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Renormalized AdS action and Critical Gravity

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Abstract: It is shown that the renormalized action for AdS gravity in even spacetime dimensions is equivalent "on shell" to a polynomial of the Weyl tensor, whose first non-vanishing term is proportional to $Weyl^2$. Remarkably enough, the coupling of this last term coincides with the one that appears in Critical Gravity.

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Contents

1 Introduction

Despite the huge success of General Relativity, it is still an insufficient theory since it does not incorporate quantum phenomena. The most promising candidate up to date to achieve such a goal seems to be String Theory that predicts non-linear terms in the curvature in the low-energy limit. In four dimensions, if one wants to modify the Einstein-Hilbert action such that Ostrogradski ghosts are absent [1] while maintaining a massless spin 2 degree of freedom, the only possibility is adding a cosmological constant. Including higher-order curvature terms is another possible modification. These terms can be added to the action in a high-energy regime, rendering the theory perturbatively renormalizable in absence of the cosmological constant [2, 3], with the drawback of appearance of ghosts in the form of massive spin-2 modes.

The issue of higher-order terms in the four-dimensional action has been recently revisited from the point of view of Critical Gravity [4], where quadratic terms in the Ricci tensor and the Ricci scalar were considered on top of the Einstein-Hilbert action with negative cosmological constant. In that line of reasoning, the presence of the cosmological constant is crucial. Indeed, perturbative analysis around anti-de Sitter (AdS) vacuum leads to constraints on the parameters of the theory when the massive spin-2 mode is rendered massless. The coupling constants are further restricted by the cancelation of the scalar excitation and they get tuned with the inverse of the cosmological constant. The on-shell energy of the remaining massless spin-2 mode becomes zero and so do the mass and the entropy of the black holes of the theory.

However, the presence of a double pole structure in the theory allows for logarithmic modes, which ruins unitarity of the theory [5–10]. Proper boundary conditions can eliminate these logarithmic modes, because their fall-off is slower than the one of modes. As a result, obtained four-dimensional unitary gravitational action contains only one term, that is the square of the Weyl tensor, and also the Gauss-Bonnet term which, in four dimensions, does not contribute to the field equations [11].

The concept of Critical Gravity with quadratic terms in the curvature was generalized to higher dimensions. Apart from Ricci-squared and Ricci scalar-squared contributions, the Gauss-Bonnet term becomes dynamical in the higher-dimensional setup [11, 12]. In the case that there is a single vacuum, a suitable choice of the couplings eliminates both the scalar mode and the mass of the massive mode. This particular point in the space of parameters provides a reconciling picture to deal with the problem that, in general, the mass of the spin-2 excitation and the one of the black holes have opposite signs: both have vanishing on-shell energy in Critical Gravity. As a result, the theory can be written in terms of the Einstein-Hilbert action and the square of the Weyl tensor [13],

$$I_{\rm CG} = \frac{1}{16\pi G}\int d^Dx\sqrt{-g}\,\left(R - 2\Lambda - \gamma_{\rm CG}\,W^{\mu\nu\alpha\beta}W_{\mu\nu\alpha\beta}\right)\,, \tag{1.1}$$

where the cosmological constant expressed in terms of AdS radius is $\Lambda = -(D-1)(D-2)/2\ell^2$. Here, the Weyl-squared term reads

$$W^{\mu\nu\alpha\beta}W_{\mu\nu\alpha\beta} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - \frac{4}{D-2}\,R^{\mu\nu}R_{\mu\nu} + \frac{2}{(D-1)(D-2)}\,R^2\,, \tag{1.2}$$

with the coupling constant for Critical Gravity given by

$$\gamma_{\rm CG} = -\frac{(D-1)(D-2)}{8\Lambda(D-3)}\,. \tag{1.3}$$

Note that the flat limit ($\Lambda = 0$) is not well-defined.

Other type of Critical Gravity with quadratic curvature contributions was discussed in ref. [14], but it does not include the Riemann square term. Extensions of Critical Gravity with cubic-curvature invariants were studied in ref. [15].

In a different line of development, it was shown that the AdS action in four dimensions evaluated on-shell is [16, 17]

$$I_{\rm ren} = \frac{\ell^2}{64\pi G}\int d^4x\sqrt{-g}\,W^{\mu\nu\alpha\beta}W_{\mu\nu\alpha\beta}\,, \tag{1.4}$$

where $I_{\rm ren}$ is the action properly renormalized by the addition of counterterms [18]. The proof in ref. [16] makes use of the renormalizing effect of topological invariants, as the addition of Gauss-Bonnet is equivalent to Holographic Renormalization procedure in asymptotically AdS gravity. Then, the bulk action becomes manifestly the one of Conformal Gravity [19–22] for Einstein spacetimes.

