs on 2 or 3-manifolds giving rise to these AdS_4 and AdS_3 geometries in the IR. We have not been able to find analytic solutions for these flows, but numerical solutions can be obtained as in other cases, see for example [20]. The results obtained in this paper are hopefully useful in the holographic study of five-dimensional SCFTs and their compactifications as well as the classification of vacua of the half-maximal gauged supergravity in six dimensions.

It would be interesting to find a possible embedding of these solutions in higher dimensions in particular in massive type IIA supergravity similar to the embedding of pure $F(4)$ gauged supergravity [23] or in type IIB supergravity as in [11] and [24]. This could give an interpretation to these solutions in terms of wrapped D4-branes. However, since there is only one class of known AdS_6 solutions, as shown in [10], embedding the AdS_6 solutions with different $\mathrm{SU}(2)$ gauge coupling constants (if possible) might not be straightforward in massive type IIA theory.

It is also interesting to find dual field theories to the AdS_4 and AdS_3 critical points identified here. Another investigation would be to study other types of gauge groups such as non-compact gauge groups to the matter-coupled $F(4)$ gauged supergravity and classify all possible gauge groups that admit supersymmetric AdS_6 vacuum similar to the recent analysis in seven dimensions [25, 26]. Finally, gravity solutions with a non-vanishing two-form field could be of interest. A simple $AdS_3 \times R^3$ solution with only the two-form and the dilaton turned on has been studied in [18]. It might be interesting to study this type of solutions and a more general twist involving $B_{\mu\nu}$ field within the framework of the matter coupled gauged supergravity. We leave these issues for future works.

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JHEP09(2015)034

A Supersymmetric AdS_6 critical points and holographic RG flows

In this appendix, we review a description of supersymmetric AdS_6 critical points with $\mathrm{SU}(2) \times \mathrm{SU}(2)$ and $\mathrm{SU}(2)_{\mathrm{diag}}$ symmetries. We consider only the $\mathrm{SU}(2)_{\mathrm{diag}}$ singlet scalar corresponding to the non-compact generator Y_s defined by

$$Y_s = Y_{11} + Y_{22} + Y_{33}\,. \tag{A.1}$$

The $\mathrm{SO}(4,3)/\mathrm{SO}(4) \times \mathrm{SO}(3)$ coset representative is accordingly parametrized by

$$L = e^{\phi Y_s}\,. \tag{A.2}$$

The scalar potential can be computed to be

$$\begin{aligned} V = & \frac{1}{16} e^{2\sigma} \left[(g_1^2 + g_2^2)[\cosh(6\phi) - 9\cosh(2\phi)] + 8(g_2^2 - g_1^2) + 8 g_1 g_2 \sinh^3(2\phi) \right] \\ & + e^{-6\sigma} m^2 - 4 e^{-2\sigma} m (g_1 \cosh^3\phi - g_2 \sinh^3\phi). \end{aligned} \tag{A.3}$$

We are mainly interested in supersymmetric critical points. Therefore, we set up the BPS equations from supersymmetry transformations of fermions given in (2.11), (2.12) and (2.13) with all but the metric and scalars σ and ϕ vanishing. The six-dimensional matric is taken to be the standard domain wall

$$ds^2 = e^{2A(r)} dx_{1,4}^2 + dr^2 \tag{A.4}$$

where $dx_{1,4}^2$ is the metric on five-dimensional Minkowski space.

With the projection condition $\gamma_{\hat{r}} \epsilon_A = \epsilon_A$, the resulting BPS equations are given by

$$\phi' = -\frac{1}{4} e^{\sigma - 3\phi} (e^{4\phi} - 1) \left[(1 + e^{2\phi}) g_1 + (1 - e^{2\phi}) g_2 \right], \tag{A.5}$$

$$\sigma' = -\frac{1}{16} e^{\sigma - 3\phi} \left[(1 + e^{2\phi})^3 g_1 + (1 - e^{2\phi})^3 g_2 \right] + \frac{3}{2} m e^{-3\sigma}, \tag{A.6}$$

