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On scalaron decay via the trace of energy-momentum tensor

Ayuki Kamada

Center for Theoretical Physics of the Universe,
Institute for Basic Science (IBS), Daejeon 34126, Korea

E-mail: ayuki.kamada@gmail.com

Abstract: In some inflation scenarios such as R^2 inflation, a gravitational scalar degrees of freedom called scalaron is identified as inflaton. Scalaron linearly couples to matter via the trace of energy-momentum tensor. We study scenarios with a sequestered matter sector, where the trace of energy-momentum tensor predominantly determines the scalaron coupling to matter. In a sequestered setup, heavy degrees of freedom are expected to decouple from low-energy dynamics. On the other hand, it is non-trivial to see the decoupling since scalaron couples to a mass term of heavy degrees of freedom. Actually, when heavy degrees of freedom carry some gauge charge, the amplitude of scalaron decay to two gauge bosons does not vanish in the heavy mass limit. Here a *quantum* contribution to the trace of energy-momentum tensor plays an essential role. This quantum contribution is known as trace anomaly or Weyl anomaly. The trace anomaly contribution from heavy degrees of freedom cancels with the contribution from the *classical* scalaron coupling to a mass term of heavy degrees of freedom. We see how trace anomaly appears both in the Fujikawa method and in dimensional renormalization. In dimensional renormalization, one can evaluate the scalaron decay amplitude in principle at all orders, while it is unclear how to process it beyond the one-loop level in the Fujikawa method. We consider scalaron decay to two gauge bosons via the trace of energy-momentum tensor in quantum electrodynamics with scalars and fermions. We evaluate the decay amplitude at the leading order to demonstrate the decoupling of heavy degrees of freedom.

Keywords: Anomalies in Field and String Theories, Cosmology of Theories beyond the SM

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Contents

1 Introduction

Inflation is a cosmological paradigm that solves issues of big bang cosmology, such as the horizon, flatness, and monopole problems [1–8]. It also provides an almost scale-invariant density contrast over homogeneous and isotropic background [9–15]. The inflation paradigm has been strongly supported by the deviation of the scalar spectral index from unity observed in cosmic microwave background anisotropies [16]. Among various inflation models [17], R^2 inflation (R: Ricci scalar) [2, 18–22] is a good benchmark. Its plateau potential predicts a tensor-to-scalar ratio sufficiently small to be consistent with the Planck data [16] but within a reach of future searches of cosmic microwave background B-mode anisotropies [23–25].

Identifying a reheating temperature T_R in R^2 inflation is important for theoretical prediction of the scalar spectral index and tensor-to-scalar ratio [26]. It also plays an important role in production mechanisms of dark matter and baryon asymmetry [27, 28]. For example, $T_R \gtrsim 10^9$ GeV (e.g., refs. [29–32]) is required for thermal leptogenesis [33] to work. Furthermore it is imprinted in the primordial gravitational wave spectrum when the energy density of Universe is transferred from oscillating inflaton to radiation [34]. Such an imprint could be seen in ultimate gravitational wave experiments [35].

In $f(R)$ gravity including R^2 inflation, a gravitational scalar degrees of freedom called scalaron is identified as inflaton. To determine the reheating temperature, we need to study scalaron coupling to matter. $f(R)$ gravity generically can be rewritten as a scalar-tensor theory through a Weyl transformation (local rescaling of the metric and fields) that is a function solely of scalaron [19, 36]. This Weyl transformation manifests scalaron coupling to

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the trace of matter energy-momentum tensor in the *scalaron frame* [37].[1] Similar situations can also be seen in a broader class of inflation models based on a scalar-tensor theory. One example is $f(\sigma)R$ gravity (let us also refer to a scalar field σ as scalaron) [38–42]. The trace of energy-momentum tensor predominantly determines scalaron coupling to matter, when scalaron direct coupling to matter in the Jordan frame is suppressed for some reason. In this paper, we consider such scenarios where a matter sector communicates with the scalaron sector only gravitationally in the Jordan frame.

Scalaron decay[2] is dominated by decay channels to two scalars if their non-minimal coupling to Ricci curvature deviates from the conformal coupling. With the conformally coupled scalars, loop-induced decay to two gauge bosons becomes relevant. The decay amplitude is proportional to the β function of the corresponding gauge coupling. Ref. [34] uses the β function at the energy scale of the scalaron mass ($\simeq 3 \times 10^{13}\,\mathrm{GeV}$ for the R^2 inflation model), which virtually counts *light* degrees of freedom. Refs. [43, 44], which study inflaton decay in $f(\sigma)R$ gravity, also virtually counts *light* degrees of freedom.

On the other hand, it is less manifest at first sight if heavy degrees of freedom do not contribute to scalaron decay. In the scalaron frame, scalaron couples to matter via mass terms. Loop-induced decay to two gauge bosons does not vanish in the heavy mass limit. It leaves scalaron coupling to gauge bosons for low-energy effective theory.[3] Meanwhile the decoupling of heavy degrees of freedom may be apparent in the Jordan frame, where scalaron does not have any direct coupling to matter. Matter fields decouple in the heavy mass limit without leaving any non-decoupling effects for low-energy effective theory. This raises an issue on the "frame equivalence" (see also ref. [58] for a related discussion).

What plays an essential role is a *quantum* contribution to the trace of energy-momentum tensor, known as Weyl anomaly or trace anomaly.[4] Trace anomaly is intensively investigated both in the flat spacetime [59–70] and in a curved spacetime [71–80] (see also ref. [81] for a review). The trace anomaly contribution from heavy degrees of freedom cancels with the contribution from the *classical* scalaron coupling to a mass term of heavy degrees of freedom. Because of the cancellation between classical and quantum contributions, the scalaron coupling to matter via the trace of energy-momentum tensor is ultraviolet insensitive.[5]

[1]Note that the Weyl transformation consists solely of scalaron. Therefore this is not the Einstein frame since scalar fields in a matter section (not scalaron) can still have a non-minimal coupling to the Ricci scalar.

[2]In this paper, we consider *perturbative* scalaron decay. We assume that non-perturbative effects associated with non-zero field values of scalaron and matter scalars are negligible. This could be true since decay proceeds only gravitationally and occurs long after inflation.

[3]This is the case for Higgs [45, 46] or axion [47–50] (see also refs. [51, 52] and [53, 54] for popular ultraviolet realizations). One famous example is coupling of Higgs [45, 46] or axion [47–50] (see also refs. [51, 52] and [53, 54] for popular ultraviolet realizations) to light gauge bosons such as photon or gluon in low-energy effective theory. With this observation, ref. [55] argues that one should count *heavy* degrees of freedom as well as *light* degrees of freedom for the β function. This result is taken from ref. [56], which studies scalaron decay in $f(\sigma)R$ gravity. A similar calculation on scalaron coupling to the standard model particles has been made in ref. [57]. Their stance on the frame equivalence is different from the present study.

[4]The trace of energy-momentum tensor and trace anomaly are often not distinguished. In this paper we use the former to refer to the whole (classical + quantum) contribution, while we use the latter to refer to only a quantum contribution.

[5]This is analogous to an anomaly mediation contribution to a sparticle mass in supersymmetric theories [82, 83], which boasts its ultraviolet insensitivity. A quantum contribution to a gaugino mass from

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This paper is organized as follows. In the next section we describe scenarios with a sequestered matter sector, where scalaron couples to matter predominantly via the trace of energy-momentum tensor. We demonstrate how the trace of energy-momentum tensor receives a quantum contribution, by employing the Fujikawa method [87–89] (see also ref. [90] for a comprehensive summary). The Fujikawa method is illustrating trace anomaly, but not convenient in practical calculations such as perturbative renormalization. Instead, in section 3, we use dimensional renormalization, i.e., the minimal subtraction (MS) or modified minimal subtraction ($\overline{\text{MS}}$) scheme [91–93], where we can compute perturbative renormalization in principle at all orders.[6] We see how trace anomaly appears in dimensional renormalization. Furthermore, we compute the leading amplitude of scalaron decay into two gauge boson in quantum electrodynamics (QED) with scalars and fermions. We see that heavy degrees of freedom do not contribute to the amplitude. section 4 is devoted to a summary and further remarks. We use a notation of ref. [94], where the four-dimension metric has the signature of $(+,-,-,-)$.

2 Gravitational coupling of scalaron to matter

We consider a class of inflation models where a scalaron sector communicates with a matter sector only gravitationally as

$$S_{\text{grav}}\left[g'_{\mu\nu},\sigma'\right]+S_{\text{mat}}\left[\{\phi'_i\},g'_{\mu\nu};\{\lambda_a\}\right]\,, \tag{2.1}$$

in the Jordan frame. $g_{\mu\nu}$ is the metric and a prime denotes the quantity in the Jordan frame. $\{\phi_i\}$ and $\{\lambda_a\}$ collectively denote matter fields and parameters, respectively. Note that scalaron in $f(R)$ gravity is not manifest in the Jordan frame. For example, in the R^2 inflation model,

$$S_{\text{grav}}=-\frac{M_{\text{pl}}^2}{2}\int d^4x\sqrt{-g'}\left(R'-\frac{R'^2}{6\mu^2}\right)\,, \tag{2.2}$$

with the reduced Planck mass $M_{\text{pl}}\simeq 2.435\times 10^{18}\,\text{GeV}$ and a mass parameter μ.