From a different point of view, the use of appropriate boundary conditions in the infrared regime of the theory led to the same conclusion in ref. [17]. Curiously enough, the relation between the cosmological constant and the coupling of Weyl square is exactly the same one that appears in Critical Gravity, while the boundary conditions are those which eliminate the logarithmic modes [10].

The remarkable feature of eq. (1.4) is that the coefficient of $Weyl^2$ term is exactly the same as the coupling $\gamma_{\rm CG}$ that appears in the Critical Gravity action (1.1).

In this paper, we extend this result to higher even dimensions along the line of the argument presented in ref. [16]. Indeed, we show that the renormalized AdS action becomes on-shell a polynomial of the Weyl tensor, whose first term is always $\gamma_{\rm CG} Weyl^2$.

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2 Renormalized AdS action in even dimensions, Kounterterms and topological invariants

In the context of AdS/CFT correspondence, the gravity action requires the addition of a counterterm series $\mathcal{L}$, which are surface terms constructed only with intrinsic quantities of the boundary, in order to cancel the divergences that appear in the asymptotic region. In doing so, the action and its variation are functionals only of a given conformal structure at the boundary, $[g_{(0)ij}]$, which is the source of the dual CFT.

Throughout the paper, we will use the radial foliation of the manifold M

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = N^2\,(\rho)\; d\rho^2 + h_{ij}(\rho, x)\, dx^i dx^j\,, \tag{2.1}$$

where x^i and h_{ij} are the coordinates and the metric at the boundary ∂M, respectively.

In the even-dimensional case, the renormalized AdS action reads [18]

$$\begin{aligned} I_{\text{ren}} = I_{\text{EH}} - \frac{1}{8\pi G}\int\limits_{\partial M} d^{2n-1}x\,\sqrt{-h}\,K + \frac{1}{8\pi G}\int\limits_{\partial M} d^{2n-1}x\,\sqrt{-h}\,\Bigg[\frac{2n-2}{\ell} + \\ + \frac{\ell\mathcal{R}}{2(2n-3)} + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}_{ij}\mathcal{R}^{ij} - \frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right) + \cdots\Bigg]\,, \end{aligned} \tag{2.2}$$

where the second term in the first line is the Gibbons-Hawking-York (GHY) term, which ensures a well-posed action principle for $\delta h_{ij} = 0$ as a boundary condition.

We have defined the extrinsic curvature as

$$K_{ij} = \frac{1}{2N}\,\partial_\rho h_{ij}\,, \tag{2.3}$$

and $\mathcal{R}^i_{jkl}(h)$ is the intrinsic curvature.

However, in asymptotically AdS (AAdS) spacetimes, the behavior of δh_{ij} at the boundary is divergent such that –strictly speaking– the counterterms are also needed for the variational problem [23]. Variations of the extrinsic curvature are equally ill-defined at the boundary, because the leading order in the expansion of δK_{ij} is the same as the leading order in δh_{ij} due to the conformal structure of the boundary. This fact motivates the inclusion of counterterms which depend on the extrinsic curvature instead of the standard series given above.

Indeed, extrinsic counterterms for Einstein-Hilbert AdS gravity were proposed in refs. [24] and [25].

In $D = 2n$ dimensions, we consider the AdS action, renormalized with the addition of this alternative counterterm series (a.k.a. Kounterterms)

$$\tilde{I}_{\text{ren}} = I_{\text{EH}} + c_{2n-1}\int\limits_{\partial M} d^{2n-1}x\, B_{2n-1}(h, K, \mathcal{R})\,, \tag{2.4}$$

where the boundary terms in the even-dimensional case are given by

$$\begin{aligned} B_{2n-1}(h, K, \mathcal{R}) = 2n\sqrt{-h}\int\limits_0^1 dt\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\,K^{j_1}_{i_1}\left(\frac{1}{2}\,\mathcal{R}^{j_2j_3}_{i_2i_3}(h) - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots \\ \cdots\times\left(\frac{1}{2}\,\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}}(h) - t^2 K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right)\,, \end{aligned} \tag{2.5}$$

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with a coefficient given in terms of the AdS radius as

$$c_{2n-1} = \frac{1}{16\pi G}\,\frac{(-1)^n\ \ell^{2n-2}}{n\,(2n-2)!}\,. \tag{2.6}$$

Above, the totally antisymmetric Kronecker delta of order m is defined as the determinant of single-index Kronecker deltas.

A relation between this extrinsic counterterm series and the standard one was sketched in ref. [16]. Here, a comparison to standard renormalization procedure is given in more detail.