$$A' = \frac{1}{16} e^{\sigma - 3\phi} \left[(1 + e^{2\phi})^3 g_1 + (1 - e^{2\phi})^3 g_2 \right] + \frac{1}{2} m e^{-3\sigma} \tag{A.7}$$

where the r-derivative is denoted by $'$. From these equations, it is clearly seen that there are two supersymmetric critical points namely

$$\phi = 0, \qquad \sigma = \frac{1}{4} \ln \left[\frac{3m}{g_1} \right], \qquad V_0 = -20 m^2 \left(\frac{g_1}{3m} \right)^{\frac{3}{2}} \tag{A.8}$$

and

$$\begin{aligned} \phi &= \frac{1}{2} \ln \left[\frac{g_1 + g_2}{g_2 - g_1} \right], \qquad \sigma = \frac{1}{4} \ln \left[-\frac{3m\sqrt{g_2^2 - g_1^2}}{g_1 g_2} \right], \\ V_0 &= -20 m^2 \left[-\frac{g_1 g_2}{3m\sqrt{g_2^2 - g_1^2}} \right]^{\frac{3}{2}}. \end{aligned} \tag{A.9}$$

This critical point is valid for $g_2 < -g_1$ when $g_1 > 0$. For $g_1 < 0$, we need to take $g_2 < g_1$. The full scalar mass spectrum at these two critical points can be found in [8].

JHEP09(2015)034

To find an RG flow solution interpolating between these two critical points, we solve the above BPS equations for $\phi(r)$, $\sigma(r)$ and $A(r)$. By defining a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{\sigma-3\phi}$, we can solve equation (A.5) for $\phi(\tilde{r})$. The solution is given implicitly by

$$\begin{aligned}\tilde{r} = &\frac{4}{g_1+g_2}\phi - \frac{1}{2g_1}\ln(1-e^{2\phi}) - \frac{1}{2g_2}\ln(1+e^{2\phi}) \\ &+\frac{(g_1-g_2)^2}{g_1g_2(g_1+g_2)}\ln\left[(1+e^{2\phi})g_1+(1-e^{2\phi})g_2\right].\end{aligned} \tag{A.10}$$

In the above solution, we have omitted an additive integration constant which can be removed by shifting the coordinate $\tilde{r}$.

Combining equations (A.5) and (A.6), we find

$$\frac{d\sigma}{d\phi} = -\frac{e^{3\phi-4\sigma}\left[24m - e^{4\sigma-3\phi}[(1+e^{2\phi})^3 g_1 + (1-e^{2\phi})^3 g_2]\right]}{4(e^{4\phi}-1)\left[(1+e^{2\phi})g_1+(1-e^{2\phi})g_2\right]} \tag{A.11}$$

which can be solved to give a solution for σ

$$\sigma = \frac{1}{4}\ln\left[\frac{e^{-\phi}(6m + C_1(e^{4\phi}-1))}{(1+e^{2\phi})g_1+(1-e^{2\phi})g_2}\right]. \tag{A.12}$$

In order to make this solution interpolate between the two critical points (A.8) and (A.9), we choose the constant C_1 to be

$$C_1 = -\frac{3m(g_1-g_2)^2}{2g_1g_2} \tag{A.13}$$

which gives the solution for σ

$$\sigma = \frac{1}{4}\ln\left[\frac{3me^{-\phi}[(1-e^{2\phi})g_1+(1+e^{2\phi})g_2]}{2g_1g_2}\right]. \tag{A.14}$$

By the same procedure, we find the solution for $A(r)$ up to an additive integration constant that can be absorbed by rescaling the coordinates in $dx^2_{1,4}$

$$A = \frac{1}{4}\phi - \frac{1}{3}\ln(1-e^{2\phi}) - \frac{1}{4}\ln(1+e^{2\phi}) + \frac{1}{3}\ln[(1+e^{2\phi})g_1+(1-e^{2\phi})g_2]. \tag{A.15}$$

It should be noted that the critical points and the flow solution have a similar structure to those in seven dimensions [19].

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JHEP09(2015)034

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JHEP09(2015)034