We assume that the matter sector is minimally coupled to gravity, while maintaining renormalizability[7] up to graviton loops that are suppressed by $1/M_{\text{pl}}^2$. In particular we require renormalizablity of energy-momentum tensor that is defined as a linear response of the matter action to the metric. For example, QED with a scalar ϕ is described by

$$\begin{aligned}S_{\text{mat}}=\int d^4x\sqrt{-g'}\Big(&-\frac{1}{4}g'^{\mu\lambda}g'^{\nu\kappa}F'_{\mu\nu}F'_{\lambda\kappa}+g'^{\mu\nu}D'_\mu\phi'^*D'_\nu\phi'\\&+\xi_{\text{grav}}R'|\phi'|^2-m_s^2|\phi'|^2-\frac{1}{4}\lambda|\phi'|^4\Big)+S_{\text{fix}}\,,\end{aligned} \tag{2.3}$$

heavy degrees of freedom cancels with a classical contribution from a coupling of a compensator field to a mass term of heavy degrees of freedom. Indeed superconformal anomaly is correctly taken into account in supersymmetric inflation setups [84–86].

[6] Ref. [44] sketches the derivation of trace anomaly at the one-loop order in Wilsonian renormalization.

[7] This does not mean the matter sector consists solely of a finite number of renormalizable terms. Non-renormalizable terms are allowed when an infinite number of non-renormalizable terms are introduced for renormalization in the usual sense of effective field theory.

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with D_μ being the gauge and diffeomorphism covariant derivative and $F_{\mu\nu}$ being the field strength of A_μ. m_s is a scalar mass and λ is a quartic coupling. A non-minimal coupling ξ_{grav}, which provides an improvement term of energy-momentum tensor [59, 60], should be kept to maintain renormalizability of energy-momentum tensor. We devote section A to the gauge fixing term S_{fix}, whose contribution to energy-momentum tensor can be omitted for physical states.

Via the Weyl transformation of

$$g'_{\mu\nu} = e^{2\omega(\sigma)} g_{\mu\nu} \,, \tag{2.4}$$

the scalaron + gravity sector turns into $S_{\text{grav}} = S_{\text{E-H}} + S_\sigma$:

$$\begin{aligned} S_{\text{E-H}} &= -\frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} R \,, \\ S_\sigma &= \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma - V(\sigma) \right) \,, \end{aligned} \tag{2.5}$$

with ∇_μ being the diffeomorphism covariant derivative. For example, in the R^2 inflation model,

$$\omega = -\frac{1}{\sqrt{6}} \frac{\sigma}{M_{\text{pl}}} \tag{2.6}$$

and

$$V(\sigma) = \frac{3}{4} \mu^2 M_{\text{pl}}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{\sigma}{M_{\text{pl}}} \right) \right]^2 \,. \tag{2.7}$$

The matter fields transform under the Weyl transformation as

$$\phi'_i = e^{-d_i \omega(\sigma)} \phi_i \,, \tag{2.8}$$

with d_i denoting the Weyl weight of the field ϕ_i. The linear variation of the matter action is responsible for the leading coupling of scalaron to matter:

$$S_{\text{mat}} \left[\{\phi'_i\}, g'_{\mu\nu}; \{\lambda_a\} \right] \simeq S_{\text{mat}} \left[\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\} \right] - \int d^4x \sqrt{-g} \omega(\sigma) A_{\text{lin}} \left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\} \right) \,. \tag{2.9}$$

When we treat fields as classical objects, it is given by

$$A_{\text{lin}}^{\text{class}} = -\sum_i d_i (\text{e.o.m.})_i + \left(g_{\mu\nu} T^{\mu\nu} \left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\} \right) \right)_{\text{class}} \,, \tag{2.10}$$

and

$$(\text{e.o.m.})_i = -\phi_i \frac{1}{\sqrt{-g}} \frac{\delta S_{\text{mat}} \left[\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\} \right]}{\delta \phi_i} \,. \tag{2.11}$$

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The second term of $A_{\rm lin}^{\rm class}$ is the *classical* trace of energy-momentum tensor, in which we treat fields as classical objects. We define energy-momentum tensor by a functional derivative of

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm mat}\left[\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right]}{\delta g_{\mu\nu}}\,. \tag{2.12}$$

For example, in scalar QED,

$$\begin{aligned} T_{\mu\nu} =& -g^{\lambda\kappa}F_{\mu\lambda}F_{\nu\kappa} + 2D_\mu\phi^* D_\nu\phi + 2\xi_{\rm grav}R_{\mu\nu}|\phi|^2 - 2\xi_{\rm grav}\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}g^{\lambda\kappa}\nabla_\lambda\nabla_\kappa\right)|\phi|^2 \\ & - g_{\mu\nu}\left(-\frac{1}{4}g^{\lambda\rho}g^{\kappa\sigma}F_{\lambda\kappa}F_{\rho\sigma} + g^{\lambda\kappa}D_\lambda\phi^* D_\kappa\phi + \xi_{\rm grav}R|\phi|^2 - m^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4\right)\,. \end{aligned} \tag{2.13}$$

When we treat fields as quantum operators, the linear variation $A_{\rm lin}$ receives an additional contribution $A_{\rm anom}$. To see it, let us take a path integral formalism with path integral measure of $\mathcal{D}\{\phi'_i\}[g'_{\mu\nu}]$. Note that the path integral measure depends on the metric such that the path integral is diffeomorphism invariant [90]. For example, for scalar QED, $\mathcal{D}\phi[g_{\mu\nu}] = \mathcal{D}(-g)^{1/4}\phi$ and $\mathcal{D}A_\mu[g_{\mu\nu}] = \mathcal{D}(-g)^{1/4}e_m^{\ \mu}A_\mu$, where $e^m_{\ \mu}$ is the vierbein. We change the variables from $\{\phi'_i\}$ in the left hand side to $\{\phi_i\}$ in the right hand side of eq. (2.9). This results in a Jacobian of path integral measure:

$$\mathcal{D}\{\phi'_i\}[g'_{\mu\nu}] \simeq \mathcal{D}\{\phi_i\}[g_{\mu\nu}]\exp\left(-i\int d^4x\sqrt{-g}\,\omega A_{\rm Jacob}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right)\right) \tag{2.14}$$

in the linear variation. One may evaluate $A_{\rm Jacob}$ by using heat kernel regularization, which is used in Fujikawa's derivation of chiral anomaly [95]. It provides a one-loop contribution to $A_{\rm anom}$, which is proportional to the Weyl tensor squared, the Gauss-Bonnet density, and a gauge field strength squared if $\{\phi_i\}$ is charged. One can identify $A_{\rm Jacob} = A_{\rm anom}$, which is Fujikawa's derivation of trace anomaly [90]. It follows that the linear variation $A_{\rm lin}$ is given by the *quantum* trace of energy-momentum tensor (see section B):

$$\begin{aligned} A_{\rm lin} &= -\sum_i d_i({\rm e.o.m.})_i + \left(g_{\mu\nu}T^{\mu\nu}\right)_{\rm class} + A_{\rm anom}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right) \\ &= -\sum_i d_i({\rm e.o.m.})_i + g_{\mu\nu}T^{\mu\nu}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right)\,. \end{aligned} \tag{2.15}$$

In the above discussion, we have taken into account a Jacobian of path integral measure associated with $\{\phi'_i\} \to \{\phi_i\}$ under a background metric. One also needs to care a Jacobian of path integral measure associated with $g'_{\mu\nu} \to g_{\mu\nu}$ in eq. (2.4). On the other hand, it is intricate to compute the gravitational Jacobian. Thus we just assume that it does not give rise to any relevant coupling between scalaron and matter. For example, in the R^2 inflation model, the scalaron coupling to matter in eq. (2.9) reads

$$S_{\sigma\text{-mat}} = \int d^4x\sqrt{-g}\frac{1}{\sqrt{6}}\frac{\sigma}{M_{\rm pl}}g_{\mu\nu}T^{\mu\nu}\,. \tag{2.16}$$

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Our assumption on the gravitational Jacobian reads that it only leads to couplings suppressed by a higher power of $1/M_{\rm pl}$. This could be true since the graviton-loop contribution is suppressed by $1/M_{\rm pl}^2$.

In the rest of this paper, we restrict our discussion within the flat spacetime. The trace of flat-spacetime energy-momentum tensor is enough to evaluate scalaron decay since the scalaron decay amplitude into graviton is further suppressed by $1/M_{\rm pl}$.

3 Trace of energy-momentum tensor

In the last section we have shown that in the scalaron frame the scalaron couples to matter via the *quantum* trace of energy-momentum tensor, by employing the Fujikawa method. Here we should remark that once we use some regularization, we need to use it throughout, for example, to calculate the renormalization of couplings $\{\lambda_a\}$. On the other hand, heat kernel regularization in the Fujikawa method is not practical for perturbative renormalization, for which dimensional renormalization is a usual choice.[8] In dimensional renormalization, we consider $d = 4 - \epsilon$ dimension instead of four dimension to make loop diagrams finite. Then we subtract divergences in the four-dimension limit such that counter terms compose solely of poles of ϵ.