We start by adding and subtracting the GHY term,

$$\tilde{I}_{\rm ren} = I_{\rm EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^{2n-1}x\sqrt{-h}\,K + \int\limits_{\partial M} d^{2n-1}x\,\mathcal{L}(h,K,\mathcal{R})\,, \tag{2.7}$$

where

$$\mathcal{L}(h,K,\mathcal{R}) = c_{2n-1}B_{n-1} - GHY. \tag{2.8}$$

It is easier to manipulate $\mathcal{L}(h,K,\mathcal{R})$ if we write down the last term in the above relation as a totally antisymmetric object, that is,

$$\begin{aligned}\mathcal{L}(h,K,\mathcal{R}) = \frac{(-1)^n\,\ell^{2n-2}\sqrt{-h}}{8\pi G(2n-2)!}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\,K^{j_1}_{i_1}\int\limits_0^1 dt\ \Bigg[\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots \\ \cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2K^{j_{2n-2}}_{i_{2n-2}}K^{j_{2n-1}}_{i_{2n-1}}\right) + \frac{(-1)^n}{\ell^{2n-2}}\,\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\Bigg]\,.\end{aligned} \tag{2.9}$$

On the other hand, for any AAdS spacetime, the asymptotic expansion of the extrinsic curvature is given by

$$K^i_j = \frac{1}{\ell}\,\delta^i_j + \ell S^i_j(h) + \mathcal{O}(\mathcal{R}^2)\,, \tag{2.10}$$

up to second-derivative terms. Here, the quantity S^i_j is the Schouten tensor of the boundary metric, that is,

$$S^i_j(h) = \frac{1}{2n-3}\left(\mathcal{R}^i_j(h) - \frac{1}{4(n-1)}\,\delta^i_j\mathcal{R}(h)\right). \tag{2.11}$$

This expansion implies that $\mathcal{L}(h,K,\mathcal{R})$ is expressible in terms of intrinsic quantities of the boundary at least up to quadratic terms in the curvature. Direct substitution of relation (2.10) in the general boundary term (2.9) produces a rather complicated expression

$$\begin{aligned}\mathcal{L}(h,K,\mathcal{R}) = \frac{(-1)^n\ell^{2n-2}}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\left(\frac{1}{\ell}\,\delta^{j_1}_{i_1} + \ell S^{j_1}_{i_1} + \cdots\right)\times \\ \times\int\limits_0^1 dt\ \Bigg[\left(-\frac{t^2}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3} + \frac{1}{2}\left(\mathcal{R}^{j_2j_3}_{i_2i_3} - 4t^2S^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) - t^2\ell^2S^{j_2}_{i_2}S^{j_3}_{i_3} + \cdots\right)\times\cdots \\ \times\left(-\frac{t^2}{\ell^2}\,\delta^{j_{2n-2}}_{i_{2n-2}}\delta^{j_{2n-1}}_{i_{2n-1}} + \frac{1}{2}\left(\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - 4t^2S^{j_{2n-2}}_{i_{2n-2}}\delta^{j_{2n-1}}_{i_{2n-1}}\right) - t^2\ell^2S^{j_{2n-2}}_{i_{2n-2}}S^{j_{2n-1}}_{i_{2n-1}} + \cdots\right) \\ + \frac{(-1)^n}{\ell^{2n-2}}\,\delta^{j_1}_{i_1}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\Bigg]\,.\end{aligned} \tag{2.12}$$

Using the definition of the Weyl tensor of the boundary metric in terms of the Riemann and the Schouten tensor, and the skew symmetry of its indices, whenever the boundary Weyl tensor enters in a totally antisymmetric formula as the one above, we have that

$$\delta^{[\cdots i_p i_{p+1}\cdots]}_{[\cdots j_p j_{p+1}\cdots]}\,\mathcal{W}^{j_p j_{p+1}}_{i_p i_{p+1}} = \delta^{[\cdots i_p i_{p+1}\cdots]}_{[\cdots j_p j_{p+1}\cdots]}\left(\mathcal{R}^{j_p j_{p+1}}_{i_p i_{p+1}} - 4S^{j_p}_{i_p}\,\delta^{j_{p+1}}_{i_{p+1}}\right). \tag{2.13}$$