In dimensional renormalization, $A_{\rm Jacob}$ does not depend on fields unlike that in the Fujikawa method with heat kernel regularization. Thus $A_{\rm anom}$ has a different origin in dimensional renormalization. The trace of energy-momentum tensor takes a form of

$$T^\mu_{\ \mu} = \lim_{\epsilon\to 0}\left(-\sum_i d_i(\text{e.o.m.})_i + (T^\mu_{\ \mu})_{\rm class}\right) . \tag{3.1}$$

In the right-hand side, a quantity inside the parenthesis is calculated in $d = 4-\epsilon$ dimension and then taken to the four-dimension limit of $\epsilon \to 0$. A key observation is that as $\epsilon \to 0$, the second term does not coincide with the four-dimension *classical* trace of energy-momentum tensor. This is because of renormalization (normal product) of the bare (composite) operators such as $F_{\mu\nu}^2$ and $|\phi|^4$ [102–105]. The renormalization coefficients, including the multiplicative renormalization of bare couplings such as λ, compose of subtracted poles of ϵ in the MS or $\overline{\rm MS}$ scheme. They lead to terms proportional to the β function of the renormalized couplings [65, 67] such as $\beta_e[\bar{F}_{\mu\nu}^2]$ and $\beta_\lambda[|\bar{\phi}|^4]$. A bar denotes the renormalized (not composite) fields and parameters and a square bracket denotes the renormalized composite operator. These contributions provide $A_{\rm anom}$. Also note that $T_{\mu\nu}$ is conserved and thus solely improvement terms arising from non-minimal couplings are renormalized. As a result $T^\mu_{\ \mu}$ is already finite up to renormalization of improvement terms. In this article we do not go into further detail about renormalization of improvement terms, since it does not change the result at the leading order.

[8] Here is a big difference between chiral anomaly and trace anomaly. Chiral anomaly takes a one-loop exact form [96, 97] up to the divergence of some gauge invariant current [98] due to its topological property, i.e., it counts a number of zero modes in an instanton background [99–101]. Thus one can use the result from heat kernel regularization even though one uses dimensional regularization for perturbative renormalization. On the other hand, it does not hold for trace anomaly.

For scalar QED, the Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + |D_\mu\phi|^2 - m_s^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4\,, \tag{3.2}$$

with $D_\mu = \partial_\mu - iqeA_\mu$ being the gauge covariant derivative for a charge q. We have integrated out the Nakanishi-Lautrup [106, 107] and (anti-)ghost fields (see section A). ξ is a gauge fixing parameter.[9] d-dimension flat-spacetime energy-momentum tensor is obtained from eq. (2.13) as

$$\begin{aligned} T_{\mu\nu} =& - g^{\lambda\kappa}F_{\mu\lambda}F_{\nu\kappa} + 2D_\mu\phi^* D_\nu\phi - 2\left(\xi^c_{\rm grav} + \frac{\eta}{d-1}\right)(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)|\phi|^2 \\ &- g_{\mu\nu}\left(-\frac{1}{4}F_{\lambda\kappa}^2 + |D_\mu\phi|^2 - m_s^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4\right)\,. \end{aligned} \tag{3.3}$$

where we rewrite $\xi_{\rm grav} = \xi^c_{\rm grav} + \eta/(d-1)$ with $\xi^c_{\rm grav} = (d-2)/(4(d-1))$ in d dimension. We remark that η is renormalized in a non-multiplicative manner to make $T_{\mu\nu}$ finite, although we do not go into further detail. Taking a *classical* trace, one finds

$$(T^\mu_{\ \mu})_{\rm class} = \epsilon\left(-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{4}\lambda|\phi|^4\right) + 2m_s^2|\phi|^2 + 2\eta\partial^2|\phi|^2 + \left(1 - \frac{\epsilon}{2}\right)(\text{e.o.m})\,, \tag{3.4}$$

where the last term with

$$(\text{e.o.m}) = \phi^*\left(D^2\phi + m_s^2\phi + \frac{2}{4}\lambda|\phi|^2\phi\right) + \left(D^2\phi^* + m_s^2\phi^* + \frac{2}{4}\lambda|\phi|^2\phi^*\right)\phi \tag{3.5}$$

cancels with $-\sum_i d_i(\text{e.o.m.})_i$ in eq. (2.15). The first term of $(T^\mu_{\ \mu})_{\rm class}$ vanishes at the *classical* level as $\epsilon \to 0$, but not at the *quantum* level. This contribution provides $A_{\rm anom}$.

We calculate a $T^\mu_{\ \mu}$-$\bar{A}_\lambda$-$\bar{A}_\kappa$ ($\bar{A}_\mu$: renormalized gauge field) correlation function in the scalaron frame by using the $\overline{\rm MS}$ scheme. More specifically, we calculate the amputated amplitude $\mathcal{M}_{TAA}$ with incoming momentum k through $T^\mu_{\ \mu}$ and outgoing momentum k_1 and k_2 through gauge bosons with helicity ϵ_1 and ϵ_2, respectively. section C is devoted to details of the computations. For example, in the R^2 inflation model, the invariant amplitude of scalaron decay into two gauge bosons is given by

$$\mathcal{M}_{\rm dec} = \frac{1}{\sqrt{6}}\frac{1}{M_{\rm pl}}\mathcal{M}_{TAA} \times Z_3^{\rm pole}\,. \tag{3.6}$$

Here, the last term $Z_3^{\rm pole}$, which is the residue of the mass pole of the gauge field, arises from the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula [108].

For scalar QED (see section C.1), the leading contribution to $\mathcal{M}_{TAA}$ arises from the following terms of the trace of energy-momentum tensor:[10]

$$T^\mu_{\ \mu} \supset \frac{1}{6}\frac{q^2\bar{e}^2}{16\pi^2}\bar{F}_{\mu\nu}^2 + 2\bar{m}^2|\bar{\phi}|^2 + 2\bar{\eta}\partial^2|\bar{\phi}|^2\,. \tag{3.7}$$

[9] Note that a gauge fixing parameter ξ is different from a non-minimal coupling $\xi_{\rm grav}$.
[10] Note that in general $\bar{F}_{\mu\nu}^2 \neq [F_{\mu\nu}^2]$, although they coincide with each other at this order.

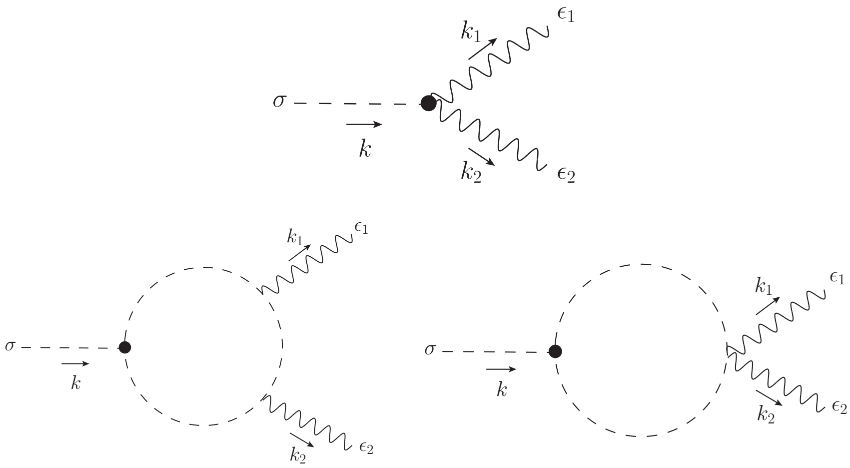

Figure 1. The leading contributions to $\mathcal{M}_{TAA}$, i.e., T^{μ}_{μ}-$\bar{A}_\lambda$-$\bar{A}_\kappa$ correlation function (σ decay into two gauge bosons). [Top] $\mathcal{M}_{F^2}$: the gauge kinetic term in eq. (3.7) is inserted. [Bottom] $\mathcal{M}_{|\phi|^2}$: the scalar mass term and η term in eq. (3.7) is inserted. There is the other contribution from the left diagram with the external gauge bosons exchanged.

The first term arises from the gauge kinetic term proportional to ϵ in eq. (3.4). Its coefficient is obtained from the leading contribution to the wave function renormalization of the gauge field [see eq. (C.9)]. Meanwhile the leading contribution to the wave function renormalization of the gauge field also determines the leading contribution to the β function [see eq. (C.10)] as

$$\beta_e = \frac{1}{3}\frac{q^2\bar{e}^3}{16\pi^2}\,. \tag{3.8}$$

The matrix element has two contributions*

$$\mathcal{M}_{TAA} = \mathcal{M}_{F^2} + \mathcal{M}_{|\phi|^2}\,. \tag{3.9}$$

The first term arises from the tree-level diagram with the gauge kinetic term inserted (see the top diagram of figure 1):

$$\mathcal{M}_{F^2} = -\frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*\right)\,. \tag{3.10}$$

The second term arises from the one-loop diagram with the scalar mass term and η term inserted (see the bottom diagrams of figure 1):

$$\mathcal{M}_{|\phi|^2} = \frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\frac{\bar{m}^2-\bar{\eta}k^2}{\bar{m}^2}I_s\left(\frac{k^2}{\bar{m}^2}\right)\left(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*\right)\,, \tag{3.11}$$

where[11]

$$\begin{aligned} I_s(r) &= 24\int_0^1 dx\int_0^{1-x}dy\,\frac{xy}{-rxy+1-i\epsilon_{\mathrm{ad}}} \\ &= \begin{cases} \dfrac{12}{r}\left(-1+\dfrac{4}{r}\arcsin^2\dfrac{\sqrt{r}}{2}\right) & (\text{for } r<4) \\ \dfrac{12}{r}\left(-1-\dfrac{4}{r}\left[\operatorname{arccosh}\dfrac{\sqrt{r}}{2}-i\dfrac{\pi}{2}\right]^2\right) & (\text{for } r>4) \end{cases}\,. \end{aligned} \tag{3.12}$$

[11] This definition is different from the one in ref. [56] by a factor of 6.

For $r > 4$, one needs to take into account an adiabatic parameter $\epsilon_{\rm ad} > 0$ properly.[12] This arises from the fact that the loop scalar can be real. Collecting the two contributions, one obtains

$$\mathcal{M}_{TAA} = -\frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(1 - \frac{\bar{m}^2 - \bar{\eta}k^2}{\bar{m}^2} I_s\left(\frac{k^2}{\bar{m}^2}\right)\right)(k_1 \cdot k_2\, \epsilon_1^* \cdot \epsilon_2^* - k_2 \cdot \epsilon_1^*\, k_1 \cdot \epsilon_2^*)\ . \tag{3.13}$$

We remark that $I_s(0) = 1$ and thus a heavy ($\bar{m}^2 \gg k^2$) scalar does not contribute to $\mathcal{M}_{TAA}$. Meanwhile, $I_s(r \to \infty) \to -12/r$ and thus a light ($\bar{m}^2 \ll k^2$) scalar indeed contributes to $\mathcal{M}_{TAA}$.

It is straightforward to generalize to the case with N_s scalars and N_f Dirac fermions (see section C.2 for QED with a Dirac fermion) since the quartic and Yukawa coupling do not matter at this order. The β function is given by

$$\beta_e = \frac{1}{3}\left(\sum_s q_s^2 + 4\sum_f q_f^2\right)\frac{\bar{e}^3}{16\pi^2}\ . \tag{3.14}$$

Note that this counts contributions from both *heavy* and *light* degrees of freedom.

Meanwhile, the matrix element is given by

$$\begin{aligned}\mathcal{M}_{TAA} =& \frac{2}{3}\frac{\bar{e}^2}{16\pi^2}\left(\sum_s q_s^2\left(1 - \frac{\bar{m}_s^2 - \bar{\eta}_s k^2}{\bar{m}_s^2} I_s\left(\frac{k^2}{\bar{m}_s^2}\right)\right) + 4\sum_f q_f^2\left(1 - I_f\left(\frac{k^2}{\bar{m}_f^2}\right)\right)\right)\\ &\times (k_1 \cdot k_2\, \epsilon_1^* \cdot \epsilon_2^* - k_2 \cdot \epsilon_1^*\, k_1 \cdot \epsilon_2^*)\ ,\end{aligned} \tag{3.15}$$

where

$$\begin{aligned}I_f(r) &= 3\int_0^1 dx \int_0^{1-x} dy\, \frac{-4xy + 1}{-rxy + 1 - i\epsilon_{\rm ad}}\\ &= \begin{cases} \dfrac{6}{r}\left(1 + \left(1 - \dfrac{4}{r}\right)\arcsin^2\dfrac{\sqrt{r}}{2}\right) & (\text{for } r < 4)\\ \dfrac{6}{r}\left(1 - \left(1 - \dfrac{4}{r}\right)\left[\operatorname{arccosh}\dfrac{\sqrt{r}}{2} - i\dfrac{\pi}{2}\right]^2\right) & (\text{for } r > 4)\end{cases}\ .\end{aligned} \tag{3.16}$$

Here $I_f(0) = 1$ and $I_f(\infty) = 0$.[13] For $r > 4$, one needs to take into account $\epsilon_{\rm ad}$ properly. This arises from the fact that the loop fermion can be real. The matrix element is approximated by

$$\mathcal{M}_{TAA} \approx \frac{2}{3}\frac{\bar{e}^2}{16\pi^2}\left(\sum_{\text{light } s} q_s^2\,(1 - 12\bar{\eta}_s) + 4\sum_{\text{light } f} q_f^2\right)(k_1 \cdot k_2\, \epsilon_1^* \cdot \epsilon_2^* - k_2 \cdot \epsilon_1^*\, k_1 \cdot \epsilon_2^*)\ . \tag{3.17}$$

The summation runs over solely *light* scalars or fermions with $m^2 < k^2$. The light scalar contribution is not only from the β function q_s^2, but also from the non-minimal coupling $\bar{\eta}$. This is because the classical contribution to T_μ^μ from the non-minimal coupling (the last term in eq. (3.7)) does not vanish for a light scalar.

[12] Note that an adiabatic parameter $\epsilon_{\rm ad}$ associated with a Wick rotation is different from $\epsilon = 4 - d$ for dimensional regularization.

[13] This definition is different from the one in ref. [56] by a factor of 3.

4 Conclusion and remarks

In this article, we have revisited scalaron decay via the trace of energy-momentum tensor. In particular we have studied scenarios with a sequestered matter sector, where the trace of energy-momentum tensor gives a dominant contribution to scalaron-matter coupling. We have referred to the R^2 inflation mode to be concrete. On the other hand, one can straightforwardly apply our results and discussions to more general $f(R)$ and $f(\sigma)R$ gravity models. One just modifies the relation between the Weyl parameter and scalaron accordingly. We have shown how trace anomaly arises by employing the Fujikawa method and dimensional renormalization. For perturbative renormalization beyond the one-loop level, the dimensional renormalization is more convenient than the Fujikawa method.

Trace anomaly plays an important role in ensuring that the trace of energy-momentum tensor is predictive in terms of low-energy effective theory. We have explicitly calculated the scalaron decay amplitude at the leading order in QED with scalars and fermions. The contribution of heavy degrees of freedom from trace anomaly cancels with the one from the mass term, in the heavy mass limit of the scalars and fermions. It is straightforward to generalize the discussion to quantum chromodynamics (QCD).

There are two caveats on the predictability of the trace of energy-momentum tensor: a non-minimal coupling of matter scalars to Ricci curvature; and the renormalization-scale dependence. They only appear in energy-momentum tensor and thus one cannot be determined its renormalized value through usual experiments unless graviton is involved in a process. Since a non-minimal coupling is required to renormalize energy-momentum tensor, one should keep it even when one considers a matter sector minimally coupled to gravity. In addition, it may not be clear how we can see that the scalaron decay amplitude is independent of the renormalization scale, since the trace of energy-momentum tensor is a composite operator. We will give a detailed discussion on these caveats somewhere else.

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A Gauge fixing term

In this section we discuss the gauge fixing term $S_{\rm fix}$ in non-Abelian gauge theory, while we consider Abelian gauge theory (QED) in the main text. The gauge fixing term takes a Becchi-Rouet-Stora-Tyutin (BRST) form [109–113] of

$$S_{\rm fix} = \int d^d x \sqrt{-g} \left(\frac{\xi}{2} B^a B^a - g^{\mu\nu} \nabla_\mu B^a A^a{}_\nu + g^{\mu\nu} \nabla_\mu \bar{c}^a D_\nu c^a \right) , \tag{A.1}$$

with ξ being a gauge fixing parameter. The superscript a runs over gauge group generators T^a [$T^a = \mathbb{I}$ (identity matrix) in QED]. D_μ is the gauge and diffeomorphism covariant derivative, while ∇_μ is the diffeomorphism (not gauge) covariant derivative. We have introduced a bosonic auxiliary Nakanishi-Lautrup field $B^a = B^{a\dagger}$, fermionic (ghost and anti-ghost) fields, $c^a = c^{a\dagger}$ and $\bar{c}^a = -\bar{c}^{a\dagger}$.

The BRST transformation is defined by the following fermionic global transformation:

$$\begin{aligned} QA_\mu &= D_\mu c\,, \\ Qc &= \frac{i}{2}e[c,c]\,, \\ Q\bar{c} &= B\,, \\ QB &= 0\,, \end{aligned} \tag{A.2}$$

with e being a gauge coupling. We have used the matrix notation of $A_\mu = A^a{}_\mu T^a$ and $D_\mu c = \partial_\mu c - ie[c, A]$, and so on. These are understood as $[Q, A_\mu] = iD_\mu c$ (commutator), $\{Q, \bar{c}\} = iB$ (anti-commutator), and so on in the operator formalism with $Q^\dagger = Q$. An operator or state is called BRST closed when it vanishes under the BRST transformation. Gauge invariant operators, such as a gauge invariant part of an action and its contribution to energy-momentum tensor [see eq. (2.13)], are BRST closed. Meanwhile an operator or state is called BRST exact when it can be written as the BRST transformation of some operator or state. Notably the gauge fixing term is BRST exact:

$$S_{\text{fix}} = \int d^dx\sqrt{-g}Q\left(\frac{\xi}{2}\bar{c}^aB^a - g^{\mu\nu}\nabla_\mu\bar{c}^aA^a{}_\nu\right). \tag{A.3}$$

S_{fix} contribution to energy-momentum tensor is also BRST exact:

$$T^{\text{fix}}_{\mu\nu} = Q\left(-\nabla_\mu\bar{c}^aA^a{}_\nu - \nabla_\nu\bar{c}^aA^a{}_\mu - g_{\mu\nu}\left(\frac{\xi}{2}\bar{c}^aB^a - g^{\lambda\kappa}\nabla_\lambda\bar{c}^aA^a{}_\kappa\right)\right). \tag{A.4}$$

One can see that the BRST transformation is nilpotent: $Q^2 = 0$. Thus a BRST-exact operator or state is BRST closed. We can introduce an equivalence class on the set of BRST-closed operators or states $\mathcal{H}_{\text{closed}}$ as $\mathcal{H}_{\text{closed}} \sim \mathcal{H}_{\text{closed}} + \mathcal{H}_{\text{exact}}$ with the set of BRST-exact operators or states $\mathcal{H}_{\text{exact}} \subset \mathcal{H}_{\text{closed}}$. The physical operator or state is defined by the quotient set of $\mathcal{H}_{\text{closed}}/\mathcal{H}_{\text{exact}}$ [114–116]. Since $T^{\text{fix}}_{\mu\nu}$ is BRST exact, one can chose a physical representative such that $T^{\text{fix}}_{\mu\nu} = 0$.