We use the last relation to eliminate the dependence of the Riemann tensor in eq. (2.12), such that it can be rewritten as

$$\begin{aligned}
\mathcal{L} = {} & \frac{(-1)^n\,\ell^{2n-2}}{8\pi G(2n-2)!}\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\left(\frac{1}{\ell}\delta^{j_1}_{i_1} + \ell S^{j_1}_{i_1} + \ldots\right)\times \\
& \times \int\limits_0^1 dt\,\left[\left(-\frac{t^2}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3} + \frac{1}{2}\left(\mathcal{W}^{j_2 j_3}_{i_2 i_3} + 4\left(1-t^2\right)S^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) - t^2\ell^2 S^{j_2}_{i_2}S^{j_3}_{i_3} + \cdots\right)\times\cdots\right. \\
& \times\left(-\frac{t^2}{\ell^2}\,\delta^{j_{2n-2}}_{i_{2n-2}}\delta^{j_{2n-1}}_{i_{2n-1}} + \frac{1}{2}\left(\mathcal{W}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} + 4\left(1-t^2\right)S^{j_{2n-2}}_{i_{2n-2}}\delta^{j_{2n-1}}_{i_{2n-1}}\right) - t^2\ell^2 S^{j_{2n-2}}_{i_{2n-2}}S^{j_{2n-1}}_{i_{2n-1}} + \cdots\right) \\
& \left. + \frac{(-1)^n}{\ell^{2n-2}}\,\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\right].
\end{aligned} \tag{2.14}$$

Notice that the term $\left(-\frac{t^2}{\ell^2}\,\delta\delta+\frac{1}{2}\left(\mathcal{W}+4\left(1-t^2\right)S\delta\right)-t^2\ell^2 SS+\cdots\right)$ appears $(n-1)$ times.

The key point to generate the standard counterterm series from the above formula is to identify the contributions coming from $\mathcal{L}$ as an expansion in powers of the boundary curvature. Symbolically, the lowest-order terms in the expansion of the trinomial to the $(n-1)$-th power are

$$\begin{aligned}
& \left(-\frac{t^2}{\ell^2}\delta\delta + \frac{1}{2}\left(\mathcal{W} + 4\left(1-t^2\right)S\delta\right) - t^2\ell^2 SS + \cdots\right)^{n-1} \\
= {} & \left(-\frac{t^2}{\ell^2}\right)^{n-1}(\delta)^{2n-2} + \frac{(n-1)}{2}\left(-\frac{t^2}{\ell^2}\right)^{n-2}(\delta)^{2n-4}\left(\mathcal{W} + 4\left(1-t^2\right)S\delta\right) \\
& + \frac{(n-1)(n-2)}{8}\left(-\frac{t^2}{\ell^2}\right)^{n-3}(\delta)^{2n-6}\left(\mathcal{W} + 4\left(1-t^2\right)S\delta\right)^2 \\
& - (n-1)\left(-\frac{t^2}{\ell^2}\right)^{n-2} t^2\,(\delta)^{2n-4}\,SS + \cdots.
\end{aligned} \tag{2.15}$$

The term with no curvatures comes just from the multiplication of Kronecker deltas,[1]

$$\begin{aligned}
\mathcal{O}(1) &= \frac{(-1)^n}{8\pi G(2n-2)!}\frac{\sqrt{-h}}{\ell}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\delta^{j_1}_{i_1}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\int\limits_0^1 dt\,\left[\left(-t^2\right)^{n-1} + (-1)^n\right] \\
&= \frac{(2n-1)}{8\pi G}\frac{\sqrt{-h}}{\ell}\left(1 - \frac{1}{2n-1}\right) \\
&= \frac{\sqrt{-h}}{8\pi G}\,\frac{2n-2}{\ell}.
\end{aligned} \tag{2.16}$$

[1]If N is the range of indices, a contraction of k indices in the Kronecker delta of order m produces a delta of order $m-k$,

$$\delta^{[i_1\cdots i_k\cdots i_m]}_{[j_1\cdots j_k\cdots j_m]}\,\delta^{j_1}_{i_1}\cdots\delta^{j_k}_{i_k} = \frac{(N-m+k)!}{(N-m)!}\,\delta^{[i_{k+1}\cdots i_m]}_{[j_{k+1}\cdots j_m]}, \quad 1\le k\le m\le N.$$

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The term linear in the curvature comes from linear terms in the Schouten tensor, when the rest of the indices are saturated with Kronecker deltas,

$$
\begin{aligned}
\mathcal{O}(\mathcal{R}) &= \frac{(-1)^n\,\ell}{8\pi G(2n-2)!}\,\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}S^{j_1}_{i_1}\delta^{j_2}_{i_2}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\times\\
&\quad\times\int_0^1 dt\,\left[(-1)^n\left(1-t^{2n-2}\right)+2(-1)^{n-2}(n-1)t^{2n-4}\left(1-t^2\right)\right]\\
&= \frac{\ell}{8\pi G}\,\sqrt{-h}\,S\int_0^1 dt\,\left[1-(2n-3)t^{2n-2}+2(n-1)t^{2n-4}\right]\\
&= \frac{\ell}{8\pi G}\,\sqrt{-h}\,S\,\frac{2(n-1)}{2n-3}=\frac{\sqrt{-h}}{8\pi G}\,\frac{\ell\mathcal{R}}{2(2n-3)}\,.
\end{aligned}
\tag{2.17}
$$

Terms linear in the Weyl tensor vanish because they involve traces of it.