B Path integral derivation of eq. (2.15)

We consider a correlation function in the path integral formalism:

$$\begin{aligned} &\int \mathcal{D}\{\phi_i\}[g'_{\mu\nu}]\exp\left(iS_{\text{mat}}\left[\{\phi_i\}, g'_{\mu\nu}; \{\lambda_a\}\right]\right)\prod\{\phi_i\} \\ &\qquad\simeq \left(1 + \int d^4x\sqrt{-g}\,\omega g_{\mu\nu}\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\right) \\ &\qquad\quad\times\int \mathcal{D}\{\phi_i\}[g_{\mu\nu}]\exp\left(iS_{\text{mat}}\left[\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right]\right)\prod\{\phi_i\}\,. \end{aligned} \tag{B.1}$$

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Meanwhile,

$$\begin{aligned}
&\int \mathcal{D}\{\phi_i\}[g'_{\mu\nu}] \exp\left(iS_{\rm mat}\left[\{\phi_i\}, g'_{\mu\nu}; \{\lambda_a\}\right]\right) \prod \{\phi_i\} \\
&\quad = \int \mathcal{D}\{\phi'_i\}[g'_{\mu\nu}] \exp\left(iS_{\rm mat}\left[\{\phi'_i\}, g'_{\mu\nu}; \{\lambda_a\}\right]\right) \prod \{\phi'_i\} \\
&\quad \simeq \int \mathcal{D}\{\phi_i\}[g_{\mu\nu}] \exp\left(iS_{\rm mat}\left[\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right]\right) \\
&\quad\quad \times \left(1 + i\int d^4x\sqrt{-g}\,\omega\left[\sum_i d_i(\text{e.o.m.})_i - g_{\mu\nu}T^{\mu\nu}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right)\right.\right. \\
&\quad\quad \left.\left. - A_{\rm Jacob}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right)\right] - \int d^4x\sqrt{-g}\,\omega\sum_i \frac{d_i\phi_i}{\sqrt{-g}}\frac{\delta}{\delta\phi_i}\right) \prod\{\phi_i\}\,.
\end{aligned} \tag{B.2}$$

In the first equality, we change a notation of the integration variable $\{\phi_i\}$, which has no physical effect. From this Ward-Takahashi identity, one finds

$$-g_{\mu\nu}T^{\mu\nu} = -(g_{\mu\nu}T^{\mu\nu})_{\rm class} - A_{\rm Jacob}\left(\{\phi_i\}, g_{\mu\nu}; \{\lambda_a\}\right)\,, \tag{B.3}$$

by ignoring the contact terms.

C One-loop calculations in QED

In the following calculations, we use the $\overline{\rm MS}$ scheme with a spacetime dimension of $d = 4-\epsilon$ and a renormalization scale of μ, while compensating a mass dimension by a modified renormalization scale $\tilde{\mu}$ defined by

$$\tilde{\mu}^2 = \mu^2\frac{e^{\gamma_E}}{4\pi} \tag{C.1}$$

with $\gamma_E \simeq 0.577$ being Euler's constant. One-loop functions are summarized in section C.3.

C.1 Scalar

The Lagrangian density is given by[14]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + |D_\mu\phi|^2 - m^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4\,, \tag{C.2}$$

with $D_\mu = \partial_\mu - iqeA_\mu$ being the gauge covariant derivative for a charge q. We have integrated out the NL and (anti-)ghost fields. Parameters are a gauge coupling e, a scalar mass m, a quartic coupling λ, and a gauge fixing parameter ξ. Multiplicative renormalization is set for fields as $\phi = Z_2^{1/2}\bar{\phi}$ and $A_\mu = Z_3^{1/2}\bar{A}_\mu$ and for parameters as $Z_2 Z_3^{1/2} e = Z_1\tilde{\mu}^{\epsilon/2}\bar{e}$,

[14] The following procedure is simplified with the hep-th notation since A_μ has a mass dimension 1 and is not renormalized due to the Ward-Takahashi identity. In this case, one needs to multiply e^2 when translating $\mathcal{M}_{TAA}$ to $\mathcal{M}_{\rm dec}$ since the gauge field is not canonically normalized.

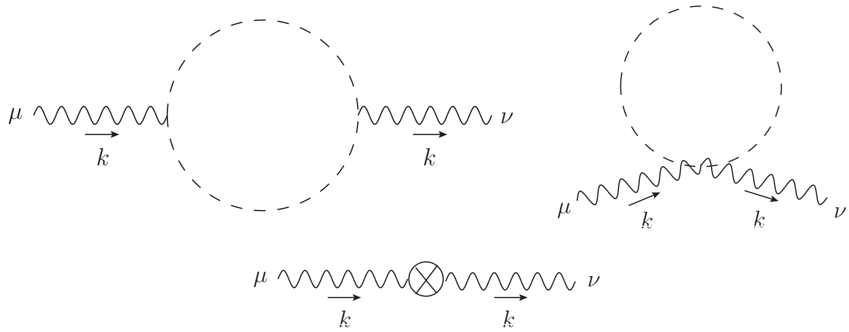

Figure 2. Leading contributions to the vacuum polarization $i\bar{\Pi}^{\mu\nu}$. [Top] scalar loop. [Bottom] counter term.

$Z_2 Z_3 e^2 = Z_4 \tilde{\mu}^\epsilon \bar{e}^2$ (i.e., $Z_2 Z_4 = Z_1^2$), $Z_2 m^2 = Z_m \bar{m}^2$, $Z^2\lambda = Z_\lambda \tilde{\mu}^\epsilon \bar{\lambda}$, and $Z_3/\xi = Z_5/\bar{\xi}$. The Lagrangian density can be written in the form of renormalized perturbation theory as

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}\bar{F}_{\mu\nu}^2 - \frac{1}{2\bar{\xi}}(\partial_\mu \bar{A}^\mu)^2 + |\partial_\mu \bar{\phi}|^2 - \bar{m}^2|\bar{\phi}|^2 \\ & - \frac{1}{4} Z_\lambda \tilde{\mu}^\epsilon \bar{\lambda} |\bar{\phi}|^4 + iq Z_1 \tilde{\mu}^{\epsilon/2} \bar{e} \bar{A}^\mu (\bar{\phi}^* \partial_\mu \bar{\phi} - \partial_\mu \bar{\phi}^* \bar{\phi}) + q^2 Z_4 \tilde{\mu}^\epsilon \bar{e}^2 \bar{A}_\mu^2 |\bar{\phi}|^2 \\ & - \frac{1}{4}(Z_3 - 1)\bar{F}_{\mu\nu}^2 - \frac{1}{2\bar{\xi}}(Z_5 - 1)(\partial_\mu \bar{A}^\mu)^2 + (Z_2 - 1)|\partial_\mu \bar{\phi}|^2 - (Z_m - 1)\bar{m}^2 |\bar{\phi}|^2 \,.\end{aligned} \tag{C.3}$$

The Ward-Takahashi identity warrants that $Z_1 = Z_2 = Z_4$, Z_3 is independent of $\bar{\xi}$, and $Z_5 = 1$. It follows that

$$\begin{aligned}\beta_e^\epsilon =& -\frac{\bar{e}}{2}\epsilon \left(1 - \frac{\bar{e}}{2}\frac{\partial \ln Z_3}{\partial \bar{e}}\right)^{-1}, \\ \beta_\lambda^\epsilon =& -\bar{\lambda}\epsilon \left(1 - 2\bar{\lambda}\frac{\partial \ln Z_2}{\partial \bar{\lambda}} + \bar{\lambda}\frac{\partial \ln Z_\lambda}{\partial \bar{\lambda}}\right)^{-1}, \\ \beta_m =& \frac{\bar{m}}{2}\beta_e^\epsilon \left(\frac{\partial \ln Z_2}{\partial \bar{e}} - \frac{\partial \ln Z_m}{\partial \bar{e}}\right) + \frac{\bar{m}}{2}\beta_\lambda^\epsilon \left(\frac{\partial \ln Z_2}{\partial \bar{\lambda}} - \frac{\partial \ln Z_m}{\partial \bar{\lambda}}\right) \\ & + \frac{\bar{m}}{2}\beta_\xi \left(\frac{\partial \ln Z_2}{\partial \bar{\xi}} - \frac{\partial \ln Z_m}{\partial \bar{\xi}}\right), \\ \beta_\xi =& -\bar{\xi}\beta_e^\epsilon \frac{\partial \ln Z_3}{\partial \bar{e}} - \bar{\xi}\beta_\lambda^\epsilon \frac{\partial \ln Z_2}{\partial \bar{\lambda}} \,.\end{aligned} \tag{C.4}$$

$Z_3 - 1$ and $Z_5 - 1$ can be determined via loop corrections to the two point correlation function of the gauge boson:

$$i\bar{\Pi}^{\mu\nu} = i\Pi^{\mu\nu} - i(Z_3 - 1)(k^2 g^{\mu\nu} - k^\mu k^\nu) - i\frac{1}{\bar{\xi}}(Z_5 - 1)k^\mu k^\nu \,, \tag{C.5}$$

where k denotes the gauge boson momentum. The one-loop vacuum polarization is given by (see the top diagrams of figure 2)

$$\begin{aligned}i\Pi^{\mu\nu} =& (iq\bar{e})^2 i^2 \tilde{\mu}^\epsilon \int \frac{d^d\ell}{(2\pi)^d} \frac{(2\ell + k)^\mu (2\ell + k)^\nu}{[\ell^2 - \bar{m}^2][(\ell + k)^2 - \bar{m}^2]} + (2iq^2\bar{e}^2 g^{\mu\nu}) i\tilde{\mu}^\epsilon \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{[\ell^2 - \bar{m}^2]} \\ =& \frac{iq^2\bar{e}^2}{16\pi^2}\left([4B_{22} - 2A]\, g^{\mu\nu} + [4B_{21} + 4B_1 + B_0]\, k^\mu k^\nu\right) \,.\end{aligned} \tag{C.6}$$

Noting that

$$4B_{21}+4B_1+B_0=\frac{4}{3k^2}\left[A-\bar{m}^2B_0+\frac{k^2}{4}B_0-\bar{m}^2+\frac{k^2}{6}\right]$$
$$=-\frac{1}{k^2}[4B_{22}-2A]\,, \tag{C.7}$$

which ensures the Ward-Takahashi identity, one finds $i\Pi^{\mu\nu}=(k^2g^{\mu\nu}-k^\mu k^\nu)i\Pi$ and

$$i\Pi=i\frac{4}{3k^2}\frac{q^2\bar{e}^2}{16\pi^2}\left[-A+\bar{m}^2B_0-\frac{k^2}{4}B_0+\bar{m}^2-\frac{k^2}{6}\right]. \tag{C.8}$$

The pole of ϵ is canceled with (see the bottom diagram of figure 2)

$$Z_3-1=-\frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\frac{1}{\epsilon} \tag{C.9}$$

and $Z_5-1=0$. Thus the four-dimension β function is given by

$$\beta_e=-\frac{\bar{e}^2}{4}\frac{\partial}{\partial\bar{e}}\left(\ln Z_3\right)^{\text{residue}}_{\text{of }\epsilon=0}=\frac{1}{3}\frac{q^2\bar{e}^3}{16\pi^2}\,. \tag{C.10}$$

The residue of the mass pole ($k^2=0$) of the gauge field is given by

$$Z_3^{\text{pole}}-1=\Pi-(Z_3-1)=\frac{1}{3}\frac{q^2\bar{e}^2}{16\pi^2}\ln\left(\frac{\bar{m}^2}{\mu^2}\right). \tag{C.11}$$

The contribution from the one-loop diagram with the scalar mass term and η term inserted is given by

$$\mathcal{M}_{|\phi|^2}=2(\bar{m}^2-\bar{\eta}k^2)\Bigg((iq\bar{e})^2i^3\int\frac{d^4\ell}{(2\pi)^4}\frac{(2\ell+k_1)\cdot\epsilon_1^*\,(2\ell+2k_1+k_2)\cdot\epsilon_2^*}{[\ell^2-\bar{m}^2][(\ell+k_1)^2-\bar{m}^2][(\ell+k_1+k_2)^2-\bar{m}^2]}$$
$$+[1\leftrightarrow 2]+2iq^2\bar{e}^2\epsilon_1^*\cdot\epsilon_2^*\,i^2\int\frac{d^4\ell}{(2\pi)^4}\frac{1}{[\ell^2-\bar{m}^2][(\ell+k_1+k_2)^2-\bar{m}^2]}\Bigg)$$
$$=2(\bar{m}^2-\bar{\eta}k^2)\frac{q^2\bar{e}^2}{16\pi^2}$$
$$\times\left(\left[-4C_{24}+B_0(k^2)\right]\epsilon_1^*\cdot\epsilon_2^*+\left[-4C_{23}-4C_{12}\right]k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*+[1\leftrightarrow 2]\right). \tag{C.12}$$

Noting that

$$-4C_{23}-4C_{12}=\frac{1}{2k^2}\left[2m^2C_0+1\right]$$
$$=-\frac{1}{2k^2}\left[-4C_{24}+B_0(k^2)\right], \tag{C.13}$$

one finds

$$\mathcal{M}_{|\phi|^2}=-4\frac{\bar{m}^2-\bar{\eta}k^2}{k^2}\frac{q^2\bar{e}^2}{16\pi^2}\left[2\bar{m}^2C_0+1+[1\leftrightarrow 2]\right](k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^*-k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*)\,. \tag{C.14}$$

With

$$2\bar{m}^2C_0+1=-\frac{k^2}{12\bar{m}^2}I_s\left(\frac{k^2}{\bar{m}^2}\right), \tag{C.15}$$

the matrix element is

$$\mathcal{M}_{|\phi|^2} = \frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\frac{\bar{m}^2 - \bar{\eta}k^2}{\bar{m}^2} I_s\left(\frac{k^2}{\bar{m}^2}\right)\left(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*\right) . \tag{C.16}$$

Let us see how we obtain the above result in Pauli-Villars regularization. Above the Pauli-Villars mass scale, $\beta_e = 0$ and the trace of energy-momentum tensor is replaced by

$$T^\mu_{\ \mu} \supset 2\bar{m}^2|\bar{\phi}|^2 + 2\bar{\eta}\partial^2|\bar{\phi}|^2 + 2\bar{m}_{\rm PV}^2|\bar{\phi}_{\rm PV}|^2 + 2\bar{\eta}_{\rm PV}\partial^2|\bar{\phi}_{\rm PV}|^2 , \tag{C.17}$$

where $\bar{\phi}_{\rm PV}$ is a Pauli-Villars partner with a wrong statistics. As a result, the matrix element is replaced by

$$\begin{aligned}\mathcal{M}_{TAA} =& -\frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(\frac{\bar{m}_{\rm PV}^2 - \bar{\eta}_{\rm PV}k^2}{\bar{m}_{\rm PV}^2} I_s\left(\frac{k^2}{\bar{m}_{\rm PV}^2}\right) - \frac{\bar{m}^2 - \bar{\eta}k^2}{\bar{m}^2} I_s\left(\frac{k^2}{\bar{m}^2}\right)\right)\\ &\times\left(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_1\cdot\epsilon_2^*\,k_1\cdot\epsilon_2^*\right) .\end{aligned} \tag{C.18}$$

After integrating out the Pauli-Villars partner, i.e., $\bar{m}_{\rm PV}^2 \to \infty$, one reproduces the above result.

C.2 Fermion

The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}iD_\mu\overline{\psi}\gamma^\mu\psi + \frac{1}{2}\overline{\psi}\gamma^\mu iD_\mu\psi - m\overline{\psi}\psi , \tag{C.19}$$

with $D_\mu = \partial_\mu - iqeA_\mu$ being the gauge covariant derivative for a charge q. We have integrated out the Nakanishi-Lautrup and (anti-)ghost fields. Parameters are a gauge coupling e, a fermion mass m, and a gauge fixing parameter ξ.

Multiplicative renormalization is set for fields as $\psi = Z_2^{1/2}\bar{\psi}$,[15] and $A_\mu = Z_3^{1/2}\bar{A}_\mu$ and for parameters as $Z_2 Z_3^{1/2} e = Z_1\tilde{\mu}^{\epsilon/2}\bar{e}$, $Z_2 m = Z_m\bar{m}$, and $Z_3/\xi = Z_4/\bar{\xi}$. The Lagrangian density can be written in the form of renormalized perturbation theory as

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}\bar{F}_{\mu\nu}^2 - \frac{1}{2\bar{\xi}}(\partial_\mu\bar{A}^\mu)^2 - \frac{1}{2}iD_\mu\overline{\bar{\psi}}\gamma^\mu\bar{\psi} + \frac{1}{2}\overline{\bar{\psi}}\gamma^\mu iD_\mu\bar{\psi} - \bar{m}\overline{\bar{\psi}}\bar{\psi} + qZ_1\bar{e}\tilde{\mu}^{\epsilon/2}\bar{A}_\mu\overline{\bar{\psi}}\gamma^\mu\bar{\psi}\\ &-\frac{1}{4}(Z_3-1)\bar{F}_{\mu\nu}^2 - \frac{1}{2\bar{\xi}}(Z_4-1)(\partial_\mu\bar{A}^\mu)^2 - (Z_2-1)\frac{1}{2}iD_\mu\overline{\bar{\psi}}\gamma^\mu\bar{\psi}\\ &+(Z_2-1)\frac{1}{2}\overline{\bar{\psi}}\gamma^\mu iD_\mu\bar{\psi} - (Z_m-1)\bar{m}\overline{\bar{\psi}}\bar{\psi} .\end{aligned} \tag{C.20}$$