One can show that $\mathcal{O}(\mathcal{R}^2)$ terms in the expansion of the extrinsic curvature will not affect quadratic-curvature terms in $\mathcal{L}(h,K,\mathcal{R})$. On the other hand, contractions of the Weyl tensor with a single Schouten tensor will again involve traces of $\mathcal{W}$, such that products between S and $\mathcal{W}$ are not present. Summing up the rest of the quadratic contributions in $\mathcal{R}$, we arrive at the expression

$$
\begin{aligned}
\mathcal{O}(\mathcal{R}^2) &= \frac{(-1)^n\,\ell^3}{16\pi G(2n-3)!}\,\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}S^{j_1}_{i_1}S^{j_2}_{i_2}\delta^{j_3}_{i_3}\cdots\delta^{j_{2n-1}}_{i_{2n-1}}\times\\
&\quad\times\int_0^1 dt\,\left(-t^2\right)^{n-3}\left[-2t^2\left(1-t^2\right)+2(n-2)\left(1-t^2\right)^2+t^4\right]\\
&= \frac{\ell^3}{16\pi G(2n-5)}\,\sqrt{-h}\,\delta^{[i_1 i_2]}_{[j_1 j_2]}S^{j_1}_{i_1}S^{j_2}_{i_2}\,.
\end{aligned}
\tag{2.18}
$$

In order to obtain the standard form of the curvature-squared counterterms, we use the identity

$$
\begin{aligned}
\delta^{[i_1 i_2]}_{[j_1 j_2]}S^{j_1}_{i_1}S^{j_2}_{i_2} &= S^2-S^{ij}S_{ij}\\
&= -\frac{1}{(2n-3)^2}\left(\mathcal{R}_{ij}\mathcal{R}^{ij}-\frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\,,
\end{aligned}
\tag{2.19}
$$

such that

$$
\mathcal{O}(\mathcal{R}^2)=\frac{\sqrt{-h}}{8\pi G}\,\frac{\ell^3}{2(2n-3)^2(2n-5)}\left(\mathcal{R}_{ij}\mathcal{R}^{ij}-\frac{(2n-1)}{4(2n-2)}\mathcal{R}^2\right)\,.
\tag{2.20}
$$

We also provide the expression for the Weyl-squared term, which is

$$
\begin{aligned}
\mathcal{O}(\mathcal{W}^2) &= \frac{(-1)^n\ell^3}{256\pi G(2n-3)(2n-5)!}\,\sqrt{-h}\,\delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]}\\
&\quad\times\int_0^1 dt\,\left(-t^2\right)^{n-3}\mathcal{W}^{j_1 j_2}_{i_1 i_2}\mathcal{W}^{j_3 j_4}_{i_3 i_4}\,\delta^{i_5}_{j_5}\cdots\delta^{i_{2n-1}}_{j_{2n-1}}
\end{aligned}
$$

$$= -\frac{\ell^3}{256\pi G(2n-3)(2n-5)}\sqrt{-h}\,\delta^{[i_1 i_2 i_3 i_4]}_{[j_1 j_2 j_3 j_4]}\mathcal{W}^{j_1 j_2}_{i_1 i_2}\mathcal{W}^{j_3 j_4}_{i_3 i_4}$$

$$= -\frac{\ell^3}{64\pi G(2n-3)(2n-5)}\sqrt{-h}\,\mathcal{W}^{ijkl}\mathcal{W}_{ijkl}\,. \tag{2.21}$$

This is just for the purpose of completing the computation, because the quadratic piece in the boundary Weyl tensor has a faster asymptotic fall-off, such that it is not considered to be part of the standard counterterm series.

In sum, in this section we have provided a nontrivial checking that the action defined by the addition of Kounterterms in eq. (2.4) is equal to the renormalized AdS action,

$$\tilde{I}_{\text{ren}} = I_{\text{ren}}. \tag{2.22}$$

On the other hand, the boundary term B_{2n-1} appears as a boundary correction to the Euler characteristic $\chi(M)$ in the Euler theorem in $2n$ dimensions,

$$\int\limits_M \mathcal{E}_{2n} = (4\pi)^n\, n!\,\chi(M) + \int\limits_{\partial M} B_{2n-1}(h,K,\mathcal{R})\,, \tag{2.23}$$

where $\mathcal{E}_{2n}$ is the Euler term in that dimension. This simply means that the GHY term plus the standard counterterm series in I_{ren} can be generated from the addition of a single topological invariant in the bulk.