The Ward-Takahashi identity warrants that $Z_1 = Z_2$, Z_3 is independent of $\bar{\xi}$, and $Z_4 = 1$. It follows that

$$\begin{aligned}\beta_e^\epsilon =& -\frac{\bar{e}}{2}\epsilon\left(1 - \frac{\bar{e}}{2}\frac{\partial\ln Z_3}{\partial\bar{e}}\right)^{-1} ,\\ \beta_m =& \bar{m}\beta_e^\epsilon\left(\frac{\partial\ln Z_2}{\partial\bar{e}} - \frac{\partial\ln Z_m}{\partial\bar{e}}\right) + \bar{m}\beta_\xi\left(\frac{\partial\ln Z_2}{\partial\bar{\xi}} - \frac{\partial\ln Z_m}{\partial\bar{\xi}}\right) ,\\ \beta_\xi =& -\beta_e^\epsilon\frac{\partial\ln Z_3}{\partial\bar{e}}\bar{\xi} .\end{aligned} \tag{C.21}$$

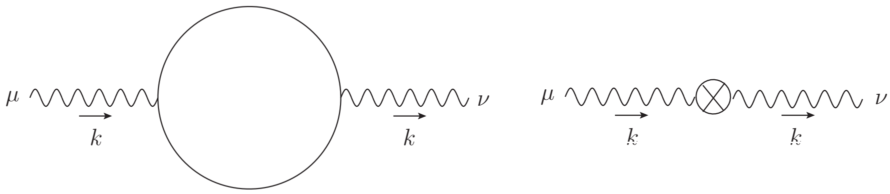

Figure 3. Leading contributions to the vacuum polarization $i\bar{\Pi}^{\mu\nu}$. [Left] fermion loop $i\Pi^{\mu\nu}$. [Right] counter term.

$Z_3 - 1$ and $Z_4 - 1$ can be determined via loop corrections to the two point correlation function of the gauge boson:

$$i\bar{\Pi}^{\mu\nu} = i\Pi^{\mu\nu} - i(Z_3 - 1)(k^2 g^{\mu\nu} - k^\mu k^\nu) - i\frac{1}{\bar{\xi}}(Z_4 - 1)k^\mu k^\nu \,, \tag{C.22}$$

where k denotes the gauge boson momentum. The one-loop vacuum polarization is given by (see the left diagram of figure 3)

$$\begin{aligned} i\Pi^{\mu\nu} &= (iq\bar{e})^2(-1)i^2\tilde{\mu}^\epsilon \int \frac{d^d\ell}{(2\pi)^d} \frac{\mathrm{tr}\left[\gamma^\mu(\not\ell + \not k + \bar{m})\gamma^\nu(\not\ell + \bar{m})\right]}{[\ell^2 - \bar{m}^2][(\ell + k)^2 - \bar{m}^2]} \\ &= -\frac{iq^2}{16\pi^2} 4\left(\left[(-2+\epsilon)B_{22} - k^2\left(B_{21} + B_1\right) + \bar{m}^2 B_0\right] g^{\mu\nu} + \left[2B_{21} + 2B_1\right] k^\mu k^\nu\right) . \end{aligned} \tag{C.23}$$

Noting that

$$\begin{aligned} 2B_{21} + 2B_1 &= \frac{2}{3k^2}\left[A - \bar{m}^2 B_0 - \frac{k^2}{2}B_0 - \bar{m}^2 + \frac{k^2}{6}\right] \\ &= -\frac{1}{k^2}\left[(-2+\epsilon)B_{22} - k^2\left(B_{21} + B_1\right) + \bar{m}^2 B_0\right] , \end{aligned} \tag{C.24}$$

which ensures the Ward-Takahashi identity, one finds $i\Pi^{\mu\nu} = (k^2 g^{\mu\nu} - k^\mu k^\nu) i\Pi$ and

$$i\Pi = i\frac{8}{3k^2}\frac{q^2\bar{e}^2}{16\pi^2}\left[A - \bar{m}^2 B_0 - \frac{k^2}{2}B_0 - \bar{m}^2 + \frac{k^2}{6}\right] . \tag{C.25}$$

The pole at $\epsilon = 0$ is canceled with (see the right diagram of figure 3)

$$Z_3 - 1 = -\frac{8}{3}\frac{q^2\bar{e}^2}{16\pi^2}\frac{1}{\epsilon} \tag{C.26}$$

and $Z_4 - 1 = 0$. Thus the four-dimension β function is given by

$$\beta_e = -\frac{\bar{e}^2}{4}\frac{\partial}{\partial\bar{e}}\left(\ln Z_3\right)^{\text{residue}}_{\text{of } \epsilon=0} = \frac{4}{3}\frac{q^2\bar{e}^3}{16\pi^2} . \tag{C.27}$$

The residue of the mass pole ($k^2 = 0$) of the gauge field is given by

$$Z_3^{\text{pole}} - 1 = \Pi - (Z_3 - 1) = \frac{4}{3}\frac{q^2\bar{e}^2}{16\pi^2}\ln\left(\frac{\bar{m}^2}{\mu^2}\right) . \tag{C.28}$$

[15]Note that a bar for a renormalized quantity is different from an overline for a Dirac bar.

d-dimension flat-spacetime energy-momentum tensor is given by[16]

$$\begin{aligned}T_{\mu\nu} =& - g^{\lambda\kappa} F_{\mu\lambda} F_{\nu\kappa} - \frac{1}{4}\left(iD_\mu\overline{\psi}\gamma_\nu + iD_\nu\overline{\psi}\gamma_\mu\right)\psi + \frac{1}{4}\overline{\psi}\left(iD_\mu\gamma_\nu + iD_\nu\gamma_\mu\right)\psi \\ &- g_{\mu\nu}\left(-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}iD_\mu\overline{\psi}\gamma^\mu\psi + \frac{1}{2}\overline{\psi}\gamma^\mu iD_\mu\psi - m\overline{\psi}\psi\right).\end{aligned} \tag{C.29}$$

Taking a *classical* trace, one finds

$$(T^\mu_{\ \mu})_{\rm class} = -\frac{1}{4}\epsilon F_{\mu\nu}^2 + m\overline{\psi}\psi + \left(\frac{3}{2} - \frac{\epsilon}{2}\right)(\text{e.o.m})\,, \tag{C.30}$$

where

$$(\text{e.o.m.}) = -\left(-i\not{D}\overline{\psi} - m\overline{\psi}\right)\psi - \overline{\psi}\left(i\not{D} - m\right)\psi\,. \tag{C.31}$$

The first term of $(T^\mu_{\ \mu})_{\rm class}$ vanishes at the *classical* level as $\epsilon \to 0$, but not at the *quantum* level. This contribution provides $A_{\rm anom}$. The leading contribution to $\mathcal{M}_{TAA}$ arises from the following trace of energy-momentum tensor:

$$T^\mu_{\ \mu} \supset \frac{2}{3}\frac{q^2\bar{e}^2}{16\pi^2}\bar{F}_{\mu\nu}^2 + \bar{m}\overline{\bar{\psi}}\bar{\psi}\,. \tag{C.32}$$

The first term arises from the gauge kinetic term proportional to ϵ in eq. (C.30). Its coefficient is obtained from the leading contribution to the wave function renormalization Z_3 of the gauge field [see eq. (C.26)]. Note that the leading contribution to Z_3 also determines the leading contribution to the β function β_e [see eq. (C.27)] The all-order form that is often quoted,

$$T^\mu_{\ \mu} = \frac{\beta_e}{2\bar{e}}[\bar{F}_{\mu\nu}^2] + (\bar{m} - \beta_m)[\overline{\bar{\psi}}\bar{\psi}]\,, \tag{C.33}$$

is obtained after renormalization of composite operators [65].

The matrix element is

$$\mathcal{M}_{TAA} = \mathcal{M}_{F^2} + \mathcal{M}_{\overline{\psi}\psi}\,. \tag{C.34}$$

The first term arises from the tree-level diagram with the gauge kinetic term inserted (see the left diagram of figure 4):

$$\mathcal{M}_{F^2} = -\frac{8}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*\right). \tag{C.35}$$

The second term is a one-loop contribution from the fermion mass term inserted (see the right diagram of figure 4):

$$\begin{aligned}\mathcal{M}_{\overline{\psi}\psi} =& \bar{m}\left((iq\bar{e})^2(-1)i^3\int\frac{d^4\ell}{(2\pi)^4}\frac{\text{tr}\left[(\not{\ell}+\not{k}_1+\not{k}_2+\bar{m})\not{\epsilon}_2^*(\not{\ell}+\not{k}_1+\bar{m})\not{\epsilon}_1^*(\not{\ell}+\bar{m})\right]}{[\ell^2-\bar{m}^2][(\ell+k_1)^2-\bar{m}^2][(\ell+k_1+k_2)^2-\bar{m}^2]} + [1\leftrightarrow 2]\right) \\ =& 4\bar{m}^2\frac{q^2\bar{e}^2}{16\pi^2\tilde{\mu}^\epsilon}\left(\left[\epsilon C_{24} - k^2\,C_{23} - k^2\,C_{12} - \frac{k^2}{2}\,C_0 + \bar{m}^2 C_0\right]\epsilon_1^*\cdot\epsilon_2^*\right. \\ &\left. + \left[4C_{23} + 4C_{12} + C_0\right]k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^* + [1\leftrightarrow 2]\right).\end{aligned} \tag{C.36}$$

[16]Curved-spacetime energy-momentum tensor takes the same form with D_μ being the gauge, Local Lorentz, and diffeomorphism covariant derivative.

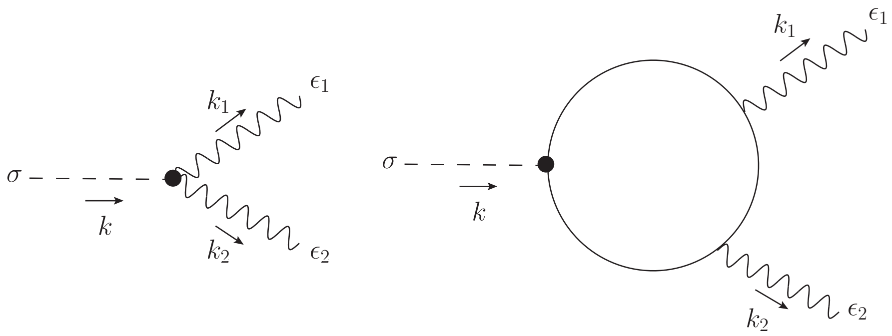

Figure 4. Leading contributions to $\mathcal{M}_{TAA}$, i.e., T^μ_μ-$\bar{A}_\lambda$-$\bar{A}_\kappa$ correlation function (σ decay into two gauge bosons). [Left] $\mathcal{M}_{F^2}$: the gauge kinetic term in eq. (C.33) is inserted. [Right] $\mathcal{M}_{\overline{\psi}\psi}$: the fermion mass term in eq. (C.33) is inserted. There is the other contribution with the external gauge bosons exchanged.

Noting that

$$\begin{aligned}4C_{23}+4C_{12}+C_0 &= -\frac{1}{2k^2}\left[2\bar{m}^2C_0+1-\frac{k^2}{2}\,C_0\right]\\ &= -\frac{1}{2k^2}\left[\epsilon C_{24}-k^2\,C_{23}-k^2\,C_{12}-\frac{k^2}{2}\,C_0+\bar{m}^2C_0\right]\,,\end{aligned}\tag{C.37}$$

one finds

$$\mathcal{M}_{\overline{\psi}\psi}=8\frac{\bar{m}^2}{k^2}\frac{q^2\bar{e}^2}{16\pi^2}\left[2\bar{m}^2C_0+1-\frac{k^2}{2}C_0+[1\leftrightarrow 2]\right](k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^*-k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*)\,.\tag{C.38}$$

With

$$2\bar{m}^2C_0+1-\frac{k^2}{2}C_0=\frac{k^2}{6\bar{m}^2}I_f\left(\frac{k^2}{m^2}\right)\,,\tag{C.39}$$

the matrix element is

$$\mathcal{M}_{\overline{\psi}\psi}=\frac{8}{3}\frac{q^2\bar{e}^2}{16\pi^2}I_f\left(\frac{k^2}{\bar{m}^2}\right)(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^*-k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*)\,.\tag{C.40}$$

Collecting the two contributions, one obtains

$$\mathcal{M}_{TAA}=\frac{8}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(1-I_f\left(\frac{k^2}{\bar{m}^2}\right)\right)(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^*-k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*)\,.\tag{C.41}$$

We remark that $I_f(0)=1$ and thus a heavy ($\bar{m}^2\gg k^2$) fermion does not contribute to $\mathcal{M}_{TAA}$. Meanwhile, $I_f(\infty)=0$ and thus a light ($\bar{m}^2\ll k^2$) fermion indeed contributes to $\mathcal{M}_{TAA}$.

Let us see how we obtain the above result in Pauli-Villars regularization. Above the Pauli-Villars mass scale, $\beta_e=0$ and the trace of energy-momentum tensor is replaced by

$$T^\mu_\mu\supset\bar{m}\overline{\bar{\psi}}\bar{\psi}+\bar{m}_{\rm PV}\overline{\bar{\psi}}_{\rm PV}\bar{\psi}_{\rm PV}\,,\tag{C.42}$$

where $\bar{\psi}_{\rm PV}$ is a Pauli-Villars partner with a wrong statistics. As a result, the matrix element is replaced by

$$\mathcal{M}_{TAA} = \frac{8}{3}\frac{q^2\bar{e}^2}{16\pi^2}\left(I_f\left(\frac{k^2}{\bar{m}_{\rm PV}^2}\right) - I_f\left(\frac{k^2}{\bar{m}^2}\right)\right)(k_1\cdot k_2\,\epsilon_1^*\cdot\epsilon_2^* - k_2\cdot\epsilon_1^*\,k_1\cdot\epsilon_2^*)\,. \tag{C.43}$$

After integrating out the Pauli-Villars partner, i.e., $\bar{m}_{\rm PV}^2 \to \infty$, one reproduces the above result.

C.3 Summary of one-loop functions

One-loop functions are based on refs. [117, 118] (see also appendix F of ref. [119]). One point integral is defined as

$$\tilde{\mu}^\epsilon \int \frac{d^d\ell}{(2\pi)^d}\frac{1}{\ell^2 - m^2} = \frac{i}{16\pi^2}A(m^2)\,. \tag{C.44}$$

The explicit form is

$$A(m^2) = m^2\left(\frac{2}{\epsilon} - \ln\left(\frac{m^2}{\mu^2}\right) + 1\right)\,. \tag{C.45}$$

Two point integrals are defined as

$$\tilde{\mu}^\epsilon \int \frac{d^d\ell}{(2\pi)^d}\frac{1;\ell_\mu;\ell_\mu\ell_\nu}{[\ell^2 - m_1^2][(\ell+k)^2 - m_2^2]} = \frac{i}{16\pi^2}B_{0;\mu;\mu\nu}(k^2;m_1^2,m_2^2)\,, \tag{C.46}$$

where

$$\begin{aligned} B_\mu &= k_\mu B_1\,,\\ B_{\mu\nu} &= g_{\mu\nu}B_{22} + k_\mu k_\nu B_{21}\,. \end{aligned} \tag{C.47}$$

For our purpose, we can take $m_1 = m_2 = m$:

$$\begin{aligned} B_1 &= -\frac{1}{2}B_0\,,\\ B_{22} &= \frac{1}{6}\left[A + 2\bar{m}^2 B_0 - \frac{k^2}{2}B_0 + 2\bar{m}^2 - \frac{k^2}{3}\right]\,,\\ B_{21} &= \frac{1}{3k^2}\left[A - \bar{m}^2 B_0 + k^2 B_0 - \bar{m}^2 + \frac{k^2}{6}\right]\,. \end{aligned} \tag{C.48}$$

The explicit form with a Feynman parameter integral is

$$B_0 = \frac{2}{\epsilon} - \int_0^1 dx\ln\left(\frac{m^2 - x(1-x)k^2 - i\epsilon_{\rm ad}}{\mu^2}\right)\,. \tag{C.49}$$

Three point integrals are defined as

$$\begin{aligned} \tilde{\mu}^\epsilon \int \frac{d^d\ell}{(2\pi)^d}&\frac{1;\ell_\mu;\ell_\mu\ell_\nu}{[\ell^2 - m_1^2][(\ell+k_1)^2 - m_2^2][(\ell+k_1+k_2)^2 - m_3^2]}\\ &= \frac{i}{16\pi^2}C_{0;\mu;\mu\nu}(k_1^2,k_2^2,k^2;m_1^2,m_2^2,m_3^2)\,, \end{aligned} \tag{C.50}$$

where $k + k_1 + k_2 = 0$ and

$$\begin{aligned} C_\mu &= k_{1\mu} C_{11} + k_{2\mu} C_{12} \,, \\ C_{\mu\nu} &= g_{\mu\nu} C_{24} + k_{1\mu} k_{1\nu} C_{21} + k_{2\mu} k_{2\nu} C_{22} + \left(k_{1\mu} k_{2\nu} + k_{2\mu} k_{1\nu}\right) C_{23} \,. \end{aligned} \tag{C.51}$$

For our purpose, again we can take $m_1 = m_2 = m_3 = m$:

$$\begin{aligned} C_{11} &= \frac{1}{k^2} \left[B_0(k_1^2) - B_0(k^2) - k^2 \, C_0 \right] \,, \\ C_{12} &= \frac{1}{k^2} \left[B_0(k^2) - B_0(k_2^2) \right] \,, \\ C_{24} &= \frac{1}{4} \left[B_0(k^2) + 2\bar{m}^2 C_0 + 1 \right] \,, \\ C_{21} &= -\frac{1}{2k^2} \left[3B_0(k^2) - 3B_0(k^2) - 2k^2 \, C_0 \right] \,, \\ C_{23} &= -\frac{1}{2k^2} \left[2B_0(k^2) - 2B_0(k_2^2) + 2\bar{m}^2 C_0 + 1 \right] \,, \\ C_{22} &= -\frac{1}{2k^2} \left[B_0(k^2) - B_0(k_2^2) \right] \,. \end{aligned} \tag{C.52}$$

The explicit form with Feynman parameter integrals is

$$C_0 = -\int_0^1 dx \int_0^{1-x} dy \, \frac{1}{-k^2 \, xy + m^2 - i\epsilon_{\rm ad}} \,. \tag{C.53}$$

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