In the next section, we exploit this remarkable feature of I_{ren} to work out a general property of the on-shell value of the renormalized AdS action in even dimensions.

3 Renormalized action and Critical Gravity

Let us consider the Einstein-Hilbert action with negative cosmological constant in $D = 2n$ dimensions,

$$I_{\text{ren}} = \frac{1}{16\pi G}\int d^{2n}x\sqrt{-g}\left[R - 2\Lambda + \alpha_{2n}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\,R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\right]. \tag{3.1}$$

It was shown in ref. [16] that the addition of the Euler term to the even-dimensional AdS gravity action is equivalent to the Holographic Renormalization program if the coupling constant is chosen as

$$\alpha_{2n} = (-1)^n\frac{\ell^{2n-2}}{2^n n(2n-2)!}\,. \tag{3.2}$$

That is the reason why, from now on, we will call it renormalized action. We can cast it in the alternative form,

$$I_{\text{ren}} = \frac{1}{2^{n+4}\pi G(2n-2)!}\int d^{2n}x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}\,\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}\right.$$
$$\left.+\frac{n-1}{n\ell^2}\,\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]} + \frac{(-1)^n}{n}\,\ell^{2n-2}\,R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\right]. \tag{3.3}$$

Now, we use the fact that, on-shell, the Weyl tensor is

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\,\delta^{[\alpha\beta]}_{[\mu\nu]}\,, \tag{3.4}$$

such that we replace this relation in I_{ren} and we get

$$
\begin{aligned}
I_{\text{ren}} = {} & \frac{1}{2^{n+4}\pi G(2n-2)!}\int d^{2n}x\,\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\times \\
& \times\left[W^{\mu_1\mu_2}_{\nu_1\nu_2}\,\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}-\frac{1}{n\ell^2}\,\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}+\right. \\
& \left.+\,\frac{(-1)^n}{n}\,\ell^{2n-2}\left(W^{\mu_1\mu_2}_{\nu_1\nu_2}-\frac{1}{\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\right)\cdots\left(W^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}-\frac{1}{\ell^2}\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}\right)\right]. \qquad (3.5)
\end{aligned}
$$

Expanding the binomial in the last line, we obtain

$$
\begin{aligned}
& \frac{(-1)^n}{n}\,\ell^{2n-2}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\left(W^{\mu_1\mu_2}_{\nu_1\nu_2}-\frac{1}{\ell^2}\,\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\right)\cdots\left(W^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}-\frac{1}{\ell^2}\,\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}\right) \\
&= \delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\left(\frac{1}{n\ell^2}\,\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}-W^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}+\right. \\
&\quad \left.+\,\frac{\ell^2}{2}\,(n-1)W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}\right)+\mathcal{O}(W^3)\,. \qquad (3.6)
\end{aligned}
$$

The first term in the above expansion cancels the second term in the first line of eq. (3.5). All terms linear in the Weyl tensor vanish because they involve traces of it. As a consequence, the first non-vanishing contribution in the renormalized action is quadratic in W,

$$
I_{\text{ren}} = \frac{\ell^2}{2^{n+6}\pi G(2n-3)!}\int d^{2n}x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}+\mathcal{O}(W^3). \qquad (3.7)
$$

We can also write it as

$$
I_{\text{ren}} = \frac{\gamma_{\text{CG}}}{16\pi G}\int d^{2n}x\sqrt{-g}\,W^{\alpha\beta\mu\nu}\,W_{\alpha\beta\mu\nu}+\mathcal{O}(W^3)\,, \qquad (3.8)
$$

because the coupling,

$$
\gamma_{\text{CG}} = \frac{\ell^2}{4(2n-3)} = -\frac{(2n-1)(2n-2)}{8\Lambda(2n-3)}\,, \qquad (3.9)
$$

is the same one that appears in the Critical Gravity action (1.1).

4 Conclusions

We have shown that, in even spacetime dimensions, the renormalized AdS action is on-shell equivalent to a polynomial of the Weyl tensor, whose first nonvanishing contribution is $Weyl^2$. The coupling of this term is the same as the one that appears in Critical Gravity, where $Weyl^2$ term is added on top of the Einstein-Hilbert Lagrangian.

We stress that this equivalence is at the level of the action evaluated for Einstein spacetimes and, by no means, we imply a dynamic equivalence between the corresponding theories.

We also emphasize that the fact $I_{\text{ren}} = \frac{\gamma_{\text{CG}}}{16\pi G}\,Weyl^2+\cdots$ is a consequence of a *topological* regularization of AAdS gravity. This can only be seen once one shows that the addition of

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topological invariants and Holographic Renormalization program in even-dimensional AdS gravity provide the same result.

Because of this argument, it is difficult to think of a similar result for odd-dimensional case.

We can go back to four-dimensional example worked out by Lu and Pope in ref. [4], in order to see what the above claim implies in that case. In 4D Critical Gravity, the action has the form

$$I_{\rm CG} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\left(R+\frac{1}{\alpha}\right)+\alpha R^2-3\alpha R_{\mu\nu}R^{\mu\nu}\right]. \tag{4.1}$$

When the cosmological term adopts the standard value of Einstein-AdS gravity ($\alpha=\ell^2/6$), the action can be rewritten as

$$I_{\rm CG} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\left(R+\frac{6}{\ell^2}\right)-\frac{\ell^2}{2}\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)\right]. \tag{4.2}$$

Quadratic terms in the curvature are given just as the difference between $Weyl^2$ and Gauss-Bonnet terms

$$I_{\rm CG} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\left(R+\frac{6}{\ell^2}\right)-\frac{\ell^2}{4}\left(W^2-GB\right)\right]. \tag{4.3}$$

The particular coupling of Gauss-Bonnet term, as originally pointed out in ref. [16], leads to the renormalized AdS action given by eq. (3.3), such that the total action for Critical Gravity is

$$I_{\rm CG} = I_{\rm ren} - \frac{\ell^2}{64\pi G}\int d^4x\,\sqrt{-g}\,W^2\,. \tag{4.4}$$

Notice that this automatically implies that $I_{\rm CG}=0$ for Einstein spaces, what seems to indicate that the critical point defines a new vacuum state of the theory.

In higher even dimensions, going beyond quadratic terms in the Weyl tensor in the expansion of the renormalized action (3.5), we get

$$\begin{aligned} I_{\rm ren} = {} & \frac{\gamma_{\rm CG}}{16\pi G}\int d^{2n}x\sqrt{-g}\,W^{\alpha\beta\mu\nu}W_{\alpha\beta\mu\nu} - \\ & -\frac{\ell^4\,(n-2)}{2^{n+6}3\pi G(2n-3)!}\int d^{2n}x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2n}]}_{[\mu_1\cdots\mu_{2n}]}\Bigg[W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6}\,\delta^{[\mu_7\mu_8]}_{[\nu_7\nu_8]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]} \\ & +6\,(n-3)!\sum_{p\geq 4}^{n}\frac{(-1)^{p+1}\,\ell^{2p-6}}{p!\,(n-p)!}W^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots W^{\mu_{2p-1}\mu_{2p}}_{\nu_{2p-1}\nu_{2p}}\,\delta^{[\mu_{2p+1}\mu_{2p+2}]}_{[\nu_{2p+1}\nu_{2p+2}]}\cdots\delta^{[\mu_{2n-1}\mu_{2n}]}_{[\nu_{2n-1}\nu_{2n}]}\Bigg]. \end{aligned} \tag{4.5}$$

Note that this action is not Weyl invariant even though it is expressed on-shell in terms of the Weyl tensor. Namely, under the Weyl transformations $g_{\mu\nu}\to\Omega^2(x)\,g_{\mu\nu}$, the tensor $W^{\mu}{}_{\nu\alpha\beta}$ is invariant, but $W^{\mu\nu}_{\alpha\beta}$ changes as $W^{\mu\nu}_{\alpha\beta}\to\Omega^{-2}W^{\mu\nu}_{\alpha\beta}$. Taking into consideration that the volume element also transforms as $d^{2n}x\sqrt{-g}\to d^{2n}x\sqrt{-g}\,\Omega^{2n}$, we find that the p-th term of the polynomial in W transforms with the weight $n-p$,

$$\left(\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2p}]}_{[\mu_1\cdots\mu_{2p}]}W^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots W^{\mu_{2p-1}\mu_{2p}}_{\nu_{2p-1}\nu_{2p}}\right)\to\Omega^{2n-2p}\left(\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_{2p}]}_{[\mu_1\cdots\mu_{2p}]}W^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots W^{\mu_{2p-1}\mu_{2p}}_{\nu_{2p-1}\nu_{2p}}\right). \tag{4.6}$$

In particular, in $D=4$, there is only one term with $n=p=2$, thus this theory is Weyl invariant.

The expression (4.5) can be rearranged as

$$I_{\rm ren}=\frac{\gamma_{\rm CG}}{16\pi G}\int d^{2n}x\sqrt{-g}\Bigg[W^{\alpha\beta\mu\nu}W_{\alpha\beta\mu\nu}+a\,\left(W^{\mu\nu}_{\alpha\beta}W^{\alpha\beta}_{\lambda\rho}W^{\lambda\rho}_{\mu\nu}-4W^{\mu\nu}_{\rho\beta}W^{\alpha\beta}_{\nu\lambda}W^{\lambda\rho}_{\mu\alpha}\right)+ \\ +\sum_{p\geq4}^{n}b_p\,\delta^{[\nu_1\cdots\nu_{2p}]}_{[\mu_1\cdots\mu_{2p}]}\,W^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots W^{\mu_{2p-1}\mu_{2p}}_{\nu_{2p-1}\nu_{2p}}\Bigg]\,, \tag{4.7}$$

using the identity for the cubic term in W,

$$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\,W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6}=2^4\left(W^{\mu\nu}_{\alpha\beta}W^{\alpha\beta}_{\lambda\rho}W^{\lambda\rho}_{\mu\nu}-4W^{\mu\nu}_{\alpha\lambda}W^{\alpha\beta}_{\mu\rho}W^{\lambda\rho}_{\nu\beta}\right)\,. \tag{4.8}$$

The corresponding couplings of $Weyl^3$ and all higher-order terms are

$$\begin{aligned} a &= 2^4 b_3=-\frac{\ell^2}{3\,(2n-5)}\,, \\ b_p &= (-1)^p\,\ell^{2p-4}\,\frac{(n-2)!\,(2n-2p)!}{2^{p-1}\,(2n-4)!\,p!\,(n-p)!}\,,\quad p\geq 3\,. \end{aligned} \tag{4.9}$$

For the purpose of comparison with Critical Gravity with cubic-curvature contributions developed in ref. [15], we use the definition of the Weyl tensor

$$W^{\mu\nu}_{\alpha\beta}=R^{\mu\nu}_{\alpha\beta}-\left(\delta^\mu_\alpha S^\nu_\beta-\delta^\nu_\alpha S^\mu_\beta-\delta^\mu_\beta S^\nu_\alpha+\delta^\nu_\beta S^\mu_\alpha\right)\,, \tag{4.10}$$

in terms of the spacetime Schouten tensor

$$S^\nu_\mu=\frac{1}{D-2}\left(R^\nu_\mu-\frac{1}{2\,(D-1)}\,\delta^\nu_\mu\,R\right)\,. \tag{4.11}$$

In doing so, we obtain

$$\begin{aligned} \frac{1}{16}\,\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\,W^{\mu_1\mu_2}_{\nu_1\nu_2}W^{\mu_3\mu_4}_{\nu_3\nu_4}W^{\mu_5\mu_6}_{\nu_5\nu_6} &= R^{\mu\nu}_{\alpha\beta}R^{\alpha\beta}_{\lambda\rho}R^{\lambda\rho}_{\mu\nu}-4R^{\mu\nu}_{\alpha\beta}R^{\alpha\lambda}_{\mu\rho}R^{\beta\rho}_{\nu\lambda} \\ &\quad -\frac{36}{D-2}\,R^{\mu\nu}_{\alpha\beta}R^{\alpha\lambda}_{\mu\nu}R^{\beta}_{\lambda}\frac{18}{(D-1)\,(D-2)}\,RR^{\mu\nu}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} \\ &\quad +\frac{12\,(D+4)}{(D-2)^2}\,R^{\mu\nu}_{\alpha\beta}R^\alpha_\mu R^\beta_\nu+\frac{8\,(7D-8)}{(D-2)^3}\,R^\mu_\nu R^\nu_\lambda R^\lambda_\mu \\ &\quad -\frac{12\,\left(D^2+9D-16\right)}{(D-1)\,(D-2)^3}\left(RR^\mu_\nu R^\nu_\mu-\frac{1}{3\,(D-1)}\,R^3\right)\,. \end{aligned} \tag{4.12}$$

Found cubic gravity belongs to a class of cubic critical gravities discussed in ref. [15]. There, all gravitational theories with up to cubic curvature terms were classified based on the requirement of unitarity around (A)dS vacuum. However, conditions of criticality (removal of the massive spin-0 mode and also that the spin-2 mode be massless) fix only two of eight cubic coupling constants in terms of the others.

Regarding the result given by eq. (3.8), at this moment, we cannot further understand the implications of this remarkable feature of the renormalized AdS action.

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However, holographic renormalization method applied to the theory around the critical point in the action of Critical Gravity may give some insight on this problem. AAdS spaces which are solutions of the Einstein equations are described by the Fefferman-Graham metric [26]. Higher-derivative terms in the field equations imply the existence of new holographic sources at the boundary, which should appear at a given order in the asymptotic expansion of the metric. Significant progress towards a holographic description of Critical Gravity has been made in four dimensions in ref. [27], where logarithmic modes play an important role. The main result presented here, eq. (3.8), seems to indicate the exact cancelation of Einstein modes in the metric of a spacetime which is a solution to Critical Gravity. Therefore, the residual dynamics should be given just in terms of the new sources of the full theory.

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